# *Momentum space topology and non - dissipative currents*

**M.Zubkov**

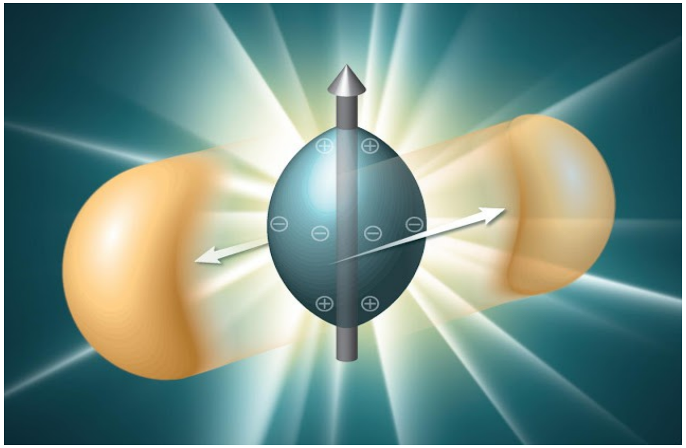
**Ariel University Israel**

7th International Conference on New Frontiers in Physics

(ICNFP 2018, 4 July 2018 - 12 July, Kolymbari, Crete, Greece)

# Non – dissipative currents may be observed experimentally in the heavy ion collisions

*chiral magnetic effect* *(does not exist in equilibrium)*

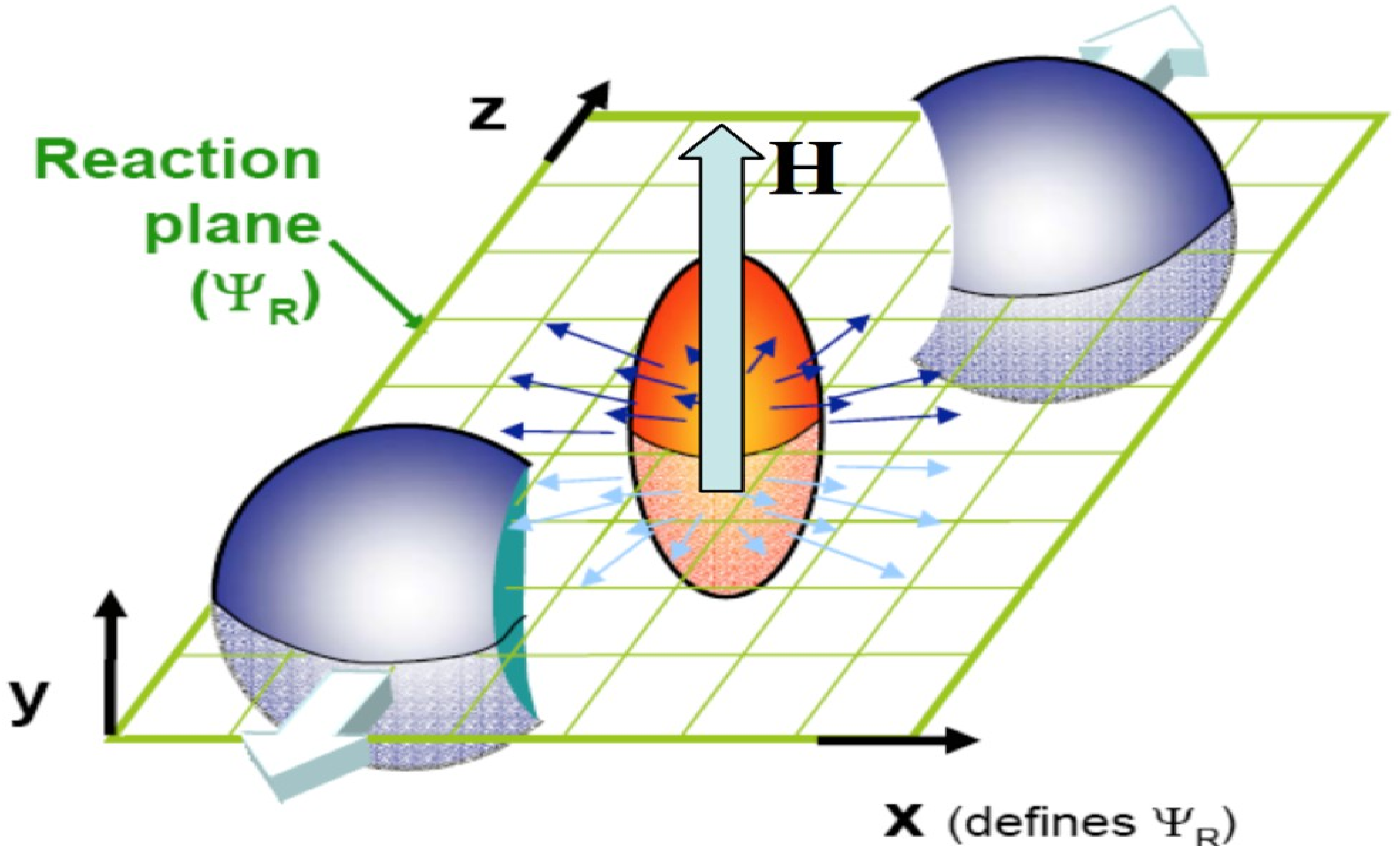


*chiral separation effect*

*chiral vortical effect*

*rotational Hall effect (new)*

# Magnetic field in the heavy ion collisions



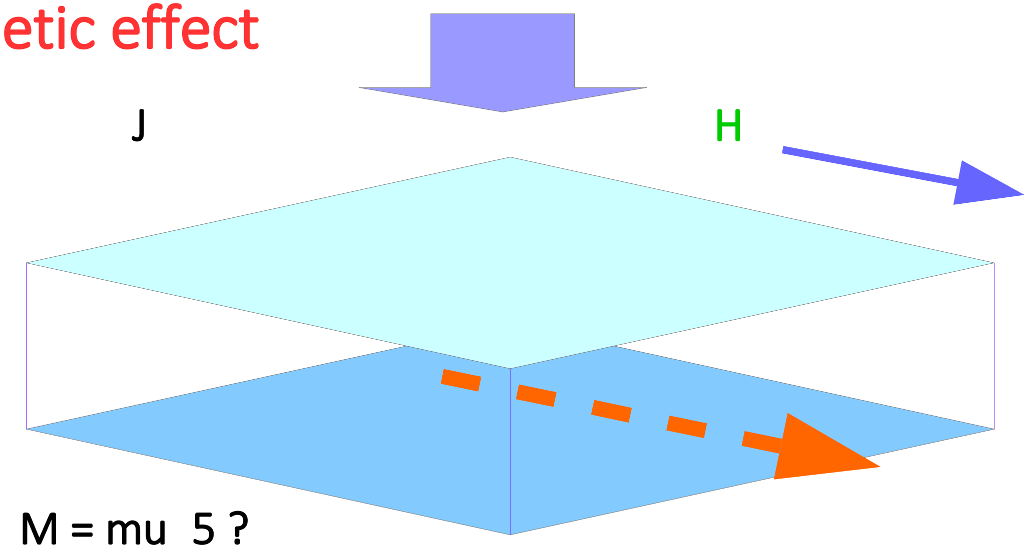
# Equilibrium chiral magnetic effect in

# **lattice regularized relativistic quantum field theory**

Nondissipative current = Topological invariant × magnetic field

**Chiral magnetic effect**

J H magnetic field direction

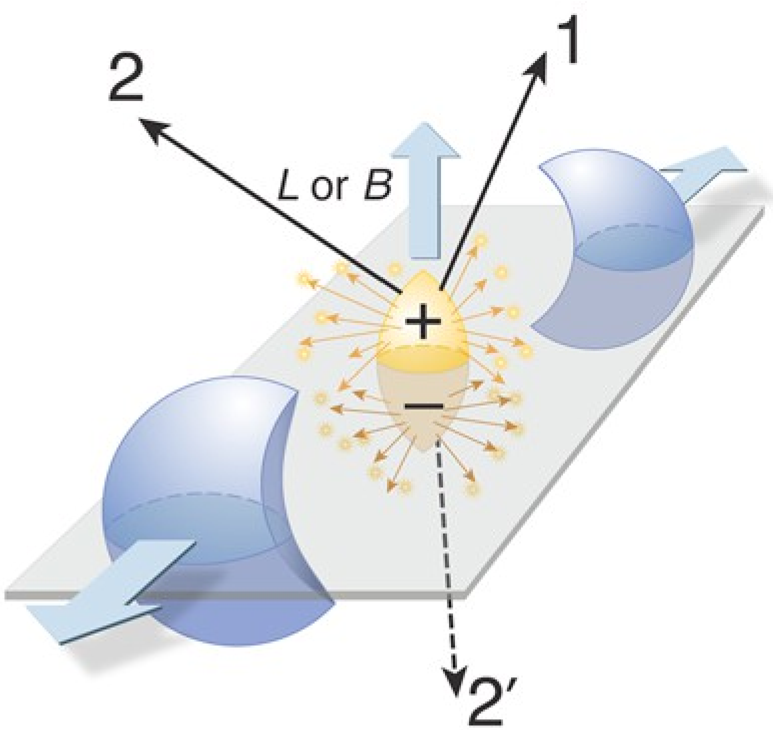


J = M /2pi^2 H M = mu_5 ?

massless relativistc fermions (heavy ion collisions)

