The Lifshitz regime in QCD

Chiral spirals and their fluctuations

1. Standard phase diagram in T & μ: critical end-point (CEP)

   Not seen from lattice at small μ

2. Quarkyonic phase at large N_c (analytic) and $N_c = 2$ (lattice)

3. Chiral Spirals in Quarkyonic matter: sigma models, SU(N) and U(1)

4. Phase diagram: *just* a 1st order line,
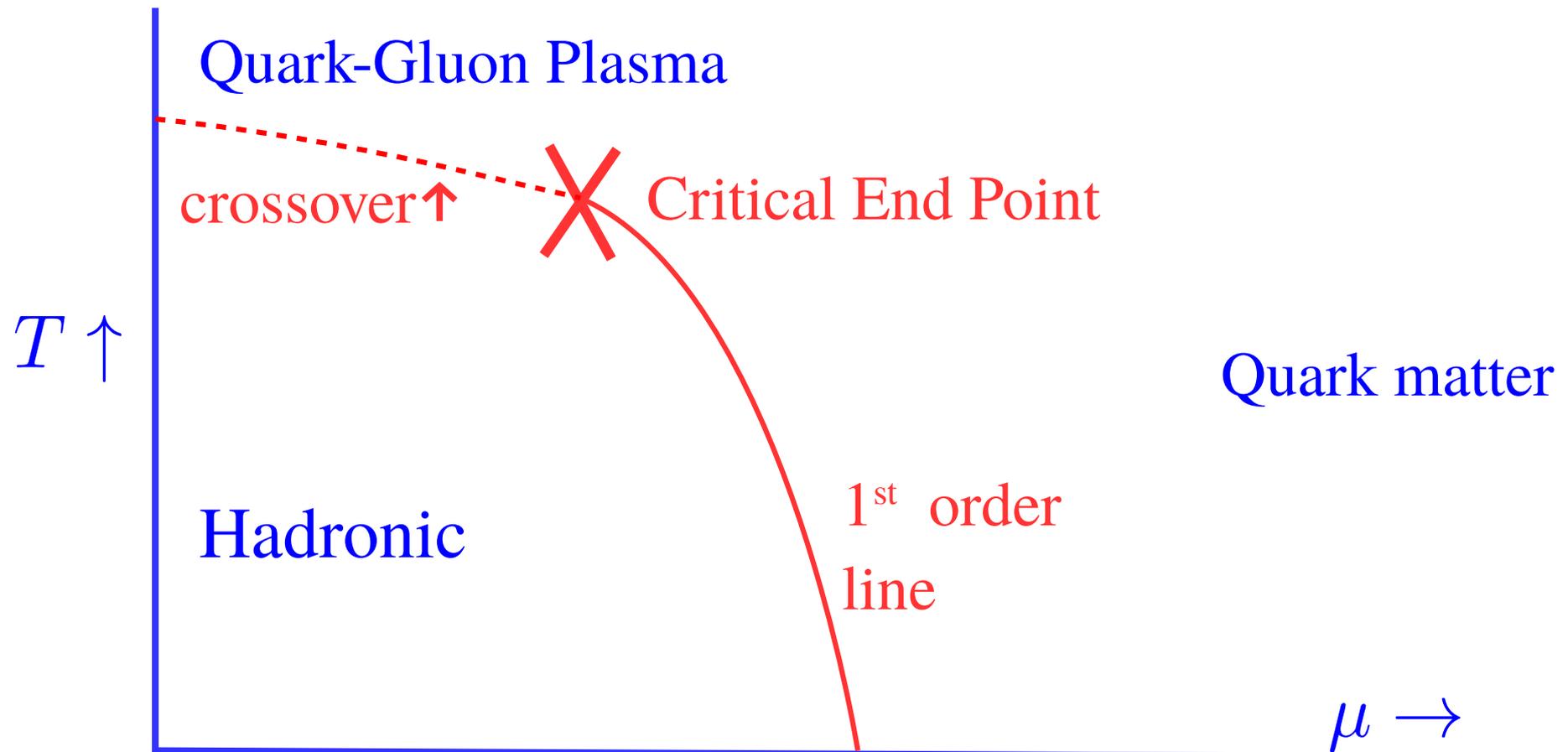   with *large* fluctuations in the Lifshitz regime

"Standard" phase diagram for QCD in T & μ: CEP?

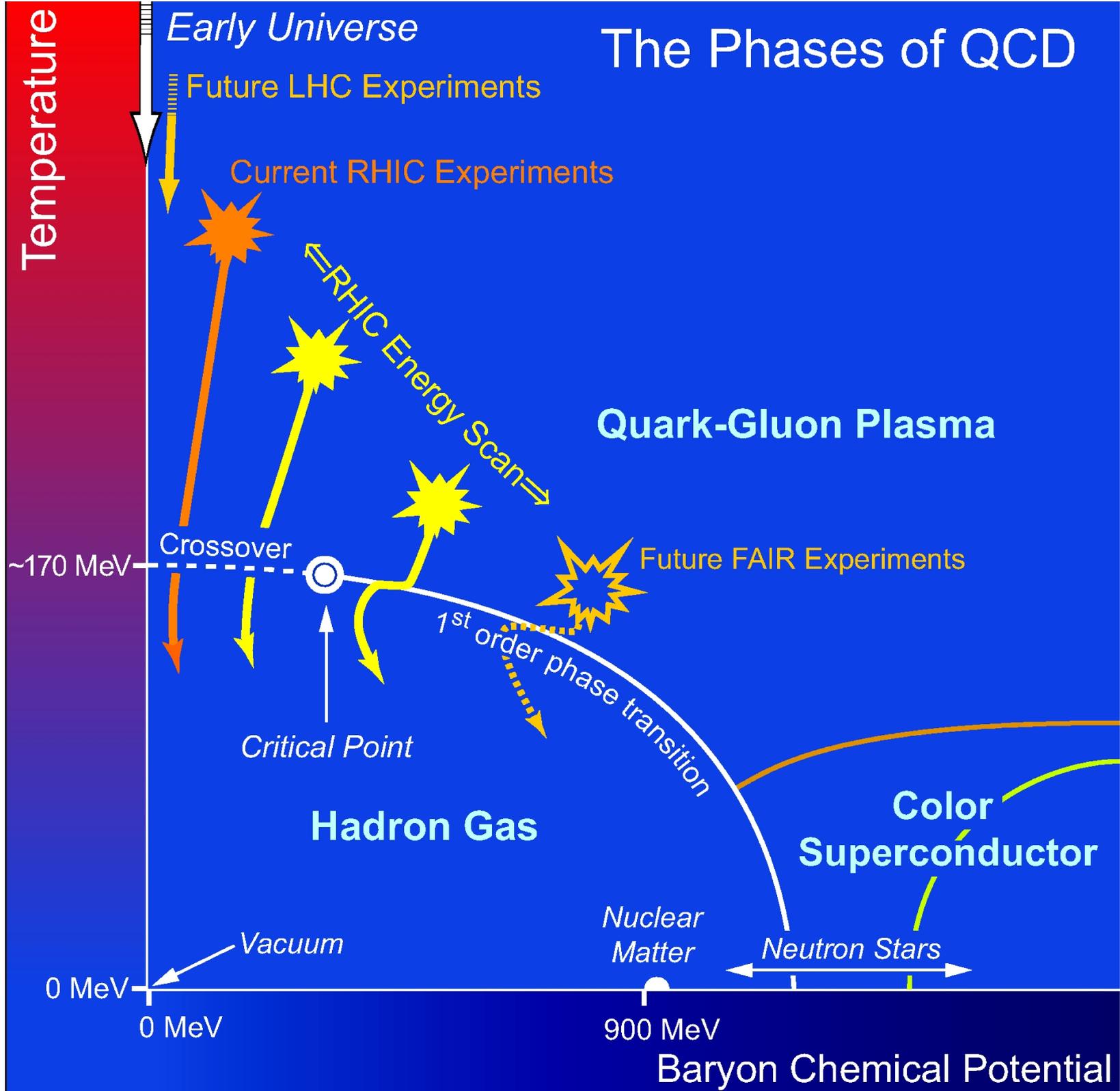Lattice: at quark chemical potential $\mu = 0$, crossover at $T_{ch} \sim 154$ MeV

At $\mu \neq 0$, quarks *might* change scalar 4-pt coupling < 0, so transition 1st order
Must meet at a Critical End Point (CEP), *true* 2nd order phase transition
Asakawa & Yazaki '89, Stephanov, Rajagopal & Shuryak '98 & '99
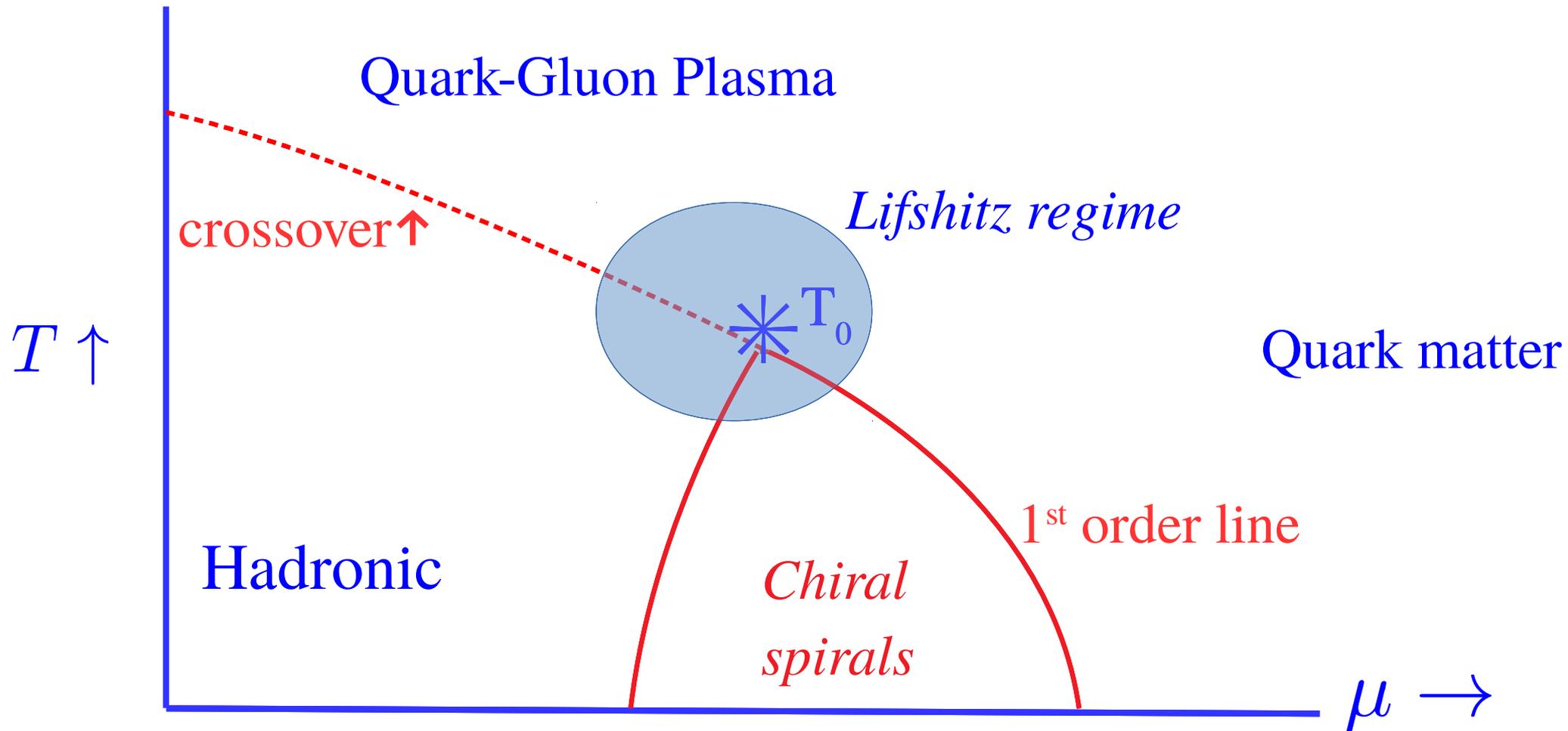
Lifshitz phase diagram for QCD

Instead: "Lifshitz regime": strongly coupled, large fluctuations

Unbroken 1st order line to spatially inhomogeneous phases = "chiral spirals"

Hints in heavy ion data?

