Taking the Muon for a Spin
AN Overview of g-2 for the Muon

Lawrence Gibbons
Cornell University
Fermilab Muon g-2
Roadmap

- A moment on history
- The current spin on g-2
- Muon g-2 experiments: theme and variations
The gyromagnetic ratio

Classical particle in orbit

\[ \vec{\mu} = I \vec{a} = \frac{q \omega}{2 \pi} \pi r^2 \hat{a} = \frac{q}{2m} m \omega r^2 \hat{a} \]

\[ = \frac{q}{2m} \vec{L} \]
The gyromagnetic ratio

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Particle with intrinsic spin

\[ \vec{\mu} = g \frac{q}{2m} \vec{S} \]
The gyromagnetic ratio

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Particle with intrinsic spin

\[ \vec{\mu} = \frac{g}{2m} \vec{S} \]
g for fermions: instructive!

$g = 2$ has been around as long as spin $\frac{1}{2}$!


Thomas precession: resolves “$g=1$” for fine structure with $g=2$ for Zeeman effect
g for fermions: instructive!

$g = 2$ has been around as long as spin $\frac{1}{2}$!


Thomas precession: resolves "$g=1$" for fine structure with $g=2$ for Zeeman effect

Now I can accept spin
g for fermions: instructive!

Dirac Eqn: framework beautifully incorporates $g = 2$

Pauli to Stern on $g$ for proton:

“If you enjoy doing difficult experiments, you can do them, but it is a waste of time and effort because the result is already known.” (Ridgen)

“Don’t you know the Dirac theory? It is obvious from Dirac’s equation that \([g_p=2]\)” (Tomonaga)

Stern and Estermann (1933)...

- $g_p \approx 5.6$
- Rabi: deuteron infers $g_n \approx -3.8$
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Kusch and Foley (enabled by WWII radar technology)

- $g_e = 2.00229 \pm 0.00008 \, \text{1947}$: inspires Schwinger

$g_e \approx 2 \left( 1 + \frac{\alpha}{2\pi} \right) \approx 2(1.00116)$

K&F : $2(1.00119 \pm 0.00005) \, \text{1948, final}$
g for fermions: instructive!

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Anomalous magnetic moment:

$$a = \frac{(g - 2)}{2}$$
The current spin on $a_e, a_\mu$

Much more happens than just virtual photon exchange

Higher order QED

Strong, Weak Contributions

Something New??!!

Contributions:

$$\delta a_\ell \sim q^2 \times \left( \frac{m_\ell}{\Lambda} \right)^2$$
The current spin on $a_e$, $a_{\mu}$

Standard Model Calculations (units $10^{-12}$)

<table>
<thead>
<tr>
<th></th>
<th>$a_e$</th>
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<tbody>
<tr>
<td>QED</td>
<td>1 159 652 180.03</td>
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<td>to 10$^{\text{th}}$ order</td>
<td>$(0.06)<em>{8}(0.04)</em>{10}(0.77)<em>{\alpha</em>{Rb}(2011)}$</td>
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Aoyama, Hayakawa, Kinoshita and Nio, PRL 109, 111807 & 111808 (2012)

12,672 diagrams at 10th order!
The current spin on $a_e$, $a_\mu$

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$$\frac{\alpha_{\text{weak}}}{4\pi} \left( \frac{m_e}{M_W} \right)^2 \sim 10^{-13}$$

$$\frac{\alpha_{\text{weak}}}{4\pi} \left( \frac{m_\mu}{M_W} \right)^2 \sim 4 \times 10^{-9}$$
The current spin on $a_e$, $a_\mu$

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Vacuum polarization:
The current spin on $a_e, a_\mu$

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Vacuum polarization:
(representative)
The current spin on $a_e, a_\mu$

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Vacuum polarization:
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The current spin on \( a_e, a_\mu \)

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Vacuum polarization:
(representative)

\[ \mu^+ \rightarrow \gamma^* \gamma^* \]

