Hamiltonian approach to QCD at finite T

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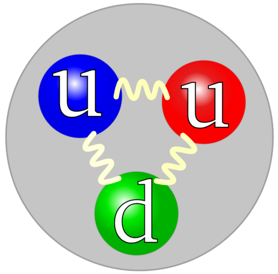
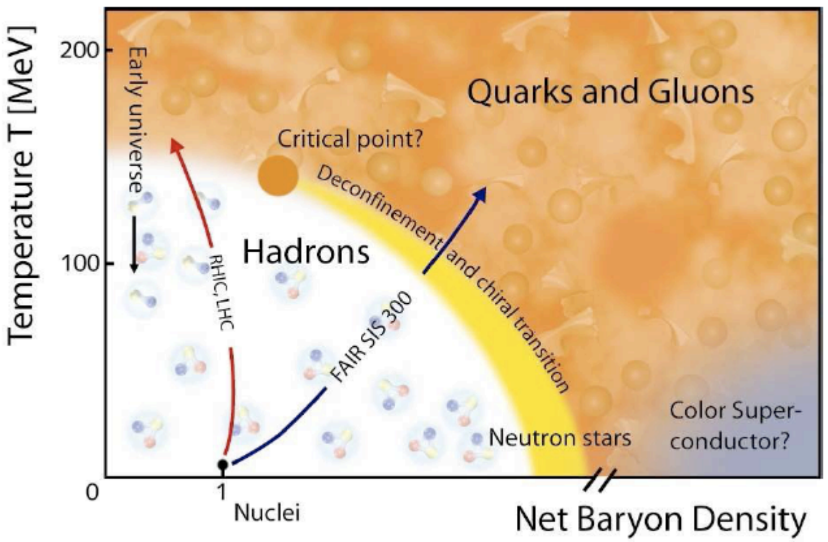
QCD

- *vacuum*
  - confinement
  - SB chiral symmetry



- *phase diagram*
  - deconfinement
  - rest. chiral symm.

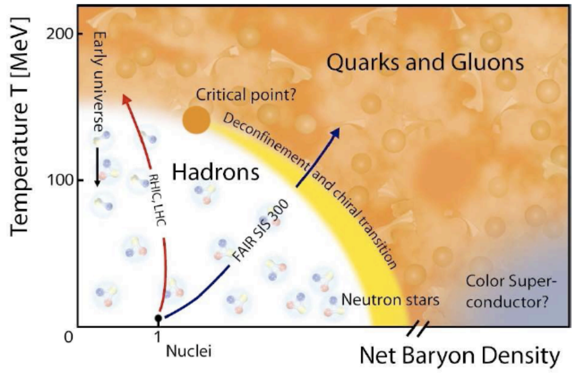


- latticeMC-fail at large chemical potential
  continuum approaches required

non-perturbative continuum approaches

- Dyson-Schwinger equations
  - Landau(+Coulomb)gauge
- FRG flow equations
  - Landau gauge
- Variational approaches
  - Covariant : Landau gauge
  - Hamiltonian: Coulomb gauge

Hamiltonian approach to finite temperature QFT



- partition function

$$Z(L) = Tr \exp(-LH) \qquad T = L^{-1}$$

- necessitates approximation to density operator $\exp(-LH)$
- common: quasiparticle approximation(Wick´s theorem)

>alternative Hamiltonian approach to finite temperature QFT:

compactification of a spatial dimension

H. R. Phys.Rev.D94(2016)045016

Outline

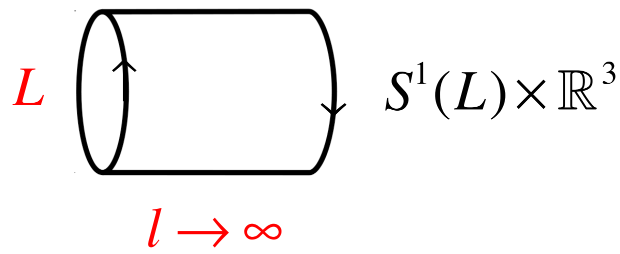
- introduction
- Hamiltonian approach at finite temperature by compactification of a spatial dimension
- basics of the Hamiltonian approach to QCD in Coulomb gauge (T=0)
    - Yang-Mills theory
    - quark sector
- QCD at finite T
    - quark condensate
    - Polyakov loop
- conclusions & outlook

Finite temperature QFT

$$Z(L) \equiv Tr \exp(-LH) = \int_{bc} D(A,\psi) \exp\left[-\int_0^L dx^0 \int d^3x L_E(A,\psi) \right]$$

$T = L^{-1}$

- compactification of (Euclidean) time
  - bc: $A(x^0 = L/2) = \mathrm{A}(x^0 = -L/2)$ Bose fields
    $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



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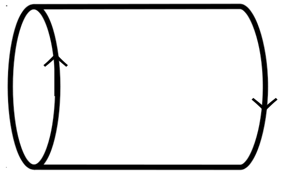
L  $S^1(L) \times \mathbb{R}^3$

$l \to \infty$

- exploit the O(4)-invariance of the Euclidean Lagrange density to rotate the time axis onto a spatial axis

$$x^0 \to x^3 \qquad \mathrm{A}^0 \to A^3 \qquad \gamma^0 \to \gamma^3$$
$$x^1 \to x^0 \qquad \mathrm{A}^1 \to A^0 \qquad \gamma^1 \to \gamma^0$$

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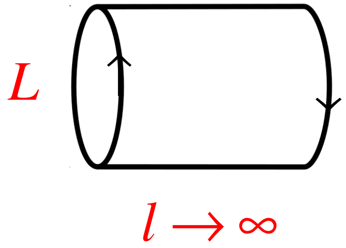

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- compactification of one spatial dimension
  - bc: $A(x^3 == L/2) = A(x^3 = -L/2)$ Bose fields
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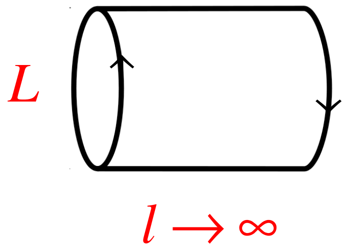

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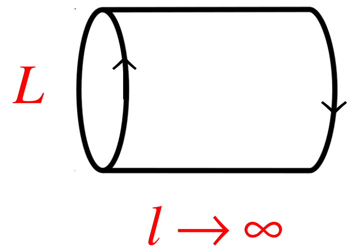

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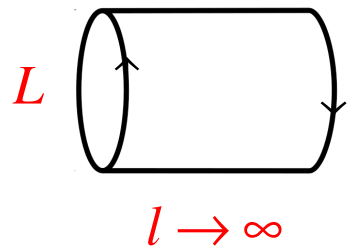

- canonical quantization on the spatial manifold $\mathbb{R}^2 \times S^1(L)$

$$Z(L) = \lim_{l\to\infty} Tr \exp(-lH(L)) = \lim_{l\to\infty} \sum_n \exp(-lE_n(L)) = \lim_{l\to\infty} \exp(-lE_0(L))$$

- *temperature is now encoded in a „spatial“dimension while „time“ has infinite extension independent of the temperature*

Hamiltonian approach to finite temperature QFT

- partition function

H. R. Phys.Rev.D94(2016)045016

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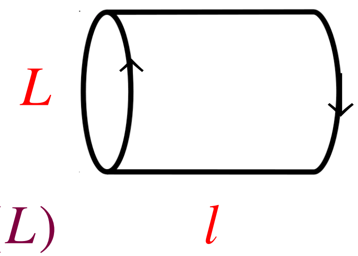

thermodynamics of a relativistic QFT is completely given
given by its vacuum state on the spatial manifold $\mathbb{R}^2 \times S^1(L)$

- ground state energy on $\mathbb{R}^2 \times S^1(L)$ $\qquad E_0(L) = l^2 L e(L)$

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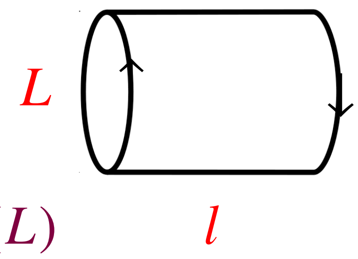

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- pressure:
- energy density:

$$P = -\partial[Ve(L)]/\partial V \qquad V = l^3$$

$$\varepsilon = \partial[Le(L)]/\partial L - \mu \partial e/\partial \mu$$

- Dirac fermions with finite chemical potential

$$h = \vec{\alpha}\cdot\vec{p} + \beta m \rightarrow h + i\mu\alpha^3$$

Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$

$\mathbb{R}^3$ $\int d^3 p f(\vec{p})$

$\mathbb{R}^2 \times S^1(L)$ $\int_L d^3 p f(\vec{p}) := \int d^2 p_\perp \frac{2\pi}{L} \sum_n f(\vec{p}_\perp, \omega_n)$ *O(3)-broken*

Matsubara frequency:

$$\omega_n = \frac{2\pi n}{L}, \quad \text{bosons} \quad n_F = 0$$

$$\omega_n = \frac{2(n+1)\pi}{L}, \quad \text{fermions} \quad n_F = 1$$

Poisson resummation:

$$\frac{1}{2\pi} \sum_{k=-\infty}^{k=\infty} e^{ikx} = \sum_{n=-\infty}^{n=\infty} \delta(x - 2\pi n)$$

$$\int_L d^3 p f(\vec{p}) := \int d^2 p_\perp dp_3 f(\vec{p}_\perp, p_3) \sum_{k=-\infty}^{\infty} (-)^{k n_F} \exp(ikLp_3)$$

vacuum (T=0): k=0 term

Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

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- energy density on $\mathbb{R}^2 \times S^1(L)$ $\qquad P = -e(L)$

$$e(L) = \frac{1}{2}\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + \omega_n^2} \qquad \omega_n = \frac{2\pi n}{L}$$

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- proper-time regularization

$$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})}\lim_{\Lambda\to\infty}\int_{1/\Lambda^2}^{\infty} d\tau\,\exp(-\tau A)$$

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$$P = -e(L) = \frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{nL}\right)^2 K_{-2}(nLm)$$

modified Bessel function

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modified Bessel function

- massles bosons: m=0

Stephan – Boltzmann – law

$$P = \frac{\zeta(4)}{\pi^2}T^4 = \frac{\pi^2}{90}T^4$$

massive bosons

$$\omega(p) = \sqrt{p^2 + m^2}$$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1}$$

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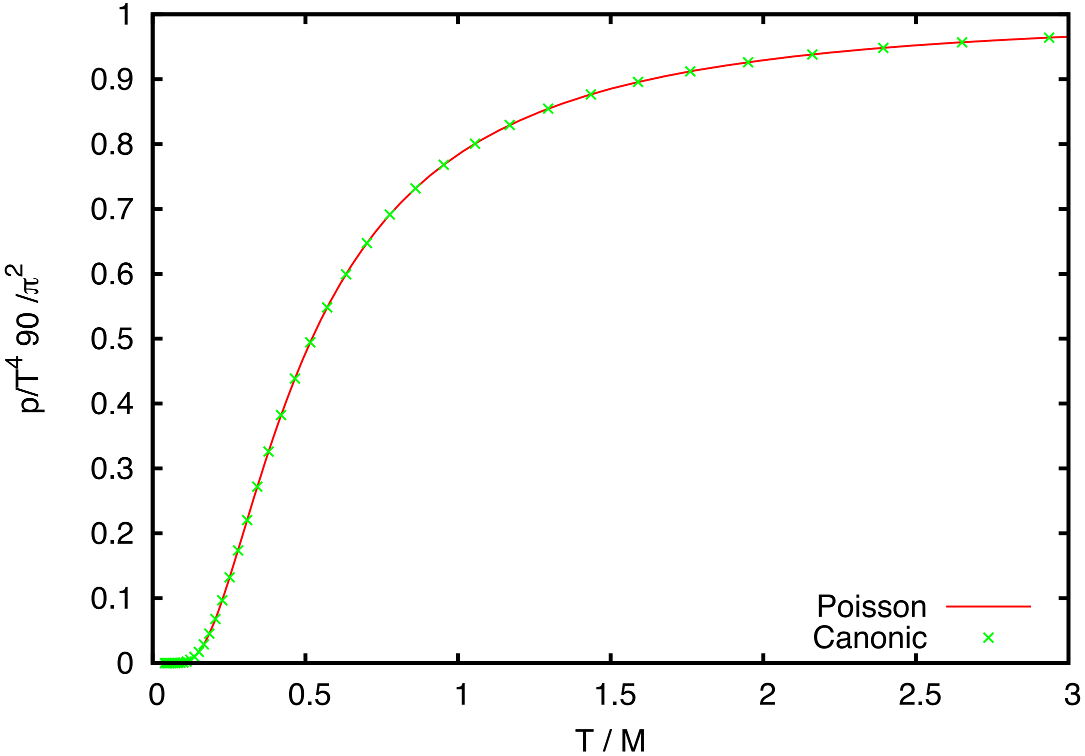
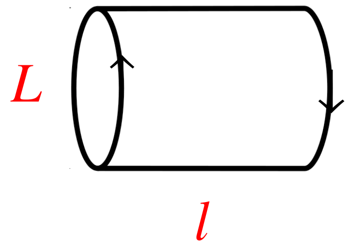


pressure of a massive relativistic Bose gas

$$e(L) = \frac{1}{2}\int d^2 p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + \omega_n^2}$$

$$\omega_n = \frac{2\pi n}{L}$$

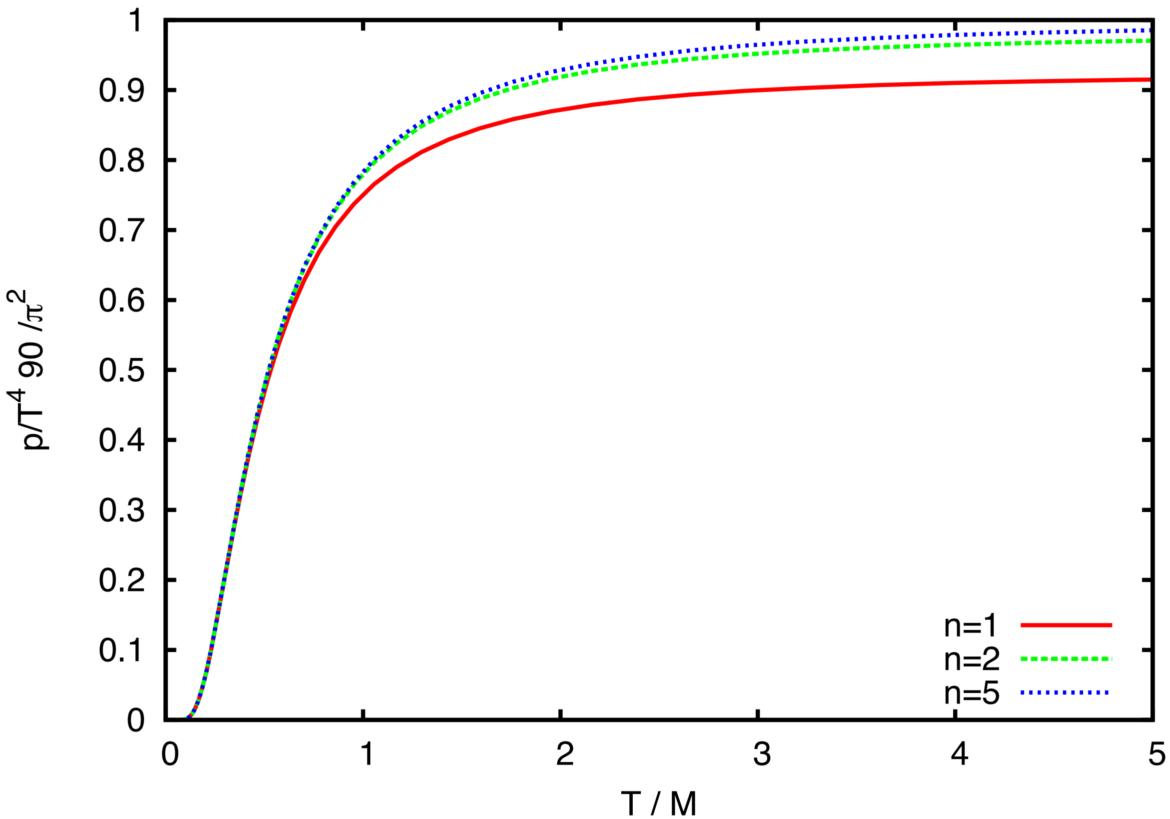


$S^1(L)\times\mathbb{R}^3$

-proper-time regularization
-Poisson resummation
-skip L-independent (div.) const.