Fundamental problem in field theory: analogies to phase diagram for polymers
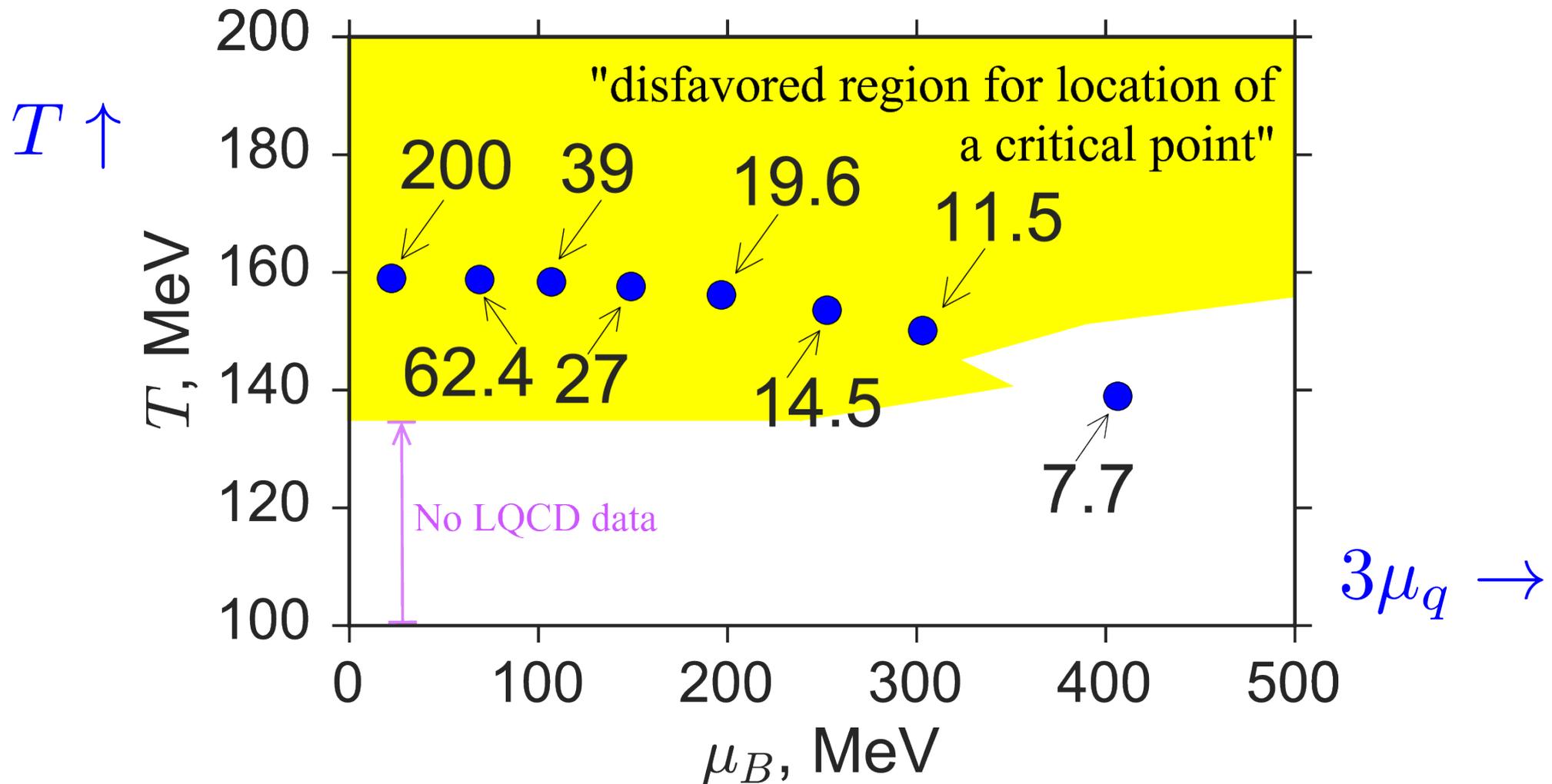
Could be CEP as well...

Lattice, hot QCD: no CEP at small μ

Lattice: Hot QCD, 1701.04325

Expand about $\mu = 0$, power series in μ^{2n}, $n = 1, 2, 3$.

Estimate radius of convergence. *No* sign of CEP by $\mu_{qk} \sim T$
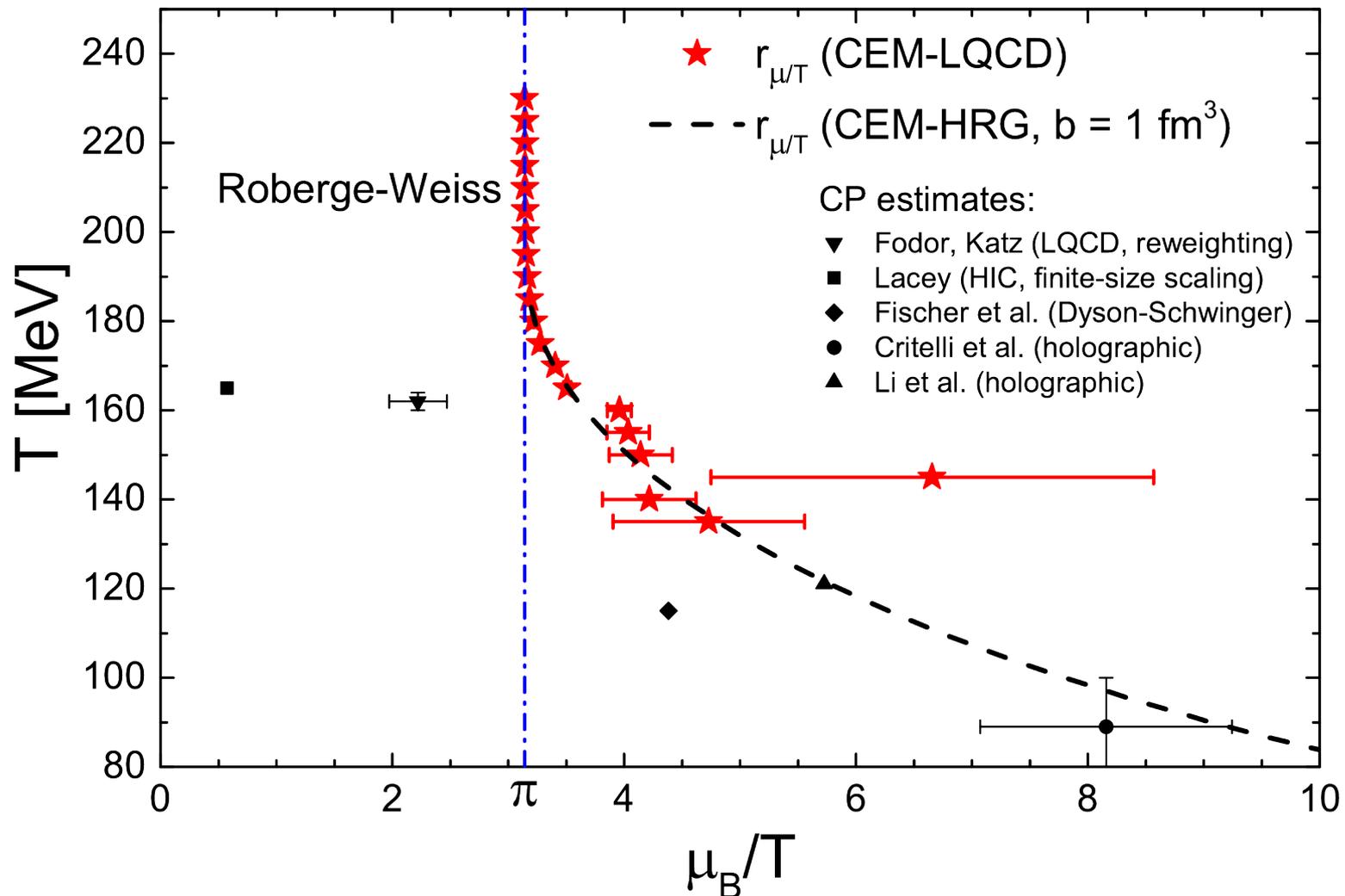
Cluster expansion: no CEP at small μ

Lattice: Vovchenko, Steinheimer, Philipsen & Stoecker, 1701.04325

Use cluster expansion method, different way of estimating power series in μ

No sign of CEP by $\mu_{qk} \sim T$
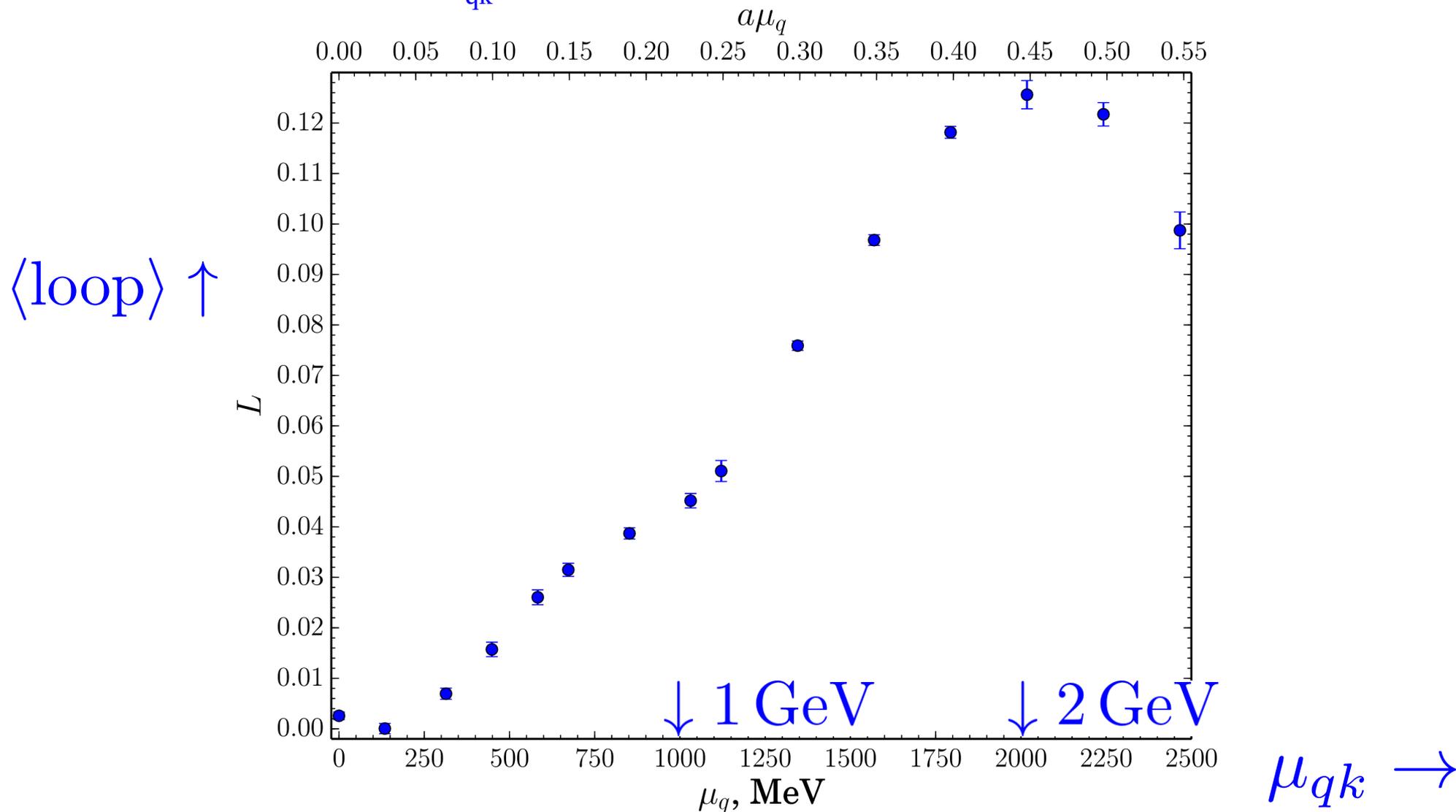

$3\mu_{qk} \rightarrow$

So if there is no critical endpoint,

what could be going on?

Lattice for T = 0, μ ≠ 0, *two* colors

Lattice: Bornyakov et al, 1711.01869. No sign problem for $N_c = 2$. Two flavors.

