Delta Gravity, Delta matter and the accelerated expansion of the Universe

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- J.A. Delta Gravity and Dark Energy, Phys. Lett B709(2012)101.
- J.A. and P. González, Cosmology in Delta Gravity, **Class.Quant.Grav. 30 (2013) 085002**.
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Two kind of fields:

$$\bar{\delta}\,\Phi_i = \Lambda_i^j(\Phi)\,\epsilon_j. \tag{1}$$

Then $\tilde{\Phi}_i = \tilde{\delta}\,\Phi_i$ transforms as:

$$\bar{\delta}\,\tilde{\Phi}_i = \tilde{\Lambda}_i^j(\Phi)\,\epsilon_j + \Lambda_i^j(\Phi)\,\tilde{\epsilon}_j, \tag{2}$$

where we used that $\tilde{\delta}\bar{\delta}\,\Phi_i = \bar{\delta}\tilde{\delta}\,\Phi_i = \bar{\delta}\,\tilde{\Phi}_i$ and $\tilde{\epsilon}_j = \tilde{\delta}\,\epsilon_j$.

$$S[\phi,\tilde{\phi}] = S_0[\phi] + \int d^4x \frac{\delta S_0}{\delta\phi_I(x)}[\phi]\,\tilde{\phi}_I(x), \tag{3}$$

the indexes I can represent any kind of indexes. This new action shows the standard structure which is used to define any modified element or function for $\tilde{\delta}$ type theories. In fact, this action is invariant under our extended general coordinate transformations developed in (1,2).

Now, we consider general coordinate transformations or diffeomorphism in its infinitesimal form.

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{2\kappa}+L_M-\frac{1}{2\kappa}\left(G^{\alpha\beta}-\kappa T^{\alpha\beta}\right)\tilde{g}_{\alpha\beta}+\tilde{L}_M\right), \tag{4}$$

where $L_M=L_M(\phi_I,\partial_\mu\phi_I)$ is the lagrangian of the matter fields ϕ_I.

$\tilde{g}_{\mu\nu}=\tilde{\delta}\,g_{\mu\nu}$ and:

$$T^{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}}\left[\sqrt{-g}\,L_M\right] \tag{5}$$

$$\tilde{L}_M=\tilde{\phi}_I\frac{\delta L_M}{\delta\phi_I}+(\partial_\mu\tilde{\phi}_I)\frac{\delta L_M}{\delta(\partial_\mu\phi_I)}, \tag{6}$$

$\tilde{\phi}_I=\tilde{\delta}\,\phi_I$ are the $\tilde{\delta}$ matter fields.

1 Symmetries

Action (4) is invariant under the following transformations(δ),

$$\begin{aligned}\delta g_{\mu\nu}&=g_{\mu\rho}\xi^\rho_{0,\nu}+g_{\nu\rho}\xi^\rho_{0,\mu}+g_{\mu\nu,\rho}\xi^\rho_0=\xi_{0\mu;\nu}+\xi_{0\nu;\mu}\\ \delta\tilde{g}_{\mu\nu}(x)&=\xi_{1\mu;\nu}+\xi_{1\nu;\mu}+\tilde{g}_{\mu\rho}\xi^\rho_{0,\nu}+\tilde{g}_{\nu\rho}\xi^\rho_{0,\mu}+\tilde{g}_{\mu\nu,\rho}\xi^\rho_0\end{aligned} \tag{7}$$

The action of a particle in a gravitational field is

$$\frac{1}{2\kappa}\int d^n y\sqrt{-g}R - m\int dt\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$$

The variation in x^μ produces the geodesic equation.

Instead in δ gravity we have:

$$S_p = m\int\frac{dt}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}\dot{x}_\mu\dot{x}_\nu(g^{\mu\nu}+\frac{1}{2}\tilde{g}^{\mu\nu}) = m\int\frac{dt}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \quad (8)$$

Far from the sources, we have the boundary conditions $g_{\mu\nu}\sim\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu}\sim 0$. In this limit we recover the action for a massive particle of mass m in Minkowsky space.

