Entanglement & wave-particle duality

Mostly a review

(in a slightly different way of seeing it)

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“Nobody understands quantum mechanics”
"Nobody understands quantum mechanics"

Figure 28

Figure: R. P. Feynman. The character of Physical law
“Nobody understands quantum mechanics”

\[ N_{12} = N_1 + N_2 \]

Result

Figure: R. P. Feynman. The character of Physical law
“Nobody understands quantum mechanics”

Result

Figure: Akira Tonomura
“Nobody understands quantum mechanics”

(not quite true.)

Individual “particles” seem to exhibit wave-behavior.

Not a wave in some medium, but a wave of ‘information’
Why do we call’em particles?

* Energy comes in discrete packets

* Spin comes in discrete packets

* Charge comes in discrete packets
Why do we call’em waves?

* Essentially interference
Particle or wave?
But it’s neither!

A monk asked, "Does a dog have a Buddha-nature or not?"

The master said, “Mu!”

Green, James. “The recorded sayings of Zen master Zhau Zhou”
Particle or wave?

But it’s neither!

We spend a lot of time trying to understand the “Mu” category
Particle or wave?

But it’s neither!

In fact, it’s just a point in projective Hilbert space.
Particle or wave?

But it’s neither!

In fact, it’s just a point in projective Hilbert space.

Spin 1/2
But, if that's so... then what's the big deal about quantum computing?

Sweetie, superposition doesn't mean "and," but it also doesn't mean "or."

It means a complex linear combination of a 0 state and a 1 state. You should think of it as a new ontological category: a way of combining things that doesn't really map onto any classical concept.
Points cannot be "two points" at the same time.

Similar considerations hold for $X$ & $P$. Just different commutation relations.
Wave suddenly get’s localized as particle.
A comment about waves and particles

\( \Box D(x - y) = \delta(x - y) \)

(a wave equation)
A comment about waves and particles

\[ \Box D(x - y) = \delta(x - y) \]

(a wave equation)

\[
D(x - y) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{1}{p^2 - m^2 + i\varepsilon}
\]

\[
= \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \int_0^\infty ds e^{is(p^2 - m^2 + i\varepsilon)}
\]
A comment about waves and particles

\[ \langle p | x \rangle = e^{ipx} \]

(standard quantum mechanics)

\[ D(x - y) = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \frac{1}{p^2 - m^2 + i\varepsilon} \]

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A comment about waves and particles

\[ \langle p | x \rangle = e^{ipx} \]

(standard quantum mechanics)

\[ D(x - y) = \int \frac{d^4p}{(2\pi)^4} \langle y | p \rangle \int_0^\infty ds e^{is(p^2 - m^2 + i\varepsilon)} \langle p | x \rangle \]
A comment about waves and particles

\[ H = -p^2 \]

*(standard quantum mechanics)*

\[ D(x - y) = \int \frac{d^4 p}{(2\pi)^4} \langle y | p \rangle \int_0^\infty ds e^{is(p^2 - m^2 + i\varepsilon)} \langle p | x \rangle \]
A comment about waves and particles

\[ H = -p^2 \]

Wave

\[ D(x - y) = \int_0^\infty ds e^{-s\varepsilon} e^{-ism^2} \langle y | e^{-isH} | x \rangle \]

Particle

End of comment.
Englert-Greenberger-Yasin relation

\[ W = |P(1) - P(2)|^2 = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2} \]

\[ V = \frac{2|A||B|}{|A|^2 + |B|^2} \]

\[ V^2 + W^2 = 1 \]

(pure states)

Figure 28

Figure: R. P. Feynman. The character of Physical law
Englert-Greenberger-Yasin relation

\[ W = |P(1) - P(2)|^2 = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2} \]

\[ V = \frac{2|A||B|}{|A|^2 + |B|^2} \]

\[ V^2 + W^2 \leq 1 \]

(mixed states)

Figure 28

Figure: R. P. Feynman. The character of Physical law
Englert-Greenberger-Yasin relation

Not only true of particles.

\[ V^2 + W^2 \leq 1 \]

Duality true of any quantum system, really.
Quantum mechanics is ‘more’ than a physical theory.

It is a sort of generalized probability theory (for amplitudes).

Like a ‘machine’ to calculate probabilities.
What is a particle?