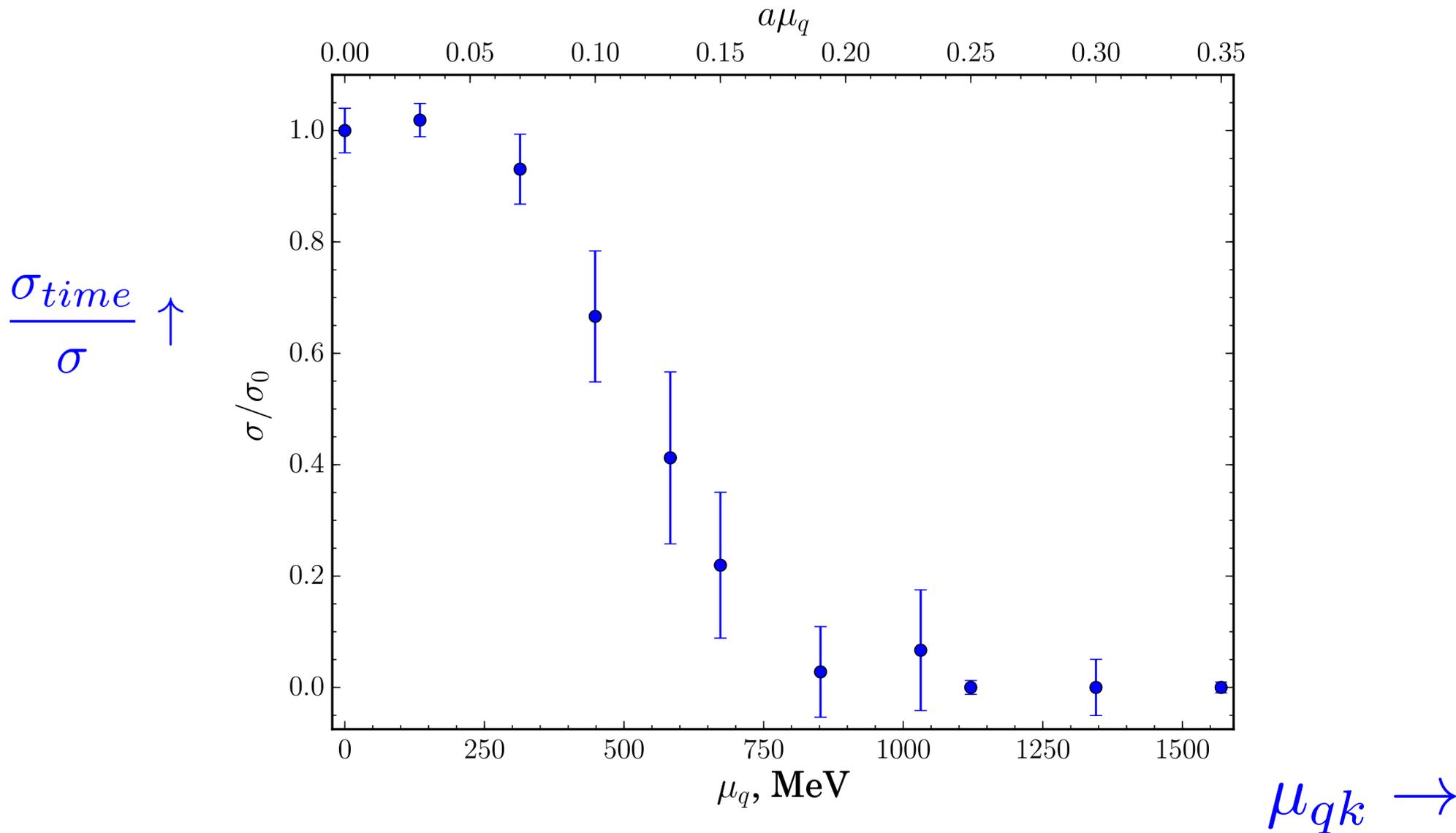Heavy pions, $m_\pi \sim 740$ MeV. $\sqrt\sigma = 470$ MeV. 32^4 lattice, $a \sim .04$ fm

Confined until very high $\mu_{qk} \sim 1$ GeV. *Bare* Polyakov loop:

Lattice for T = 0, μ ≠ 0, *two* colors

Lattice: Bornyakov et al, 1711.01869.

String tension in time: nonzero up to $\mu_{qk} \sim 750$ MeV

$\dfrac{\sigma_{time}}{\sigma}$ ↑

$\mu_{qk} \rightarrow$

Phases for $N_c = 2$, $T \sim 0$, $\mu \neq 0$

Braguta, Ilgenfritz, Kotov, Molochkov, & Nikolaev, 1605.04090 (earlier: Hands, Skellerud + …)

Lattice: $N_c = 2$, $N_f = 2$. $m_\pi \sim 400$ MeV, fixed $T \sim 50$ MeV, vary μ_{qk}.

Hadronic phase: $0 \leq \mu_{qk} < m_\pi /2 \sim 200$ MeV. Confined, independent of μ

Dilute baryons: $200 < \mu_{qk} < 350$. Bose-Einstein condensate (BEC) of diquarks.

Dense Baryons: $350 < \mu_{qk} < 600$. Pressure *not* perturbative, BEC

Quarkyonic: $600 < \mu_{qk} < 1100$: pressure $\sim$ perturbative, but excitations *confined*
(Wilson loop $\sim$ area)

Perturbative: $1100 < \mu_{qk}$, but μa too large.

Quarkyonic matter

At large N_c, $g^2 N_c \sim 1$, $g^2 N_f \sim 1/N_c$, so need to go to *large* $\mu \sim N_c^{1/2}$.

$$m_{Debye}^2 = g^2((N_c + N_f/2)T^2/3 + N_f\mu^2/(2\pi^2))$$

Doubt large N_c applicable at $N_c = 2$.

When does perturbation theory work?

$T = \mu = 0$: scattering processes computable for momentum p > 1 GeV

$T \neq 0$: p > $2\pi T$, lowest Matsubara energy

$\mu \neq 0$, $T = 0$: μ is like a scattering scale, so *perhaps* $\mu_{pert} \sim$ 1 GeV.
   At least for the pressure. Excitations determined by region near Fermi surface

Possible phases of cold, dense quarks

Confined: $0 \leq \mu_{qk} < m_{baryon}/3$. μ doesn't matter

Dilute baryons: $m_{baryon} 3 < \mu_{qk} < \mu_{dilute}$: . Effective models of baryons, pions

Dense baryons: $\mu_{dilute} < \mu_{qk} < \mu_{dense}$. Pion/kaon condensates.
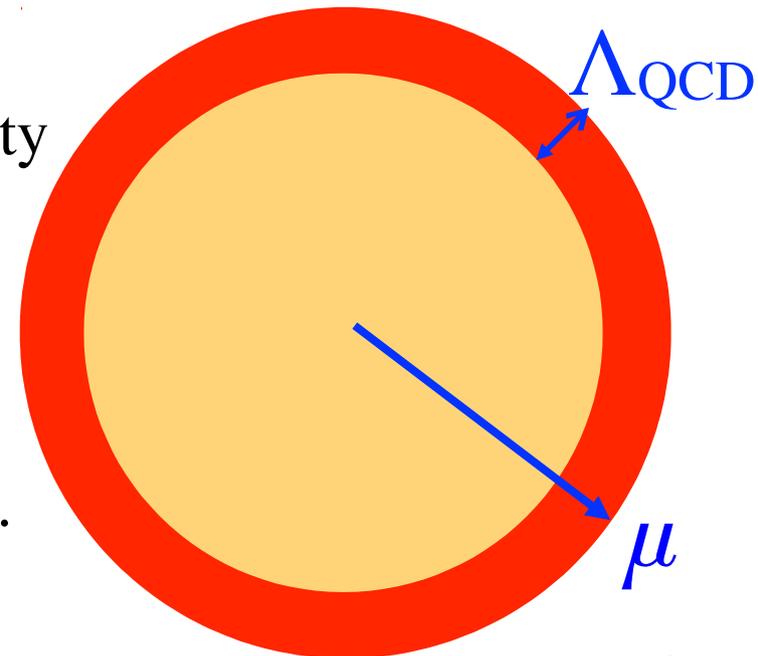
Quarkyonic: $\mu_{dilute} < \mu_{qk} < \mu_{perturbative}$. 1-dim. chiral spirals.

Perturbative: $\mu_{perturbative} < \mu_{qk}$. Color superconductivity

$\mu_{perturbative} \sim 1$ GeV?

Dense baryons and quarkyonic *continuously* related.

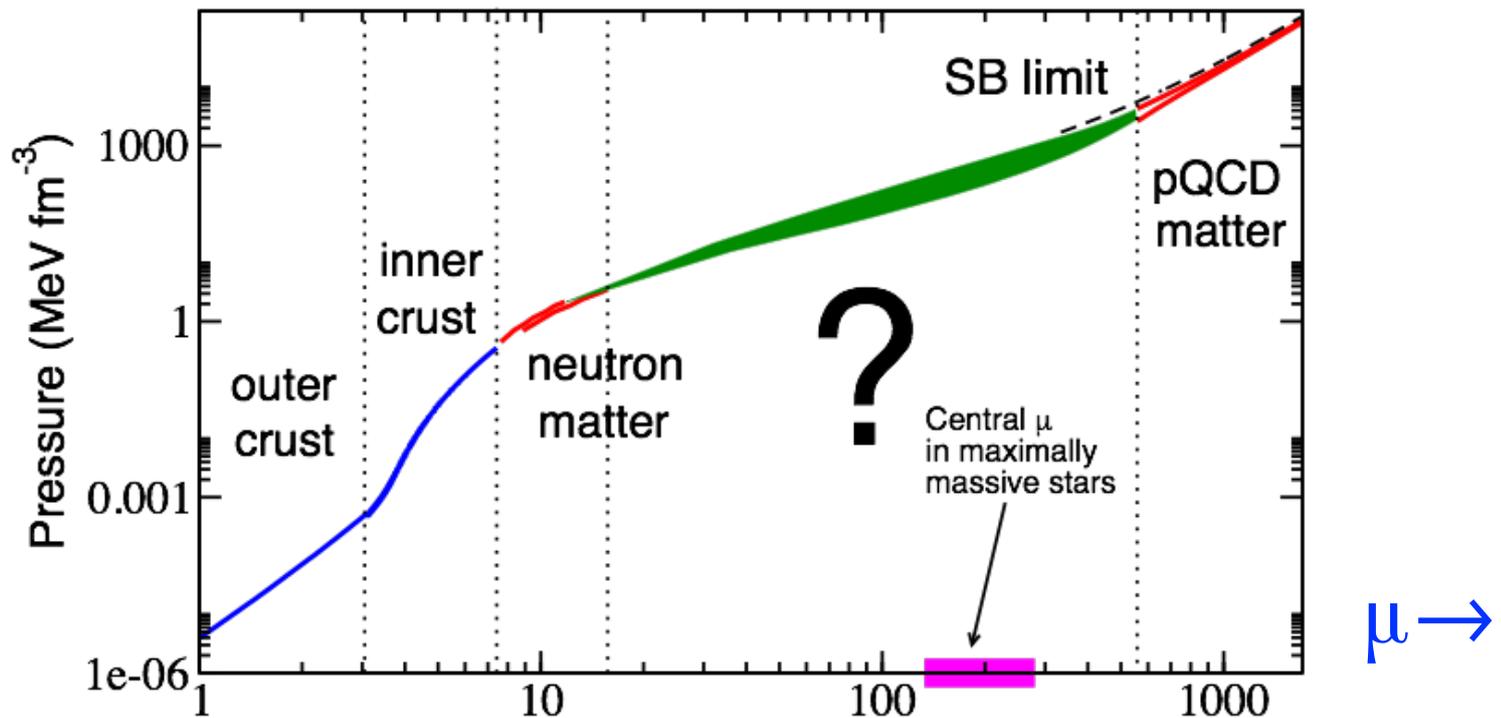*U(1) order parameter in both.*

Λ_{QCD}

μ

Relevance for neutron stars

Fraga, Kurkela, & Vuorinen 1402.6618.

Maximum μ_{qk} may reach quarkyonic (for pressure), but true perturbative?

Ghisoiu, Gorda, Kurkela, Romatschke, Säppi, & Vuorinen, 1609.04339: pressure(μ_{qk}) ~ g^6.

Will be able to compute Λ_{pert} = # μ_{qk} # ~ 1?

Quarkyonic matter: 1-dim. reduction

Kojo, Hidaka, McLerran & RDP 0912.3800: as toy model, assume confining potential

$$\Delta_{00} = \frac{\sigma_0}{(\vec{p}^2)^2} \ , \ \Delta_{ij} \sim \frac{1}{p^2}$$

Near the Fermi surface, reduces to effectively 1-dim. problem in patches.
For *either* massless or massive quarks, excitations have zero energy about
Fermi surface; just Fermi velocity $v_F < 1$ if m $\neq$ 0.

Spin in 4-dim. -> "flavor" in 1-dim., so *extended* 2N$_f$ flavor symmetry,
SU(N$_f$)$_L$xSU(N$_f$)$_R$ -> SU(2 N$_f$)$_L$xSU(2 N$_f$)$_R$. Similar to Glozman,1511.05857.

Extended 2 N$_f$ flavor sym. broken by transverse fluctuations, only approximate.