*In the presence of the fluctuation of the chiral chemical potential* **the electric current along the direction of Magnetic field is expected**

**However, the equilibrium CME does not exist!**

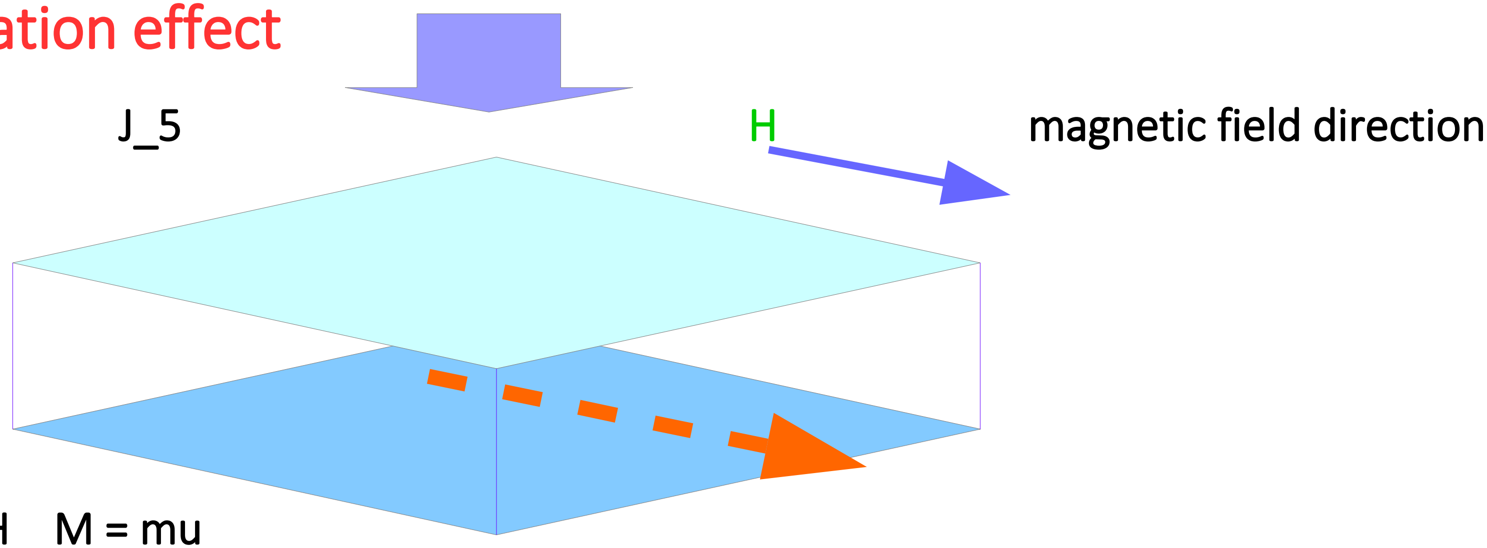


# *Equilibrium chiral separation effect in*

# lattice regularized relativistic quantum field theory

Nondissipative current = Topological invariant × magnetic field

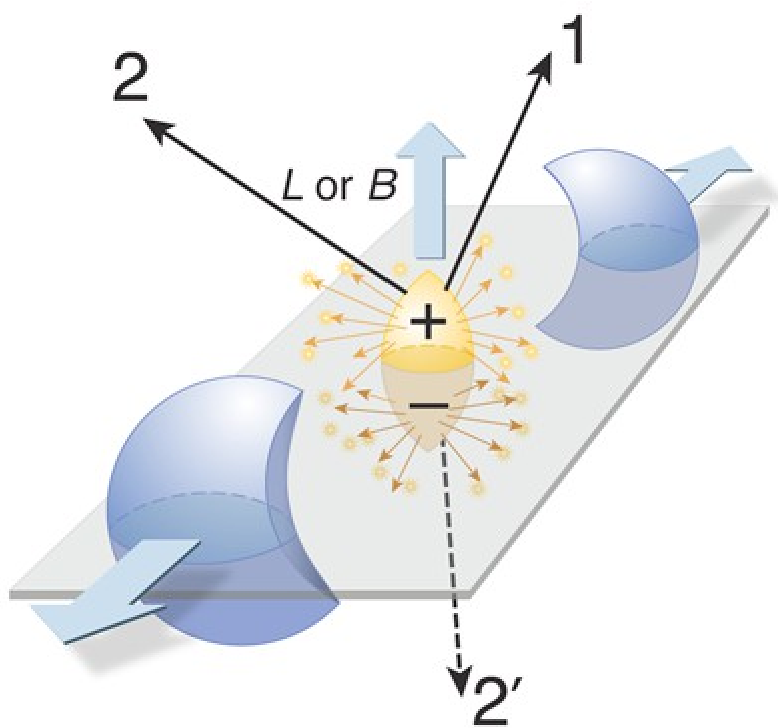
## Chiral separation effect



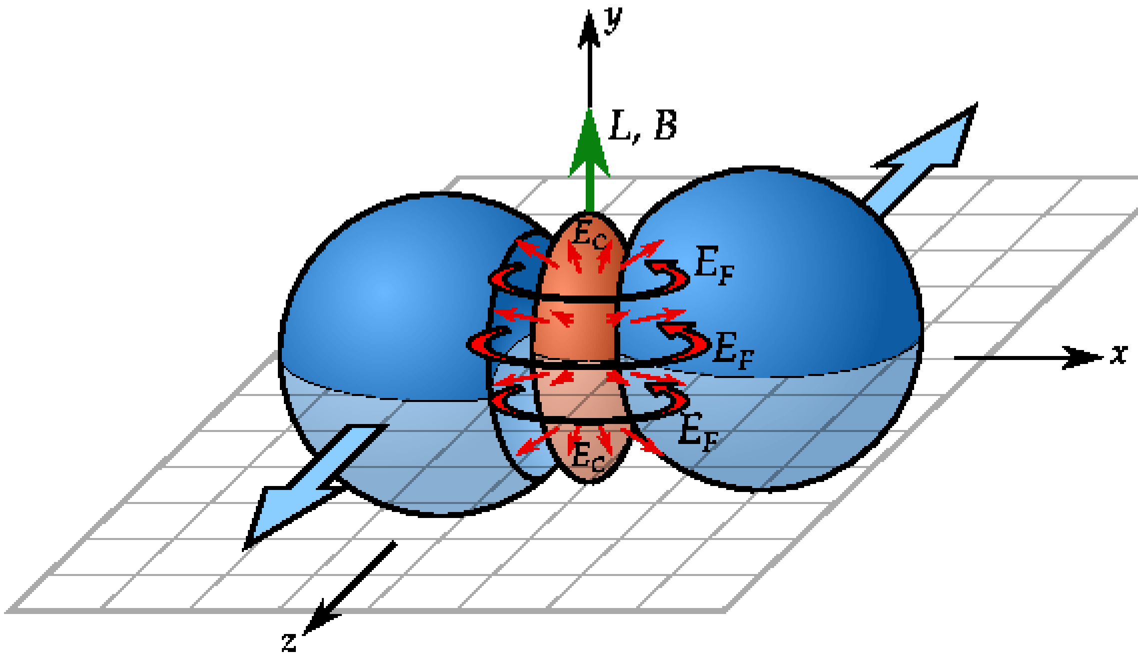

J_5 = M /2pi^2 H    M = mu

massless relativistc fermions (heavy ion collisions)

**the axial current along the direction of Magnetic field is expected ==>** *asymmetry of the left-handed and the right-handed particles after the decay of the fireball*



# Rotated fireball in the heavy ion collisions

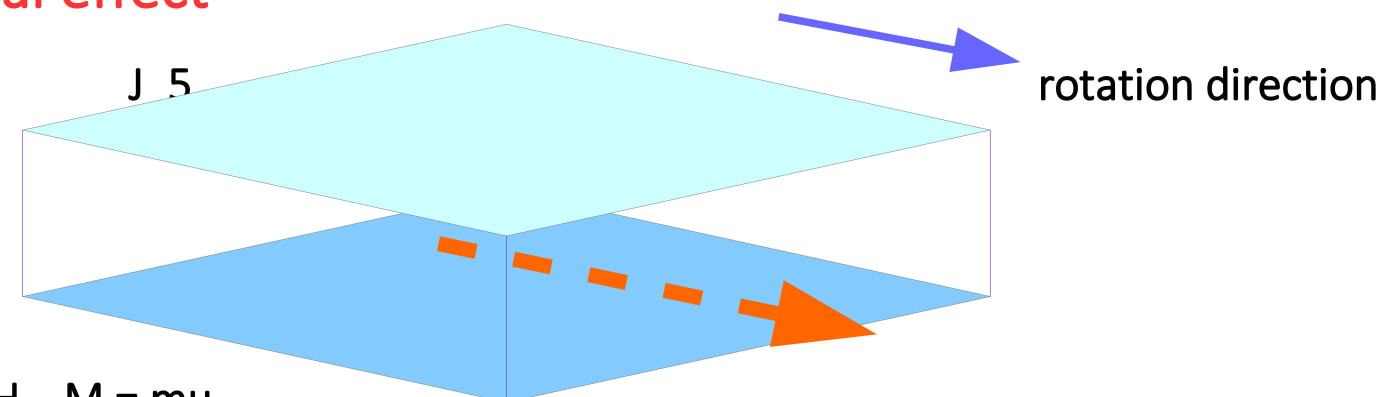


*Rotation may be introduced as the effective magnetic field (Sadofyev, Shevchenko, Zakharov)==>*