$$P = \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{m}{nL}\right)^2 K_{-2}(nLm)$$

at moderate temperatures a few terms are sufficient to reproduce the result of the usual grand canonical ensemble



Relativistic Fermi gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)}(n_+(p)+n_-(p)) \qquad n_\pm(p) = \frac{1}{e^{L(\omega(p)\mp\mu)}+1} \qquad \omega(p) = \sqrt{p^2+m^2}$$

- energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = -2\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2+p_\perp^2+(\omega_n+i\mu)^2} \qquad \omega_n = \frac{2n+1}{L}\pi$$

  - proper-time
  - Poisson resummation

$$P = -e(L) = -\frac{2}{\pi^2}\sum_{n=-\infty}^{\infty}\cos\left[nL(\tfrac{\pi}{L}-i\mu)\right]\left(\frac{m}{nL}\right)^2 K_{-2}(nLm)$$

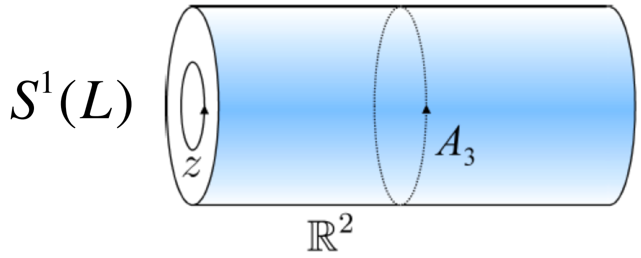
- massles Dirac fermions: m=0
  - analytic continuation for iμ⟶ x $\displaystyle\sum_{n=1}^{\infty}(-)^n\frac{\cos(nx)}{n^4} = \tfrac{1}{48}\left[-\tfrac{7}{15}\pi^4+2\pi^2x^2-x^4\right]$

$$P = \frac{1}{12\pi^2}\left[\frac{7}{15}\pi^4T^4+2\pi^2T^2\mu^2+\mu^4\right]$$

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$

  H. R. Phys.Rev.D94(2016)045016



- finite temperature is fully encoded in the vacuum

- variational solution of the Schrödinger equation for the vacuum

chiral phase transition

>quark condensate

M.Quandt, E.Ebadati, H.R. & P.Vastag
arXiv:1806.04493

deconfinement phase transition

>Polyakov loop

H. R. & J. Heffner, PRD88

M.Quandt & H.R. to be published

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\rho^a = -f^{abc}A^b\Pi^c + \rho^a_m$

$$\langle \phi | \ldots | \psi \rangle = \int DA J(A) \phi^*(A) \ldots \psi(A)$$

$$H\psi[A] = E\psi[A]$$

Variational approach to YMT

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\,\omega(x,y)\,A(y)\right]$$

gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

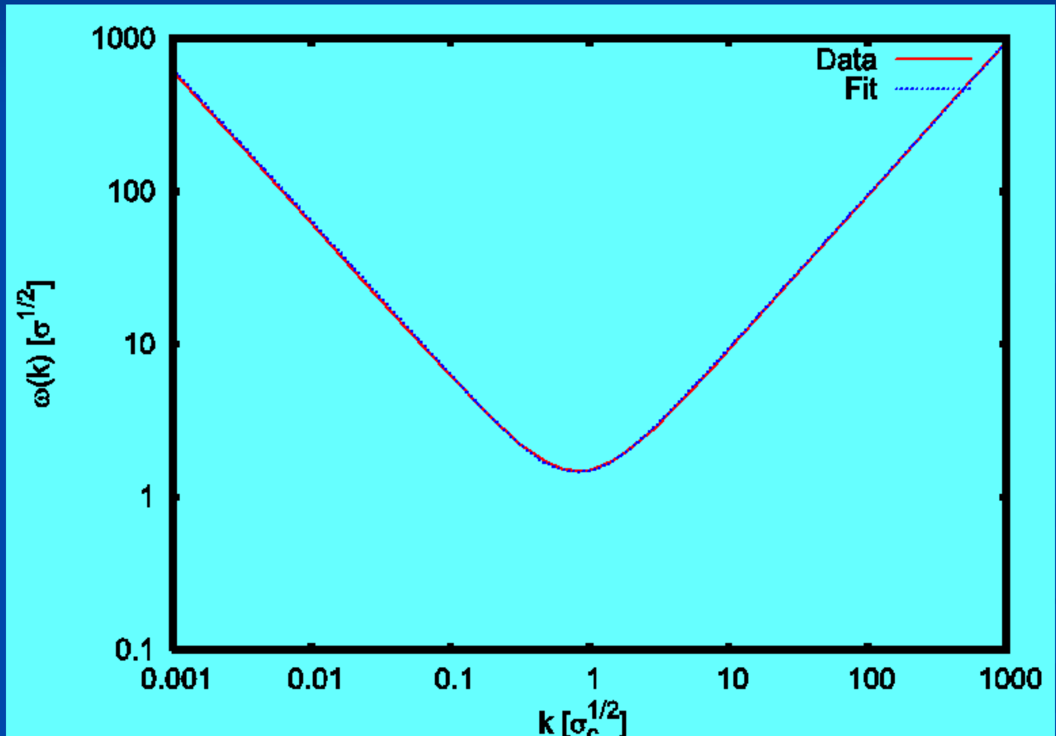
variational kernel $\omega(x,x')$ determined from

$$\langle \Psi | H | \Psi \rangle \to \min$$

Numerical results

gluon energy

D. Epple, H. R. & W.Schleifenbaum, PRD 75 (2007)



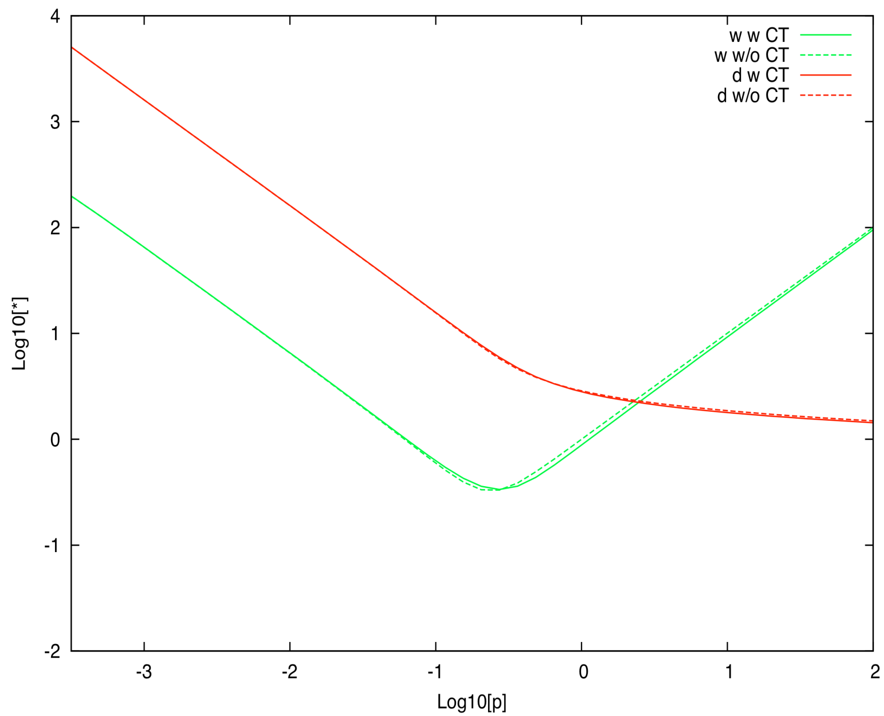
$$IR: \quad \omega(k) \sim 1/k \qquad\qquad UV: \quad \omega(k) \sim k$$

The Ghost Propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d / (-\Delta)$$

horizon condition

$$\mathrm{d}^{-1}(0) = 0$$



The color dielectric function of the QCD vacuum

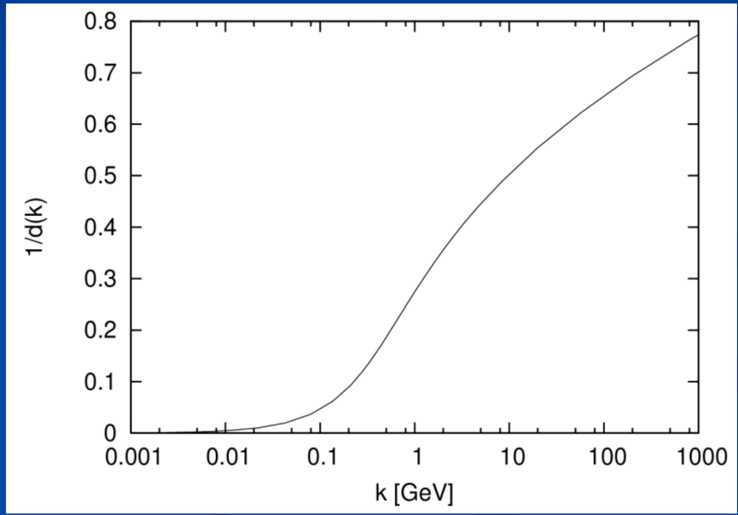
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d \,/\, (-\Delta)$$

- dielectric „constant“

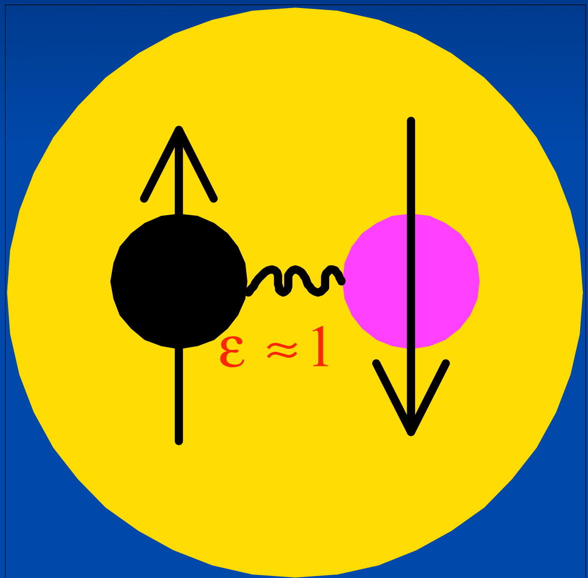
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)

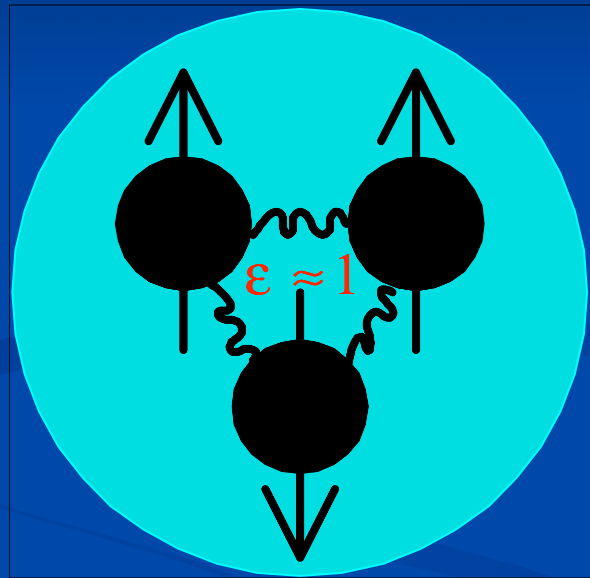


- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



$$\varepsilon = 0$$



no free color charges in the vacuum: confinement

The color dielectric function of the QCD vacuum

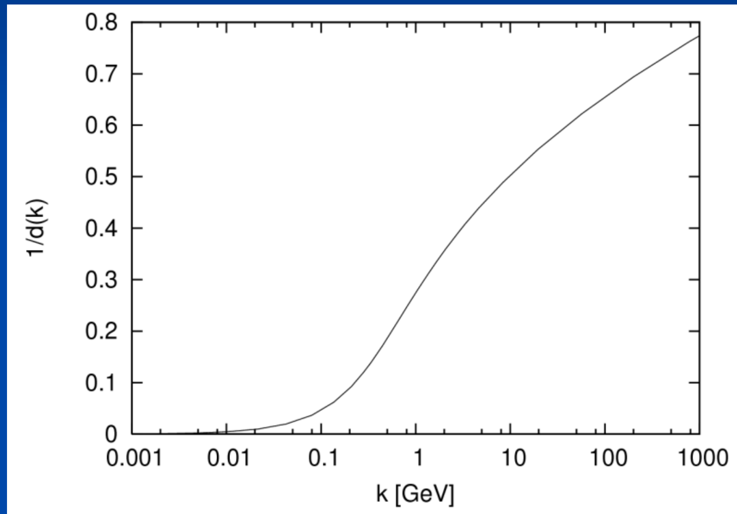
- ghost propagator

$$\langle(-D\partial)^{-1}\rangle = d/(-\Delta)$$

- dielectric „constant“

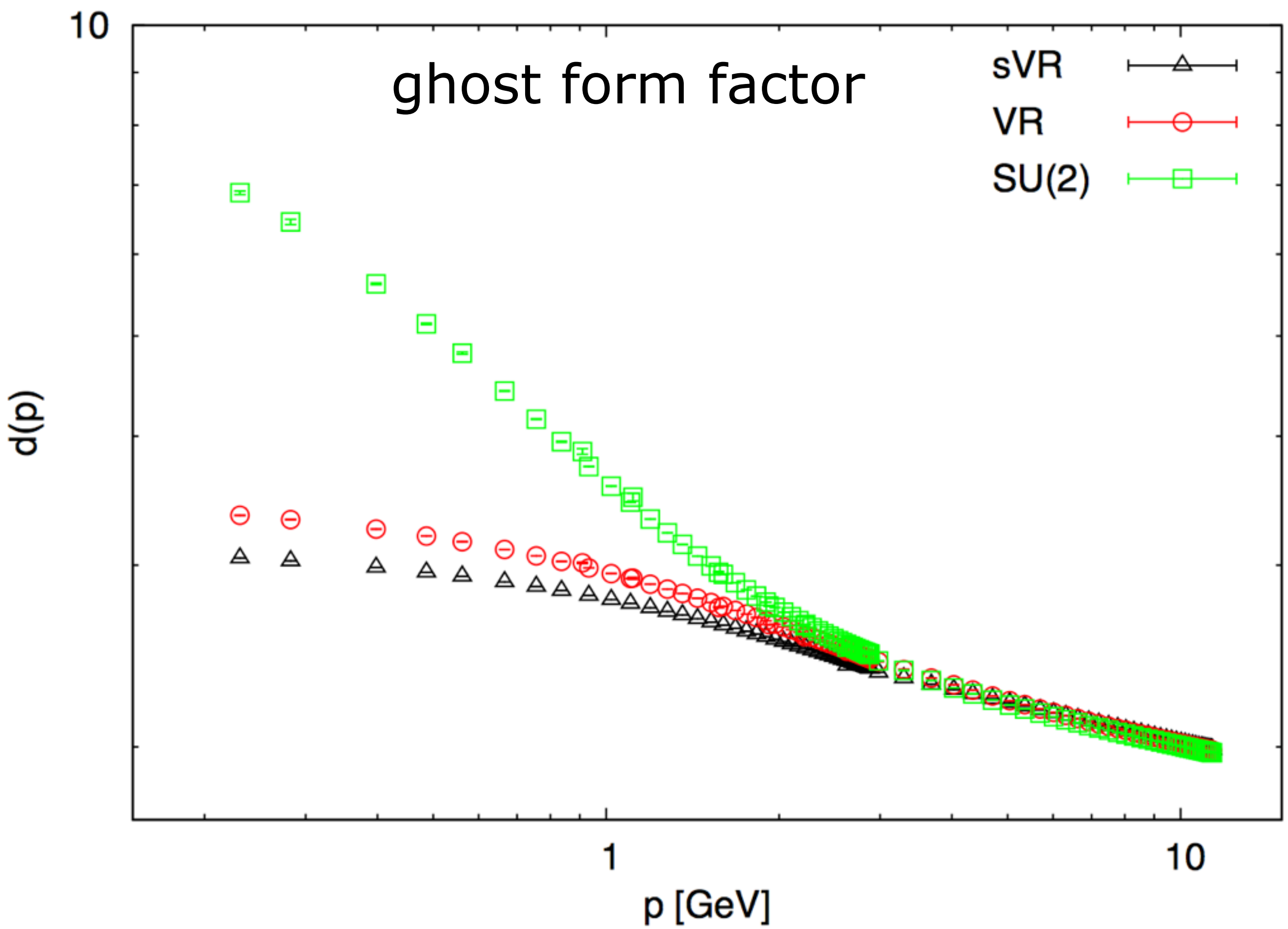
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)