Equation of motion for massive particles:

- Massive particles do not move on geodesics
- Second order in time derivatives

Massless particle moves in a null geodesic of $\mathfrak{g}_{\mu\nu} = g^{\mu\nu}+\tilde{g}^{\mu\nu}$:

$$K = \dot{x}^\mu\dot{x}^\nu\mathfrak{g}_{\mu\nu} = 0 \quad , \; L_0 = -\int dt\frac{1}{4}(\dot{x}^\mu\dot{x}^\nu\mathfrak{g}_{\mu\nu})$$

Remark 1. The equation of motion for massive particles satisfies the important property of preserving the form of the proper time in a particle in free fall. Notice that in our case the quantity that is constant using the equation of motion for massive particles is $\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}$. **This single out this definition of proper time and not other.**

Proper time:

$$d\tau = \sqrt{-g_{\mu\nu}\, dx^\mu\, dx^\nu} = \sqrt{-g_{00}}\, dx_0 \tag{9}$$

The change in x^0 for a roundtrip of a light ray from a point with coordinates x^α to a neighboring point with coordinates $x^\alpha + dx^\alpha$ is:

$$\Delta x^0 = \frac{2}{\mathfrak{g}_{00}} \sqrt{(\mathfrak{g}_{0\alpha}\mathfrak{g}_{0\beta} - \mathfrak{g}_{\alpha\beta}\mathfrak{g}_{00})}$$

$$dl = \sqrt{\frac{g_{00}}{\mathfrak{g}_{00}}} \sqrt{\mathfrak{g}_{\alpha\beta} - \frac{\mathfrak{g}_{0\alpha}\mathfrak{g}_{0\beta}}{\mathfrak{g}_{00}}}$$

$$dl^2 = \gamma_{ij} dx^i dx^j, \quad \gamma_{ij} = \frac{g_{00}}{\mathfrak{g}_{00}} (\mathfrak{g}_{ij} - \frac{\mathfrak{g}_{0i}\mathfrak{g}_{0j}}{\mathfrak{g}_{00}}) \tag{10}$$

$g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are given by:

$$g_{\mu\nu}\,d\,x^{\mu}\,d\,x^{\nu} = -c^2\,d\,t^2 + R^2(t)\,(d\,x^2 + d\,y^2 + d\,z^2) \quad (11)$$

$$\tilde{g}_{\mu\nu}\,d\,x^{\mu}\,d\,x^{\nu} = -3\,F_a(t)\,c^2\,d\,t^2 + F_a(t)\,R^2(t)\,(d\,x^2 + d\,y^2 + d\,z^2). \quad (12)$$

These expressions represent an isotropic and homogeneous universe.

To make the usual connection between redshift and the scale factor, we consider light waves traveling to $r=0$, from $r=r_1$, along the r direction with fixed θ,ϕ. Photons moves on a null geodesic of $\mathfrak{g}$:

$$-(1+3\,F_a(t))\,c^2\,d\,t^2+R^2(t)\,(1+F_a(t))\,d\,r^2=0.$$

Define the effective scale factor:

$$\tilde{R}(t)=R(t)\sqrt{\frac{1+F_a(t)}{1+3\,F_a(t)}},\qquad \text{Then } r_1=c\int_{t_1}^{t_0}\frac{d\,t}{\tilde{R}(t)},$$

where t_1 and t_0 are the emission and reception times.

A typical galaxy will have fixed r_1,θ_1,ϕ_1. If a second wave crest is emitted at $t=t_1+\delta\,t_1$ from $r=r_1$, it will reach $r=0$ at $t_0+\delta\,t_0$, where $\int_{t_1+\delta t_1}^{t_0+\delta t_0}\frac{d\,t}{\tilde{R}(t)}=r_1$

Therefore, for $\delta\,t_1,\delta\,t_0$ small, which is appropiate for light waves, we have $\frac{\delta\,t_0}{\delta\,t_1}=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. A crucial point is that $\delta\,t$ measure the change in proper time. That is:$\frac{\nu_1}{\nu_0}=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$, where ν_0 is the light frequency detected at $r=0$ corresponding to a source emission at frequency ν_1. Or in terms of the redshift parameter z, defined as the fractional increase of the wavelength λ:

$$z=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}-1=\frac{\lambda_0-\lambda_1}{\lambda_1} \tag{13}$$

Let us consider a mirror of radius b that is receiving light from a distant source. The photons that reach the mirror are inside a cone of half-angle ε with origin at the source.