Number of patches N$_{patch}$ ~ μ/σ_0 , so spherical Fermi surface recovered as σ_0 -> 0
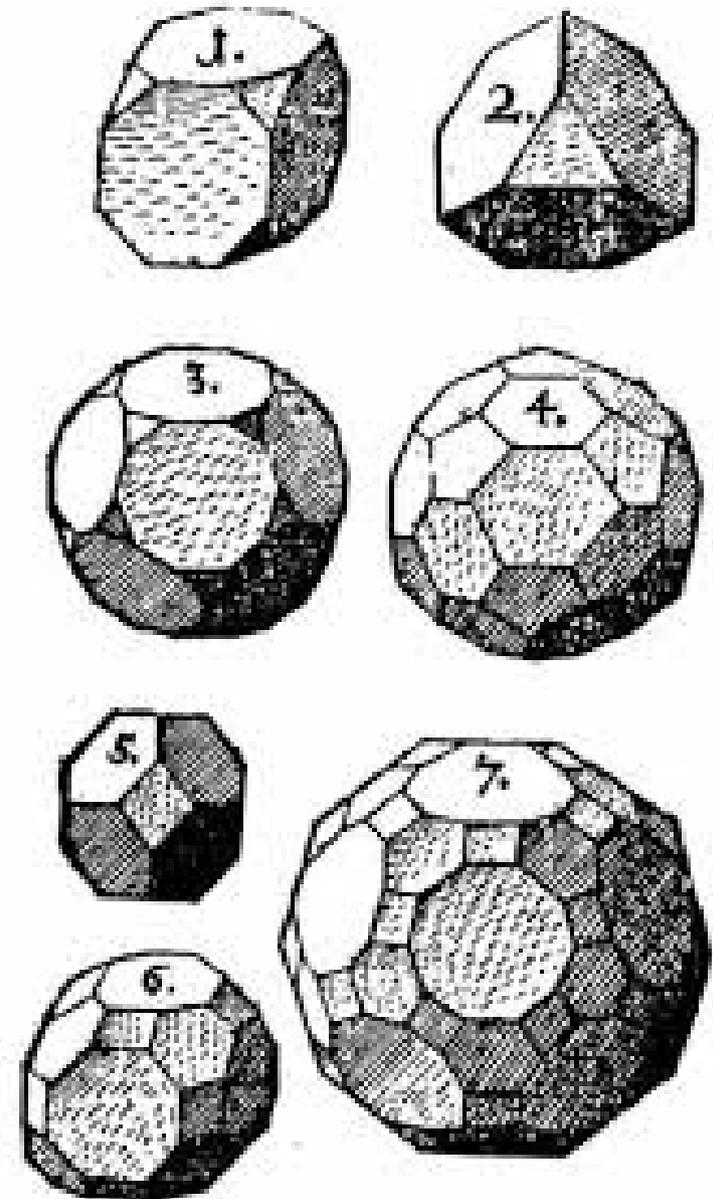
Transitions with # patches

Minimal number of patches = 6.

Probably occurs in dense baryonic phase.

In quarkyonic, presumably weak 1st order
transitions as # patches changes.

Like Keplers....

Chiral spirals in 1+1 dimensions

In 1+1 dim., can eliminate μ by chiral rotation:

$$q' = \mathrm{e}^{i\mu z \Gamma_5} q \; , \; \overline{q}(\slashed{D} + i\mu\Gamma_0)q = \overline{q}' \; \slashed{D} q' \; , \; \Gamma_5 \Gamma_z = \Gamma_0$$

Thus a constant chiral condensate automatically becomes a chiral spiral:

$$\overline{q}' q' = \cos(2\mu z)\overline{q}q + i\sin(2\mu z)\overline{q}\gamma_5 q$$

Argument is only suggestive.

N.B.: anomaly ok, gives quark number: $\langle \overline{q}\Gamma_0 q \rangle = \mu/\pi$

Pairing is between quark & quark-hole, both at edge of Fermi sea.
Thus chiral condensate varies in z as ~ 2 μ.

Bosonization in 1+1 dimensions

Do not need detailed form of chiral spiral to determine excitations.
Use bosonization. For one fermion,

$$\overline{\psi}\,\slashed{\partial}\,\psi \leftrightarrow (\partial_i \phi)^2$$

ϕ corresponds to U(1) of baryon number. In general, non-Abelian bosonization.
For flavor modes,

$$\mathcal{S}_{eff}^{flavor} = \int dt \int dz\; 3\,\frac{1}{16\pi}\,\mathrm{tr}(\partial_\mu U^\dagger)(\partial_\mu U) + \dots$$

where U is a SU(2 N_f) matrix.

Do not show Wess-Zumino-Witten terms for level 3 = # colors.
Also effects of transverse fluctuations, reduce SU(2 N_f) -> SU(N_f); quark mass

Lastly, SU(3) + level 2 N_f sigma model. Modes are gapped by confinement.

Pion/kaon condensates & U(1) phonon

Overhauser '60, Migdal '71....Kaplan & Nelson '86...

Pion/kaon condensate:

$$\langle \bar{q}_L q_R \rangle \sim \langle \Phi \rangle \sim \Phi_0 \ \exp(i(qz + \phi)t_3)$$

Condensate along σ and π^0 => t_3. Kaon condensate σ and K, etc.

Excitations are the $SU(N_f)$ Goldstone bosons *and* a "phonon", φ.

Phases with pion/kaon condensates and quarkyonic Chiral Spirals both spontaneously break U(1), have associated massless field.

Continuously connected: $SU(N_f)$ of π/K condensate => ~ $SU(2 N_f)$ of CS's.

Fluctuations same in both.

Perhaps WZW terms for π/K condensates?

Anisotropic fluctuations in Chiral Spirals

Spontaneous breaking of global symmetry =>
Goldstone Bosons have derivative interactions, $\sim \partial^2$

π/K condensates and CS's break *both* global *and* rotational symmetries

Interactions along condensate direction usual quadratic, $\sim \partial_z^2$

Those quadratic in transverse momenta, $\sim \partial_\perp^2$, *cancel, leaving quartic*, $\sim \partial_\perp^4$.

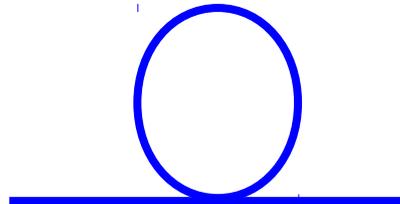$$\mathcal{L}_{eff} = f_\pi^2 |(\partial_z - ik_0)U|^2 + \kappa |\partial_\perp^2 U|^2 + \dots$$

Valid for *both* the U(1) phonon φ and Goldstone bosons U

Hidaka, Kamikado, Kanazawa & Noumi 1505.00848;
Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Nitta, Sasaki & Yokokura 1706.02938
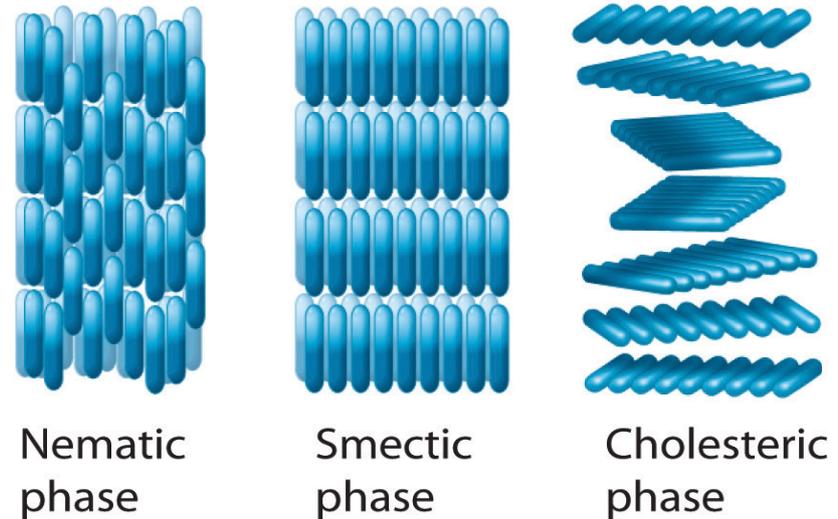
No long range order in Chiral Spirals

Consider tadpole diagram with anisotropic propagator

$$\int d^2k_\perp \, dk_z \, \frac{1}{(k_z - k_0)^2 + (k_\perp^2)^2} \sim \int d^2k_\perp \, \frac{1}{k_\perp^2} \sim \log \Lambda_{\text{IR}}$$

Old story for π/K condensates: Kleinert '81; Baym, Friman, & Grinstein, '82 .

Similar to smectic-C liquid crystals:
ordering in one direction,
liquid in transverse.
Hence anisotropic propagator

Nematic phase   Smectic phase   Cholesteric phase
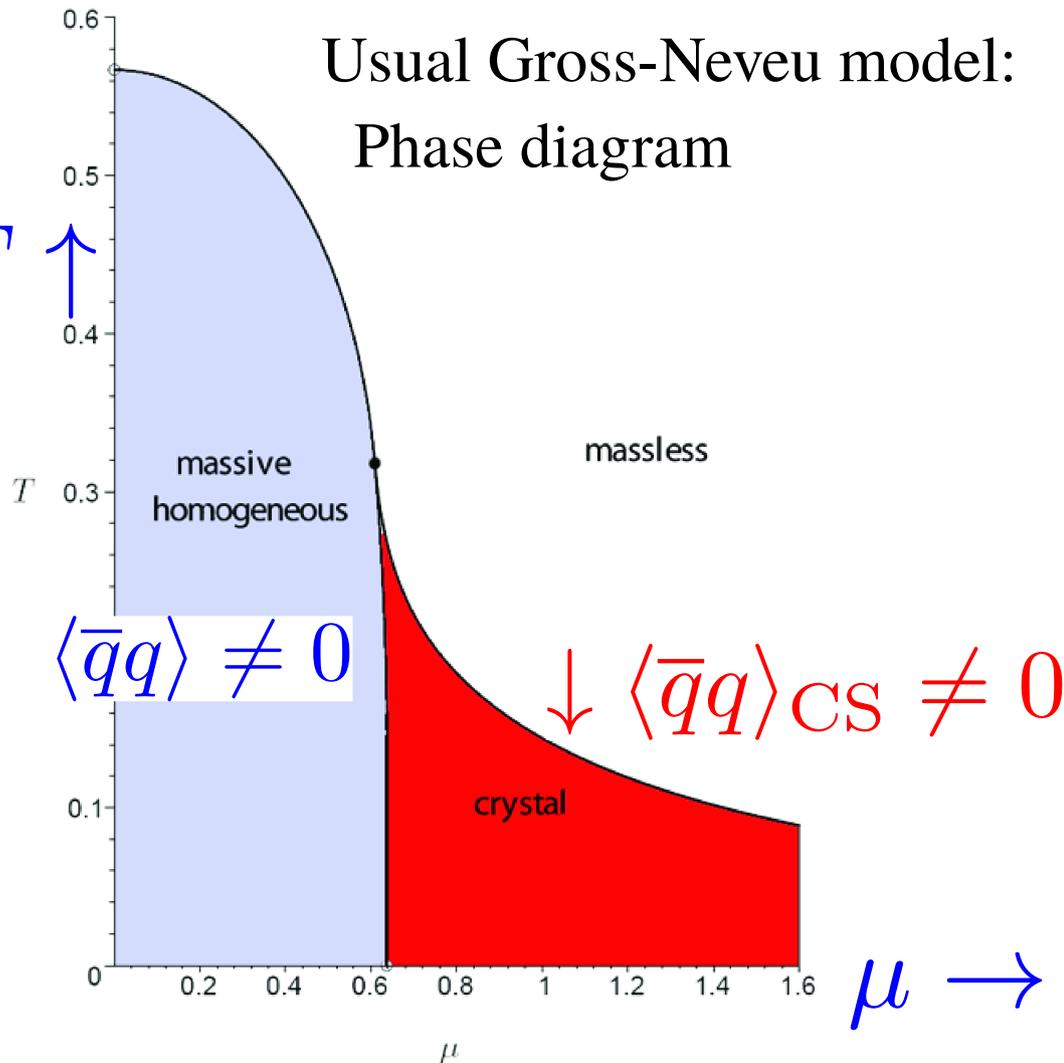
Increasing opacity

Chiral Spirals in 1+1 dimensions

Overhauser/Migdal's pion condensate: $(\sigma, \pi^0) = f_\pi(\cos(k_0 z), \sin(k_0 z))$

Ubiquitous in 1+1 dimensions: Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