- horizon condition:
  - : $d^{-1}(k=0)=0$ $\quad \varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
  - $\varepsilon(k)<1$ anti-screening

Gribov scenario & center vortex picture



G. Burgio, M. Quandt, H.R. & H.Vogt, Phys. Rev.D92(2015)

- elimination of center vortices: IR enhancement disappears
- horizon condition $d^{-1}(0) = 0$ is lost

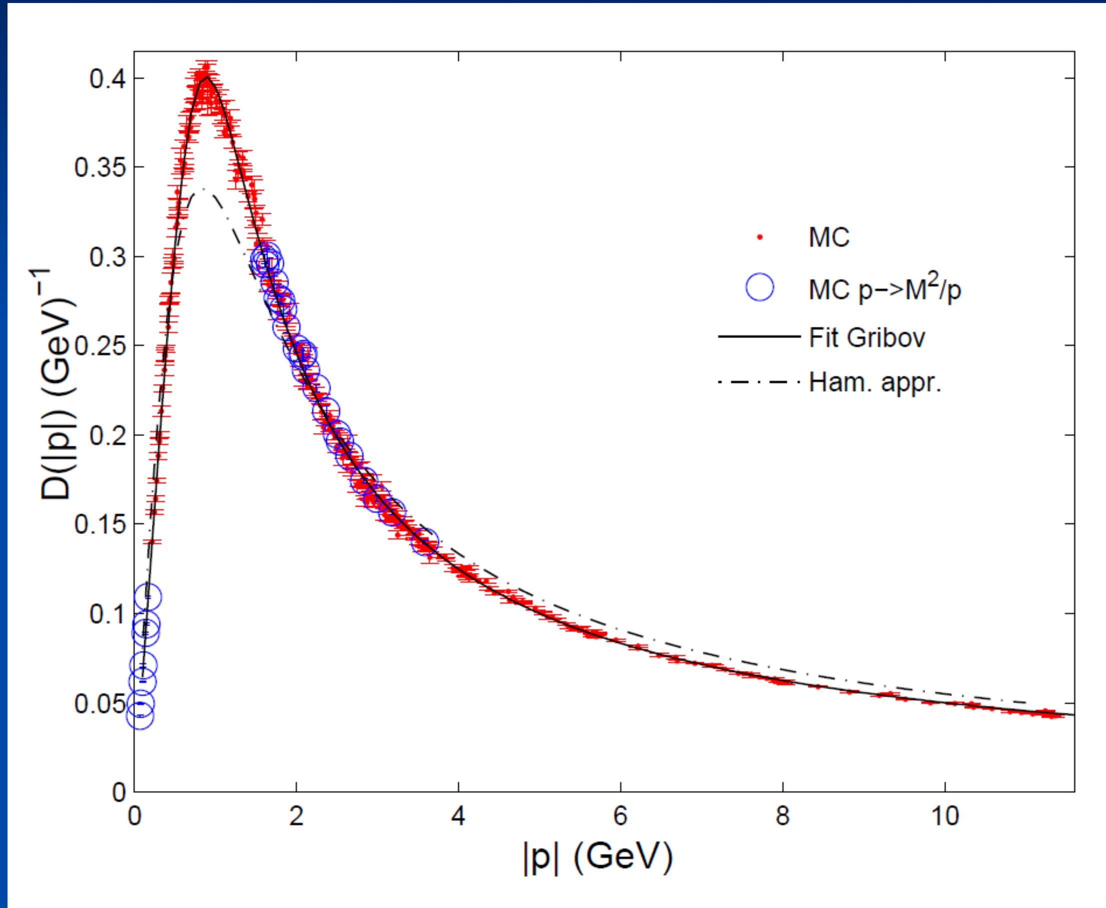
Static gluon propagator in D=3+1

$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$



missing strength in
mid momentum regime

lattice: G. Burgio, M.Quandt , H.R., **PRL102(2009)**

continuum: D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

wave functional

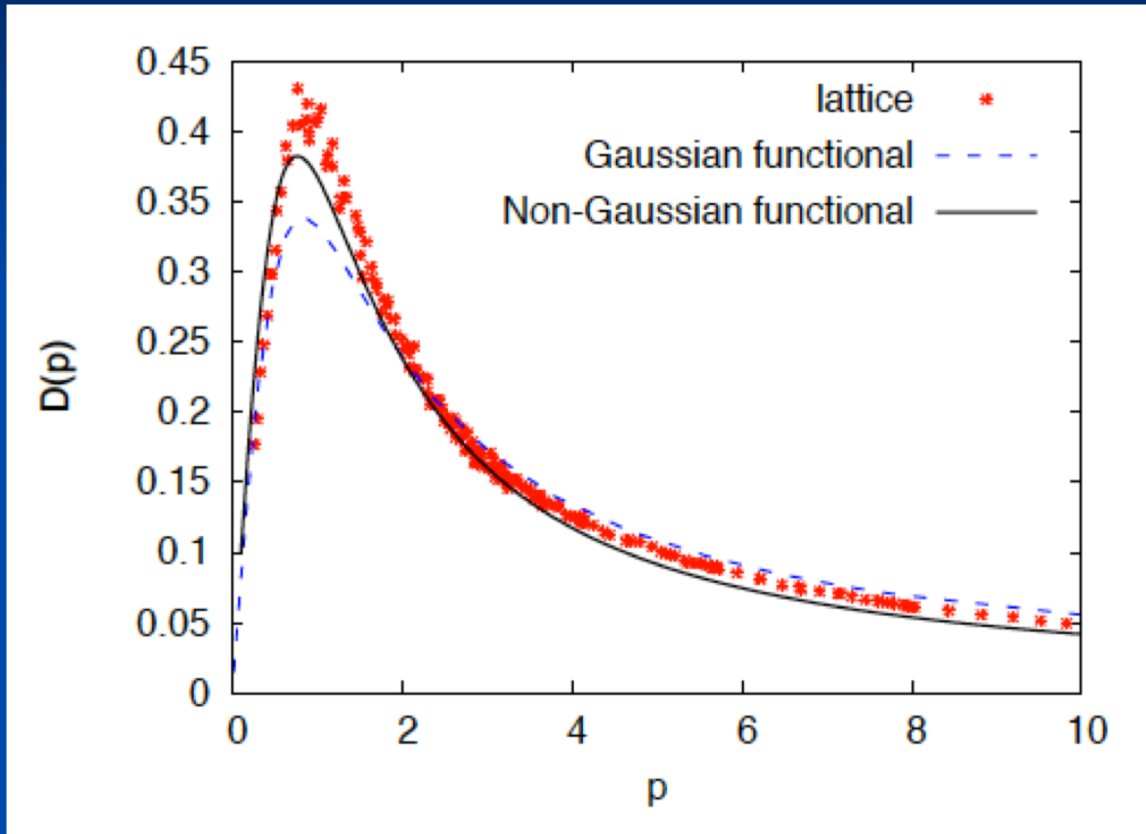
$$|\psi[A]|^2 = \exp(-S[A])$$

ansatz

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

Static gluon propagator in D=3+1



YM Hamiltonian in $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial)$$

$$D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\rho^a = -f^{abc}A^b\Pi^c + \bar{q}t^a q$

YM Hamiltonian in $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) + H_C$$

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$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

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Coulomb term

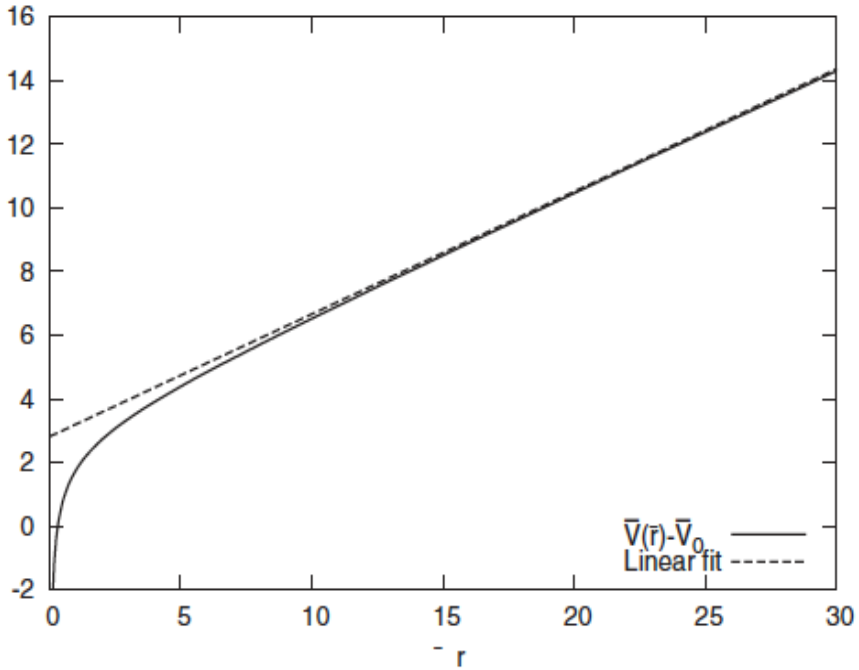
color charge density $\rho^a = -f^{abc}A^b\Pi^c + \bar{q}t^a q$

static quark potential

$$V_C(|\vec{x}-\vec{y}|) = \langle\langle \vec{x}|(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}|\vec{y}\rangle\rangle$$

Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$



D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(r) = \xrightarrow[r\to 0]{} \sim 1/r$$

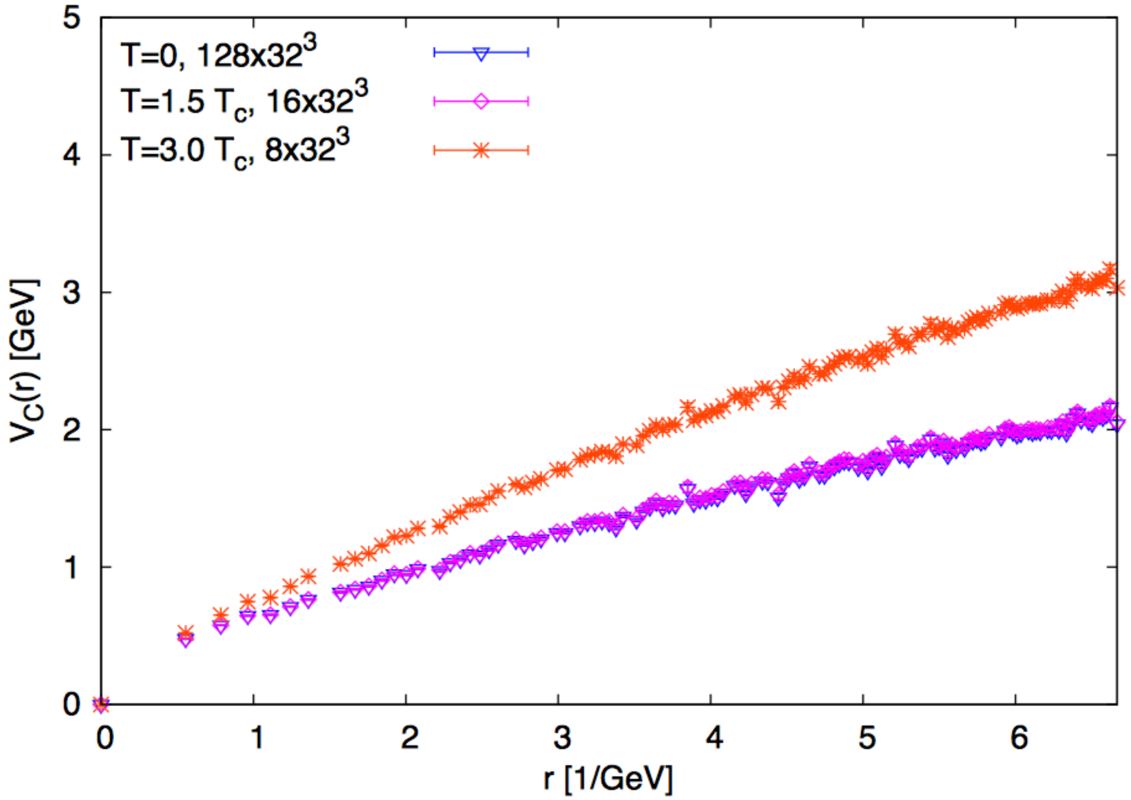
$$V(r) = \xrightarrow[r\to\infty]{} \sigma_C r,$$

lattice : $\sigma_C = 2...4\sigma_W$ strict relation $\sigma_W < \sigma_C$ *D. Zwanziger*

σ_C is bound to $\sigma_{Wspatial}$ and not to $\sigma_{Wtemporal}$

G. Burgio, M. Quandt,
H. R. & H. Vogt,
Phys.Rev.D92(2015)

Coulomb potential at finite T



G. Burgio, M. Quandt, H. R. & H. Vogt,
Phys.Rev.D92(2015)

The QCD Hamiltonian in Coulomb gauge

$$H_{QCD} = H_{YM} + H_q + H_C$$

gluon part

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) \qquad \Pi = -i\delta/\delta A \qquad J(A^\perp) = Det(-D\partial)$$

quark part

$$H_q = \int \Psi^\dagger(x)[\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0]\Psi(x) \qquad \vec{\alpha},\beta - \textit{Dirac matrices}$$

Coulomb term

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density

$$\rho^a = -f^{abc}A^b\Pi^c + \Psi^\dagger(x)t^a\Psi(x)$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha} \cdot \vec{A} + w\beta\vec{\alpha} \cdot \vec{A}) \Psi_- \right] | 0 \rangle$$

$s, v, w -$ variational kernels $\quad \vec{\alpha}, \beta -$ *Dirac matrices*

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0$: *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

$\mathrm{v}\neq 0, \mathrm{w}=0$: *quark - gluon - coupling*

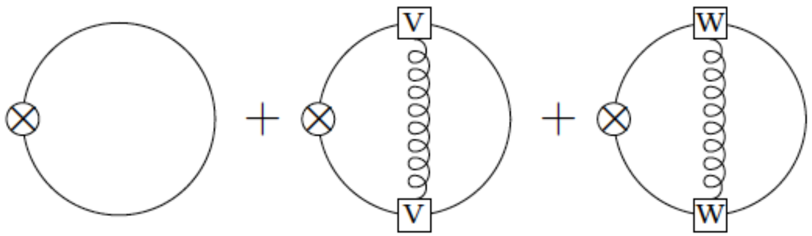
Pak & Reinhardt,

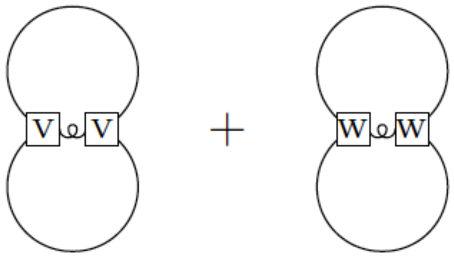
quark wave functional

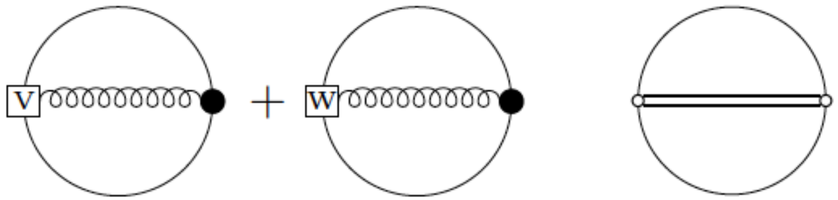
$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

> calculate $\langle H_{QCD}\rangle$ up to 2 loops







contributions from the kinetic energy of the gluons

quark wave functional

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s,v,w – variational kernels $\vec{\alpha},\beta$ – *Dirac matrices*

> calculate <H_{QCD}> up to 2 loops

> variation w.r.t. **s, v, w**

$v(p,q) = f_v[s,\omega]$ $w(p,q) = f_w[s,\omega]$

$s(p) = f_s[s,v,w;p]$ *gap equation*

cancelation of all UV-divergencies

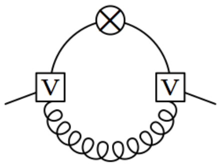
D. Campagnari, Ebadati, H.R & Vastag, PRD94(2016)

cancellation of UV-divergencies

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

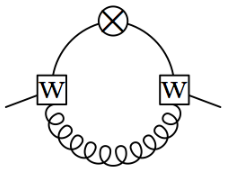
divergent loop contributions to the gap equation

> kernel **V**



$$\frac{C_{\mathrm{F}}}{16\pi^2} g^2 S(k)\left[-2\Lambda + k\ln\frac{\Lambda}{\mu}\left(-\frac{2}{3} + 4P(k)\right)\right]$$