***We can use the results on the chiral separation effect***

Nondissipative current = Topological invariant × Angular velocity

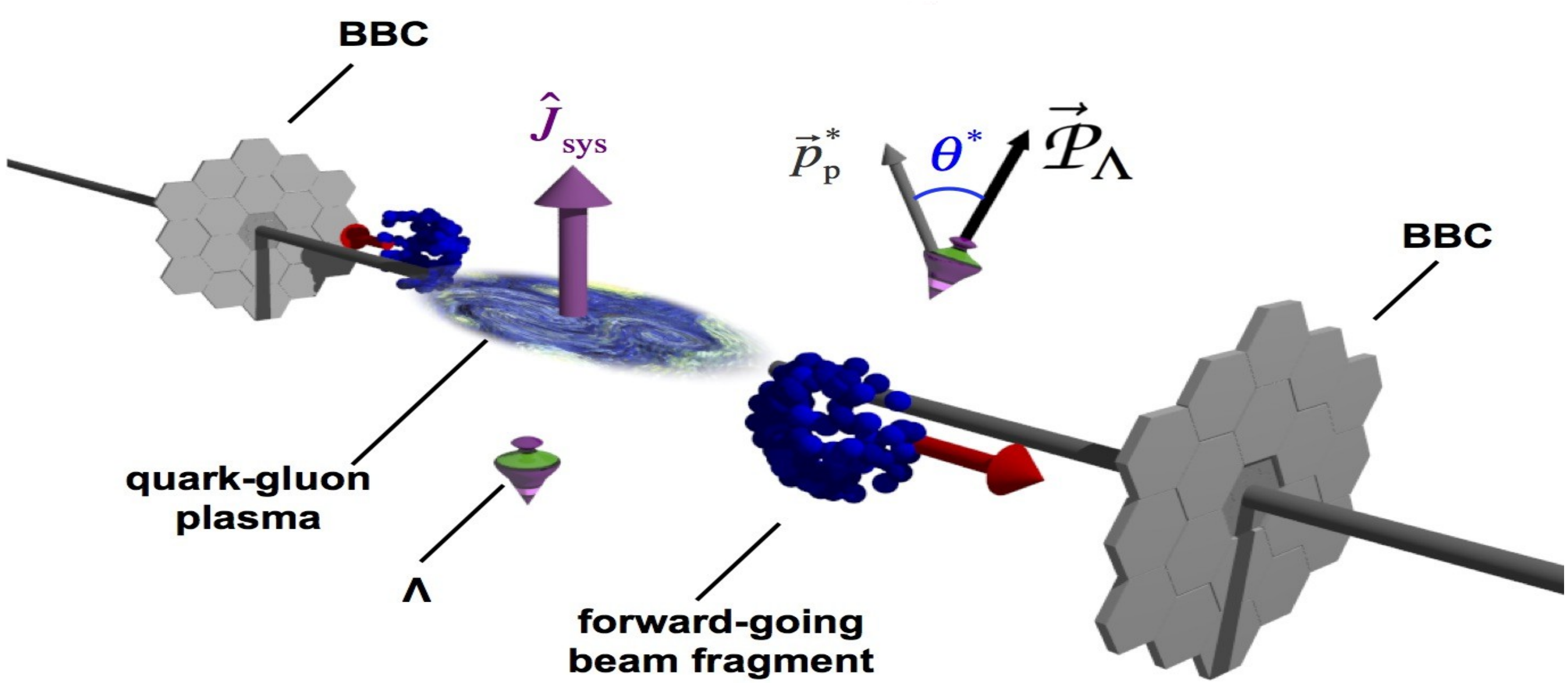
Chiral vortical effect




J_5 = M /2pi^2 H    M = mu

relativistc fermions (heavy ion collisions)

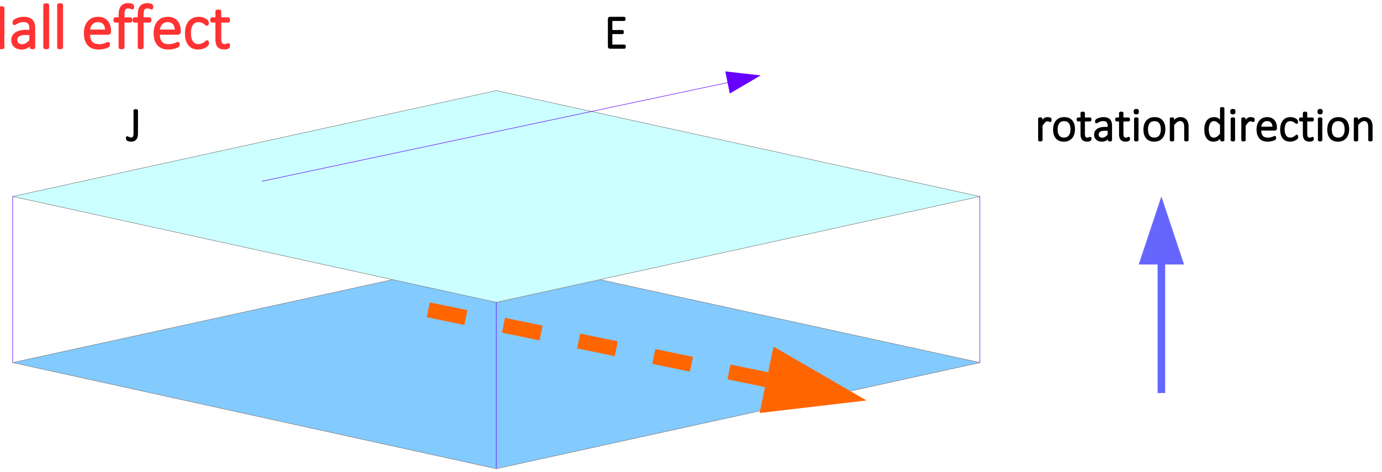
**The axial current along the direction of Rotation axis is expected ==>** *asymmetry of the left-handed and the right-handed particles after the decay of the fireball*



*Rotation may be introduced as the effective magnetic field (Sadofyev, Shevchenko, Zakharov)==>*

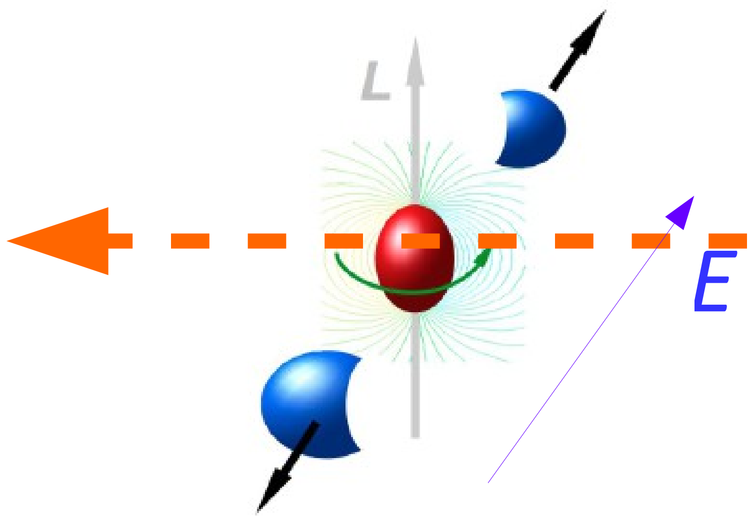
***We can use the results on the quantum Hall effect***

Rotational Hall effect




relativistc fermions (heavy ion collisions)

**The electric current orthogonal to rotation axis and external electric field is expected ==>** *asymmetry of the electric charge after the decay of the fireball*



M.A.Zubkov, « Absence of equilibrium chiral magnetic effect » arXiv:1605.08724, Physical Review D 93, 105036 (2016)

M.A.Zubkov, « Hall effect in the presence of rotation» EuroPhysLetters 121 (2018) no.4, 47001 e-Print: arXiv:1801.0536

Z.V.Khaidukov, M.A.Zubkov, "Chiral Separation effect in lattice regularization" Phys. Rev. D 95 (2017), 074502

Ruslan Abramchuk, Z.V. Khaidukov, M.A. Zubkov, “Anatomy of the chiral vortical effect» e-Print: arXiv:1806.026

F.A.Berezin

(Wigner transform, deformational quantization, and all that)

G.E.Volovik

(Momentum space topology in condensed matter and beyond)

Unusual analytical

Methods of lattice

Quantum field theory

Condensed

Matter Physics

Momentum space topology

Solid state physics (topological insulators, Weyl semimetals)

High energy physics

Lattice regularized quantum field theory

$$Z = \int D\bar{\Psi} D\Psi \exp\Big( - \int_{\mathcal{M}} \frac{d^D \mathbf{p}}{|\mathcal{M}|} \bar{\Psi}(\mathbf{p}) \hat{\mathcal{Q}}(i\partial_{\mathbf{p}}, \mathbf{p}) \Psi(\mathbf{p}) \Big)$$

gauge field as the pseudodifferential operator in momentum space

$$\hat{\mathcal{Q}} = \mathcal{G}^{-1}(\mathbf{p} - \mathbf{A}(i\partial_{\mathbf{p}}))$$

