Exploring chiral symmetry restoration in heavy-ion collisions with fluctuation observables

Krzysztof Redlich (Uni Wroclaw)

- Modelling regular part of pressure in hadronic phase: S-matrix approach:
  - charge-baryon correlations in LQCD
  - proton production yields at LHC
- Fluctuations of net-baryon charge:
  - probing chiral criticality systematics:
    - FRG-PNJL model versus STAR data
  - decoding phase structure of QCD with a Fourier expansion coefficients of net baryon density

in collaboration with: Gabor Almasi, Bengt Friman, Pok Man Lo, Kenji Morita, Chihiro Sasaki, Anton Andronic, Peter Braun-Munzinger, Johanna Stachel
Statistical operator of HRG provides good approximation of QCD thermodynamics in hadronic phase

Hadron Resonance Gas (HRG):

- Good description of particle yields data and EqS from LQCD


HRG provides 1st approximation of QCD free energy in hadronic phase,
Pressure of an interacting, \(a + b \leftrightarrow a + b\), hadron gas in an equilibrium

\[
P(T) \approx P_{\text{id}}^a + P_{\text{id}}^b + P_{\text{int}}^{ab}
\]

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

\[
P_{\text{int}}^l = \sum_{l, j} \int_{m_{\text{th}}}^{\infty} dM \ B_j^l(M) P_{\text{id}}^l(T, M)
\]

\[
B_j^l(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^l(M)
\]

**Effective weight function**  
- Interactions driven by narrow resonance of mass \(M_R\)

\[
B(M) = \delta(M^2 - M_R^2) \quad \Rightarrow \quad P_{\text{int}} = P_{\text{id}}^l(T, M_R) \quad \Rightarrow \quad \text{HRG}
\]

- For non-resonance interactions or for broad resonances the HRG is too crude approximation and \(P_{\text{int}}(T)\) should be linked to the phase shifts

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**References**
Probing non-strange baryon sector in $\pi N$-system

$\Delta \chi_{BQ} \approx \sum_{I_z,j,B} d_j BQ \int dM \int d^3 p \frac{1}{T} \frac{d\delta^I_j}{dM}
\times e^{-\beta \sqrt{p^2 + M^2}} \left(1 + e^{-\beta \sqrt{p^2 + M^2}}\right)^{-2}
$

Considering contributions of all $\pi N$ $(N^*, \Delta^*$ resonances) to $\chi_{BQ}$ within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover $0.15 < T < 0.16$ GeV

$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|) / 2$

LQT from: A. Bazavov et al. Phys. Rev. D 95, 054504 (2017), N $\Delta$, N $\Delta$


Phenomenological consequences: proton production yields

- Yields of protons in the S-matrix is suppressed relative to HRG
  For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018) P. Huovinen, poster QM2018

- S-matrix results well consistent with pp data

\[
\left\langle N_p \right\rangle_{\text{Smat.}} / \left\langle N_p \right\rangle_{\text{HRG}}
\]

\[
T_f^{\text{LHC}}
\]

\[
data/HRG
\]

Model: HRG at \( T_f = 156 \text{ MeV} \)
Model: Smat. at \( T_f = 156 \text{ MeV} \)

Yields of protons in AA collisions at LHC is consistent with S-matrix result within \( 1\sigma \)


DATA ALICE

For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018) P. Huovinen, poster QM2018

S-matrix results well consistent with pp data
Net-baryon fluctuations as a probe of chiral criticality


- An excellent observable of chiral criticality
- Modelling chiral properties of QCD in PNJL model within FRG approach.

X. Luo et al. (2015), STAR Coll.

\[ \chi_n^B = \frac{\partial^n (P/T^4)}{\partial (\mu_B/T)^n} \quad \text{and} \quad R^{n,m} = \frac{\chi_n^B}{\chi_m^B} \]


Are the above deviations an indication of the chiral criticality and the existence of the CEP?