> kernel **W**



$$\frac{C_{\mathrm{F}}}{16\pi^2} g^2 S(k)\left[2\Lambda + k\ln\frac{\Lambda}{\mu}\left(\frac{10}{3} - 4P(k)\right)\right]$$

> Coulomb term V_C

$$-\frac{C_{\mathrm{F}}}{6\pi^2} g^2 k S(k)\ln\frac{\Lambda}{\mu}$$

quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

$s, v, w -$ variational kernels $\quad \vec{\alpha}, \beta -$ *Dirac matrices*

numerical calculation

D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820,PRD94(2016)074027

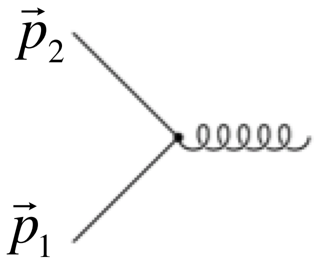
input: $\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}} \quad M = 0.88 GeV$

$lattice: \quad \sigma_C = 2.5\sigma$

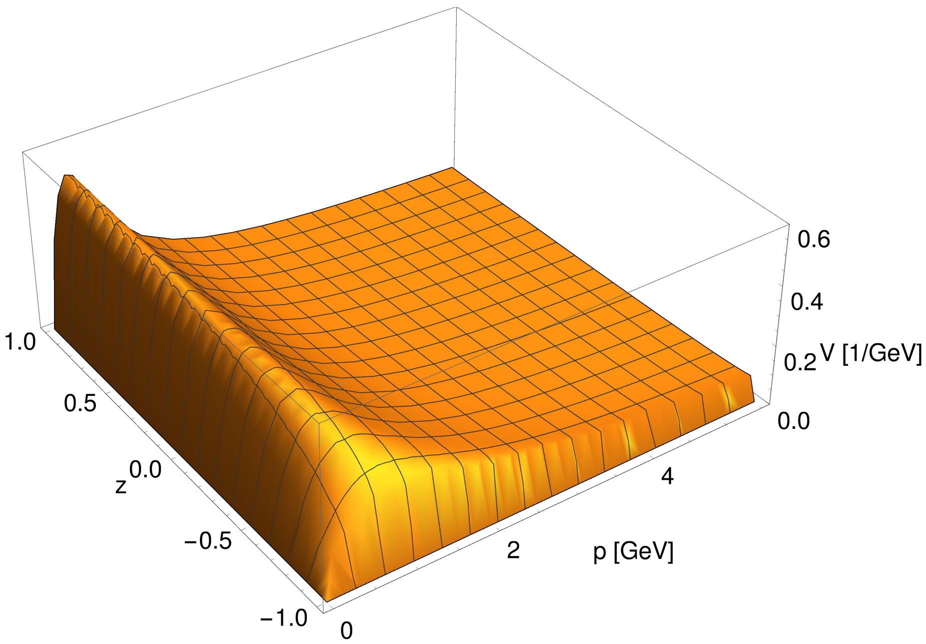
G. Burgio, M.Quandt , H.R.,
PRL102(2009)

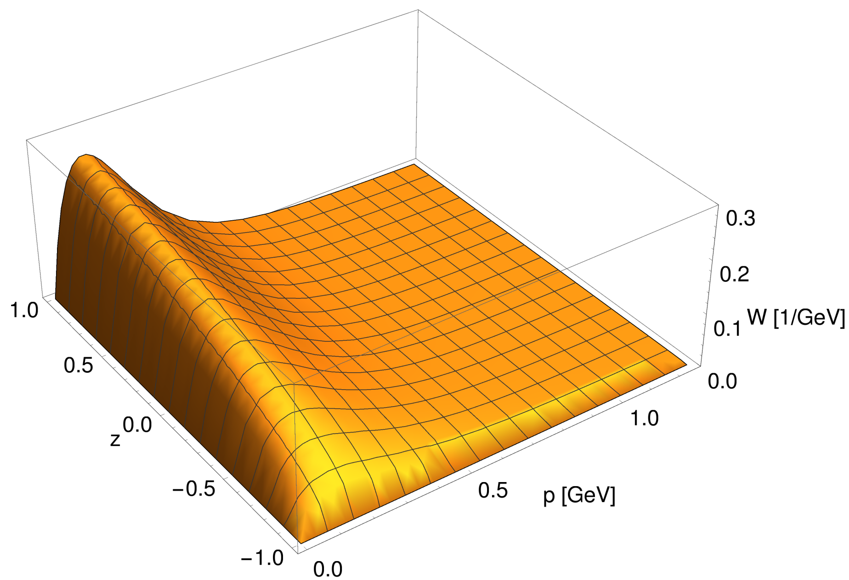
choose g *to reproduce* $\langle \bar{q}q \rangle = (-235 MeV)^3 \quad \Rightarrow g \simeq 2.1$

vector form factors v , w

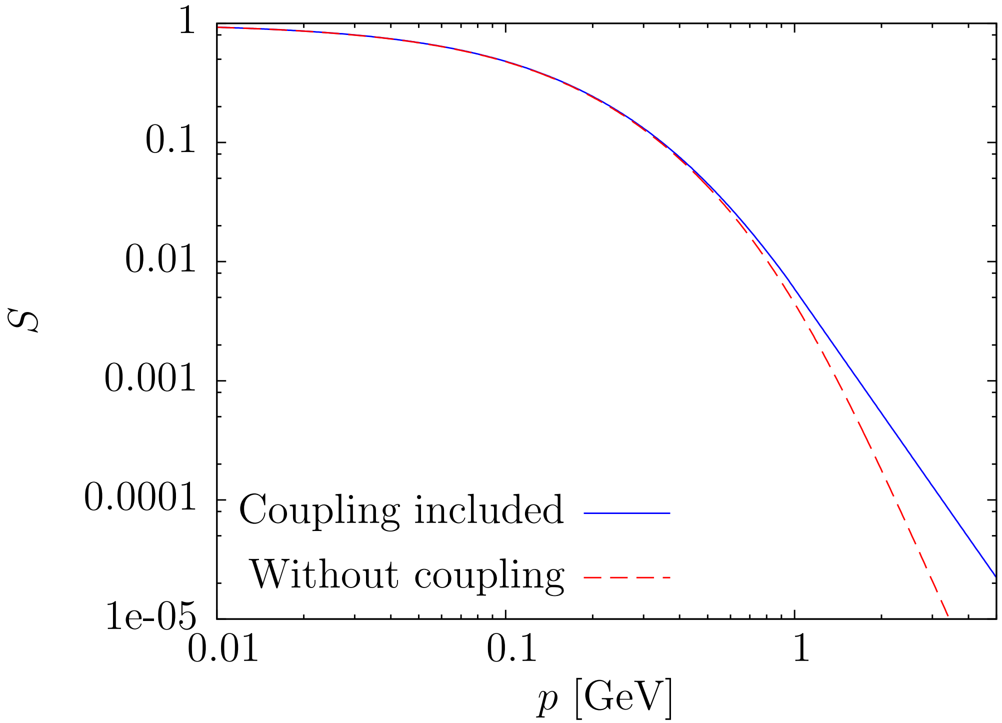


$$v,w(\vec{p}_1,\vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \;\; z = \cos\sphericalangle(\vec{p}_1,\vec{p}_2)$$





scalar form factor



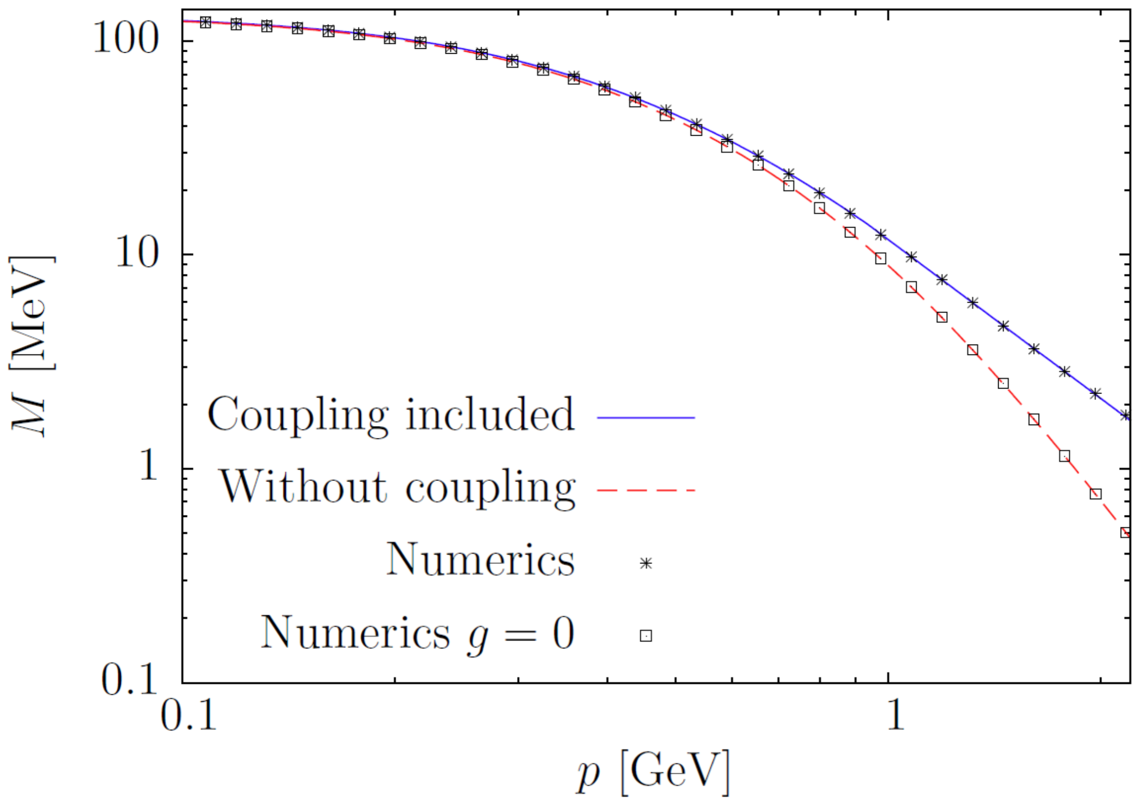
-quark-gluon coupling modifies only the mid- and high-momentum regime

-low-momentum regime is dominated by Coulomb term

effective quark mass

$$M(p) = \frac{2pS(p)}{1 - S^2(p)}$$

D.Campagniari, E.Ebadati, H. Reinhardt, P.Vastag, PRD **94** 074027 (2016)



Quark condensate

$$\langle \bar{q}q \rangle = (-236\,\text{Mev})^3$$

$$g = 2.1$$

Adler-Davis (g=0):

$$\langle \bar{q}q \rangle = (-185\,\text{Mev})^3$$

IR-mass:

M(0)=140MeV

> coupling to transversal gluons substantially increases chiral symmetry breaking

Covariant vs Constituent Quark Mass

$$S_3(p) = \int \frac{dp^4}{2\pi} S(p)$$

- ***massive Dirac particle***

$$S^{-1}(p) = \not{p} - m \qquad S(p) = \frac{\not{p} + m}{p^2 - m^2} \qquad S_3(p) = \frac{\vec{\gamma}\vec{p} - m}{2E_{\vec{p}}} \qquad E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

- ***momentum dependent mass***

$$S^{-1}(p) = \not{p}A(p^2) - B(p^2) \qquad M(p^2) = B(p^2)/A(p^2)$$

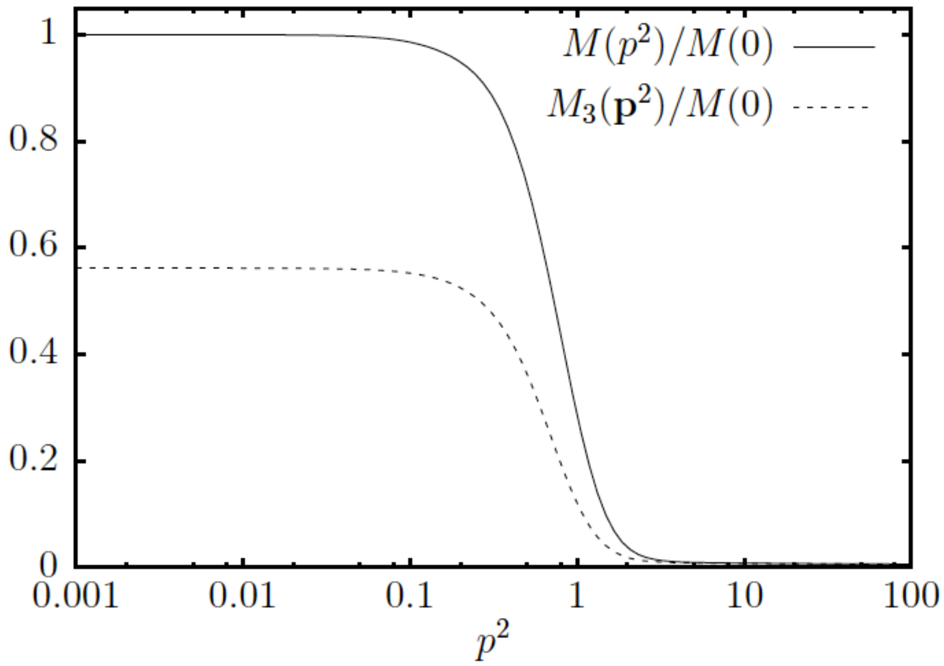
$$S_3(p) = \frac{1}{Z(p^2)} \frac{\vec{\gamma}\vec{p} - M_3(\vec{p}^2)}{2E_{\vec{p}}}$$

$$E_{\vec{p}} = \sqrt{p^2 + M_3^2(p^2)}$$

$$M_3(\mathbf{p}^2) = \frac{\displaystyle\int_0^\infty \mathrm{d}p_4 \, \frac{1}{A(p_4^2 + \mathbf{p}^2)} \frac{M(p_4^2 + \mathbf{p}^2)}{p_4^2 + \mathbf{p}^2 + M^2(p_4^2 + \mathbf{p}^2)}}{\displaystyle\int_0^\infty \mathrm{d}p_4 \, \frac{1}{A(p_4^2 + \mathbf{p}^2)} \frac{1}{p_4^2 + \mathbf{p}^2 + M^2(p_4^2 + \mathbf{p}^2)}}$$

D. Campagnari & H. R , PRD97(2018)

Covariant vs Constituent Quark Mass



D. Campagnari & H. R ,
PRD97(2018)

M(p) from DSE,
M. Huber

$$M_3(0) \approx \tfrac{1}{2} M(0)$$

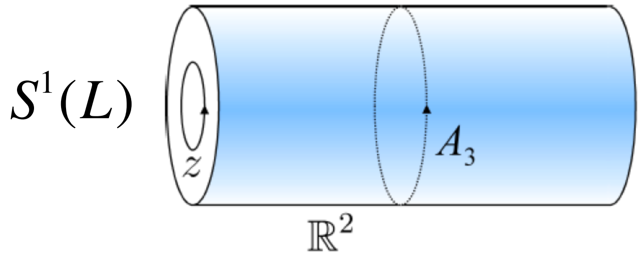
summary of T=0 calculation

- Hamiltonian approach to QCD in Coulomb gauge:
  - decent discription of the IR properties
    - confinement
    - SB of chiral symmetry
  - reasonable agreement with lattice data