Wigner transformation of the Green function, Weyl symbols of operators

$$j^k(\mathbf{R}) = \int_{\mathcal{M}} \frac{d^D \mathbf{p}}{|\mathcal{V}||\mathcal{M}|} \mathrm{Tr}\, \tilde{G}(\mathbf{R}, \mathbf{p}) \frac{\partial}{\partial p_k} \Big[\tilde{G}^{(0)}(\mathbf{R}, \mathbf{p})\Big]^{-1}$$

Derivative expansion. Iterative solution of the Groenewold equation

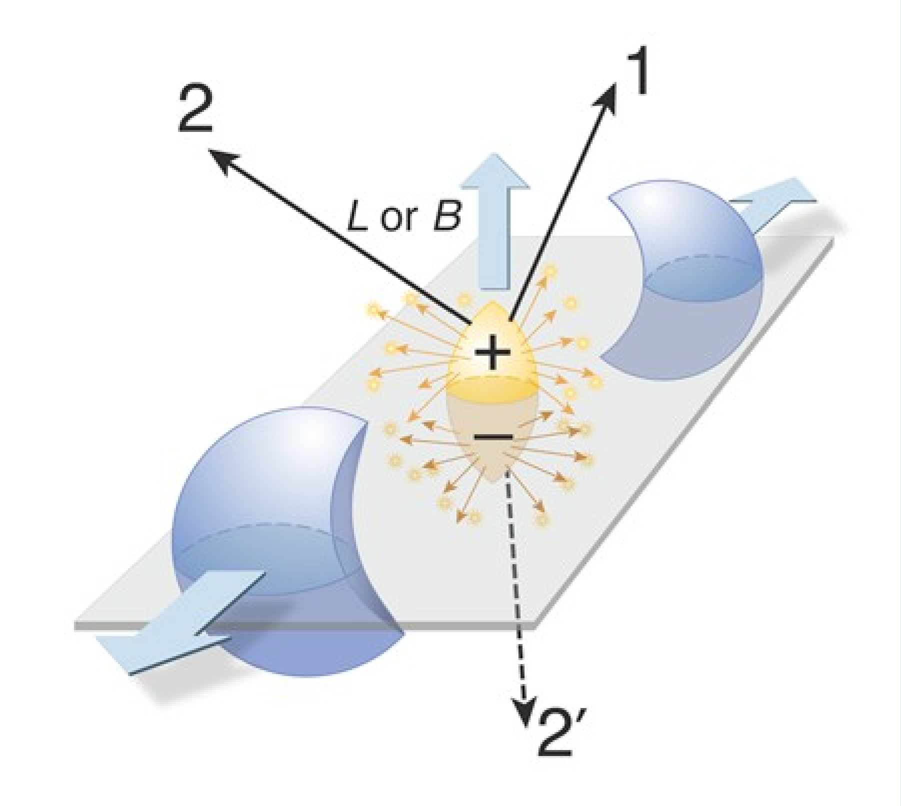
$$\begin{aligned} 1 &= \mathcal{Q}(\mathbf{R}, \mathbf{p}) * \tilde{G}(\mathbf{R}, \mathbf{p}) \\ &\equiv \mathcal{Q}(\mathbf{R}, \mathbf{p}) e^{\frac{i}{2}(\overleftarrow{\partial}_{\mathbf{R}} \overrightarrow{\partial}_{\mathbf{p}} - \overleftarrow{\partial}_{\mathbf{p}} \overrightarrow{\partial}_{\mathbf{R}})} \tilde{G}(\mathbf{R}, \mathbf{p}) \end{aligned}$$

Nondissipative current = Topological invariant × Electric or magnetic field

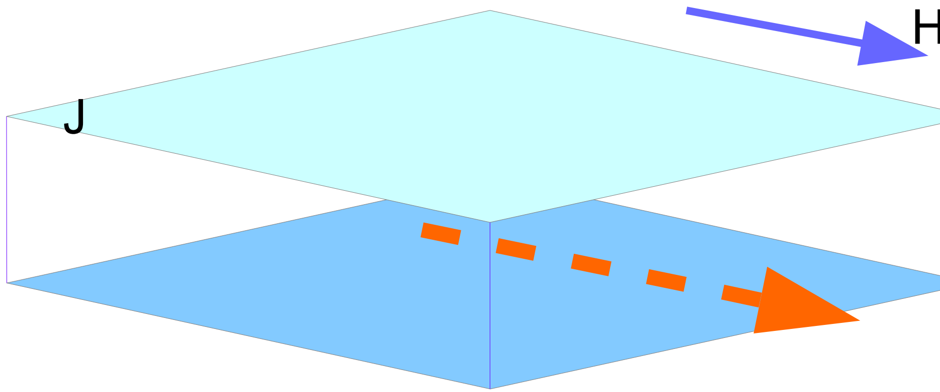
## 1. *The absence of equilibrium CME*

M.A.Zubkov, « Absence of equilibrium chiral magnetic effect »
arXiv:1605.08724, Physical Review D 93, 105036 (2016)

This work argues that the simplest version of this effect does not exist. The presented analytical proof is valid for the wide class of systems both in condensed matter physics and in high energy physics



Chiral Magnetic Effect (CME) is the appearance of electric current in the direction of the external magnetic field in the presence of chiral chemical potential




J = M /2pi^2 H    M = mu_5 ?

Pre – history: the existence of chiral magnetic effect was proposed in

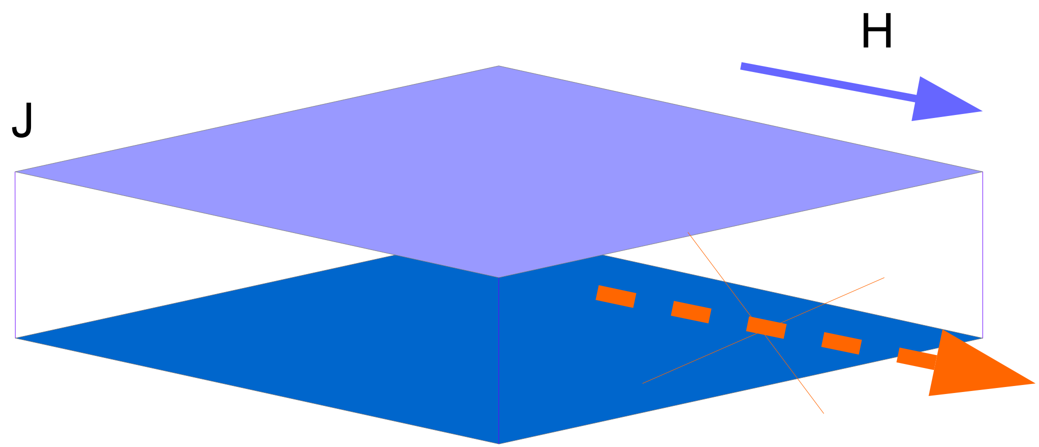
A. Vilenkin, Equilibrium parity-violating current in a magnetic field, Phys. Rev. D **22**, 3080 (1980).

This proposition was later repeated in

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).

and in the sequence of the other papers

Chiral Magnetic Effect (CME) is the appearance of electric current in the direction of the external magnetic field in the presence of chiral chemical potential




Later the existance of the equilibrium bulk static CME was questioned.

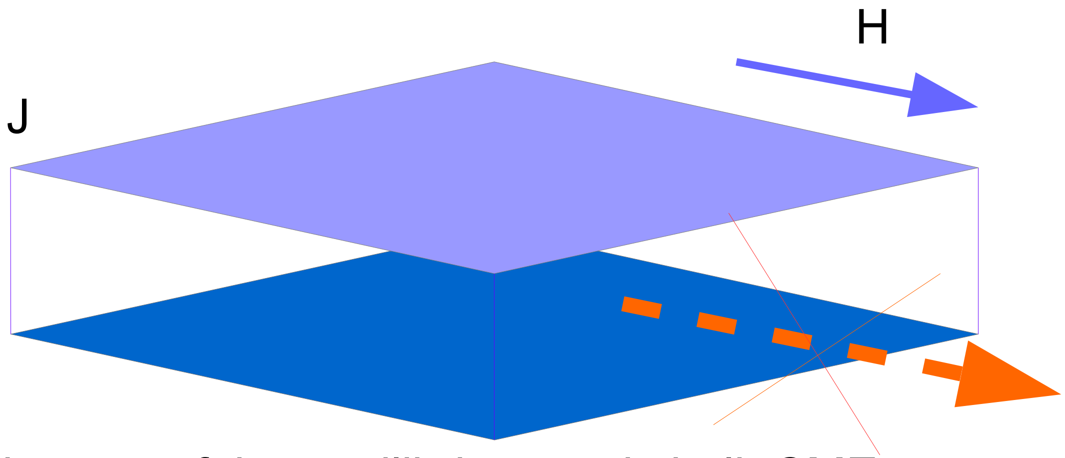
S. N. Valgushev, M. Puhr, and P. V. Buividovich, Chiral magnetic effect in finite-size samples of parity-breaking Weyl semimetals, arXiv:1512.01405.

P. V. Buividovich, M. Puhr, and S. N. Valgushev, Chiral magnetic conductivity in an interacting lattice model of parity-breaking Weyl semimetal, Phys. Rev. B **92**, 205122 (2015).

P. V. Buividovich, Spontaneous chiral symmetry breaking and the chiral magnetic effect for interacting Dirac fermions with chiral imbalance, Phys. Rev. D **90**, 125025 (2014).