Consider systematics of \( R^{n,m} \) in relation to STAR data.
Self-consistent freeze-out and STAR data

- Freeze-out line in $(T, \mu)$-plane is fixed by $\chi_B^3 / \chi_B^1$ to data
- Ratio $\chi_B^1 / \chi_B^2 \approx \tanh(\mu / T)$ $\Rightarrow$ further evidence of equilibrium and thermalisation at $7 \text{ GeV} \leq \sqrt{s} < 5 \text{ TeV}$
- Ratio $\chi_B^1 / \chi_B^2 \neq \chi_B^3 / \chi_B^2$ expected due to critical chiral dynamics
- Enhancement of $\chi_B^4 / \chi_B^2$ at $\sqrt{s} < 20 \text{ GeV}$ not reproduced

Similar conclusions as in the previous comparison of LQCD results with STAR data:
Higher order cumulants - energy dependence

- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC
- At freeze-out, the ratio $\chi_B^0 / \chi_B^2 \approx 0$ in agreement with preliminary STAR data, albeit within still very large error

However, to make final conclusions the influence of non-critical fluctuations must be analyzed:

See e.g. P. Braun-Munzinger, A. Rustamov and J. Stachel
Nucl. Phys. A 960, 114 (2017),

Anar Rustamov, talk at this conference
Fourier coefficients of $\chi^1_B(T, \mu)$ and chiral criticality


- Considering the Fourier series expansion of baryon density
  \[ \chi^1_B(T, \mu) = \sum_{k=1}^{\infty} b_k(T) \sinh(k \mu) \]
  with
  \[ b_k(T) = \frac{2}{\pi} \int_0^\pi d\theta [\text{Im} \chi^1_B(T, i\theta)] \sin(k\theta) \]
  and \( \mu = (\mu / T) \), \( \theta = \text{Im} \mu \)

- At \( \mu = 0 \), the susceptibility \( \chi^n_B(T) \) expressed by Fourier coefficients
  \[ \chi^n_B(T, \mu) = \sum_{k=1}^{\infty} b_k(T) \frac{\partial^{n-1}}{\partial \mu^{n-1}} \sinh(k \mu), \text{ thus} \]
  \[ \chi^n_B(T, \mu = 0) = \sum_{k=1}^{\infty} k^{2n-1} b_k(T) \]

- Since \( b_k(T) \) are carrying information on chiral criticality, thus their \( T \) – and \( k \) – dependence must inform about phase transition


* see also K. Kashiwa and A. Ohnishi (2017) hep-1712.06220, for \( b_k(T) \) properties related with deconfinement transition
Chiral limit: scaling of $b_k(T)$

- $T < T_c$, $P(T, \mu)$ dominated by
  \[ P(T, \mu) \approx f(m) \cosh(\mu) \]
  thus exponential damping \[ b_k(T) \approx K_2(k m) \]

- $T = T_c$, $P(T, \mu)$ dominated by $P_{\text{sing}}$
  \[ \chi_1^B \approx \theta \left| \frac{T - T_c}{T_c} - \kappa \theta^2 \right|^{1-\alpha} \]
  \[ b_k(T_c) \approx k^{2\alpha-4} \]

- $T_c < T < T_{RW}$
  \[ b_k(T > T_c) \approx k^{\alpha-2} \sin(k \theta_c - \alpha \pi / 2) \]

- $T = T_{RW}$, $P_{\text{sing}}$
  \[ \chi_1^B \approx (\pi - \theta_B)^{1/\delta} \]
  \[ b_k(T_{RW}) \approx (-1)^{k-1} k^{-1-1/\delta} \]

- $T > T_{RW}$, 1\textsuperscript{st} order transition at $\theta = \pi$
  \[ b_k(T) \approx (-1)^{k-1} k^{-1} \]
Scaling of Fourier coefficients: PNJL MF-results

- In the chiral limit, i.e. $m_\pi = 0$, the phase transition is signaled by oscillations of $b_k(T)$ just above $T_c$.

- For $m_\pi > 0$ the singularity moves to the complex $\mu -$ plane resulting in an additional dumping of oscillations

$$b_k \approx k^{-2} e^{-k \text{Re} \mu_c(m_\pi, T)} \sin(k\theta_c)$$
Conclusions:

- The S-matrix approach to hadron gas thermodynamics with empirical scattering phase shifts provides consistent description of LQCD results on (electric charge)-baryon correlations in the chiral crossover, and the proton production yields in AA and pp collisions at the LHC.

- Systematics of net-proton number fluctuations at $\sqrt{s} > 20$ GeV measured by STAR in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics and deconfinement, however, possible contributions to fluctuation observables from effects not related to critical phenomena have to be understood.

- The Fourier expansion coefficients of baryon density exhibit rich structure to probe the QCD phase diagram and chiral criticality in the complex chemical potential.