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$

  H. R. Phys.Rev.D94(2016)045016



- finite temperature is fully encoded in the vacuum

- variational solution of the Schrödinger equation for the vacuum

chiral phase transition

>quark condensate

M.Quandt, E.Ebadati, H.R. & P.Vastag
arXiv:1806.04493

deconfinement phase transition

>Polyakov loop

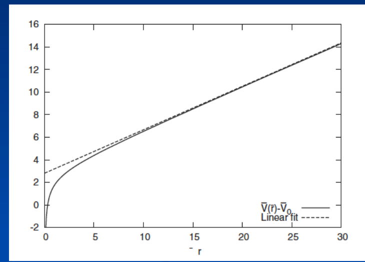
H. R. & J. Heffner, PRD88

M.Quandt & H.R. to be published

The QCD Hamiltonian in Coulomb gauge: Adler-Davis model

-neglect coupling of quarks to the spatial gluons

-keep only IR part of the Coulomb potential



$$H_{AD} = \int d^3x \Psi^\dagger(x) \vec{\alpha}\vec{p} \Psi(x) + \tfrac{1}{2}\int d^3x d^3y \rho(\vec{x}) V_C(\vec{x}-\vec{y}) \rho(\vec{y})$$

color charge density $\rho^a(x) = \Psi^\dagger(x) t^a \Psi(x)$

$$V_C(\mathbf{p}) = \frac{8\pi\sigma_C}{|\mathbf{p}|^4}$$

wave functional $|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger \beta s \Psi_-\right]|0\rangle$

s – variational kernel

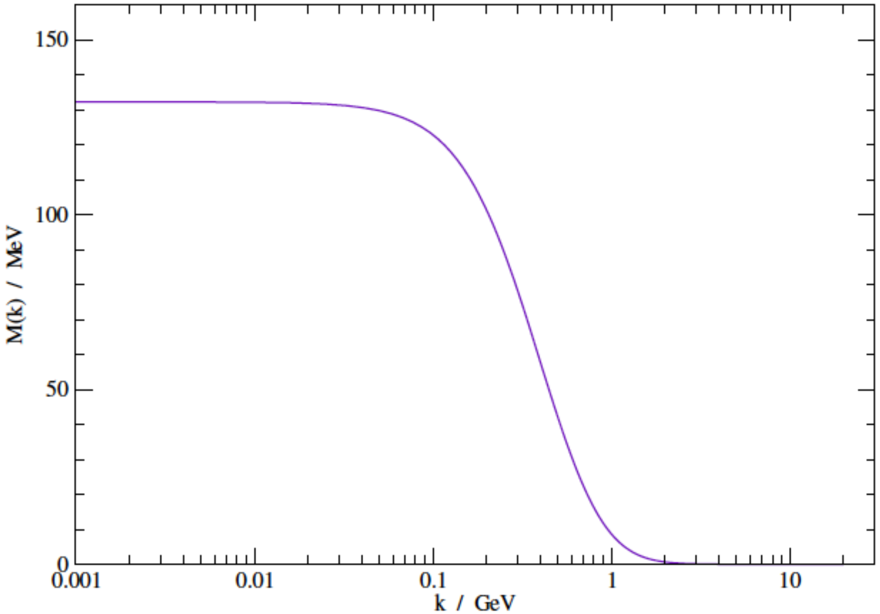
UV-finite

Adler-Davis model on $\mathbb{R}^3$ (T=0)

effective quark mass $$M(\vec{p}) = \frac{2pS(\vec{p})}{1 - S^2(\vec{p})}$$

gap equation $$M(\vec{k}) = C_F \int d^3p V_C(\vec{p} - \vec{k}) \frac{M(\vec{p}) - M(\vec{k})\hat{p}\hat{k}}{\sqrt{\vec{p}^2 + M^2(\vec{p})}}$$

$$\mathbb{R}^3 : \quad O(3) - symmetry : M(\vec{p}) = M(|\vec{p}|)$$



$$\sigma_C = 2.5\sigma$$

$$\langle \bar{q}q \rangle = (-185\,\mathrm{Mev})^3$$

Adler-Davis model on $\mathbb{R}^2 \times S^1(L)$

gap equation

$$M(\vec{k}) = C_F \int_L d^3p V_C(\vec{p}-\vec{k}) \frac{M(\vec{p}) - M(\vec{k})\hat{p}\hat{k}}{\sqrt{\vec{p}^2 + M^2(\vec{p})}}$$

$\mathbb{R}^2 \times S^1(L)$

$$\int_L d^3p f(\vec{p}) := \int d^2p_\perp \tfrac{2\pi}{L} \sum_n f(\vec{p}_{\perp,} \omega_n)$$

Matsubara frequency:

$$\omega_n = \tfrac{2\pi n}{L}, \quad \text{bosons} \quad n_F = 0$$

$$\omega_n = \tfrac{2(n+1)\pi}{L}, \quad \text{fermions} \quad n_F = 1$$

Poisson resummation:

$$\int_L d^3p f(\vec{p}) := \int d^2p_\perp dp_3 f(\vec{p}_\perp, p_3) \sum_{k=-\infty}^{\infty} (-)^{kn_F} \exp(ikLp_3)$$

Poisson resummed gap equation: oscillating integrands

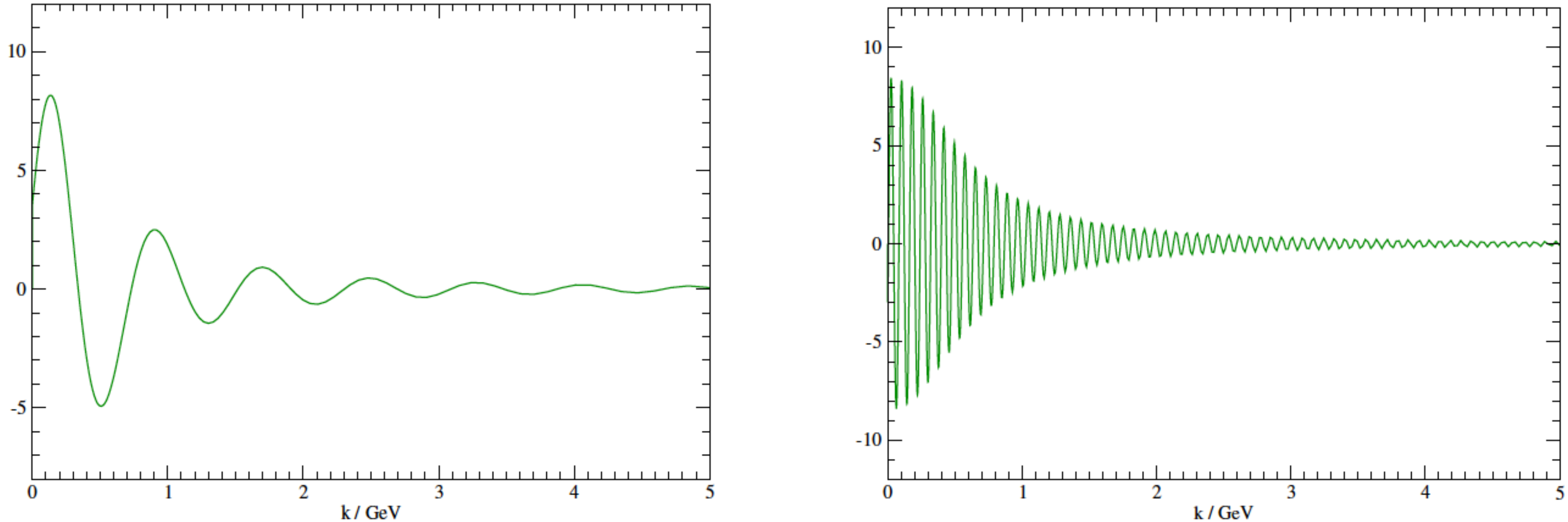


FIG. 3: Full integrand of the momentum integral in the numerator of the gap equation (44) for a temperature of $T = 50\,\mathrm{MeV}$ and Poisson index $\mathfrak{m} = 1$ (*left*) and $\mathfrak{m} = 10$ (*right*).

Convergence of alternating sums

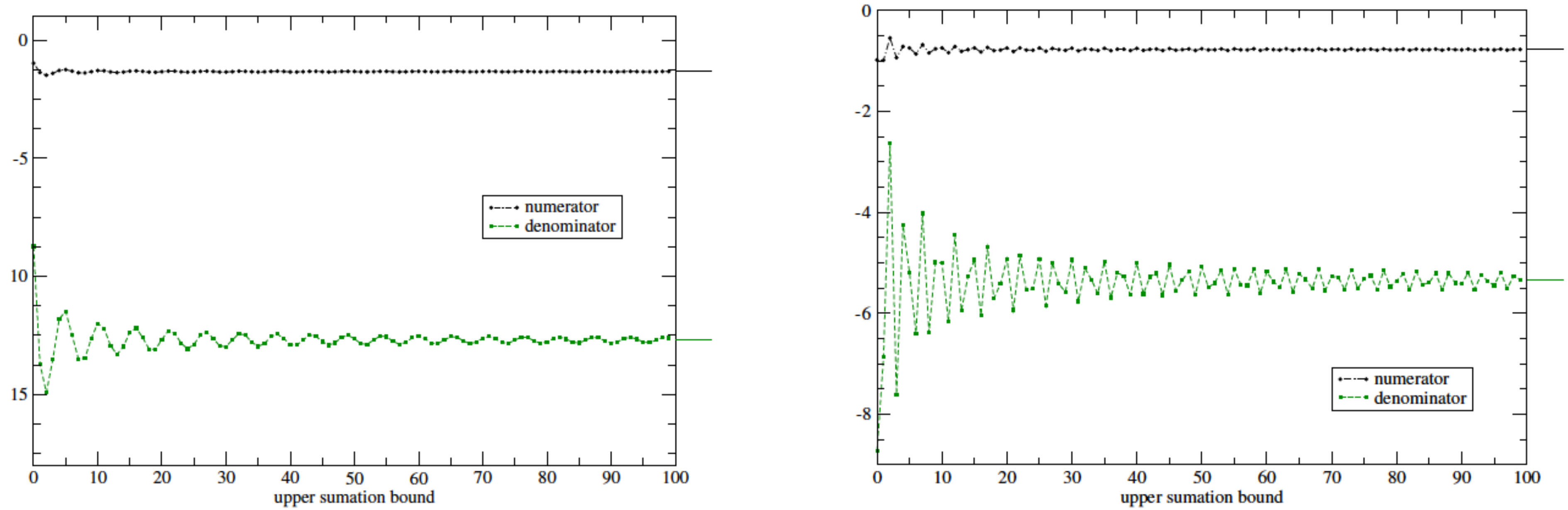
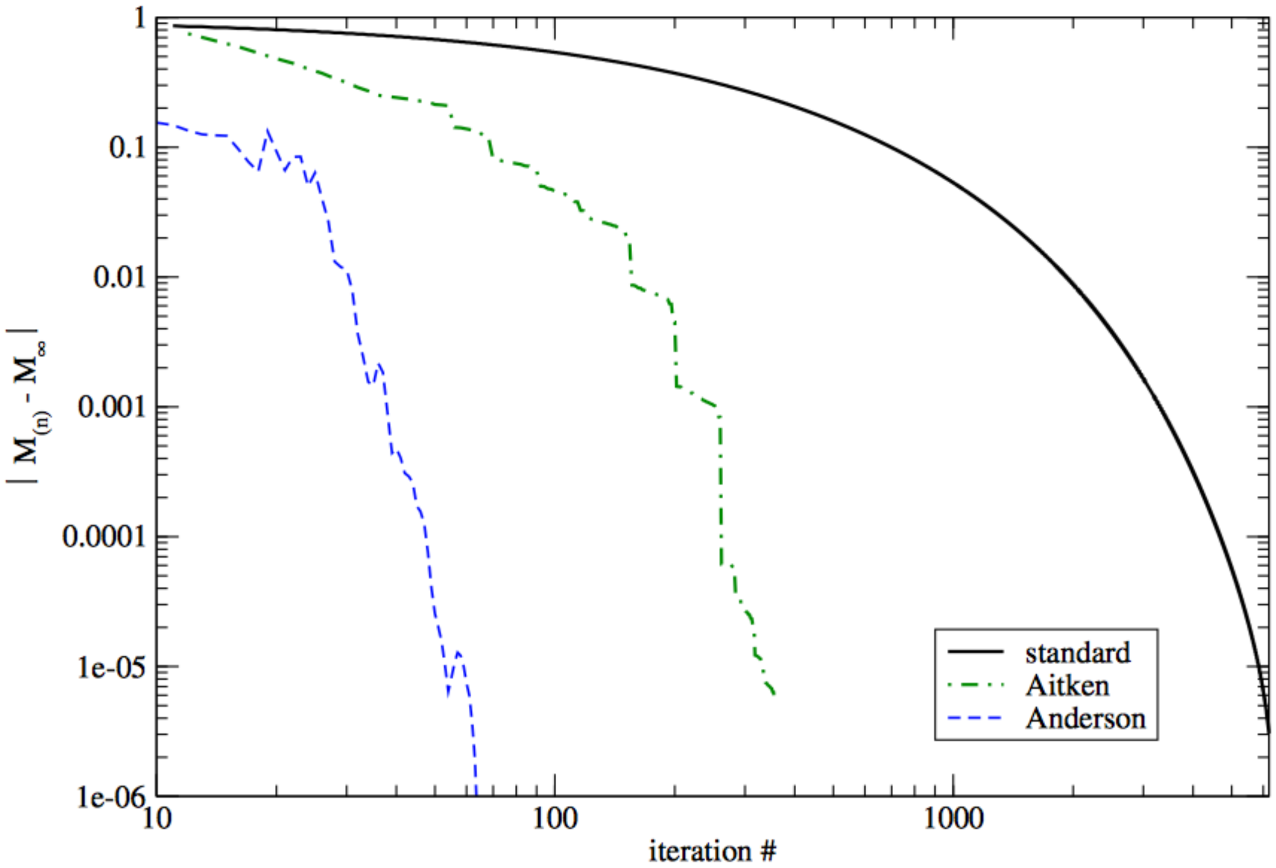


FIG. 5: Partial sums in the numerator and denominator of the gap equation (44), as a function of the upper summation bound. The small horizontal bar on the right of the coordinate box indicates the value for the infinite series predicted by the ϵ-algorithm. The left panel is for $T = 50\,\mathrm{MeV}$, while the right panel shows $T = 150\,\mathrm{MeV}$. In all cases, the external momentum was fixed to the preferred value $k = 200\,\mathrm{MeV}$ and $\xi_k = 0.5$.

Convergence history of the iteration method: standard vs accelerated

-



Effective quark mass

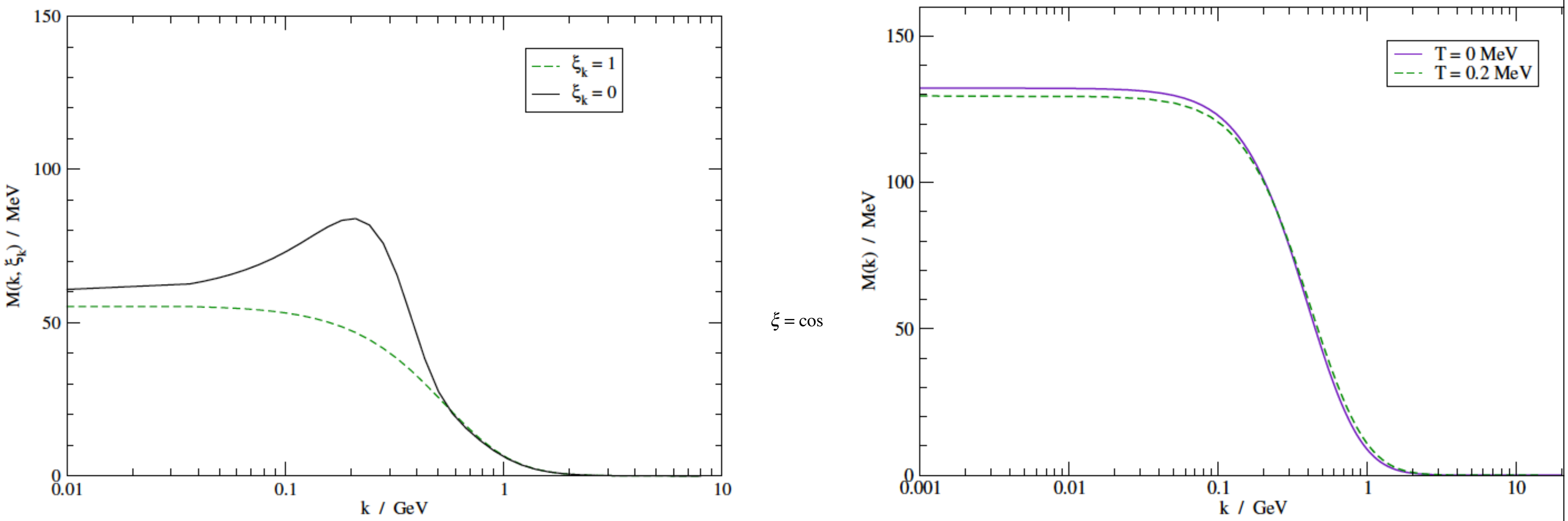
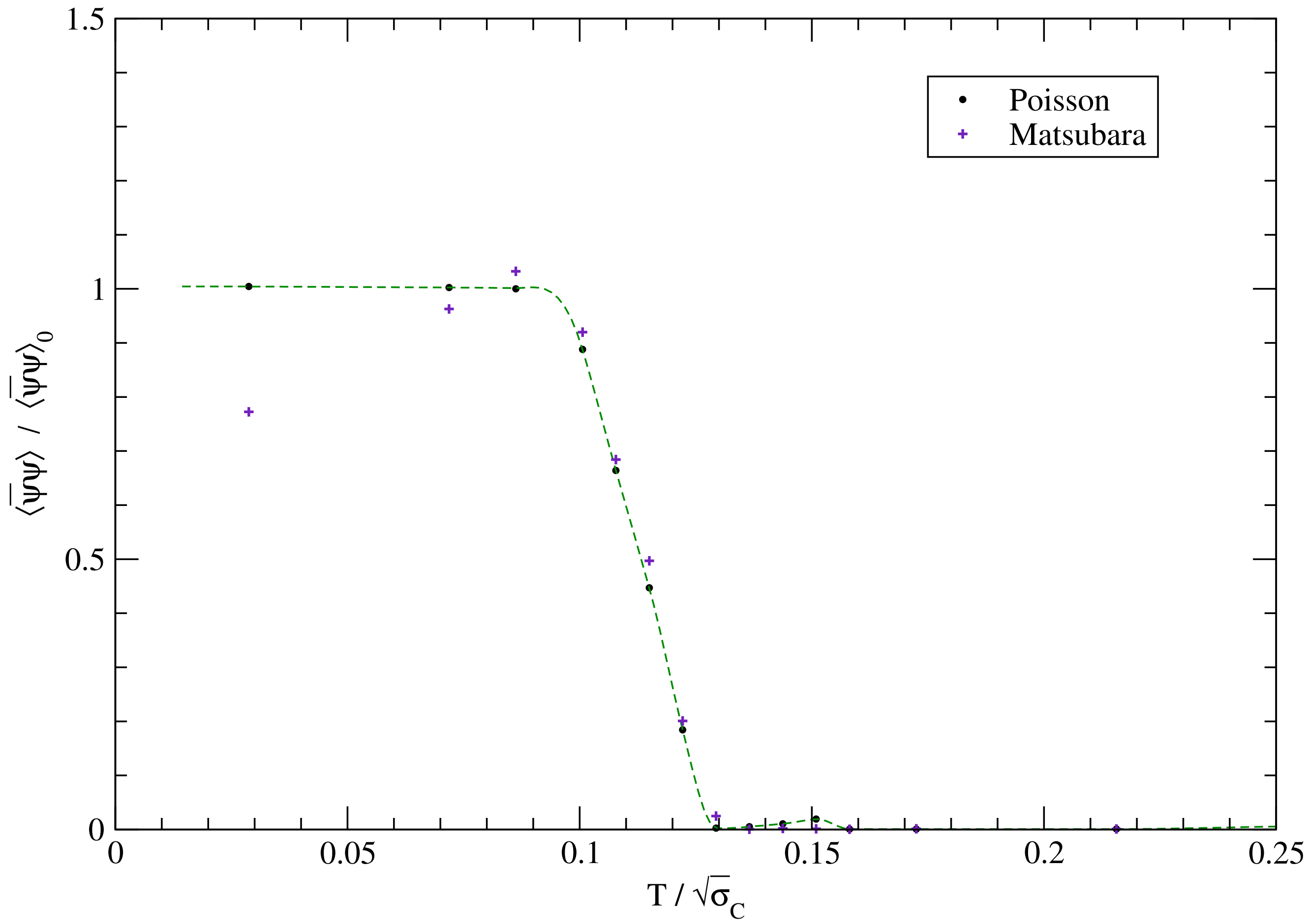