P. V. Buividovich, Anomalous transport with overlap fermions, Nucl. Phys. **A925**, 218 (2014).

Chiral Magnetic Effect (CME) is the appearance of electric current in the direction of the external magnetic field in the presence of chiral chemical potential




Later the existance of the equilibrium static bulk CME was questioned.

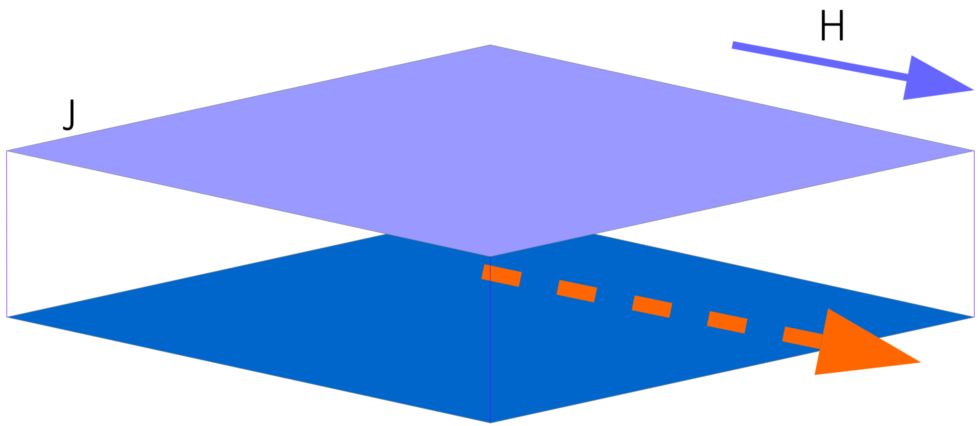
Weyl semimetals M. Vazifeh and M. Franz, Electromagnetic Response of Weyl Semimetals, Phys. Rev. Lett. **111**, 027201 (2013).

Analysis based on the attempt to apply Bloch theorem

.

N. Yamamoto, Generalized Bloch theorem and chiral transport phenomena, Phys. Rev. D **92**, 085011 (2015).

We start from the lattice model with massive fermions that describes lattice regularized quantum field theory or the insulators whose excitations are described by massive Dirac action (in solid state physics).




J = M /2pi^2 H    M = mu_5 ?

Chiral imbalance is described by the appearance of the chiral chemical potential

Green function (without external magnetic field) is:

$$\mathcal{G}(\mathbf{p}) = \Big(\sum_k \gamma^k g_k(\mathbf{p}) + i\gamma^4\gamma^5\mu_5 - im(\mathbf{p})\Big)^{-1}$$

Example : Wilson fermions

$$g_k(\mathbf{p}) = \sin p_k, \quad m(\mathbf{p}) = m^{(0)} + \sum_{a=1,2,3,4} (1-\cos p_a)$$

# 3+1 D Chiral Magnetic Effect

In lattice models
we obtain for the first time
$\mathcal{M}_4$ is responsible for the CME

$$j^{(1)k}(\mathbf{R}) = \frac{1}{4\pi^2}\epsilon^{ijkl}\mathcal{M}_l A_{ij}(\mathbf{R})$$

*In continuous models this follows trivially from Feinman diagrams 4x4 Green function*

$$\mathcal{M}_l = \int \mathrm{Tr}\, \nu_l \, d^4p$$

$$\nu_l = -\frac{i}{3!\,8\pi^2}\,\epsilon_{ijkl}\left[\mathcal{G}\frac{\partial \mathcal{G}^{-1}}{\partial p_i}\frac{\partial \mathcal{G}}{\partial p_j}\frac{\partial \mathcal{G}^{-1}}{\partial p_k}\right]$$

$$\mathcal{G}(\mathbf{p}) = \Big(\sum_k \gamma^k g_k(\mathbf{p}) + i\gamma^4\gamma^5\mu_5 - im(\mathbf{p})\Big)^{-1}$$

$$g_k(\mathbf{p}) = \sin p_k, \quad m(\mathbf{p}) = m^{(0)} + \sum_{a=1,2,3,4}(1-\cos p_a)$$

$\mathcal{M}_4$ is responsible for the CME

$$\mathcal{M}_l = \int \mathrm{Tr}\, \nu_l \, d^4 p$$

$$\nu_l = -\frac{i}{3!\, 8\pi^2}\, \epsilon_{ijkl} \left[ \mathcal{G} \frac{\partial \mathcal{G}^{-1}}{\partial p_i} \frac{\partial \mathcal{G}}{\partial p_j} \frac{\partial \mathcal{G}^{-1}}{\partial p_k} \right]$$

$$\mathcal{G}(\mathbf{p}) = \Big( \sum_k \gamma^k g_k(\mathbf{p}) + i\gamma^4\gamma^5 \mu_5 - i m(\mathbf{p}) \Big)^{-1} \qquad g_k(\mathbf{p}) = \sin p_k, \quad m(\mathbf{p}) = m^{(0)} + \sum_{a=1,2,3,4} (1 - \cos p_a)$$

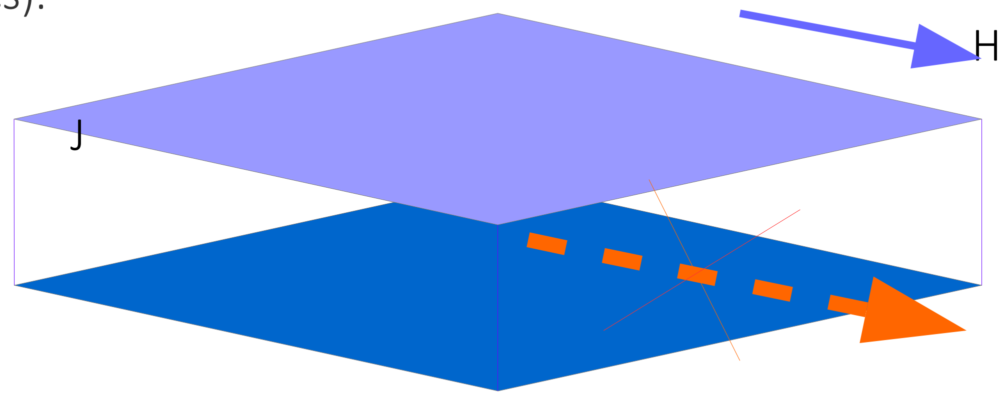
(Wilson fermions at m_0 = 0 is the simplest model of Dirac semimetal)

| m_0=0 <br> mu_5≠0 | $\Leftrightarrow$ | m_0≠0 <br> mu_5≠0 | $\Leftrightarrow$ | m_0≠0 <br> mu_5=0 |
|---|---|---|---|---|
| | | | | $\Uparrow$ |
| | | | | $\mathcal{M}_4$=0 |

Poles of the Green function may appear for the nonzero mu_5 if

$$g_4^2(\mathbf{p}) + \Big(\mu_5 \pm \sqrt{g_1^2(\mathbf{p}) + g_2^2(\mathbf{p}) + g_3^2(\mathbf{p})}\Big)^2 + m^2(\mathbf{p}) = 0$$

We considered lattice models with both massive qnd massless fermions that describe lattice regularized quantum field theory or the insulators and Dirac semimetals whose excitations are described by massive/massless Dirac action (in solid state physics).




J = M_4 /2pi^2 H    M_4 = 0 as long as mu_5 is nonzero

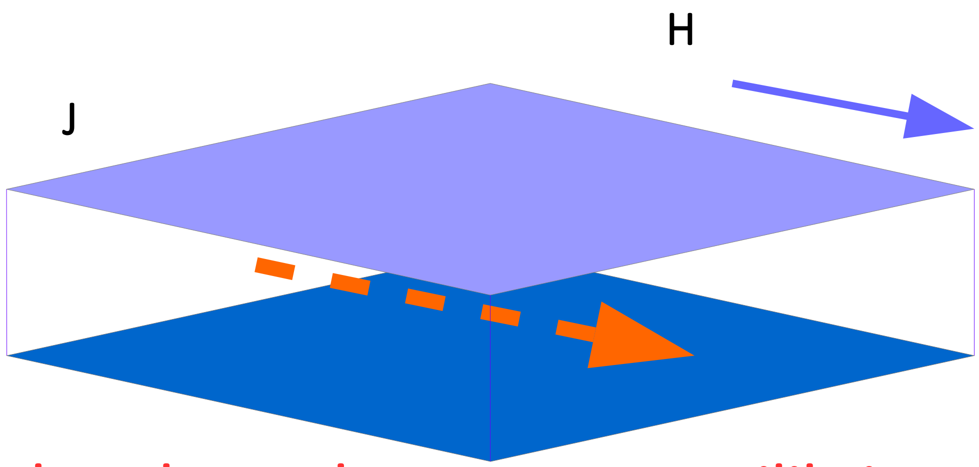
Chiral imbalance is described by the appearance of the chiral chemical potential

Green function (without external magnetic field) is:

$$\mathcal{G}(\mathbf{p}) = \Big(\sum_k \gamma^k g_k(\mathbf{p}) + i\gamma^4\gamma^5\mu_5 - im(\mathbf{p})\Big)^{-1}$$