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<td>$0.04(0)$</td>
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Hadronic “light × light”

(representative)
The current spin on $a_e, a_\mu$

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<td>$1159.6524181.76(77)_{Rb}$</td>
<td>$1165.917835(434)$</td>
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<td>$1159.6524181.57(23)_{Cs18}$</td>
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<td>$1159.6524180.73(28)$</td>
<td>$1165.920910(630)$</td>
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Hanneke, Fogwell, and Gabrielse, PRL 100, 120801 (2008)

The current spin on $a_e$, $a_\mu$

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QED

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\[ a_e = \frac{e}{\pi} - 2a_\mu = 1 159 652 181.57(23)_{\text{Cs}} \]

Hanneke, Fogwell, and Gabrielse, PRL 100, 120801 (2008)

\[ a_e = \frac{e}{\pi} - 2a_\mu = 1 159 652 180.73(28) \]

maximal


\[ a_e = \frac{e}{\pi} - 2a_\mu = 1 165 917 835(434) \]

expt

The current spin on $a_e, a_\mu$

Hadronic Vacuum polarization (QCD)
The current spin on $a_e$, $a_\mu$

Hadronic Vacuum polarization (QCD$_l$)

$\sigma(e^+e^- \rightarrow \text{hadrons})$
The current spin on $a_e$, $a_\mu$

Hadronic Vacuum polarization (QCDi)

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

\[ a_\mu(HVP) = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{m_\pi^2} R(s)\hat{K}(s) \frac{ds}{s^2} \]

Lattice QCD efforts maturing.
- HVP: optimally combining LQCD and R methods can provide best precision
- HL×L: LQCD crucial to eliminate models. Verified that HL×L estimates not responsible for discrepancy
The current spin on $a_e, a_\mu$

- experiment vs SM prediction
  \[ \Delta a_\mu \sim (271 - 306) \pm 73 \times 10^{-11} \]

- deviation $> 3.7 \sigma$!

- $\Delta a_\mu > 2 \cdot a_\mu(\text{Weak})$!

Is it real? Remeasure!
- Fermilab - running: goal 540 ppb
- BNL precision $\rightarrow$ 140 ppb
- J-PARC (Japan) - proposing new technique: goal 460 ppb
Beyond Standard Model

generally: \[ \delta a_\mu (\text{N.P.}) = \mathcal{O}(C) \left( \frac{m_\mu}{M} \right)^2, \quad C = \frac{\delta m_\mu (\text{N.P.})}{m_\mu} \]

classify new physics: \( C \) very model-dependent

<table>
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<tr>
<th>( \mathcal{O}(1) )</th>
<th>radiative muon mass generation . . .</th>
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<td>[Czarnecki, Marciano '01]</td>
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<th>( \mathcal{O}(\frac{\alpha}{4\pi}) \ldots )</th>
<th>supersymmetry (( \tan \beta )), unparticles</th>
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<td>[Cheung, Keung, Yuan '07]</td>
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<td>[Davioudasl, Hewett, Rizzo '00]</td>
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<td>[Graesser, '00][Park et al '01][Kim et al '01]</td>
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Measurement of $a_\mu$

Theme and Variations

Fermilab Muon g-2

J-PARC Muon g-2
Measurement of $a_\mu$

Motion in a B field
\[\left( \vec{\beta} \perp \vec{B}, \vec{E} = 0 \right)\] (Jackson Ch. 11.11)

- cyclotron frequency
  \[\omega_C = B \frac{e}{m_\mu c \gamma}\]

- spin precession frequency
  \[\omega_s = B \frac{g_\mu}{2} \frac{e}{m_\mu c} + B \frac{e}{m_\mu c} \left( \frac{1}{\gamma} - 1 \right)\]
Measurement of $a_\mu$

Motion in a B field
\[
(\vec{\beta} \perp \vec{B}, \vec{E} = 0) \quad \text{(Jackson Ch. 11.11)}
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\(g = 2: \omega_s = \omega_C\)
Measurement of $a_\mu$