FIG. 6: *Left:* Mass function $M(k, \xi_k)$ at $T = 80\,\text{MeV}$ with the momentum $\boldsymbol{k}$ pointing in various directions relative to the heat bath. *Right:* Mass function $M(k, 1)$ for small temperatures compared to the $T = 0$ limit.

Quark condensate



Adler-Davis model on $\mathbb{R}^2 \times S^1(L)$

-2. order transition
critical temperature:

$$T_\chi = 0.13\sqrt{\sigma_C}$$

$$lattice: \quad \sigma_C = 2...4\sigma$$

$$\sigma_C = 2.5\sigma$$

$$T_\chi = 92\,MeV$$

$$\langle \bar{q}q \rangle = (-185\,MeV)^3$$

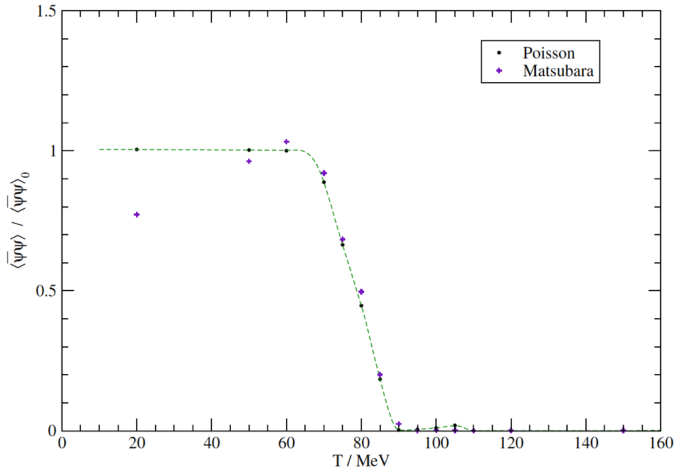


FIG. 7: Chiral condensate as a function of the temperature, from both the Matsubara and Poisson ormulation. The dashed line indicates a fit to the Poisson data from which the critical temperature is determined.

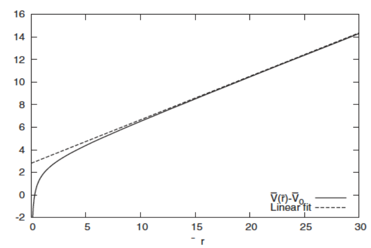
-neglect of spatial gluons

$$adjust \quad \sigma_C = 4.1\sigma$$

$$T_\chi = 115\,MeV$$

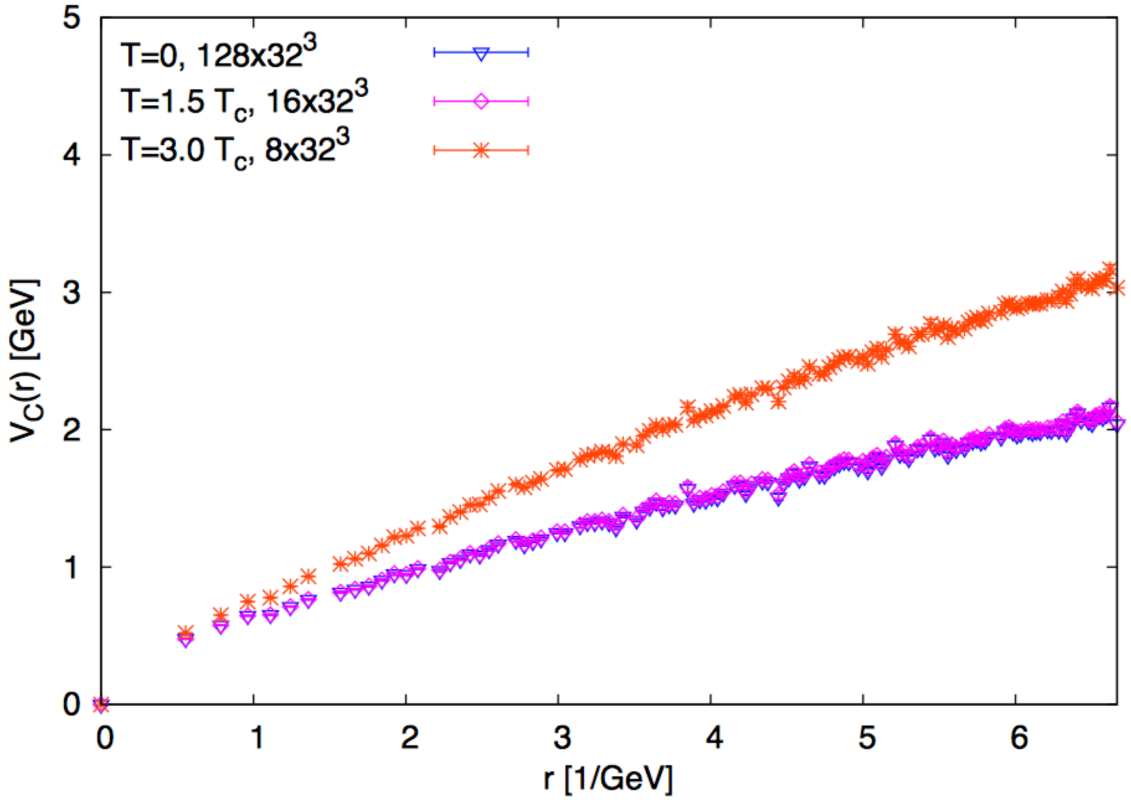
$$T_\chi^{lat} = 155\,MeV$$

$$\langle \bar{q}q \rangle = (-235\,MeV)^3$$



-neglect of UV-part of the Coulomb potential
-quenched: T=0 gluon vacuum: σ_C increases with T

Coulomb potential at finite T



G. Burgio, M. Quandt, H. R. & H. Vogt,
Phys.Rev.D92(2015)

Adler-Davis model on $\mathbb{R}^2 \times S^1(L)$

-2. order transition
critical temperature:

$$T_\chi = 0.13\sqrt{\sigma_C}$$

$lattice: \quad \sigma_C = 2...4\sigma$

$\sigma_C = 2.5\sigma$

$$T_\chi = 92\,MeV$$

$\langle \bar{q}q \rangle = (-185\,MeV)^3$

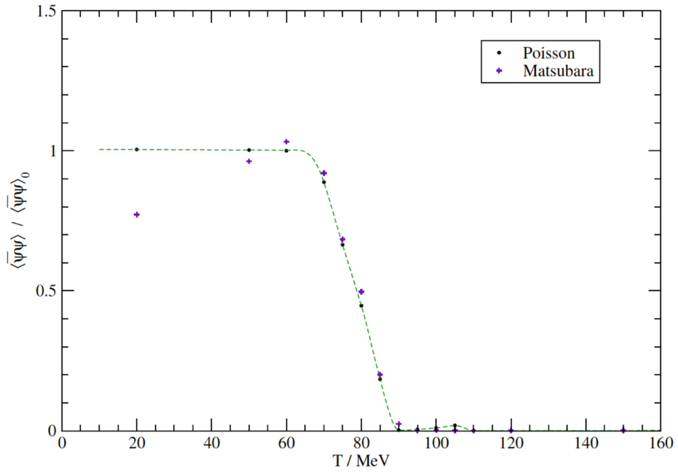


FIG. 7: Chiral condensate as a function of the temperature, from both the Matsubara and Poisson ormulation. The dashed line indicates a fit to the Poisson data from which the critical temperature is determined.

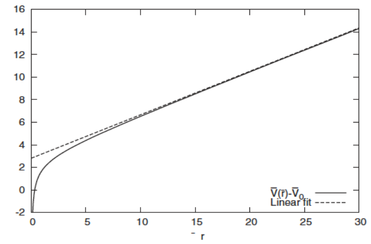
-neglect of spatial gluons

$adjust \quad \sigma_C = 4.1\sigma$

$$T_\chi = 115\,MeV$$

$$T_\chi^{lat} = 155\,MeV$$

$\langle \bar{q}q \rangle = (-235\,MeV)^3$

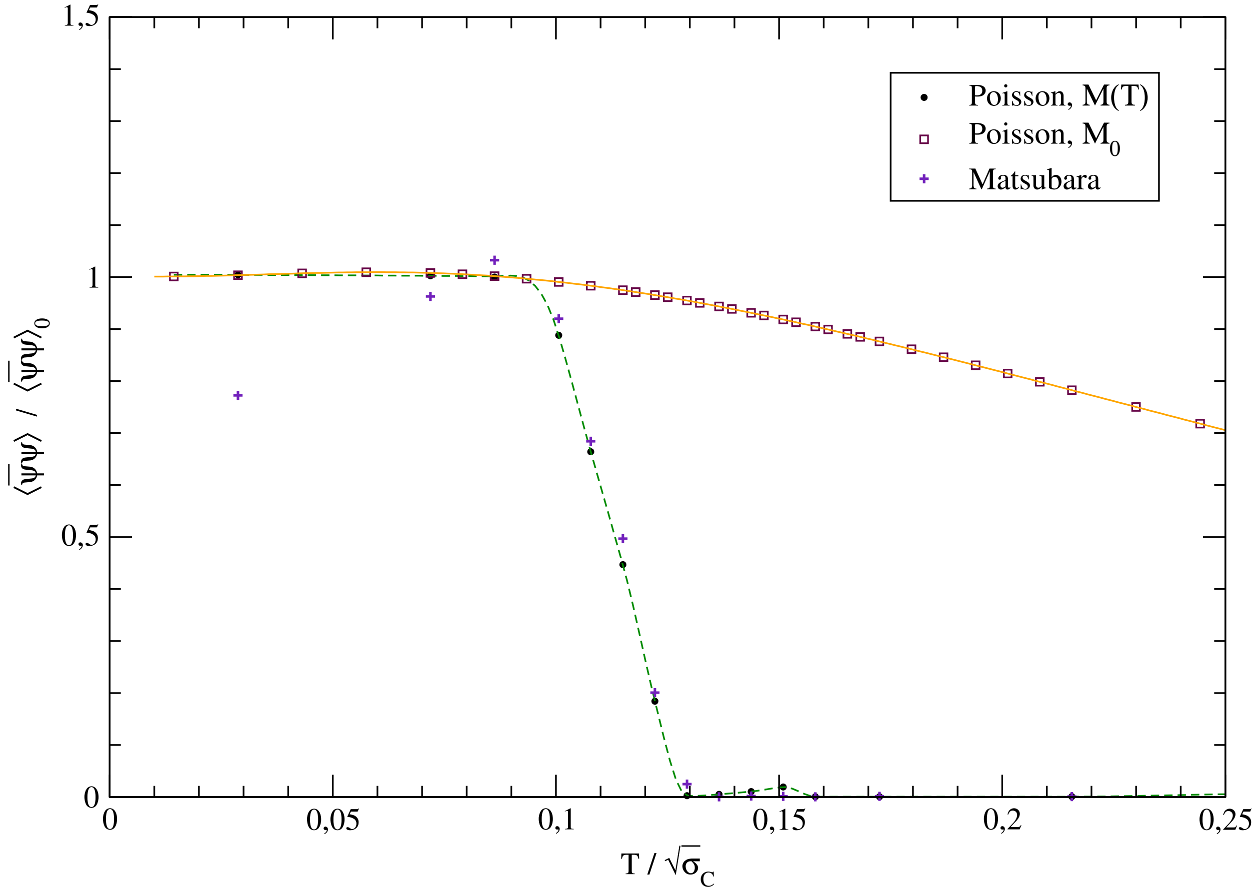


-neglect of UV-part of the Coulomb potential
-quenched: T=0 gluon vacuum: σ_C increases with T

canonical finite temperature Hamiltonian approach with quasiparticle approx. to the density operator exp(-H/T):