There is no equilibrium CME

# IN WHICH FORM THE CME MAY SURVIVE ?

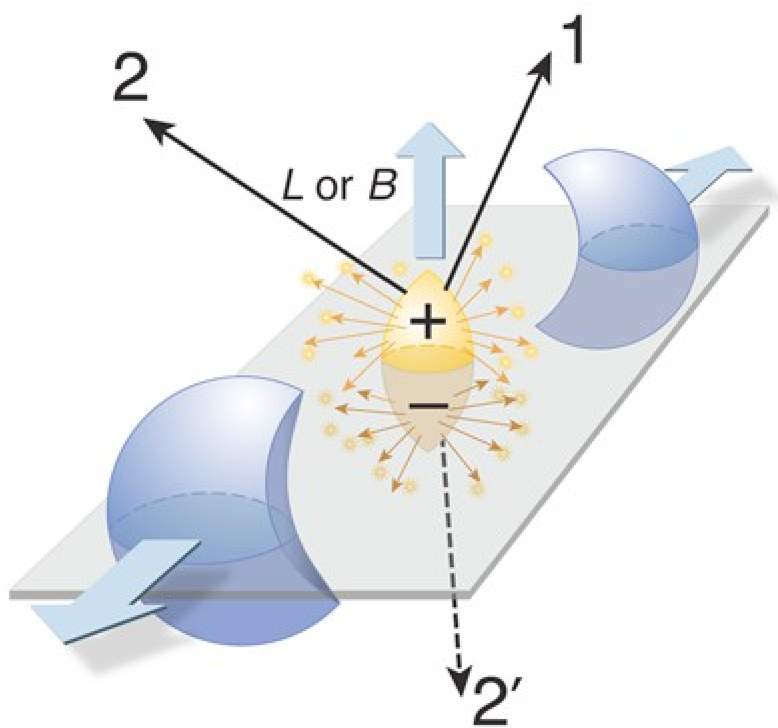



3) Quark – gluon plasma : nonequilibrium CME contributions to the kinetic equations in the presence of the chiral imbalance?

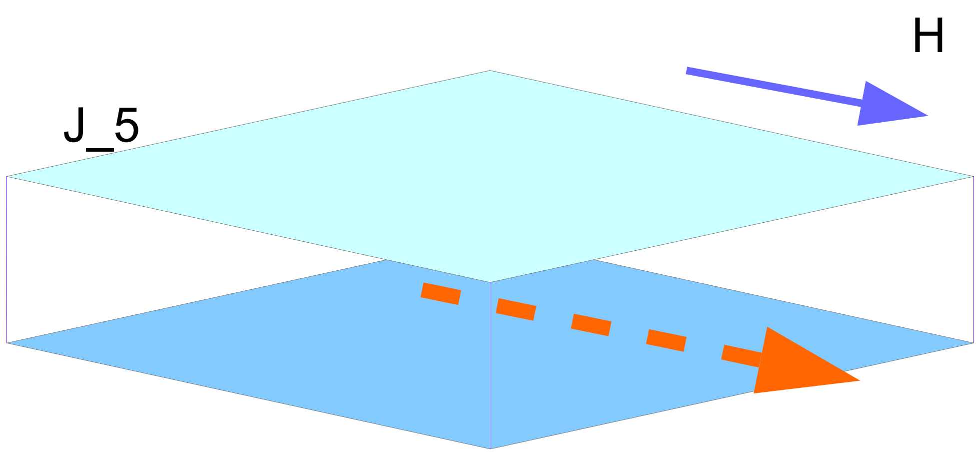
Chiral imbalance that is described by chiral density rather than the chiral chemical potential ?

## *2. Chiral Separation Effect for massless or nearly massless fermions*

Z.V.Khaidukov, M.A.Zubkov, "Chiral Separation effect in lattice regularization" Phys. Rev. D 95 (2017), 074502



Chiral Separation Effect (CME) is the appearance of axial current in the direction of the external magnetic field in the presence of chemical potential




J_5 = M /2pi^2 H    M = mu

A. Metlitski and Ariel R. Zhitnitsky,Phys. Rev. D 72 (2005), 045011

CSE is the appearance of axial current along the external magnetic field

$$\bar{\mathrm{j}}^5 = -\frac{\mu}{2\pi^2}\overline{\mathrm{B}}$$

$\mu$ - chemical potential, $\mathrm{j}^{5,\mathrm{k}} = \langle\bar{\psi}\gamma^{\mathrm{k}}\gamma^5\rangle$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

Anomalous Axion Interactions and Topological Currents in Dense Matter Max A. Metlitski and Ariel R. Zhitnitsky, Phys. Rev. D 72, 045011

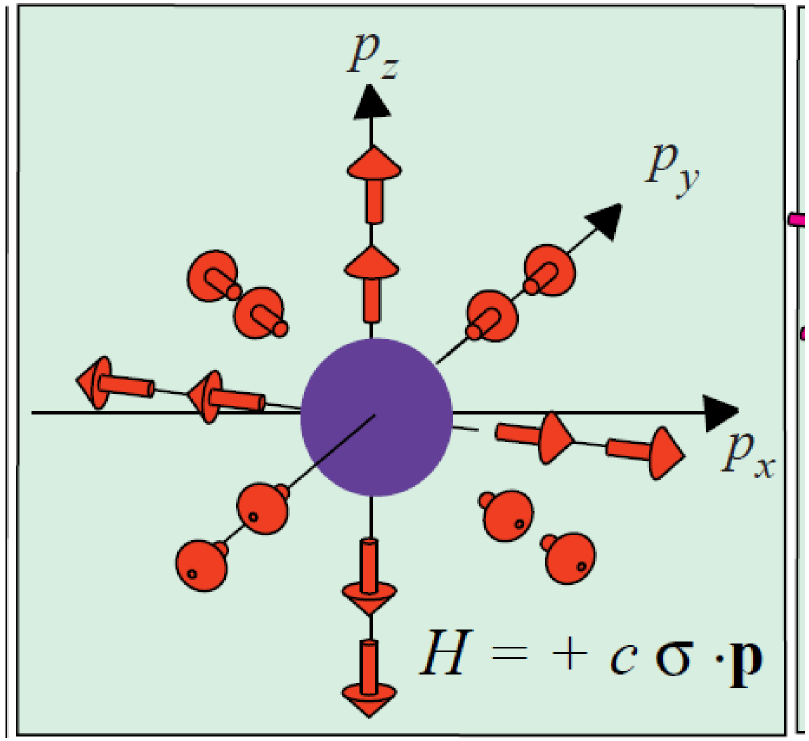
## The term linear in chemical potential

$$j^{5k} = \frac{\mathcal{N}\,\epsilon^{ijk}}{4\pi^2} F_{ij}\mu$$

Here

$$\mathcal{N} = \frac{1}{12}\int_{\Sigma} \frac{1}{(2\pi)^2} Tr\gamma^5 \mathcal{G}(\omega,\mathbf{p}) d\mathcal{G}^{-1}(\omega,\mathbf{p}) \wedge d\mathcal{G}(\omega,\mathbf{p}) \wedge d\mathcal{G}^{-1}(\omega,\mathbf{p})$$

# The surface surrounds the Fermi point in momentum space




Weyl point - hedgehog in p-space

For the case of Wilson fermions

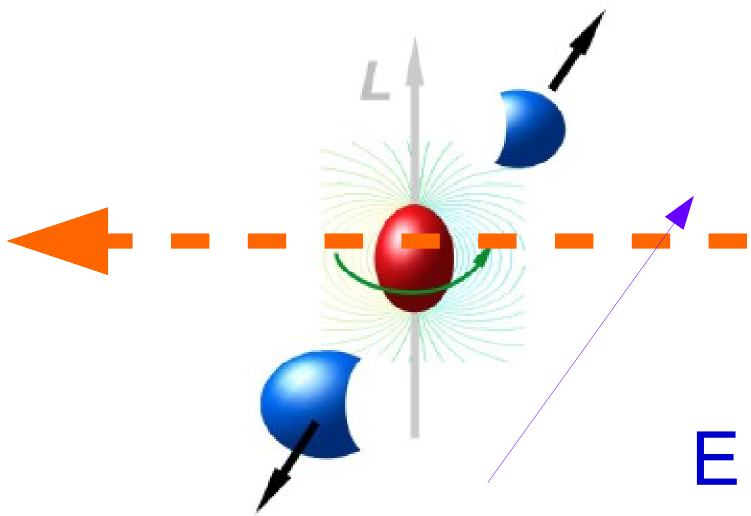
$$G^{-1}(p) = \sum_{i=1..4} \sin(p_i)\gamma^i + (\sum_{i=1..4} 2\sin^2(p_i/2) + m^{(0)})\mathbf{i}$$

Poles may appear at $\omega = 0, \pi$. For $m^{(0)} \to 0$ there is the pole only at $\omega = 0$

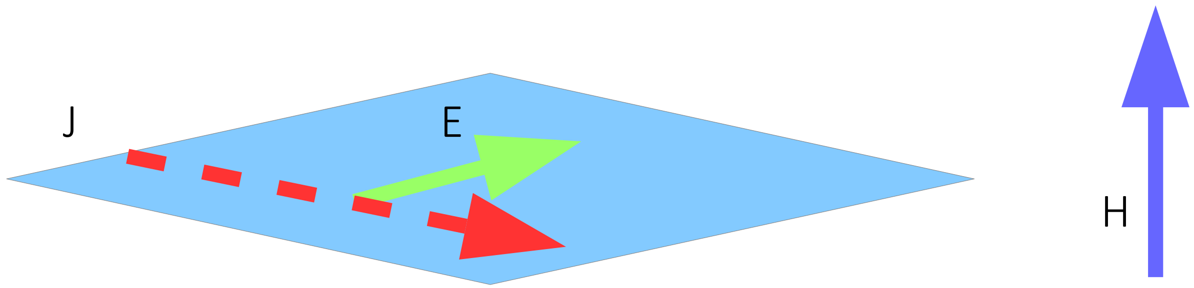
$$\mathcal{N} = 1$$

# 3. Rotational Hall effect

M.A.Zubkov, « Hall effect in the presence of rotation» EuroPhysLetters 121 (2018) no.4, 47001 e-Print: arXiv:1801.0536