'Big rock is most fundamental particle in universe!'

No, big rock is made up of small rocks.

To collider!

Somewhere in CERN...
What is a particle?

Somewhere in CERN...

WAIT! SMALL ROCK MADE UP OF VERY SMALL ROCKS! VERY SMALL ROCK IS FUNDAMENTAL.

NO SUCH THING AS VERY SMALL ROCK! TO COLLIDER!
What is a particle?

10,000 iterations later...

Professor, what's a fundamental particle?

Anything smaller than what was fundamental a generation ago.

Somewhere in CERN...
What is a particle?

I would even go further and say that particles are our current understanding of the building blocks of nature.

* Identical objects.
* Universal properties (mass, spin, charge)
* Follow relativity and quantum mechanics.
What is a particle?

In fact, what an electron is (for e.g.) depends on other fields (e.g. a Higgs field with a different v.e.v).

Particle identity is given by their behavior under spacetime symmetries (Wigner).
What is a particle?

We even have clues on how to get rid of the particle concept...

(What about string-wave duality?!)
Entanglement

\[ \mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 \]
Entanglement

\[ \mathcal{H} = \mathbb{C}^2 \otimes (\mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2) = \mathcal{H}_a \otimes \mathcal{H}_b \]

We place the 'cut' wherever we want.
Entanglement

\[ \mathcal{H} = \mathbb{C}^2 \otimes (\mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2) = \mathcal{H}_a \otimes \mathcal{H}_b \]

But it’s only really interesting when ‘a’ and ‘b’ are space-like separated.
Entanglement

\[ \mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \]

\[ \Psi = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}} \neq |\phi\rangle|\varphi\rangle \]

* Quantum correlations

* Incompatible with local hidden variables
Entanglement

I’d like to focus on a particular story about waves, particles and entanglement
Entanglement

\[ V^2 + W^2 \leq 1 \]
Entanglement

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_a |0\rangle_b - |0\rangle_a |1\rangle_b) \]

\[ = \frac{1}{\sqrt{2}} (a^\dagger - b^\dagger) |0\rangle \]
$|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_a |0\rangle_b - |0\rangle_a |1\rangle_b)$

$= \frac{1}{\sqrt{2}} (a^\dagger - b^\dagger) |0\rangle$
Entanglement

Entanglement between modes of the quantum field: can we verify it at space-like separations?

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_a |0\rangle_b - |0\rangle_a |1\rangle_b) \\
= \frac{1}{\sqrt{2}} (a^\dagger - b^\dagger) |0\rangle
\]
Entanglement

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B) \]

\[ D(\gamma) = e^{\gamma a - \gamma^* a^\dagger} \]

\[ \gamma = z e^{i\theta} \]

"Weak field homodyne"

F. Monteiro et al., PRL 114, 170504 (2015)
Entanglement

Setup

F. Monteiro et al., PRL 114, 170504 (2015)
F. Monteiro et al., PRL 114, 170504 (2015)
Displaced entanglement + single photon counting ~ oscillations!

\[ p(00|\gamma_a, \gamma_b) = e^{-2\alpha^2} (1 - \eta_a - \eta_b) + e^{-2\beta^2} (\eta_a + \eta_b) z^2 + \sqrt{\eta_a \eta_b} z^2 e^{-2\alpha^2} \cos \theta \]
Displaced entanglement + single photon counting ~ oscillations!

\[ p(00|\gamma_a, \gamma_b) = e^{-2z^2}(1 - \eta_a - \eta_b) + e^{-2z^2}(\eta_a + \eta_b)z^2 \]
\[ + \sqrt{\eta_a\eta_b}z^2e^{-2z^2}\cos \theta. \]

F. Monteiro et al., PRL 114, 170504 (2015)
Alice

Bob

HSPS

$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B)$

$V^2 + W^2 \leq 1$

$\mathcal{H} = \mathbb{C}^2 \otimes (\mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2) = \mathcal{H}_a \otimes \mathcal{H}_b$

Superposition $\sim$ entanglement

(just need space-time + measurements)
Come hear me on the 11th!
Room 4 (~16:30)
Thank you!