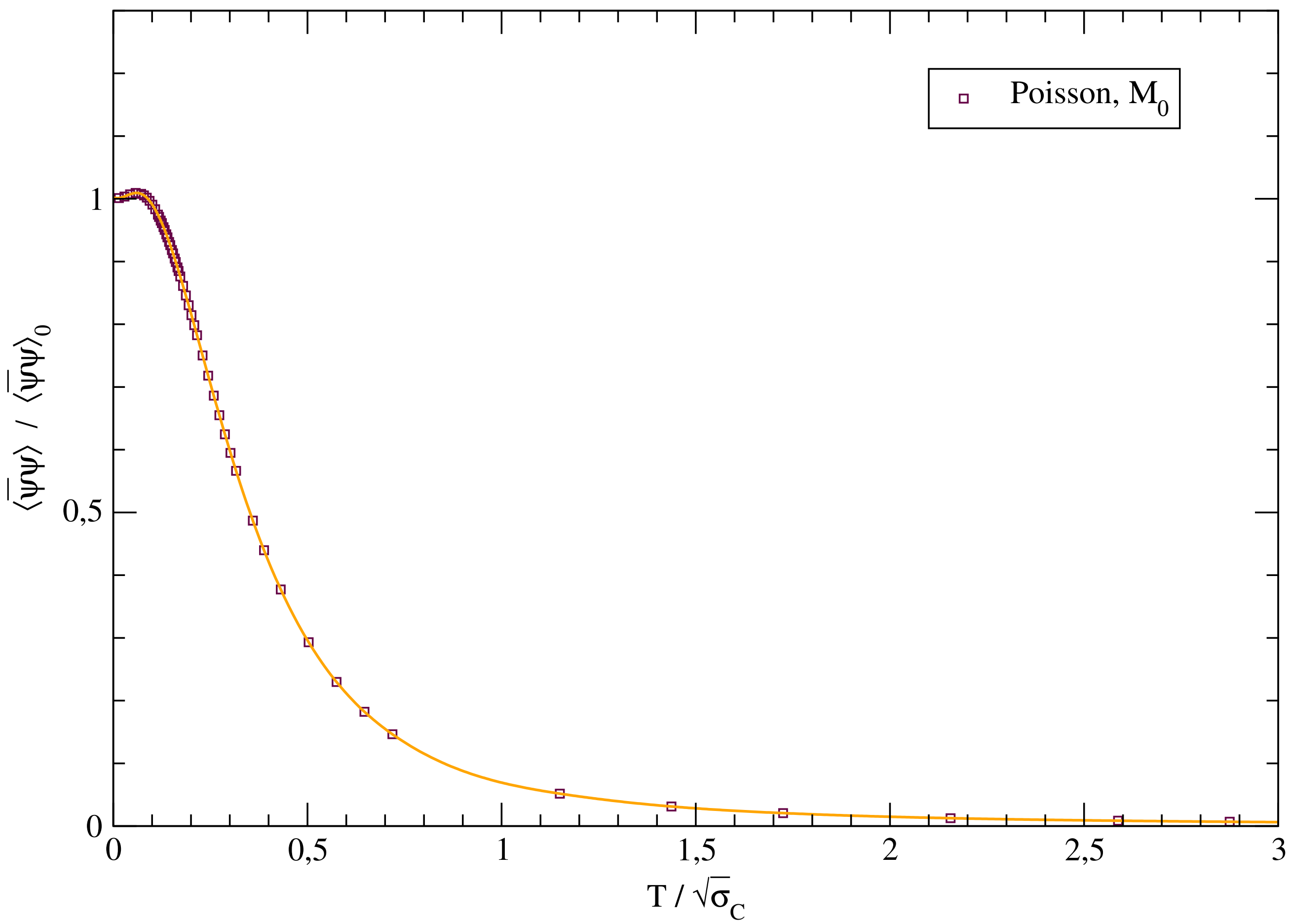
$$T_\chi = 0.091\sqrt{\sigma_C}$$

Quark condensate with T=0 solution



-no phase transition

Quark condensate with T=0 solution

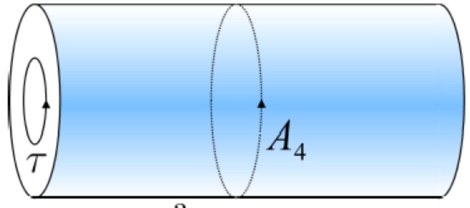


The Polyakov loop in the Hamiltonian approach

$A_0 = 0$

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$T^{-1} = L$

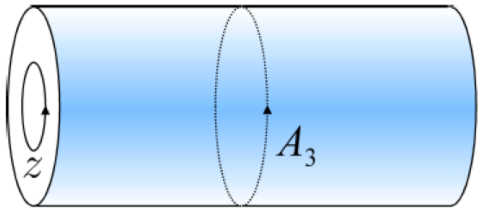


canonical Hamiltonian approach to finite T:
Polyakov loop - not accessible

alternative Hamiltonian approach to finite T
with a compactified spatial dimension:
Polyakov loop - accessible

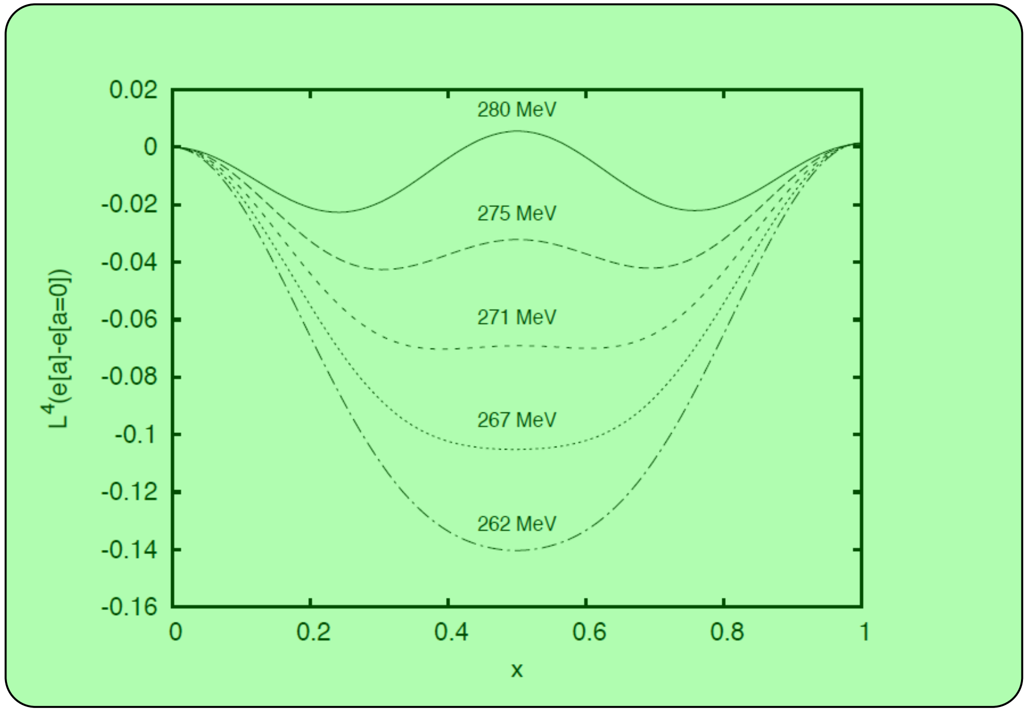
$$P[A_3](\vec{x}_\perp) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_3 A_3(\vec{x}_\perp, x_3) \right]$$

$T^{-1} = L$



The gluon effective potential *SU(2)*

variational calculation in Coulomb gauge



$$x = \frac{aL}{2\pi}$$

second order phase transition:

$$input: \quad M = 880\,MeV \qquad T_C \simeq 269\,MeV$$

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

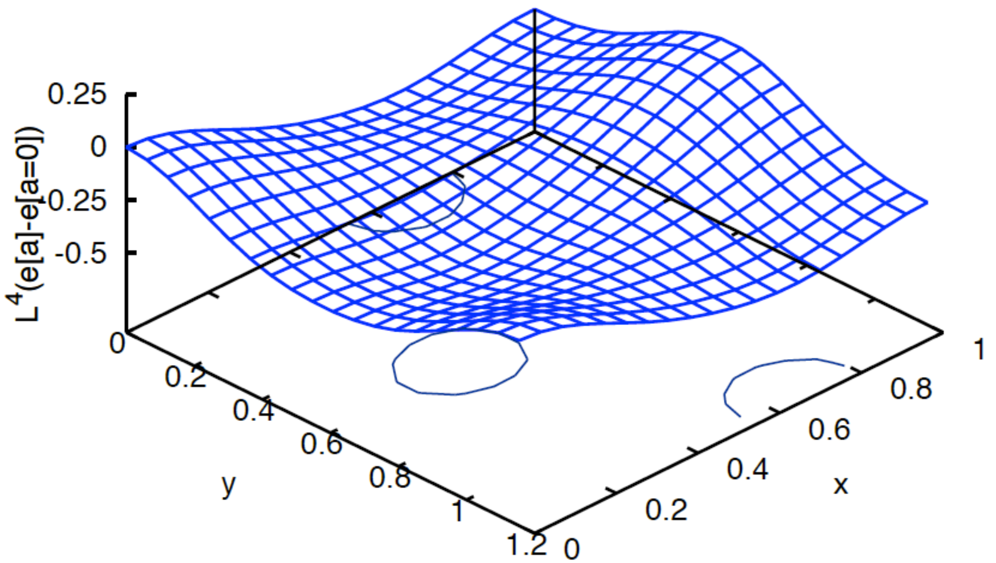
$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

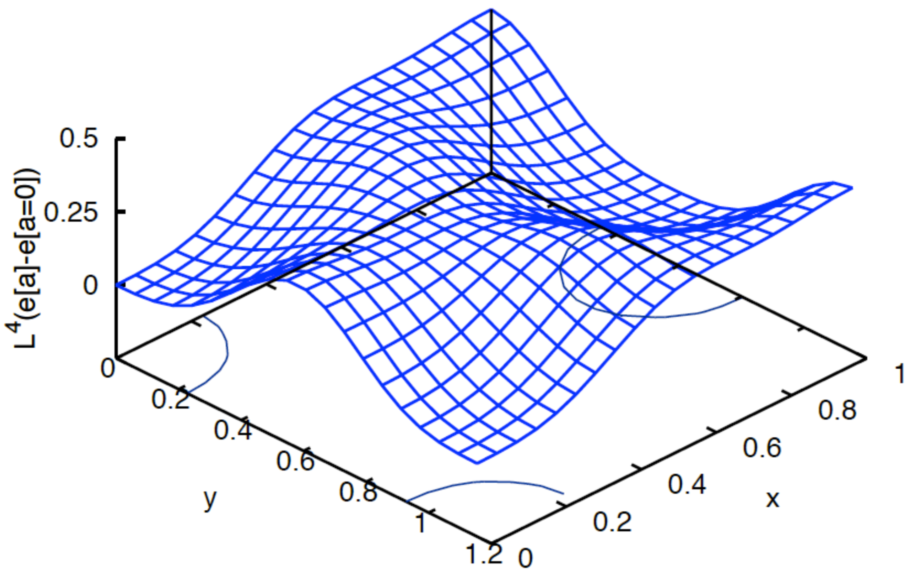
The full effective potential for SU(3)

variational calculation in Coulomb gauge

$T < T_C$



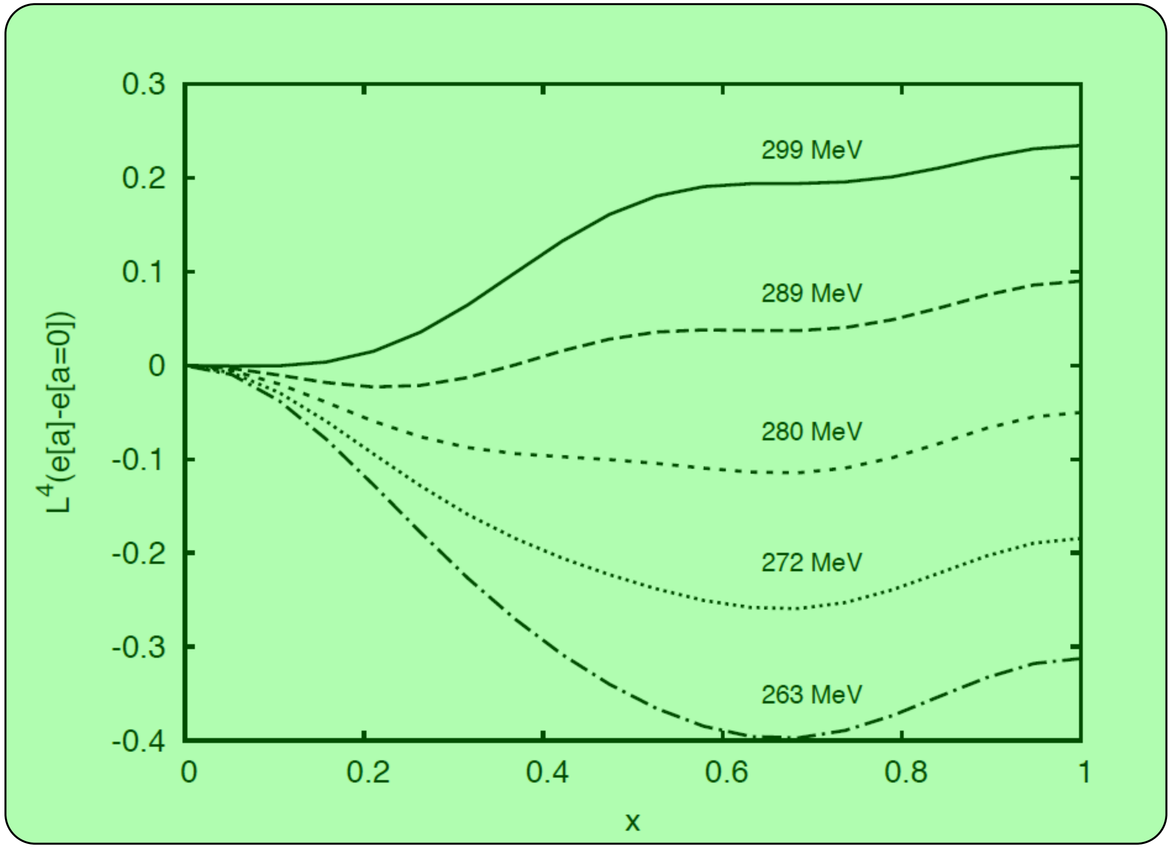
$T > T_C$



$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$$

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

Polyakov loop potential for SU(3)



$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

$$input:\ SU(2) - data:$$
$$M = 880 MeV$$

$$T_C = 283 MeV$$

critical temperature

$lattice:$ $T_C^{SU(2)} = 312\,MeV$ $T_C^{SU(3)} = 284\,MeV$

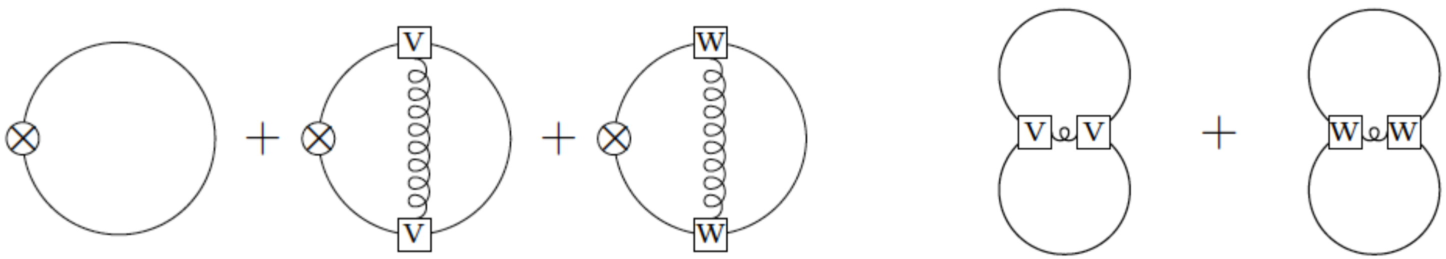
$this\ work:$ $T_C^{SU(2)} = 269\,MeV$ $T_C^{SU(3)} = 283\,MeV$

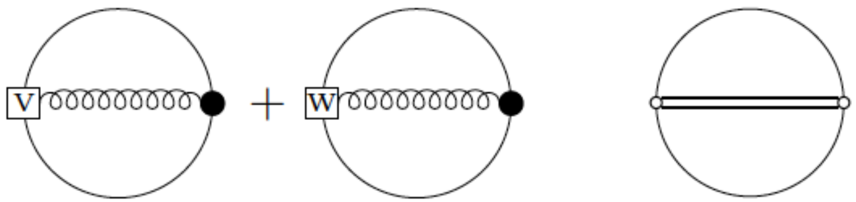
$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

lattice: *B. Lucini, M. Teper, U. Wenger, JHEP01(2004)061*

Effective potential of the Polyakov loop in full QCD

$= \langle H_{QCD} \rangle$ on $R^2 \times S^1$ in the presence of a constant background field directed along the compactified dimension