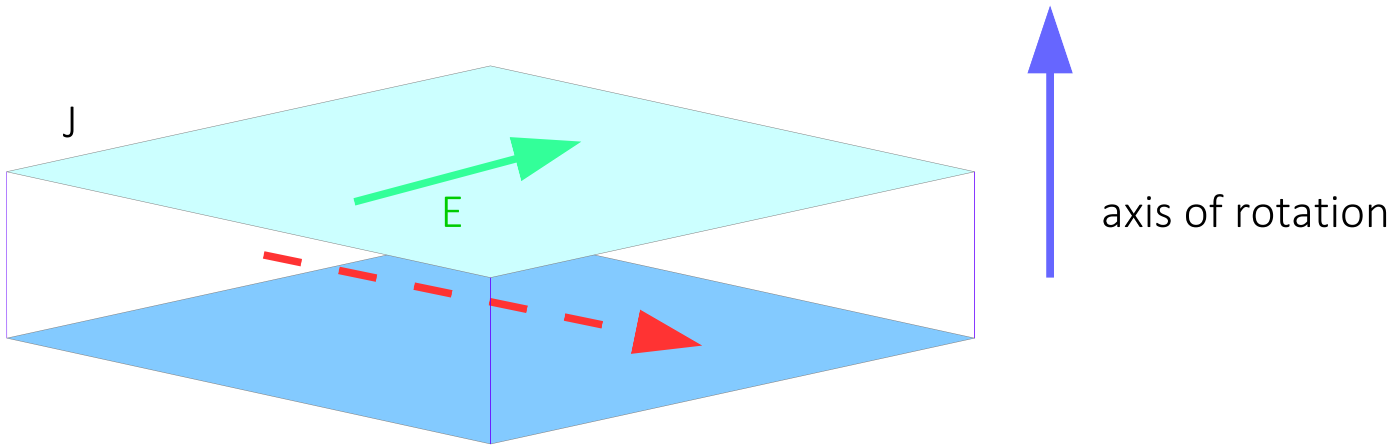
Hall effect is the appearance of electric current in the direction orthogonal to the external magnetic field and external electric field.




Qauntum Hall effect is the quantized Hall effect

J = N/2pi E

Rotational Hall effect is the appearance of electric current in the direction orthogonal to the external electric field and the axis of rotation.



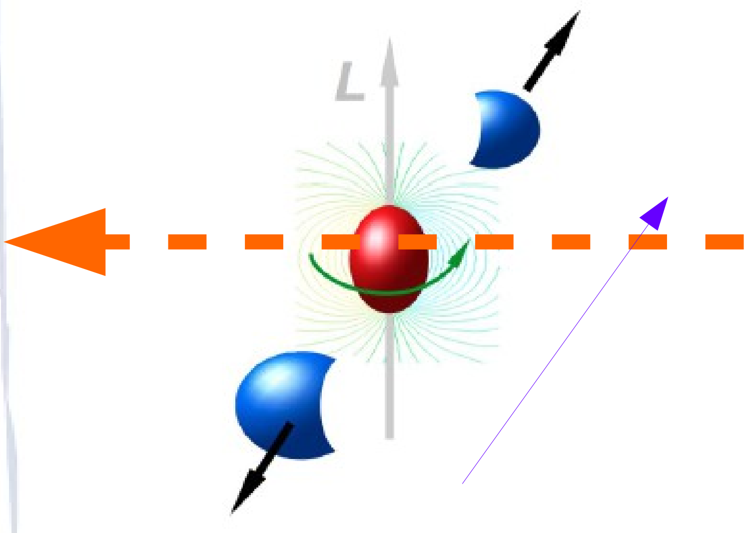

$$\mathbf{B} = (0, 0, -2\mu\Omega)$$

$$\mathbf{j}^{(HTL)QCD} = \sum_f \frac{\partial \Omega^{(HTL)QCD}}{\partial \mu_f^2} Q_f^2 \frac{\mathbf{E}_{ext} \times \mathbf{\Omega}}{\Omega^2}$$

$$\begin{aligned} \hat{m}_{q,f} &= \frac{m_{q,f}}{2\pi T}, \quad \hat{m}_D = \frac{m_D}{2\pi T} \\ d_A &= N_c^2 - 1 \end{aligned}$$

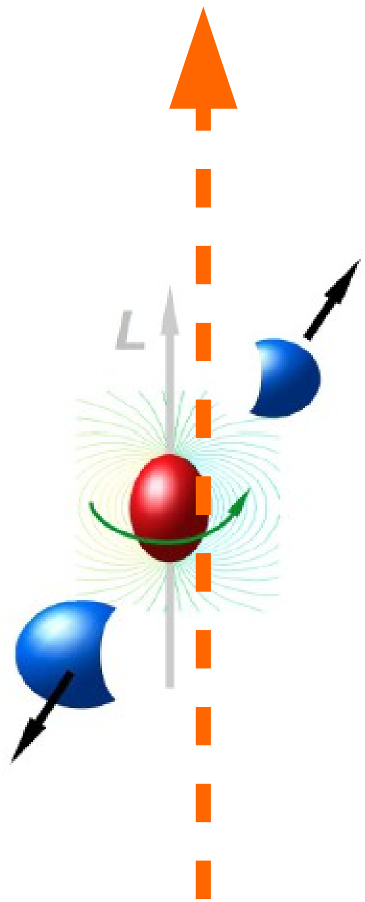
$$\begin{aligned} m_D^2 &= \frac{g^2}{3}\Big[(N_c + \frac{N_f}{2})T^2 + \frac{3}{2\pi^2}\sum_f \mu_f^2\Big] \\ m_{q,f}^2 &= \frac{g^2}{4}\frac{N_c^2 - 1}{4N_c}(T^2 + \frac{\mu_f^2}{\pi^2}) \end{aligned}$$

$$\begin{aligned} \Omega^{(HTL)QCD} &= \frac{d_A \pi^2 T^4}{45}\Big(\frac{30N_c}{d_A}\sum_f(\hat{\mu}_f^2 + 2\hat{\mu}_f^4) - \frac{15}{2}\hat{m}_D^2 \\ &- \frac{30N_c}{d_A}\sum_f(1 + 12\hat{\mu}_f^2)\hat{m}_{q,f}^2 + 30\hat{m}_D^3 \\ &+ \frac{60N_c}{d_A}(6 - \pi^2)\sum_f \hat{m}_{q,f}^4 \\ &+ \frac{45}{4}(\gamma_E - \frac{7}{2} + \frac{\pi^2}{3} + \log\frac{\bar{\Lambda}}{4\pi T})\hat{m}_D^4\Big) \quad (17) \end{aligned}$$



# 4. Chiral vortical effect

Ruslan Abramchuk, Z.V. Khaidukov, M.A. Zubkov, “Anatomy of the chiral vortical effect»
e-Print: arXiv:1806.026



On the axis of rotation.

Effective magnetic field $\mathbf{B} = (0, 0, -2\mu\Omega)$

Axial current (T=0) $j^{5k} = \frac{\mathcal{N}\epsilon^{12k}}{2\pi^2}\mu^2\Omega$

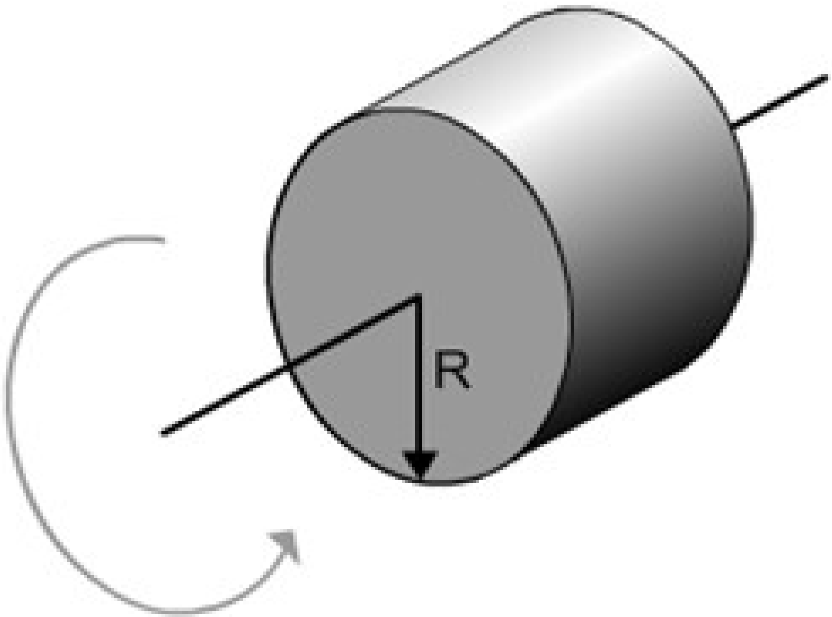
$$\mathcal{N} = \frac{1}{12}\int_{\Sigma}\frac{1}{(2\pi)^2} Tr\gamma^5 \mathcal{G}(\omega, \mathbf{k}) d\mathcal{G}^{-1}(\omega, \mathbf{k}) \wedge d\mathcal{G}(\omega, \mathbf{k}) \wedge \mathcal{G}^{-1}(\omega, \mathbf{k})$$