Wealth of exact solutions, phase diagrams at *infinite* N_f.
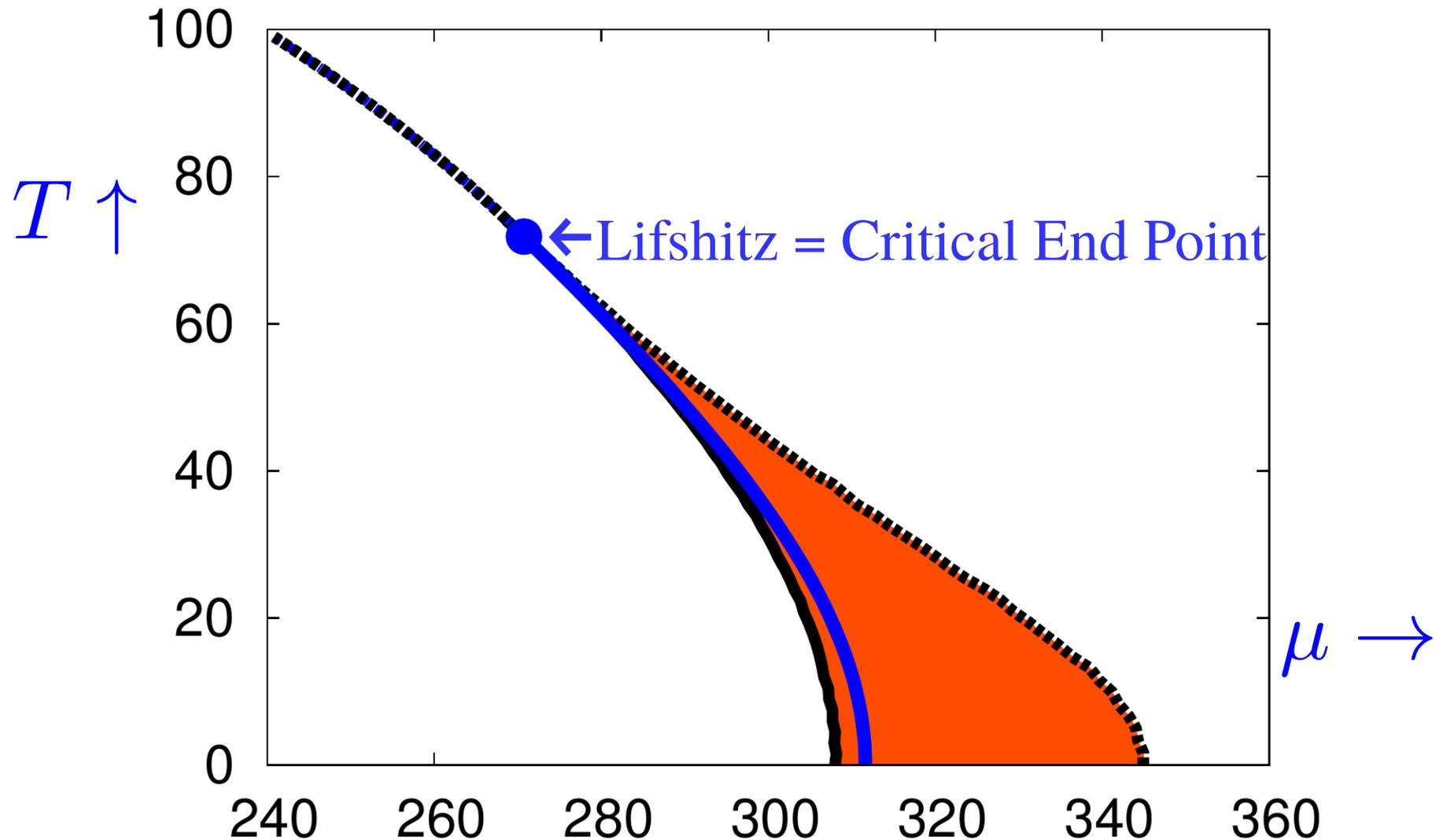
Usual Gross-Neveu model:

Phase diagram

Chiral spiral:

$T \uparrow$

massive homogeneous

massless

$\langle \overline{q}q \rangle \neq 0$

$\downarrow \langle \overline{q}q \rangle_{CS} \neq 0$

crystal

$\mu \rightarrow$

Chiral Spirals in 3+1 dimensions

In 3+1, *common* in NJL models: Nickel, 0902.1778 + ...Buballa & Carignano 1406.1367 + ...

In reduction to 1-dim, $\Gamma_5^{1\text{-dim}} = \gamma_0 \gamma_z$, so chiral spiral between $\bar{q}q$ & $\bar{q}\gamma_z\gamma_z\gamma_5 q$

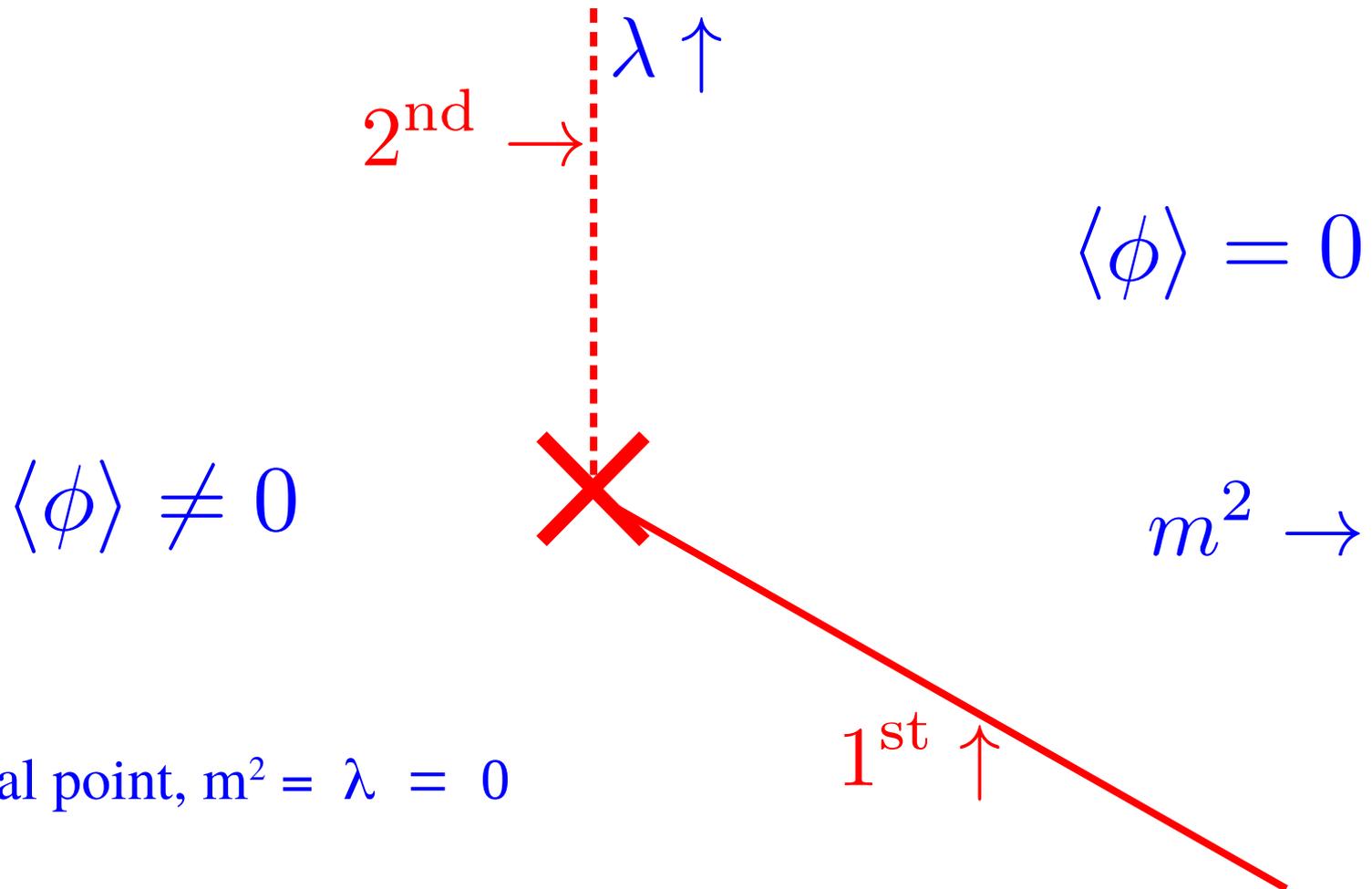Both of these phase diagrams are *dramatically* affected by fluctuations:

no Lifshitz point in 1+1 or 3+1 dimensions at finite N

there *is* a *Lifshitz regime*

Standard phase diagram

$$\mathcal{L} = (\partial_\mu \phi)^2 + m^2\,\phi^2 + \lambda\,\phi^4 + \kappa\phi^6$$

Negative quartic coupling, λ, turns a 2nd order transition into 1st order. Two phases.

$\lambda \uparrow$

$2^{\mathrm{nd}} \rightarrow$

$\langle \phi \rangle = 0$

$\langle \phi \rangle \neq 0$
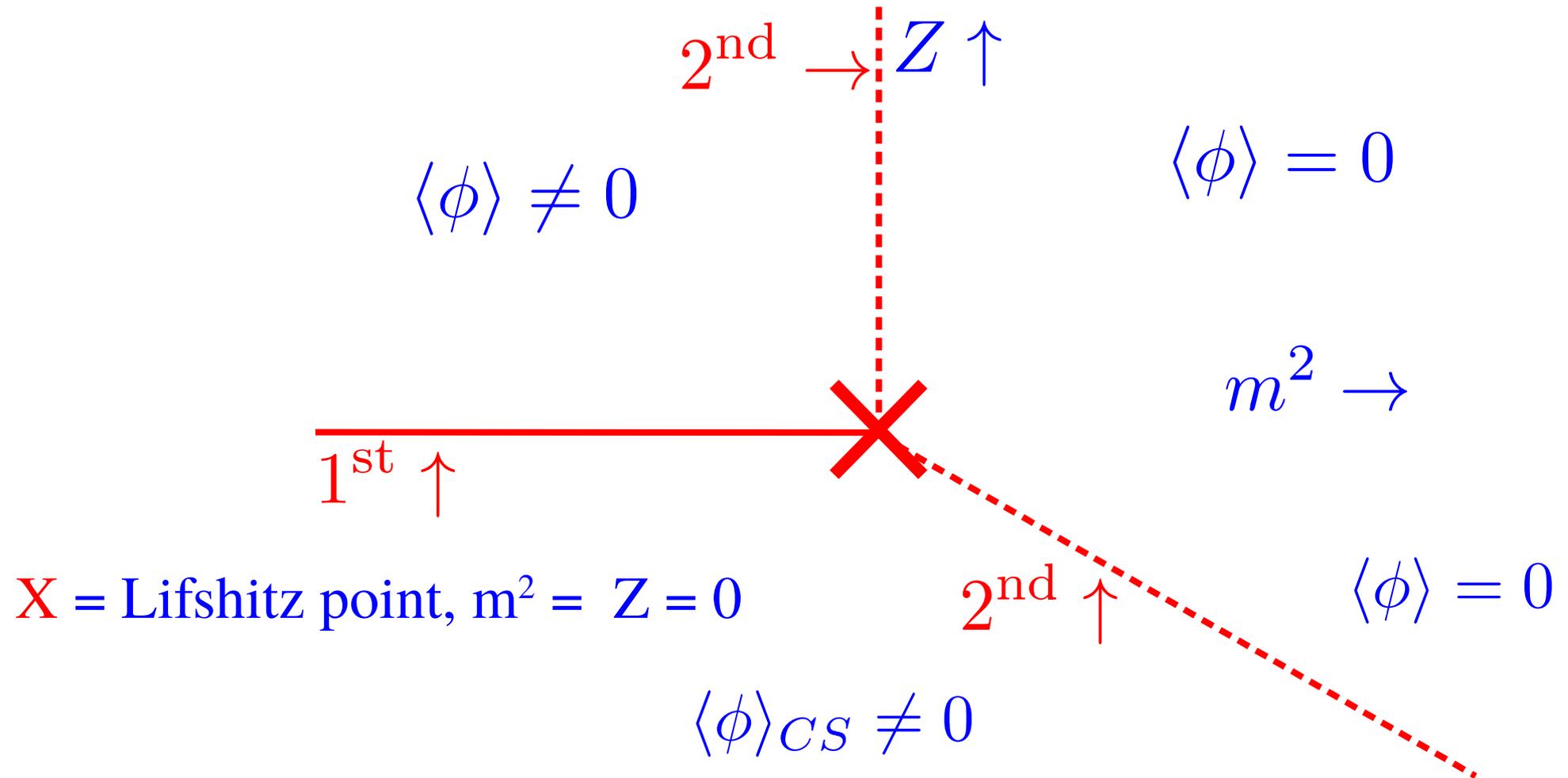
$m^2 \rightarrow$

$1^{\mathrm{st}} \uparrow$

X = tri-critical point, $m^2 = \lambda = 0$

Lifshitz phase diagram (*in mean field theory*)

$$\mathcal{L}_{\text{Lifshitz}} = (\partial_0\phi)^2 + Z(\partial_i\phi)^2 + \frac{1}{M^2}(\partial_i^2\phi)^2 + m^2\phi^2 + \lambda\phi^4$$
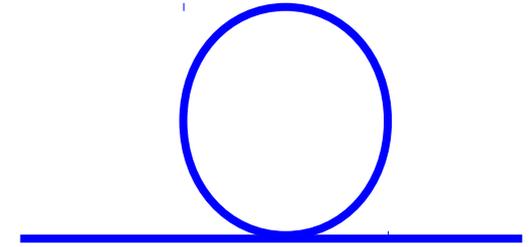
Negative kinetic term, Z < 0, generates spatially inhomogeneous phase, CS.
Three phases.

$2^{\text{nd}} \rightarrow$ $Z \uparrow$

$\langle\phi\rangle \neq 0$

$\langle\phi\rangle = 0$

$m^2 \rightarrow$

$1^{\text{st}} \uparrow$

X = Lifshitz point, m^2 = Z = 0

$2^{\text{nd}} \uparrow$

$\langle\phi\rangle = 0$

$\langle\phi\rangle_{CS} \neq 0$

No massless modes in too few dimensions

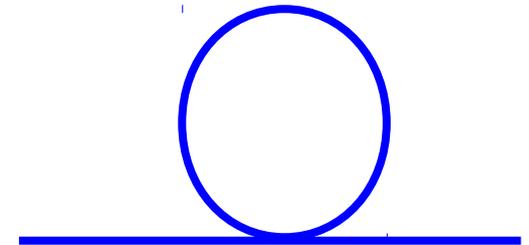No massless modes in d $\leq$ 2 dimensions:

$$\int d^2 k \, \frac{1}{k^2} \sim \log \Lambda_{\mathrm{IR}}$$

Cannot break a continuous symmetry in d $\leq$ 2 dimensions: instead of Goldstone bosons, generate a mass *non*-perturbatively.