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$g \neq 2$: relative precession

\[\omega_s - \omega_C = B \frac{e}{m_\mu c} \left(\frac{g_\mu}{2} - 1\right)\]
Measurement of $a_\mu$

Motion in a B field

\[ (\vec{\beta} \perp \vec{B}, \vec{E} = 0 ) \]  

(Jackson Ch. 11.11)

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$g \neq 2$: relative precession

$\omega_s - \omega_C = B \frac{e}{m_\mu c} \left( \frac{g_\mu}{2} - 1 \right)$
Measurement of $a_\mu$

Most energetic $e^+$ from $\mu^+$ decay

$\nu_e \leftrightarrow \nu_{\mu}$

$\mu^+ \leftrightarrow e^+$

$E_{\text{th}} \rightarrow E_e$
Measurement of $a_\mu$ 

Most energetic $e^+$ from $\mu^+$ decay 

\[ V_e \quad \mu^+ \quad e^+ \]

aligned with $\mu^+$ spin direction!
Measurement of $a_\mu$

Most energetic $e^+$ from $\mu^+$ decay

\[
\begin{align*}
V_e & \Rightarrow \mu^+ \\
\bar{V}_\mu & \Rightarrow e^+ \\
\text{aligned with } \mu^+ \text{ spin direction!}
\end{align*}
\]
Measurement of $a_\mu$

Most energetic $e^+$ from $\mu^+$ decay

\[
\begin{align*}
V_e \quad &\Rightarrow \quad \mu^+ \\
\bar{V}_\mu \quad &\Rightarrow \quad e^+
\end{align*}
\]

aligned with $\mu^+$ spin direction!

Count above fixed threshold. Rate oscillation rate $\propto g_\mu^{-2}$
Threshold $(1.8 \text{ GeV})$
Measurement of a µ Threshold (1.8 GeV)

Positron Energy [MeV]

Fraction $e^+$ above threshold

Phase of $\mu^+$ spin

Threshold (1.8 GeV)
Measurement of $a_\mu$

Theme

Polarized $\mu^+$ production: Parity Violation!!

$\nu_\mu \leftrightarrow \pi^+ \leftrightarrow \mu^+$

Fermilab: first hour of (low rate) data

J-PARC: TRIUMF muonium test beam data

Vacuum Region

$10 < z < 40$ (mm)
Measurement of $a_\mu$

and Variations

\[ \tilde{\omega}_a \equiv \tilde{\omega}_s - \tilde{\omega}_c = -\frac{q}{m} a_\mu \hat{B} \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\hat{\beta} \times \hat{E}}{c} \]

Relativistic $\mu$ beam
- high rate, polarization
- vertical focusing ($\vec{E} \neq 0$) required
- choose $\gamma_\mu^2 = 1 + 1/a_\mu$
- $O$(ppm) correction for $p_\mu$ spread
- CERN, BNL, now FNAL approach
- Goal: 140 ppb (21×BNL statistics)

Ultracold $\mu$ beam
- no transverse momentum $\leftrightarrow$ no strong focusing ($\vec{E} = 0$)
- challenging production
- lower polarization
- new J-PARC approach
- Goal (Phase 1): 460 ppb
Measurement of $a_\mu$

**Variation 1:** relativistic $\mu$

**Booster:**
$4 \times 10^{12}$ protons per batch

**Main Injector:**
rebunch to 4 batches of $10^{12}$ $ightarrow$ 12 Hz rep. rate

$\mu^+ /$ proton
$11.5 \times$ BNL
Measurement of $a_\mu$

Variation 1: relativistic $\mu$

Booster:
$4 \times 10^{12}$ protons per batch

Main Injector rebunch to 4 batches of $10^{12}$ → $12$ Hz rep. rate

$\mu^+ / \text{proton} = 11.5 \times \text{BNL}$
Measurement of $a_\mu$

Variation I: relativistic $\mu$
Measurement of $a_\mu$

Variation 1: relativistic $\mu$
Measurement of $a_{\mu}$

Variation 1: relativistic $\mu$
Measurement of $a_\mu$

Variation 1: 

```latex
\text{relativistic } \mu^+ + \pi^+ + \nu_\mu \to \mu^+ \to \nu_\mu
```