- $\varepsilon = \frac{b}{\tilde{R}(t_0)\, r_1}$.
- The solid angle of the cone is $\pi\,\varepsilon^2 = \frac{A}{r_1^2\,\tilde{R}(t_0)^2}$, where $A = \pi\, b^2$ is the proper area of the mirror.
- The fraction of all isotropically emitted photons that reach the mirror is $f = \frac{A}{4\,\pi\, r_1^2\,\tilde{R}(t_0)^2}$
- Each photon carries an energy $h\,\nu_1$ at the source and $h\,\nu_0$ at the mirror. Photons emitted at intervals $\delta\, t_1$ will arrive at intervals $\delta\, t_0$. We have $\frac{\nu_1}{\nu_0} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$, $\frac{\delta\, t_0}{\delta\, t_1} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. Therefore the power at the mirror is $P_0 = L\frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2}\, f$, where L is the luminosity of the source.
- The apparent luminosity is $l = \frac{P_0}{A} = L\frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2}\,\frac{1}{4\,\pi\, r_1^2\,\tilde{R}(t_0)^2}$.This permits to define the luminosity distance:$d_L = \sqrt{\frac{L}{4\,\pi\, l}} = \tilde{R}(t_0)^2\frac{r_1}{\tilde{R}(t_1)}$
- $d_L = (1+z)\int_0^z \frac{d\,z'}{\tilde{H}(z')}, \tilde{H} = \frac{\dot{\tilde{R}}}{\tilde{R}}$
- Angular diameter distance, $d_A = \frac{\tilde{R}^2(t_1)}{\tilde{R}^2(t_0)}\, d_L = \frac{d_L}{(1+z_1)^2}$

In cosmology, the metric $g_{\mu\nu}$ is given by (11). Moreover, we know that Einstein's equations do not change and $T_{\mu\nu}$ is conserved. Therefore, the usual cosmological solution is still valid.

- Equations of state:

  Non-relativistic matter we use $p_M(t) = 0$;

  Radiation $p_R(t) = \frac{1}{3}\rho_R(t)$

We find:

$$t(Y) = \frac{2\sqrt{C}}{3 H_0 \sqrt{\Omega_R}} \left(\sqrt{Y + C}\,(Y - 2C) + 2C^{\frac{3}{2}} \right) \tag{14}$$

$$Y = \frac{R(t)}{R_0}, \tag{15}$$

where $t(Y)$ is the time variable, R_0 is the scale factor in the present, $C = \frac{\Omega_R}{\Omega_M}$, and Ω_R and Ω_M are the radiation and non-relativistic matter density in the present respectively, with $\Omega_M = 1 - \Omega_R$.

- We know that $\Omega_R \ll 1$, so $\Omega_M \sim 1$ and $C \ll 1$.
- $Y \gg C$ describes the non-relativistic era and $Y \ll C$ describes the radiation era.

We have $\tilde{\delta}$ non-relativistic matter and radiation densities, given by $\tilde{\rho}_M$ and $\tilde{\rho}_R$ respectively.

The solution to the field equations is:

$$F_a(Y) = -L_2 \frac{Y}{3} \sqrt{\frac{Y}{C} + 1} \tag{16}$$

$$\tilde{\rho}_M(Y) = \frac{9\, H_0^2\, \Omega_R}{2\, \kappa\, c^2\, C} \frac{(-F_a(Y))}{Y^3} \tag{17}$$

$$\tilde{\rho}_R(Y) = \frac{6\, H_0^2\, \Omega_R}{\kappa\, c^2} \frac{(-F_a(Y))}{Y^4}, \tag{18}$$

- The effective scale factor $\tilde{Y} = \frac{\tilde{R}(t)}{R(t_0)}$ is:

$$\tilde{Y}(Y, L_1, L_2, C) = Y \sqrt{\frac{1 - L_2 \frac{Y}{3} \sqrt{Y + C}}{1 - L_2\, Y \sqrt{Y + C}}}. \tag{19}$$

The supernova Ia data gives, m (apparent or effective magnitude) as a function of z. This is related to distance d_L by:

$$m = M + 5log(\frac{d_L}{10pc})$$

Here M is common to all supernova and m changes with d_L alone.

We compare δ gravity to General Relativity(GR) with a cosmological constant:

Before we start the data analysis, we must define the parameters of the model.