Lifshitz point: Z = m^2 = 0, so propagator just $\sim 1/k^4$:
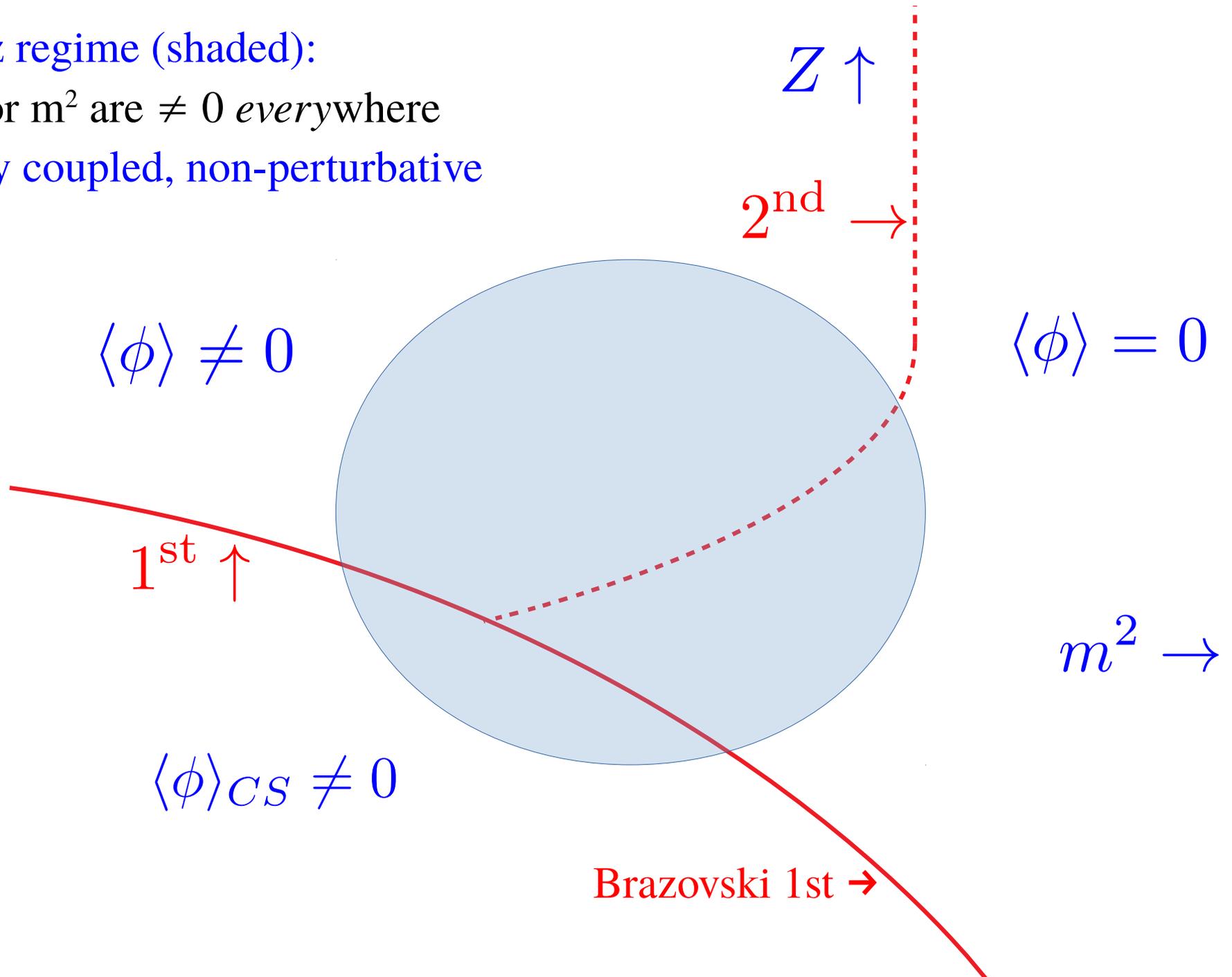
$$\int d^4 k \, \frac{1}{k^4} \sim \log \Lambda_{\mathrm{IR}}$$

Hence *no* Lifshitz point in d $\leq$ 4 (spatial) dimensions.

Must generate either a mass m^2, *or* term $\sim$ Z p$^2 \neq$ 0, *non*-perturbatively

Lifshitz regime

Lifshitz regime (shaded):
Z and/or m² are ≠ 0 *every*where
strongly coupled, non-perturbative

$Z \uparrow$

$2^{\text{nd}} \rightarrow$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

$1^{\text{st}} \uparrow$

$m^2 \rightarrow$

$\langle \phi \rangle_{CS} \neq 0$

Brazovski 1st →

Example: inhomogenous polymers

Like mixing oil & water: polymers A & B, with AB diblock copolymer ("co-AB")

homopolymer

$$A \quad A \quad A \quad A \quad A \quad A \quad A \quad A \quad A \quad A$$

AB diblock copolymer

$$A \quad A \quad A \quad A \quad B \quad B \quad B \quad B \quad B \quad B$$

Three phases: high temperature, A & B mix, symmetric phase

low temperature, little co-AB: A & B seperate, broken phase

co-AB tends to decrease interface tension between A & B phases, can turn it negative. Like Z < 0
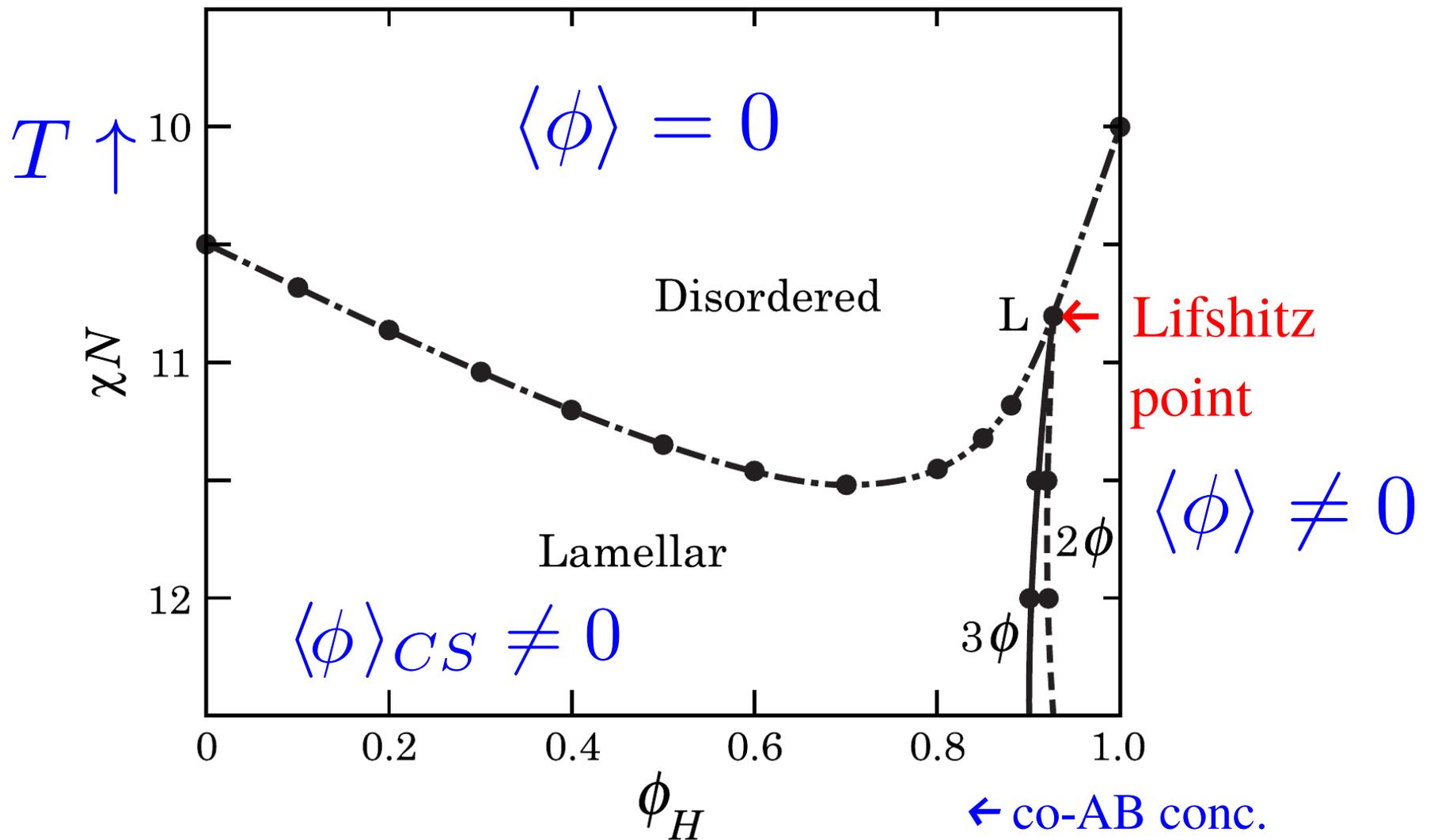
Low temperature, high concentration co-AB: "lamellar" phase, stripes of A & B. Like smectic.

Lifshitz point in inhomogenous polymers: mean field

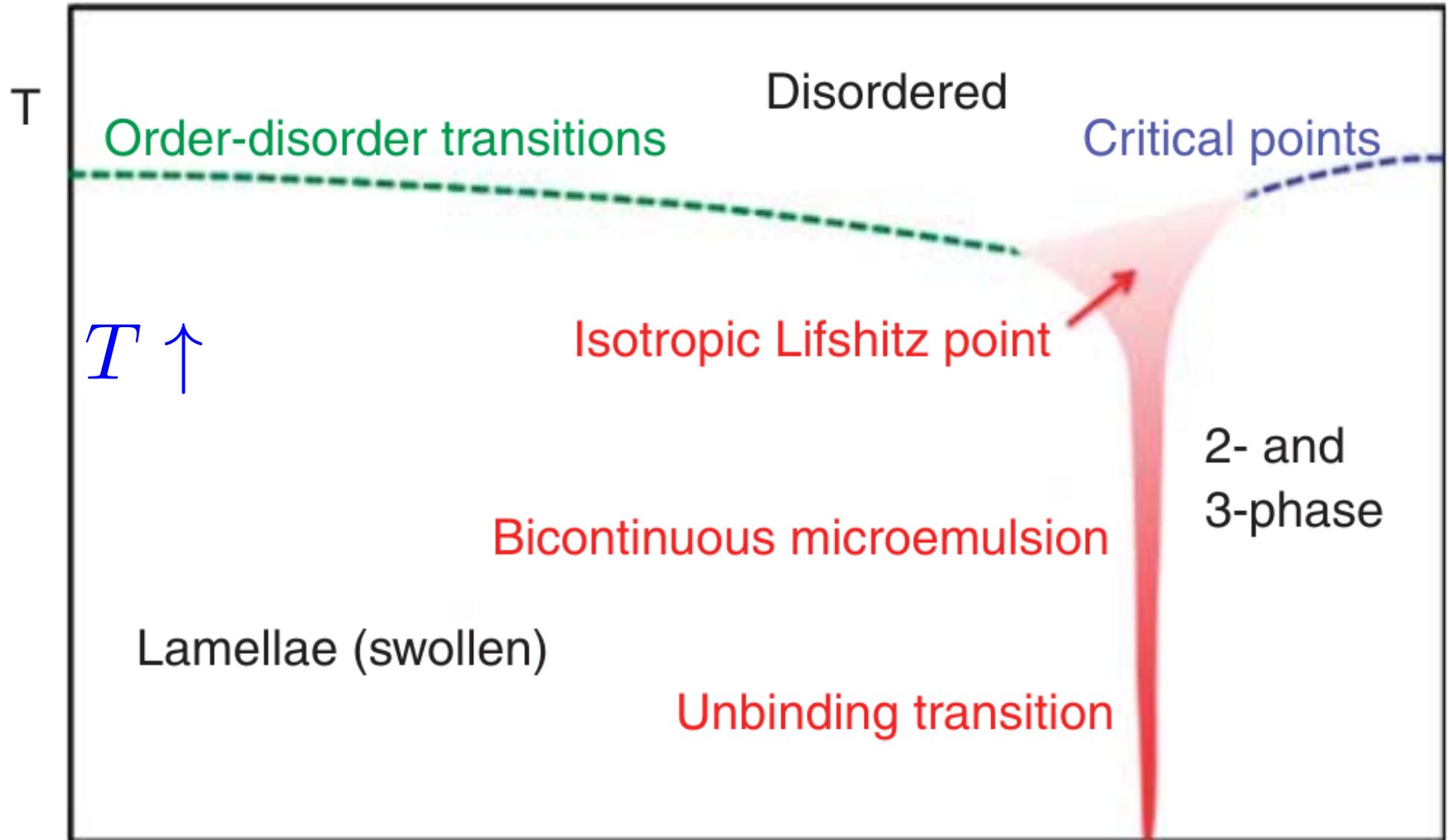Three phases, symmetric, broken, & spatially inhomogenous