*Select $\pi^+, p, \ldots$ at “magic momentum” ($\sim 3.1$ GeV)*

\[ \approx 95\% \text{ polarized at storage ring} \]
Measurement of $a_\mu$

Variation 1: $\mu^+ + \pi^+ + \nu^\mu \rightarrow \mu^+$

Select $\pi^+$, $p$, … at “magic momentum” ($\sim 3.1$ GeV)

$\cong$ 95% polarized at storage ring
Measurement of $a_\mu$

Variation 1: relativistic $\mu$
Measurement of $a_\mu$

Variation 1: relativistic $\mu$
Measurement of $a_\mu$

Variation 1: relativistic $\mu$
24 Calorimeter stations around the ring

NMR probes and electronics around the ring
Watching the spin spin \((at \, \omega_a)\)

9x6 array PbF\(_2\) crystals 
\((2.5 \times 2.5) \, \text{cm} \) 
(Čerenkov radiation)

Fast SiPM 
photodetectors

Pileup:
- distorts precession phase
- \(\varphi(t) \sim \varphi_0 + \alpha t + \ldots\)

Fast SiPM + fast digitizers = pileup id 
(much) better than 5 ns

800 MHz 12 bit digitizers

40 crystals per day.

NOA 164. The CMS glue.
UV cured. Stains crystals, because of Sulphur.
Zeiss OK 2030. Certified Sulphur free.
Repairable. The winner.
Measurement of $a_\mu$

Variation 2: ultracold $\mu$

J-PARC Facility (KEK/JAEA)

LINAC

3 GeV Synchrotron

Neutrino Beam

To Kamioka

Material and Life Science Facility

Main Ring (30 GeV)

Hadron Hall
Measurement of $a_{\mu}$

Variation 2: ultracold $\mu$

Muon g-2/EDM Experiment at J-PARC with Ultra-Cold Muon Beam

- 3 GeV proton beam (333 uA)
- Production target (20 mm)
- Surface muon beam (28 MeV/c)
- Muonium Production (300 K ~ 25 meV $\Rightarrow$ 2.3 keV/c)
- Silicon Tracker 66 cm
- Super Precision Storage Magnet (3T, ~1ppm local precision)
- Resonant Laser Ionization of Muonium ($\sim 10^6 \mu^+$/s)
Measurement of $a_\mu$

Variation 2: ultracold $\mu$

Mu production experiment at TRIUMF (June-July, 2017)

Continuous ablation

$, \, y = 41.5x + 6.1$
Measurement of $a_\mu$

Variation 2: ultracold $\mu$

Muon $g$-2/EDM Experiment at J-PARC with Ultra-Cold Muon Beam

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## Measurement of $a_\mu$

### Variation 2: ultracold $\mu$

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Beta Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
<td>0.01</td>
</tr>
<tr>
<td>0.34</td>
<td>0.08</td>
</tr>
<tr>
<td>5.1</td>
<td>0.3</td>
</tr>
<tr>
<td>42.3</td>
<td>0.7</td>
</tr>
<tr>
<td>200</td>
<td>0.94</td>
</tr>
</tbody>
</table>

- **Bunching Section**
  - RFQ (324 MHz)
  - High capture efficiency

- **Low-\(\beta\)**
  - Interdigital-H (324 MHz)
  - High shunt impedance

- **Middle-\(\beta\)**
  - DAW (1300 MHz)
  - External coupling structure for Middle-\(\beta\)

- **High-\(\beta\)**
  - Disk loaded structure (1300 MHz)
  - High gradient (4 structures)

---

*Images of experimental setup and equipment.*
Measurement of $a_\mu$

Variation 2:
ultracold $\mu$

Radial $\vec{B}$ compresses spiral
- 33.3 cm storage radius
- pulsed kicker centers orbit in storage volume
Measurement of $a_\mu$

Variation 2: ultracold $\mu$

Silicon strip tracking modules
- detect $e^+$ from $\mu^+$ decay
- inside stored $\mu^+$ orbit

First functional test module
Measurement of $a_\mu$

What about $\vec{B}$?