In the first place, d_L in GR depends upon four parameters: Y, $H_0 = 100\ h$ km s^{-1} Mpc^{-1}, Ω_M and Ω_R. However, from CMB black body spectrum we obtain the photons density in the present, Ω_γ. Now, if we assume that $\Omega_R = \Omega_\gamma + \Omega_\nu = \left(1 + 3(\frac{7}{8})(\frac{4}{11})^{4/3}\right)\Omega_\gamma$($\Omega_\nu$is the primordial neutrino density), we get $h^2\,\Omega_R = 4.15 \times 10^{-5}$. Therefore, the parameters in d_L can be reduced to three: Y, h and $h^2\,\Omega_M$.

In the same way, in $\tilde{\delta}$ Gravity with $\tilde{\delta}$ matter, d_L depends on three parameters: Y, C and $L_2(\mathrm{related\ to\ the\ density\ of}\ \delta$ matter). We will use $H_0\sqrt{\Omega_R} = 0.644$ km s^{-1} Mpc^{-1}.

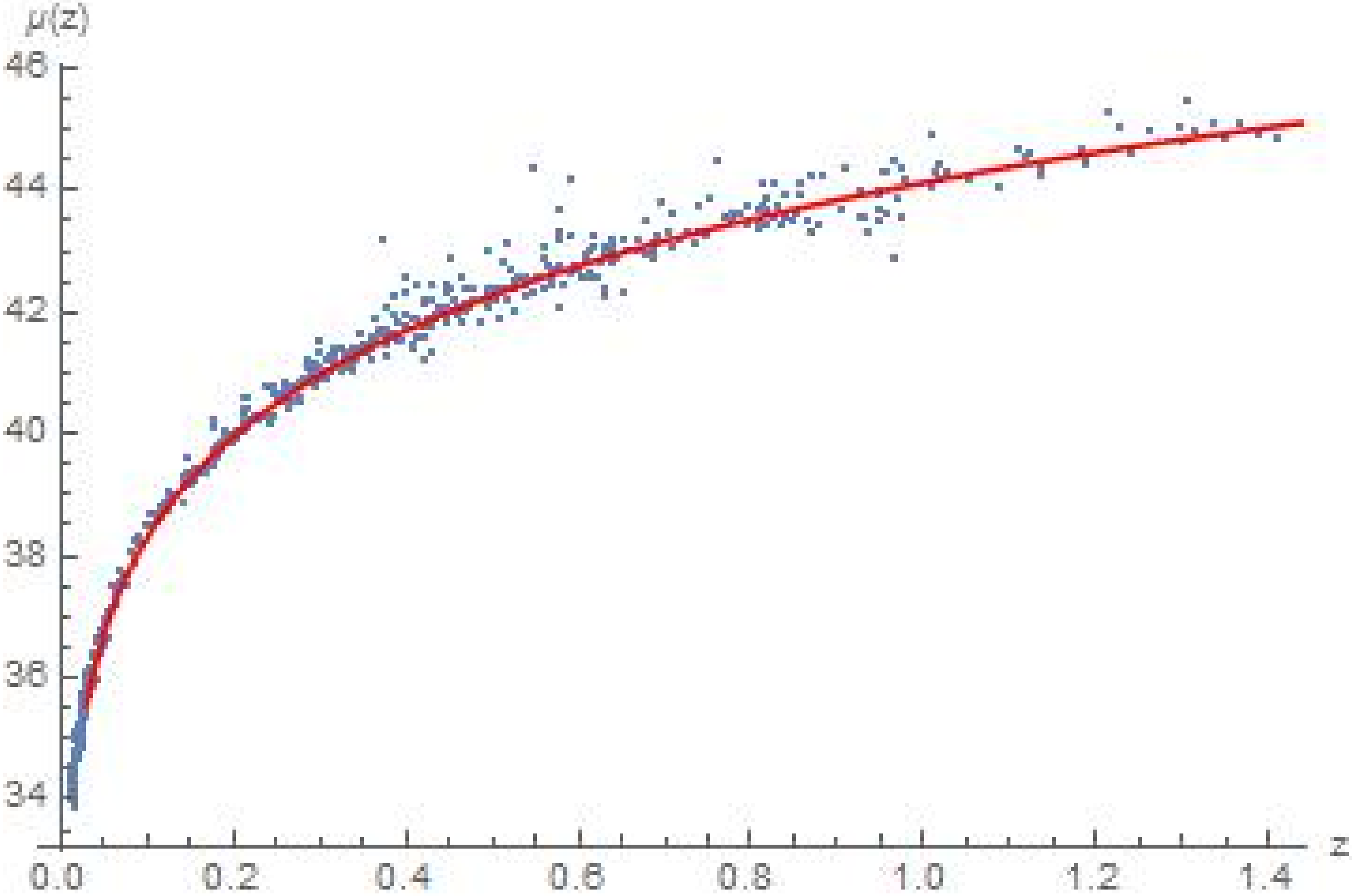


Figure 1. Distance modulus vs Redshift. We have fitted 580 supernovae to $\tilde{\delta}$ Gravity.

In GR: $h = 0.7 \pm 3.37 \times 10^{-3}$ and $h^2 \Omega_M = 0.136 \pm 8.5 \times 10^{-3}$ with $\chi^2(per point) = 0.985$.

In $\tilde{\delta}$ Gravity with $\tilde{\delta}$ matter: $L_2 = 0.457 \pm 0.0114$ and $C = 1.89 \times 10^{-4} \pm 4.92 \times 10^{-6}$ with $\chi^2(per point) = 0.985$.

Age of the Universe:1.391×10^{10} years

Big Rip:$t_{Big-Rip} = 3.042 \times 10^{10}$ years