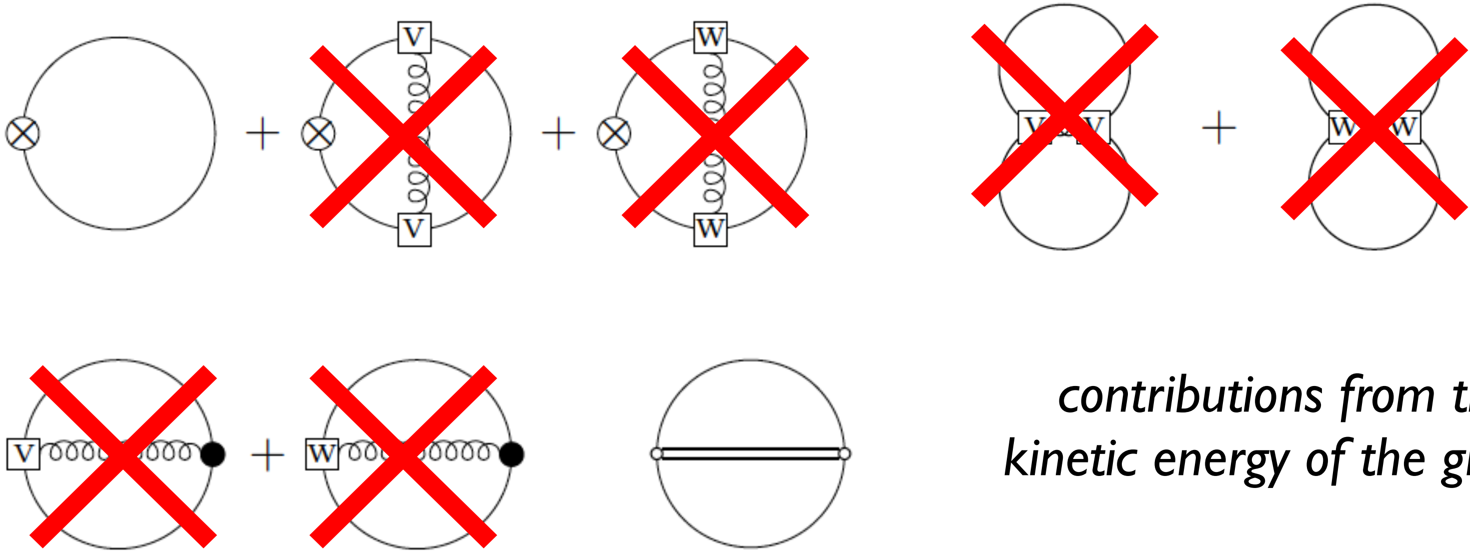




contributions from the kinetic energy of the gluons

Effective potential of the Polyakov loop in full QCD

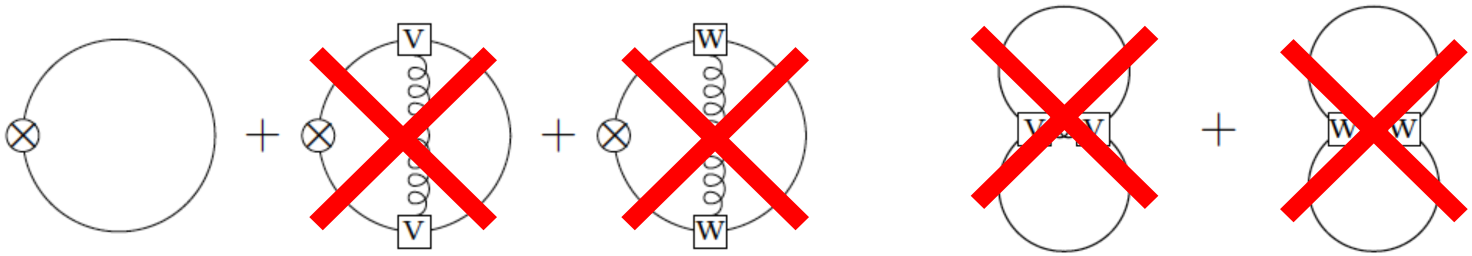
= ⟨H_{QCD}⟩ on $R^2 \times S^1$ in the presence of a constant background field directed along the compactified dimension

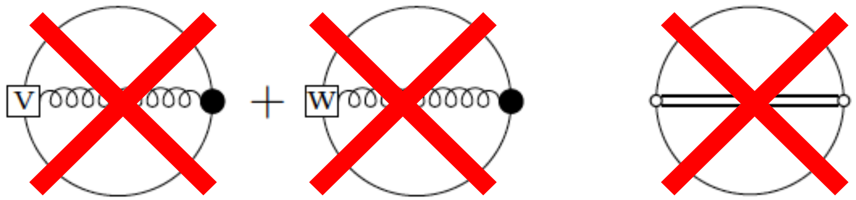


contributions from the kinetic energy of the gluons

Effective potential of the Polyakov loop in full QCD

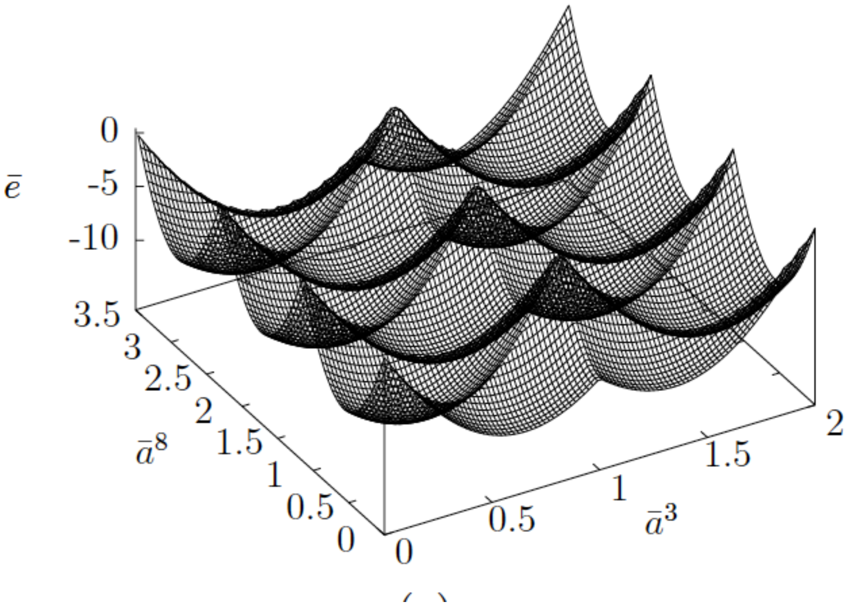
= $\langle H_{QCD} \rangle$ on $R^2 \times S^1$ in the presence of a constant background field directed along the compactified dimension



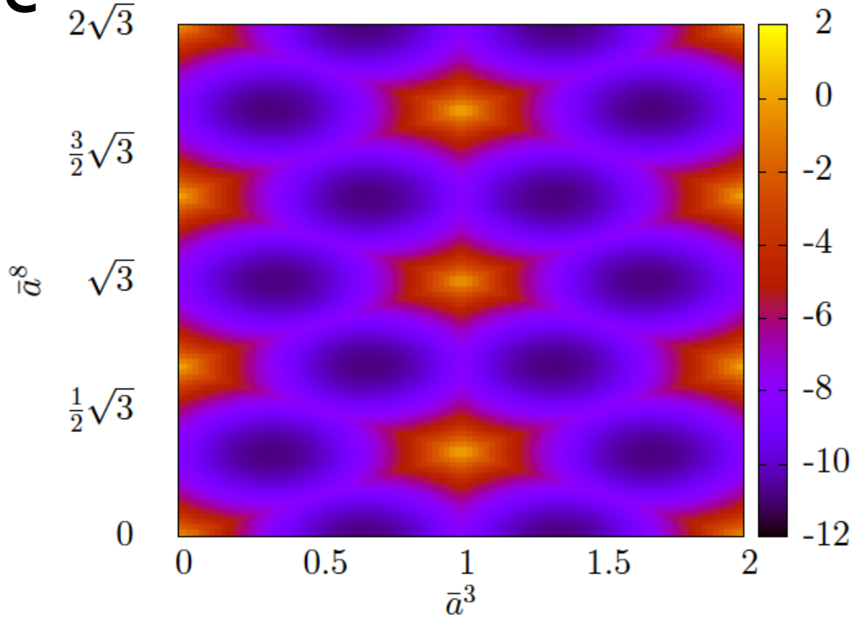


contributions from the kinetic energy of the gluons

T<Tc

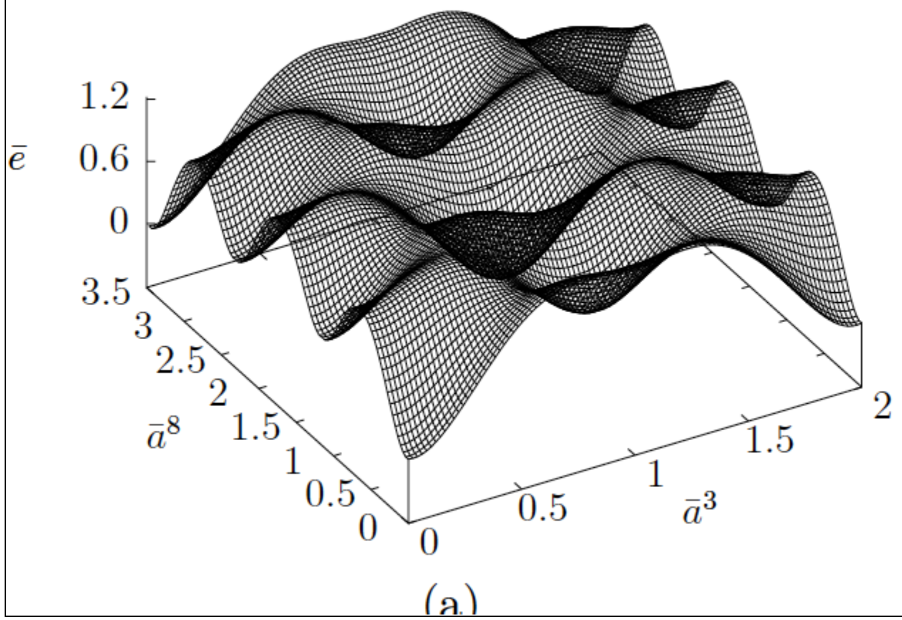


$\bar{e}$
0
-5
-10
3.5
3
2.5
2
1.5
1
0.5
0
$\bar{a}^8$
0
0.5
1
1.5
2
$\bar{a}^3$

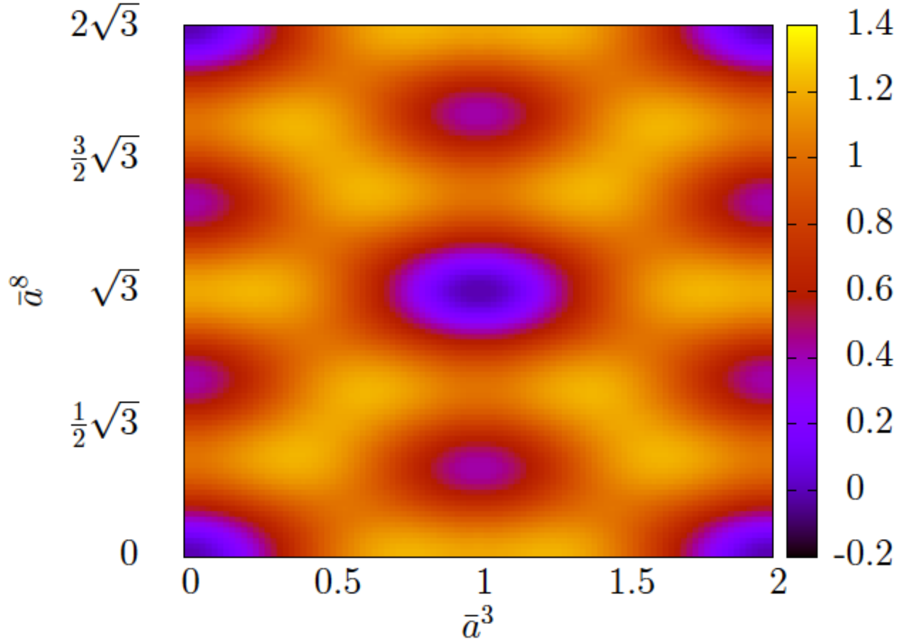


$2\sqrt{3}$
$\frac{3}{2}\sqrt{3}$
$\sqrt{3}$
$\frac{1}{2}\sqrt{3}$
0
$\bar{a}^8$
0
0.5
1
1.5
2
$\bar{a}^3$
2
0
-2
-4
-6
-8
-10
-12

T>Tc

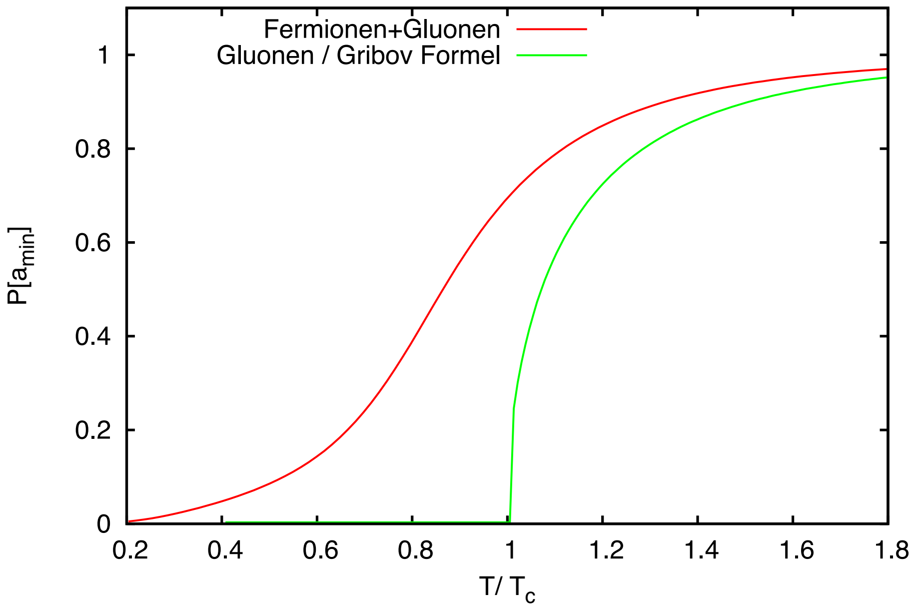


$\bar{e}$
1.2
0.6
0
3.5
3
2.5
2
1.5
1
0.5
0
$\bar{a}^8$
0
0.5
1
1.5
2
$\bar{a}^3$
(a)

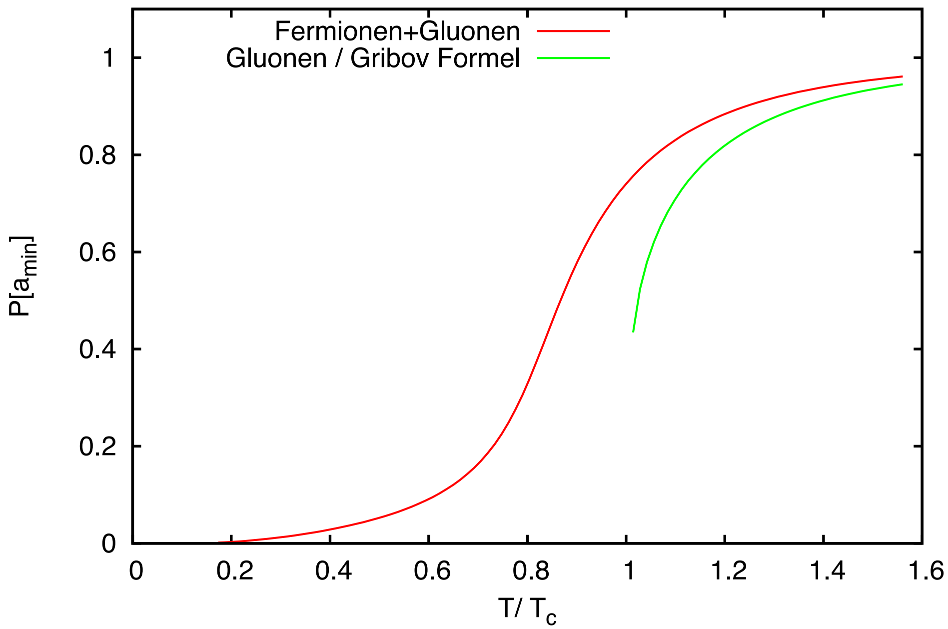


$2\sqrt{3}$
$\frac{3}{2}\sqrt{3}$
$\sqrt{3}$
$\frac{1}{2}\sqrt{3}$
0
$\bar{a}^8$
0
0.5
1
1.5
2
$\bar{a}^3$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2

The Polyakov loop



SU(2)



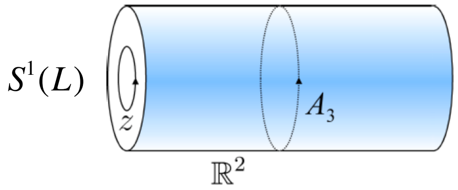
SU(3)

- *no ghost loop*
- *no Coulomb term*

M. Quandt & H. Reinhardt, to be published

Conclusions

- Hamiltonian approach to QCD in Coulomb gauge at T=0
  - decent description of the IR sector
    - confinement
    - chiral symmetry breaking
  - satisfactory agreement with lattice
- QCD at finite temperature
  - compactification of a spatial dimension



    - chiral phase transition
      - weak second order
    - effective potential of the Polyakov loop
      - deconfinement phase transition in YMT
      - SU(2): 2.order
      - SU(3): 1.order
    - inclusion of quarks:
      - deconfinement phase transition is turned into a crossover
    - dual quark condensate
- outlook: -Polyakov loop with Coulomb term
-            -finite chemical potential

Thanks for your attention