Measure $\vec{B}$ using pulsed NMR:

- $\omega_p$: proton Larmor frequency in pulsed NMR free induction decay
- Two approaches to extract $a_\mu$ from measured $\omega_a/\tilde{\omega}_p$

\[
a_\mu(\text{expt}) = \frac{g_e}{2} \frac{\omega_a}{\tilde{\omega}_p} \frac{m_\mu}{m_e} \frac{\mu_p}{\mu_e} \\
= \frac{\omega_a/\tilde{\omega}_p \mu_\mu/\mu_p}{\omega_a/\tilde{\omega}_p - \omega_a/\tilde{\omega}_p}
\]

0.26 ppt

LANL: 120 ppb

J-PARC MuSEUM: 10 ppb goal
FNAL: Reuse BNL solenoid

15 m diameter coils
1 mm vertical flex tolerance

6” clearance!
FNAL: Reuse BNL solenoid

15 m diameter coils
1 mm vertical flex
tolerance

6” clearance!
Creating the precision 1.45 T B field

1) Align the pole faces only
   • painstaking and iterative!
   • **red**: before and during shimming
   • **blue**: E821 after *all* shimming
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Creating the precision 1.45 T B field

2) Adjust **Top Hats** and **Wedges**
Creating the precision 1.45 T B field

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[Diagram showing adjustments to top hats and wedges]

[Graph showing dipole [ppm] vs. θ [deg] from Nov 2015 to Sep 2016]
Creating the precision 1.45 T B field

2) Adjust **Top Hats** and **Wedges**
Creating the precision 1.45 T B field

3) Move beyond E821: iron laminations
   - adjust effective $\mu$ locally via foil patchwork
   Azimuthal uniformity
   - meets Muon g-2 design spec
   - significant improvement over E821
Creating the precision 1.45 T B field

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Creating the precision 1.45 T B field

3) Move beyond E821: iron laminations
   - adjust effective µ locally via foil patchwork
Azimuthal uniformity
   - meets Muon g-2 design spec
   - significant improvement over E821
J-PARC: 3T MRI-style

Must shim for
• uniform $\vec{B}$ for $\mu^+$ storage
• radial (fringe) $\vec{B}$ for spiral injection scheme

Learning to shim with MuSEUM 1.7 T solenoid
Measurement of $a_\mu$

What about $\vec B$?

Fermilab

15 cm radius (expt: 33 cm)

cf. beam sigma $\sim 10$ cm

J-PARC “practice”

0.4 ppm
-0.5 ppm
Measurement of $a_\mu$

What about $\vec{B}$?

Fermilab + J-PARC Absolute Cross calibration

- Two experiments cross-calibrating absolute NMR probes using FNAL g-2 MRI magnet at ANL
- First round: agreement to 21 ppb
- Second round of testing completed March, 2018, analysis proceeds
Fermilab g-2 status

2018 run: expect $\sim 2.3 \times$ BNL stats

Many lessons learned:
- summer tune-up begins 7/7

Analysis underway!

See talk by Nandita Raha, Saturday
Summary

Muon g-2 Standard Model prediction rock solid!

- precision continues to improve
- already reached precision goal estimated for FNAL g-2 Technical Design Report (TDR)

Fermilab Muon g-2 experiment underway

- Very informative first year of running, ~2x BNL dataset in hand
- on track for 140 ppb measurement!

J-PARC Muon g-2 TDR in progress

- Complementary technique at 460 ppb in phase 1
- many critical steps have been achieved
Thanks!