Collaborators and friends:
Bruno Melo (PUC), George Svetlichny (PUC), Fernando Monteiro (Yale), Welles Morgado (PUC), Valerio Euculpi (CERN), Vieri Candelise (CERN), Enrico Schioppa (CERN), Francesco Corardeschi (Cambridge)

If you’re interested, come talk to me (theory and exp)!
Backup
Tripartite entanglement

\[ |1\rangle_A |0\rangle_B |0\rangle_C + |0\rangle_A |1\rangle_B |0\rangle_C + |0\rangle_A |0\rangle_B |1\rangle_C \]

\[ z_3^{exp} - z_{ppt}^{max} = 0.99 \pm 0.10 > 0 \]

Experiment: oscillations

Phase and amplitude control of displacement
Review: Stress-induced birefringence

Photon: Right
Coherent state: Left

Displacement intensity

\[ R_z(\theta) = e^{-i\frac{\theta}{2}Z} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \]

Transmission = \underline{0.98} \times 0.98 \times 0.98 \times 0.98 = 0.93

connector  BS  3 piezos  connector
The single photon entangled state under loss:

\[
\begin{pmatrix}
1 - \eta_a - \eta_b & 0 & 0 & 0 \\
0 & \eta_a & \sqrt{\eta_a \eta_b} & 0 \\
0 & \sqrt{\eta_a \eta_b} & \eta_b & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

Undergoes oscillations as a function of relative displacement phase

\[
p(00|\gamma_a, \gamma_b) = e^{-2z^2}(1 - \eta_a - \eta_b) + e^{-2z^2}(\eta_a + \eta_b)z^2
\]

\[
+ \sqrt{\eta_a \eta_b} z^2 e^{-2z^2} \cos \theta
\]

oscillations scales linear with losses!

We cannot know whether “vacuum” came from loss or from the original entangled state:

NO WAY TO POSTSELECT DATA
Heralded entanglement

How many photons do we REALLY need to herald entanglement?

Path entanglement

\[ |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D \ldots + \ldots + |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D \ldots \]

It’s scalable!

Experiment: entanglement witness

A bipartite entanglement witness:

$$Z_\rho = 2(\sigma_\gamma \otimes \sigma_\gamma) - \sigma_0 \otimes \sigma_0$$

$$z_\rho = 2(P_{00} + P_{cc} - P_{0c} - P_{c0})|_{\gamma} - (P_{00} + P_{cc} - P_{0c} - P_{c0})|_{\gamma=0}$$

If $$z_\rho - z_{ppt}^{\text{max}} > 0$$, we have entanglement.

Where $$z_{ppt}^{\text{max}}$$ is a function of measured local probabilities.

Experiment: entanglement witness

If $z_\rho - z_{\text{ppt}}^\text{max} > 0$, we have entanglement.

Unbalanced states

$C_1|10\rangle + C_2|01\rangle$

$\sim 8$ kHz heralded entanglement (two and three partite)

Experiment: entanglement witness

The effect of loss: linearity

Measuring single photon entanglement

The answer is simple: Displacement operations!

\[ D(\gamma) = e^{\gamma a - \gamma^* a^\dagger} \]

Probability of “no click”: given by the trace of the projection of the state onto coherent state

\[ D(\gamma)|0\rangle\langle 0|D(\gamma^*) = |\gamma\rangle\langle \gamma| \quad \text{outcome +1} \]

Probability of “click”: the complement

\[ 1 - |\gamma\rangle\langle \gamma| \quad \text{outcome -1} \]

V. C. Vivoli et. al. NJP, 17, 2015
Measuring single photon entanglement

\[ M(\gamma) = 2|\gamma\rangle\langle\gamma| - 1 \]

\[ = \begin{pmatrix}
2e^{-z^2} - 1 & 2e^{-z^2} - i\theta z \\
2e^{-z^2} + i\theta z & 2e^{-z^2} z^2 - 1
\end{pmatrix} \]

\[ \gamma = ze^{i\theta} \]

(vacuum-1 photon subspace)

Displacement allows you to access complementary basis.

Approximates Pauli matrices.

\[ \sigma_{\gamma=0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \sigma_{\gamma=1} \approx \begin{pmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{pmatrix} \]
Experiment: oscillations

Quantum mechanical prediction:

Model: fixed coherent state value, no double-pairs

V_{exp} = 0.053 \quad V_{th} = 0.050