For one nearly massless Dirac fermion N = 1

# MIT boundary conditions

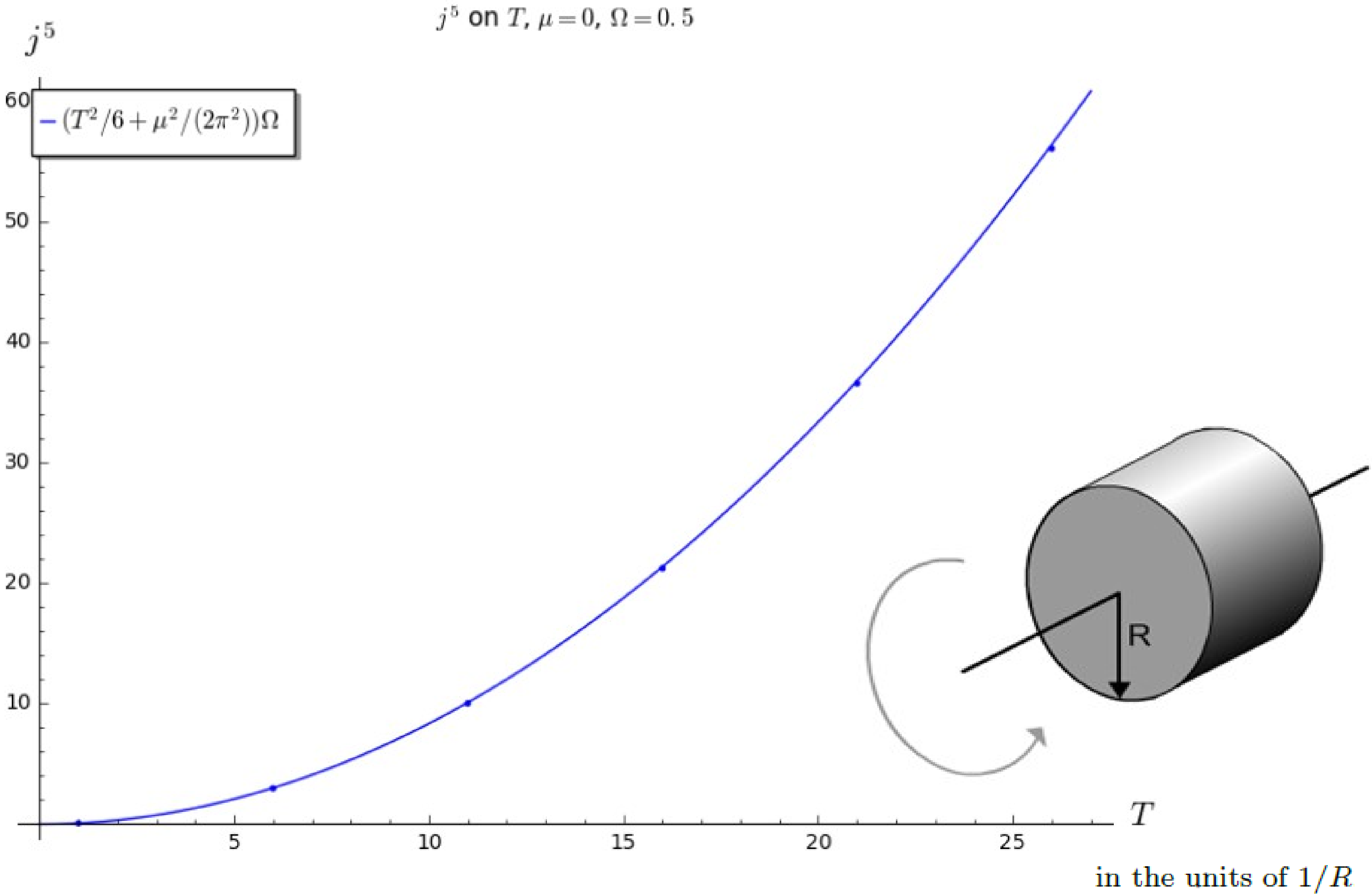
$$0 = (i\gamma^\mu n_\mu - 1)\psi|_{r=R}$$

$$j^\mu n_\mu = 0$$

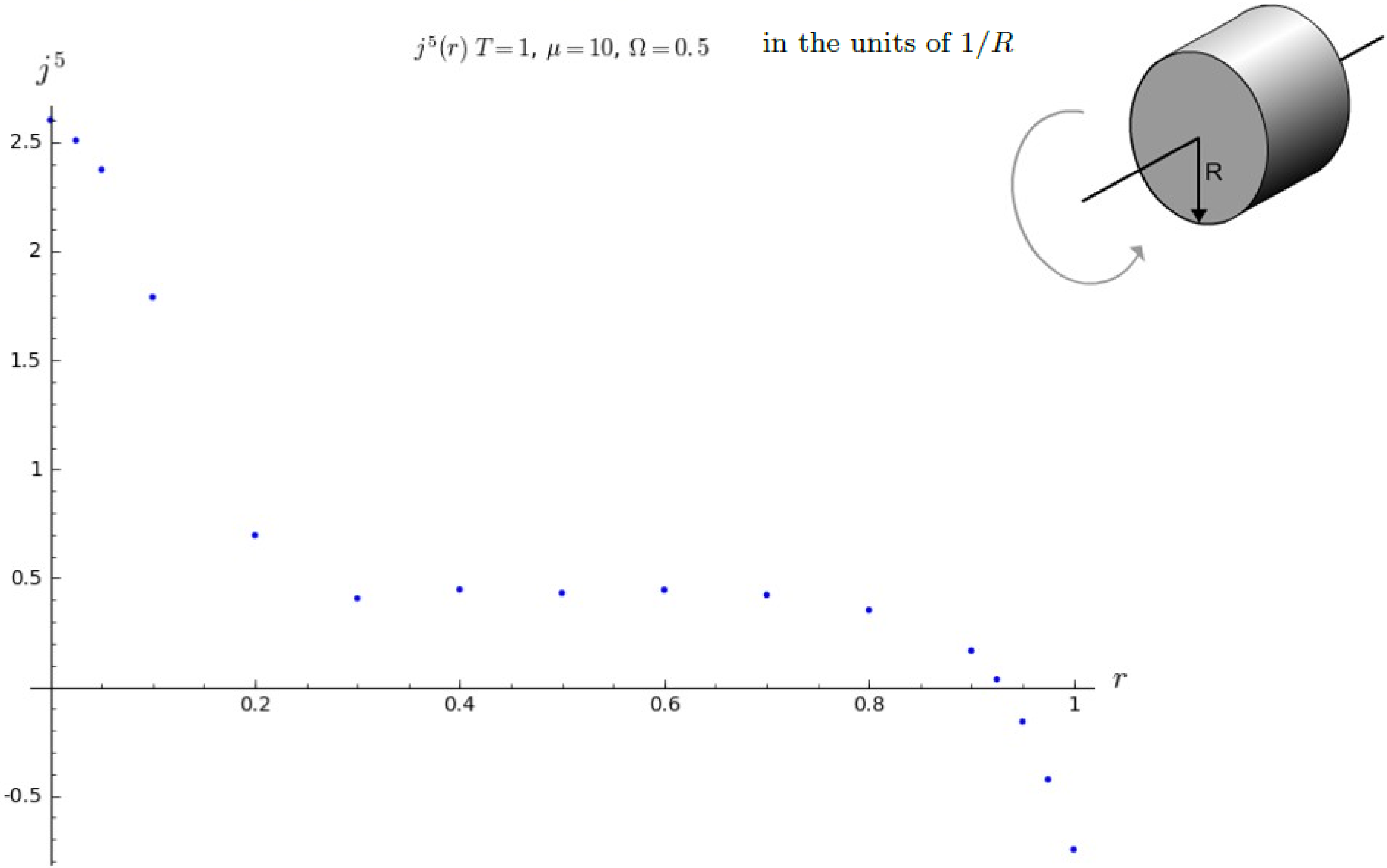



Axial current on the rotation axis is given by the standard expression

$$(T^2/6 + \mu^2/(2\pi^2))\Omega$$




Axial current on the rotation axis is given by the standard expression




Axial current out of the rotation axis deviates essentially from the standard expression

# Conclusions

1. *The equilibrium chiral magnetic effect* **does not exist** *and cannot be observed in its classical form in the heavy ion collisions.*

2. The existence of the equilibrium chiral separation effect is confirmed. *It can be observed in the heavy – ion collisions via the observation of the asymmetry in outgoing the left – handed /right handed particles.*

3. The chiral vortical effect (for the axial current) also exists. *It also may be observed in the heavy ion collisions.* We predict the essential finite volume effects.

4. **The new anomalous transport effect – the rotational Hall effect is proposed.** *It may be observed as well in the heavy ion collisions through the asymmetry of the electric charge of the outgoing particles. The extra electric field is to be added to the experimental setup*