Mean field predicts Lifshitz point at given T & concentration of co-AB

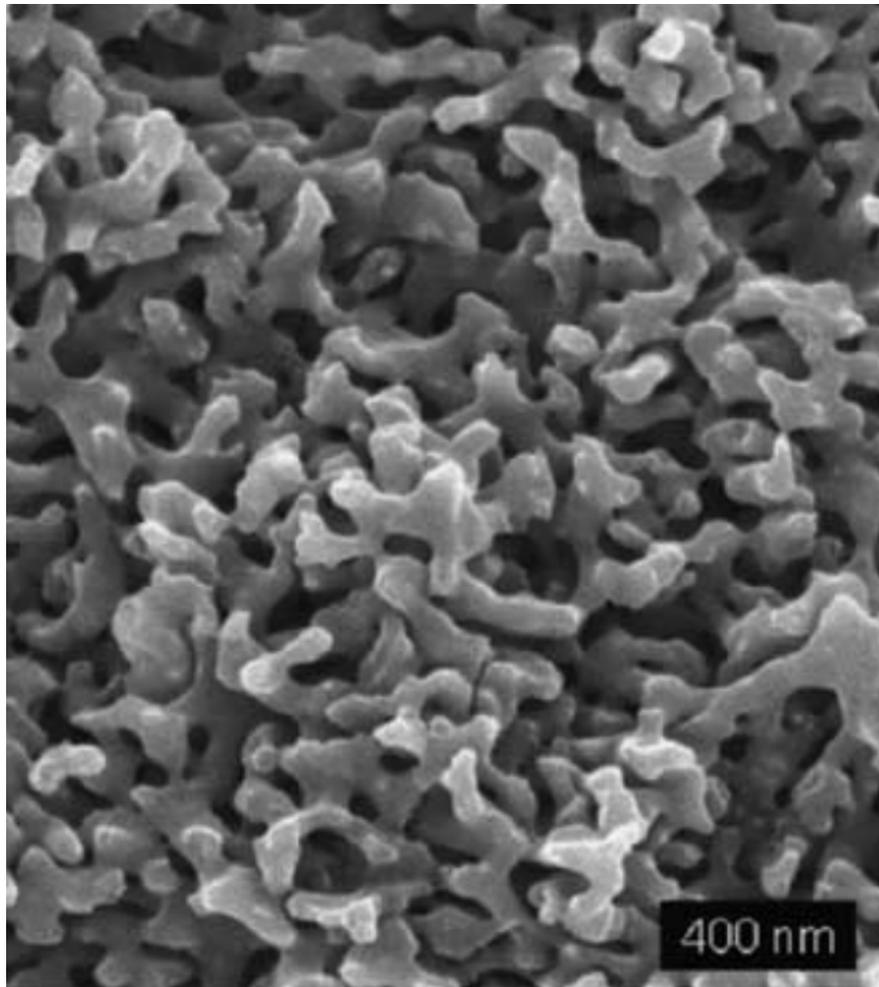Fredrickson & Bates, Jour. Polymer Sci. 35, 2775 (1997)

Lifshitz regime in inhomogenous polymers

Instead of Lifshitz point predicted by mean field theory, find

Bicontinuous microemulsion: $Z \neq 0$, $m^2 = 0$: Lifshitz regime

T

Order-disorder transitions

Disordered

Critical points

$T \uparrow$

Isotropic Lifshitz point

2- and 3-phase

Bicontinuous microemulsion

Lamellae (swollen)

Unbinding transition

0   Jones & Lodge
    Polymer Jour. 131 (44) 2012 $\phi_{homopolymer}$ ← co-AB conc.   1

Bicontinuous microemulsion: $Z \approx 0$

Experiment

Jones & Lodge,
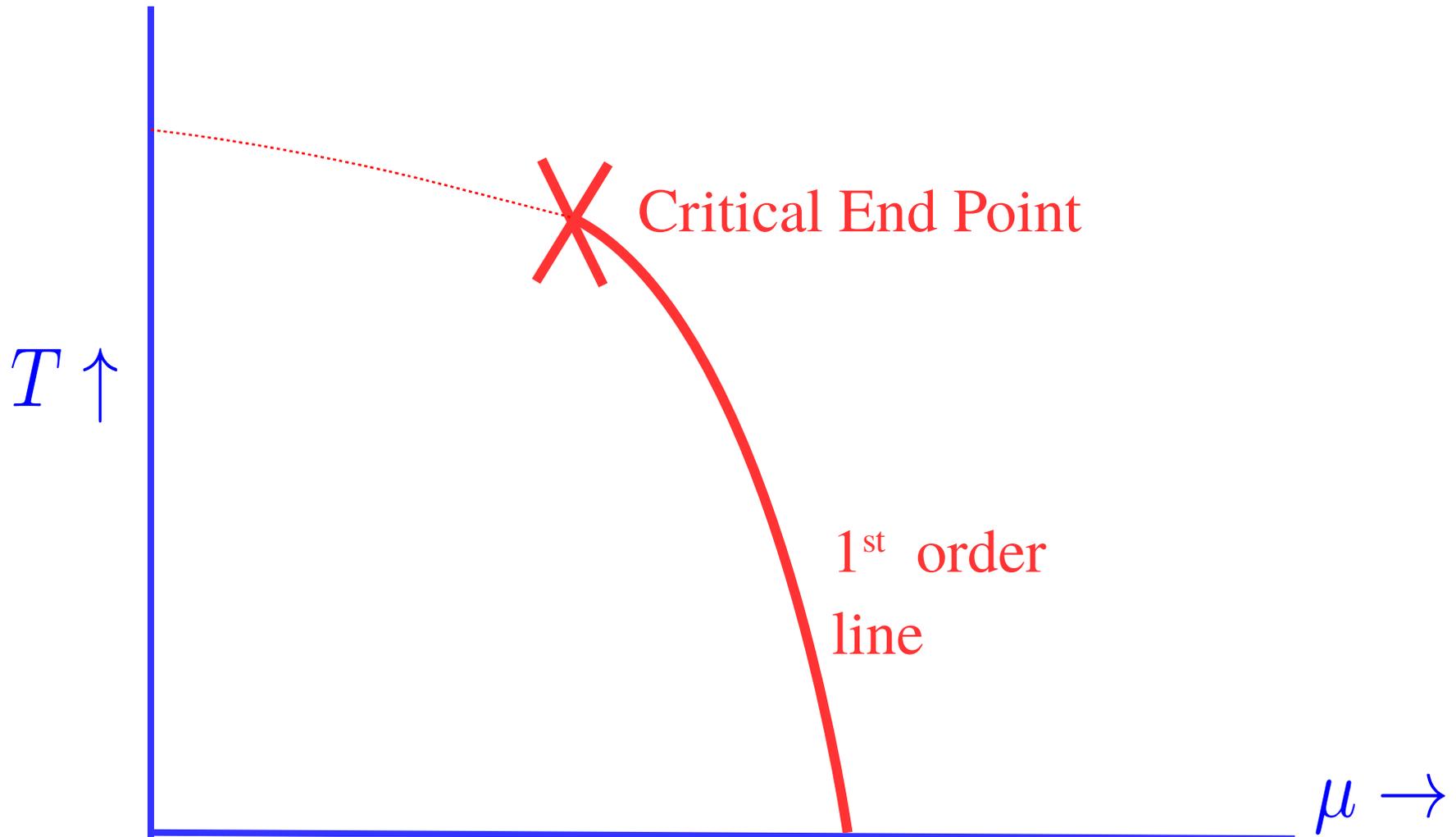Polymer Jour. 131 (44) 2012
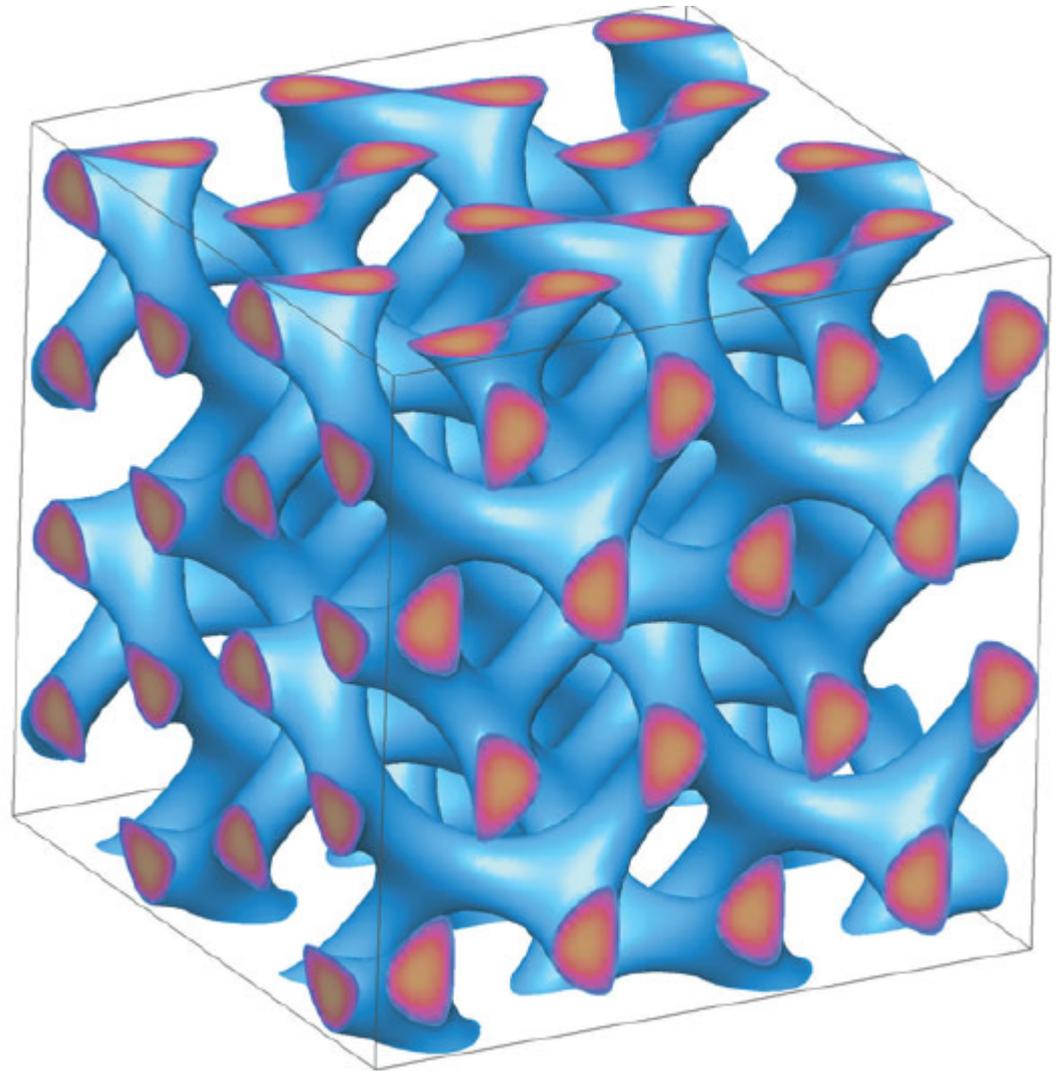
Self-consistent field theory

Fredrickson, "The equilibrium theory of
inhomogenous polymers"