Phantom fields also produce a cosmological model that have this property.

In the $\tilde{\delta}$ gravity model we can avoid a Big Rip at later time by a mechanism that give masses to all massless particles.

Some options are quantum effects (which are finite in this model) or massive photons due to superconductivity which could happen at very low temperatures, which are natural at a later stages of the expansion of the Universe.

We obtain for the normalized density of δ matter today:

$$\begin{aligned}\tilde{\Omega}_M &\approx \frac{L_2}{2}\,\Omega_M \\ &\approx 0.23\,\Omega_M \qquad (20)\\ \tilde{\Omega}_R &\approx \frac{2\,L_2}{3}\,\Omega_R \\ &\approx 0.3\,\Omega_R. \qquad (21)\end{aligned}$$

Finally, we want to point out that since for $t \to 0$, we have $w \to \frac{1}{3}$, then $\tilde{R}(t) = R(t)$.

- Age of the Universe: $1,36 \times 10^{10} \pm 1,56 \times 10^{8}$ years
- Hubble parameter today:

> Lambda CDM with 3 neutrinos and Planck data: 67.74 ± 0.46 km$/(s\,\mathrm{Mpc})$
>
> Best estimation: $73.52 \pm 1.62\ \mathrm{km}\,/\,(s\ \mathrm{Mpc})$ (Riess et al.2018), 3.4σ higher than Lambda CDM.
>
> Delta gravity : H_0=74.47$\pm$ 1.63 and Supernova data

- Desacelleration parameter and effective scale factor:

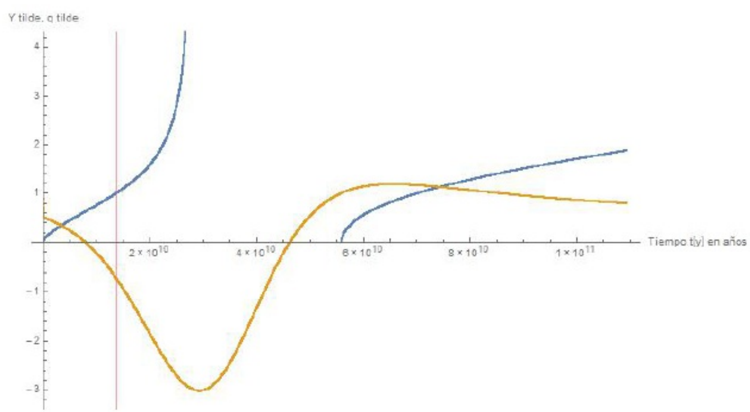
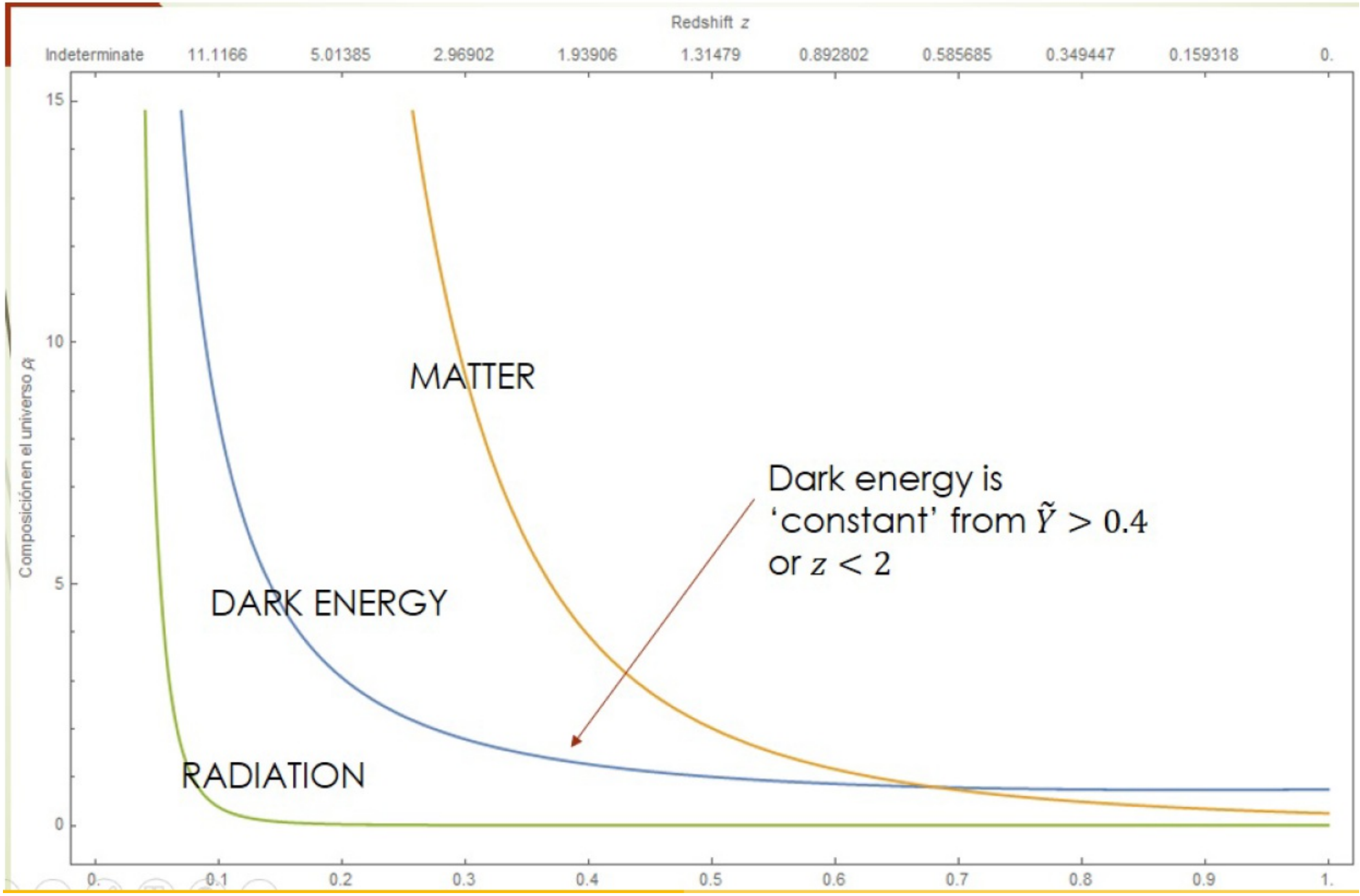
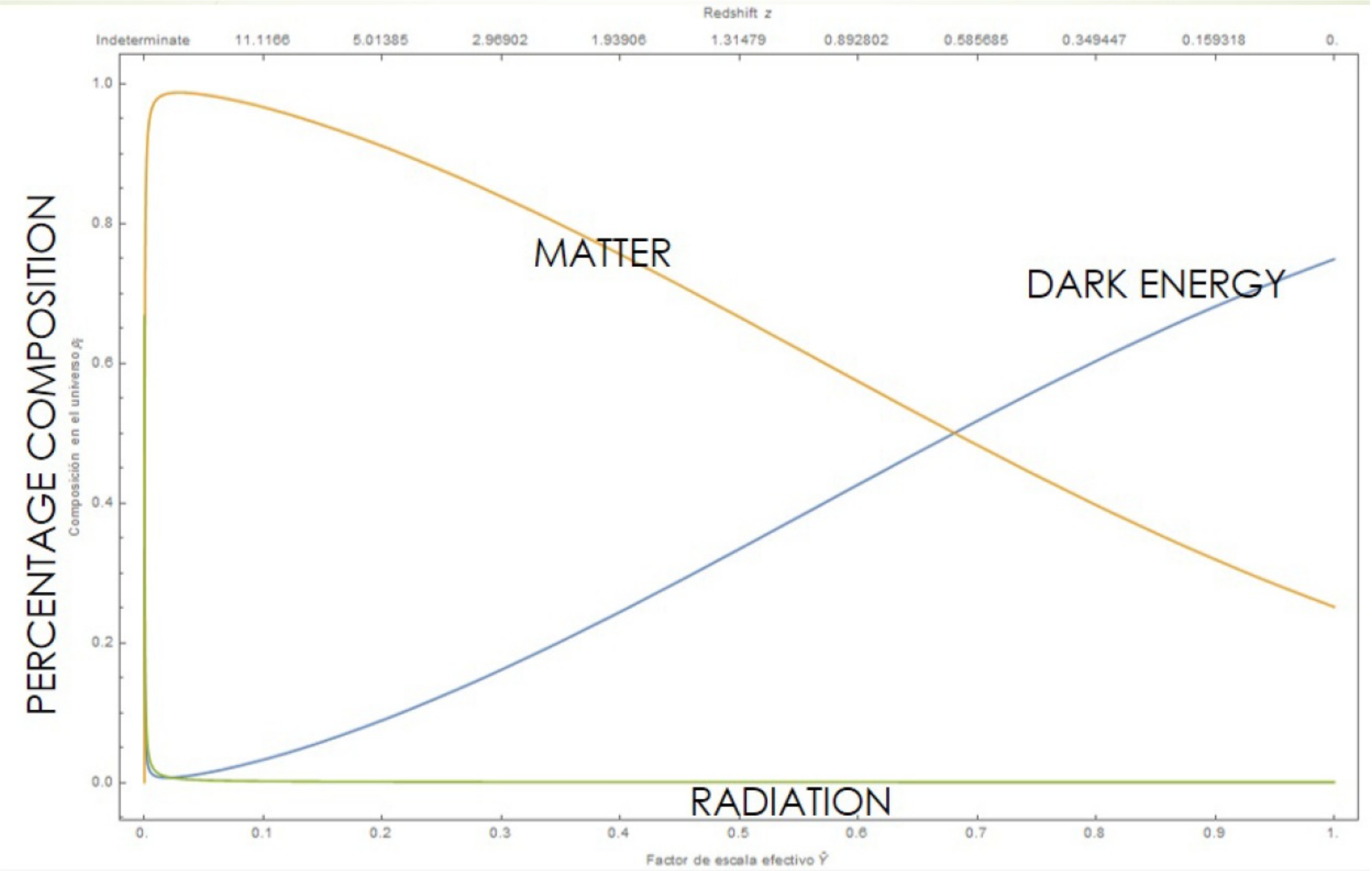


Figura 6.4: Gráfico de $\tilde{q}$ (amarillo) y $\tilde{Y}$ (azul) versus tiempo cosmológico en años.


Redshift z
Indeterminate 11.1166 5.01385 2.96902 1.93906 1.31479 0.892802 0.585685 0.349447 0.159318 0.
Composición en el universo ρ_i
15
10
5
0
0. 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.
MATTER
DARK ENERGY
RADIATION
Dark energy is
'constant' from $\tilde{Y} > 0.4$
or $z < 2$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1.0
0.8
0.6
0.4
0.2
0.0
PERCENTAGE COMPOSITION
Composición en el universo
MATTER
DARK ENERGY
RADIATION
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.
Factor de escala efectivo Ŷ

In a homogeneous and isotropic universe, we get accelerated expansion without a cosmological constant or additional scalar fields.

Delta gravity calculation of H_0 agrees with Riess et al.

Growth of Density perturbations?

Anisotropies in the CMBR?

Inflation?

Meaning of Delta matter?

THANK YOU!