Phase diagram for QCD in T & μ: usual picture

Two phases, one Critical End Point (CEP)
between crossover and line of 1st order transitions
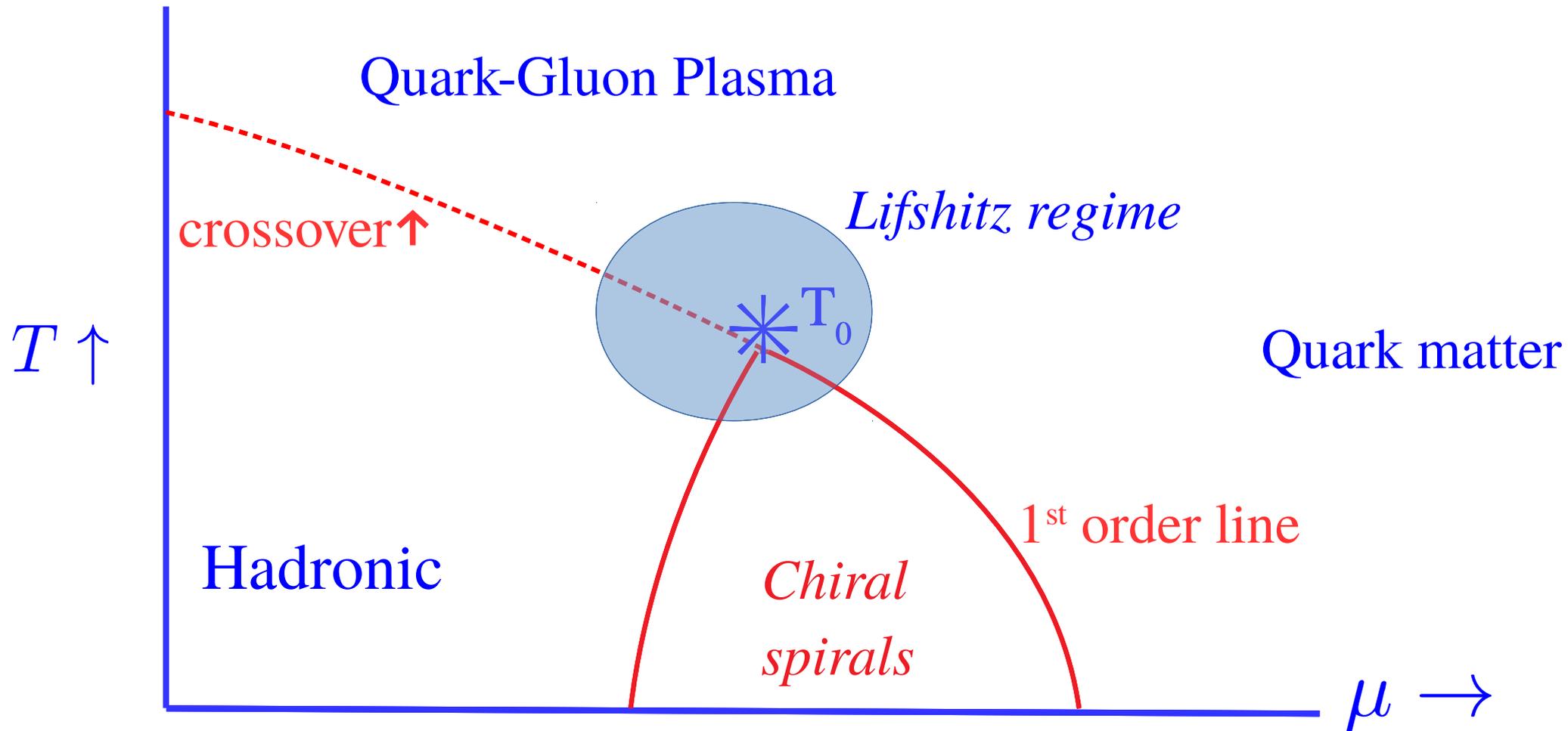Ising fixed point, dominated by *massless* fluctuations at CEP

Lifshitz phase diagram for QCD

Lifshitz regime: strongly coupled, large fluctuations

Unbroken 1st order line to spatially inhomogeneous phases = "chiral spirals"

Heavy ions: could go through *two* 1st order transitions

T_0: maximum T, point of equal concentrations (unequal entropy)

Fluctuations at 7 GeV
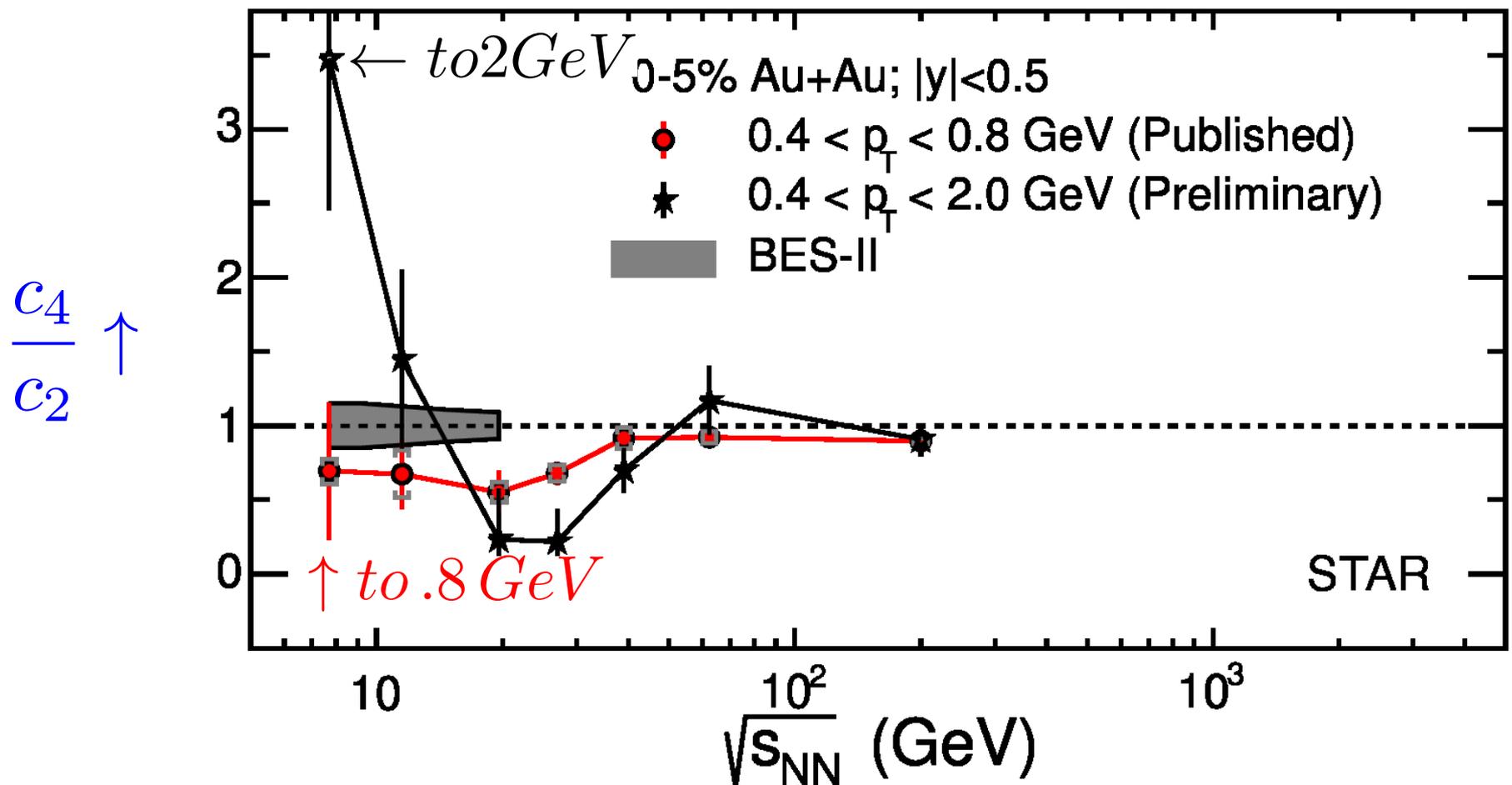
Beam Energy Scan, down to 7 GeV.

Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV

Trivial multiplicity scaling? ... or Chiral Spiral?

But fluctuations in nucleons, not pions.

X. Luo & N. Xu, 1701.02105, fig. 37; Jowazee, 1708.03364

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T, \mu)$$

0-5% Au+Au; |y|<0.5
- ● $0.4 < p_T < 0.8$ GeV (Published)
- ★ $0.4 < p_T < 2.0$ GeV (Preliminary)
- ▬ BES-II

$\frac{c_4}{c_2}$ ↑

← $to\,2\,GeV$

↑ $to\,.8\,GeV$

STAR

$\sqrt{s_{NN}}$ (GeV)

Experimentally

For *any* sort of periodic structure (1D, 2D, 3D...),

   Fluctuations concentrated about some characteristic momentum k_0

   So "slice and dice": bin in intervals, 0 to .5 GeV, .5 to 1., etc.

   If peak in fluctuations in a bin not including zero,
   may be evidence for $k_0 \neq 0$.

Signals for Lifshitz regime?

Must *measure fluctuations in pions, kaons...*

Gertrude Stein about Oakland, California, ~ 1890:

"There's no there, there."

Heavy Ions at "low" energy:
Beam Energy Scan at RHIC,
NICA, FAIR....

There *is* a there, there

But what is it?

NJL models and Lifshitz points

Consider Nambu-Jona-Lasino models.

Nickel, 0902,1778 & 0906.5295 + ... + Buballa & Carignano 1406.1367

$$\mathcal{L}_{\mathrm{NJL}} = \overline{\psi}(\slashed{\partial} + g\sigma)\psi + \sigma^2$$

Integrating over ψ,

$$\mathrm{tr}\,\log(\slashed{\partial} + g\,\sigma) \sim \ldots + \kappa_1((\partial\sigma)^2 + \sigma^4) + \ldots$$

Due to scaling, $\partial \to \lambda\partial$, $\sigma \to \lambda\sigma$.

Consequently, in NJL @ 1-loop, *tricritical = Lifshitz point.*

Special to including only σ at one loop.

Not generic: violated by the inclusion of more fields, to two loop order, etc.

Improved gradient expansion near critical point:

Carignano, Anzuni, Benhar, & Mannarelli, 1711.08607.

Symmetric to CS: 1D (Brazovski) fluctuations

Consider $m^2 > 0, Z < 0$: minimum in propagator at *non*zero momentum

Brazovski '75; Hohenberg & Swift '95 + ... ;
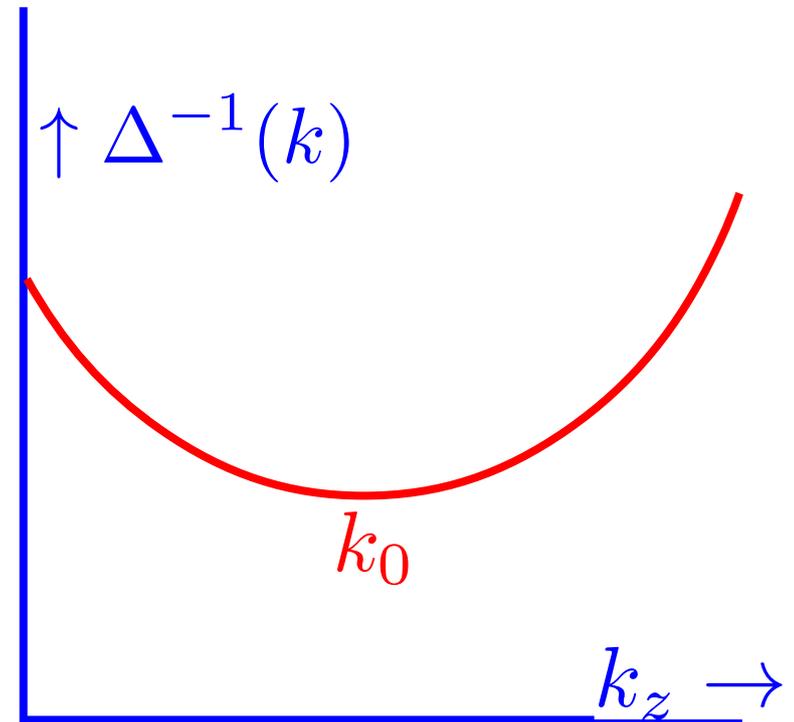Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Yoshiike, Lee & Tatsumi 1702.01511

$$\Delta^{-1} = m^2 + Z\,k^2 + k^4/M^2$$
$$= m_{\text{eff}}^2 - 2\,Z\,k_z^2 + O(k_z^3, k_z k_\perp^2)$$

$k=(k_\perp, k_z-k_0)$: *no* terms in $k_\perp^2$, *only* $(k_\perp^2)^2$.

Due to spon. breaking of rotational sym.
1-loop tadpole diagram:

$$\int d^3k\, \frac{1}{k_z^2 + m_{\text{eff}}^2 + \dots} \sim M^2 \int \frac{dk_z}{k_z^2 + m_{\text{eff}}^2} \sim \frac{M^2}{m_{\text{eff}}}$$
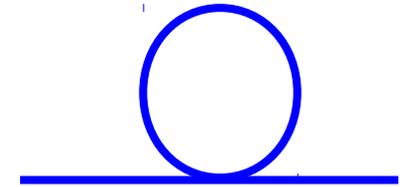
Effective reduction to 1-dim for any spatial dimension d, any global symmetry
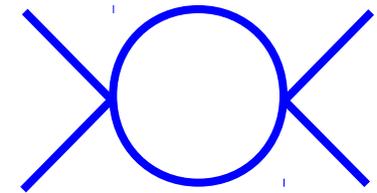
1$^{\text{st}}$ order transition in 1-dim.

Strong infrared fluctuations in 1-dim., both in the mass:

$$\Delta m^2 \sim \lambda \int d^3k \, \frac{1}{k_z^2 + m_{\text{eff}}^2 + \dots} \sim \lambda \frac{M}{m_{\text{eff}}}$$

and for the coupling constant:

$$\Delta \lambda \sim -\lambda^2 \int \frac{d^3k}{(k_z^2 + m_{\text{eff}}^2 + \dots)^2} \sim -\lambda^2 M^3 \int_{m_{\text{eff}}} \frac{dk_z}{k_z^4} \sim -\lambda \frac{M^3}{m_{\text{eff}}^3}$$

Can*not* tune m$_{\text{eff}}^2$ to 0: λ_{eff} goes negative, 1$^{\text{st}}$ order trans. induced by fluctuations

Not like other 1st order fluc-ind'd trans's: just that in 1-d, m$_{\text{eff}}^2 \neq 0$ always