Chiralspin symmetry and its implications for QCD.

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Chiralspin symmetry, L.Ya.G., EPJA 51 (2015) 27; L.Ya.G., M. Pak,PRD, 92 (2015) 016001

The Dirac Lagrangian in the chiral limit

$$\mathcal{L} = i\bar{\Psi}\gamma_\mu \partial^\mu \Psi = i\bar{\Psi}_L \gamma_\mu \partial^\mu \Psi_L + i\bar{\Psi}_R \gamma_\mu \partial^\mu \Psi_R,$$

where

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi, \quad \psi_L = \frac{1}{2}(1-\gamma_5)\psi,$$

is chirally symmetric

$$SU(N_F)_L \times SU(N_F)_R \times U(1)_A \times U(1)_V.$$

Fermion charge (Lorentz-invariant)

$$Q = \int d^3x \bar{\Psi}(x)\gamma_0 \Psi(x) = \int d^3x \Psi^\dagger(x)\Psi(x)$$

is invariant with respect to any unitary transformation. So far known unitary transformations are those which leave the Dirac Lagrangian invariant:

$$SU(N_c), SU(N_F), U(N_F)_L \times U(N_F)_R$$

Chiralspin symmetry, L.Ya.G., EPJA (2015); L.Ya.G., M. Pak,PRD, (2015)

The $SU(2)_{CS}$ chiralspin transformations and generators:

$$\Psi \to \Psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right)\Psi$$

,

$$\Sigma^n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\},$$

$n = 1, 2, 3$. γ_k is any Hermitian Euclidean gamma-matrix:

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta^{ij}; \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4.$$

The $\mathfrak{su}(2)$ algebra $[\Sigma^a, \Sigma^b] = 2i\epsilon^{abc}\Sigma^c$ is satisfied with any $k = 1, 2, 3, 4$.

The free massless Dirac Lagrangian does not have this symmetry. However, it is a symmetry of the fermion charge

The fermion charge and continuity eq. have a larger symmetry than the Dirac eq.

Chiralspin symmetry, L.Ya.G., EPJA (2015); L.Ya.G., M. Pak,PRD, (2015)

The chiralspin transformation and generators can be presented in an equivalent form. With $k = 4$ they are

$$\Sigma^n = \{\mathbb{1} \otimes \sigma^1,\ \mathbb{1} \otimes \sigma^2,\ \mathbb{1} \otimes \sigma^3\}.$$

Here the Pauli matrices σ^i act in the space of spinors

$$\Psi = \begin{pmatrix} R \\ L \end{pmatrix}, \tag{1}$$

where R and L represent the upper and lower components of the right- and left-handed Dirac bispinors

The $SU(2)_{CS}$, $k = 4$ transformation can then be rewritten as

$$\Psi \to \Psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right)\Psi = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)\begin{pmatrix} R \\ L \end{pmatrix}. \tag{2}$$

A fundamental irreducible representation of $SU(2)_{CS}$ is two-dimensional and the $SU(2)_{CS}$ transformations mix the R and L components of fermions.

Chiralspin symmetry, L.Ya.G., EPJA (2015); L.Ya.G., M. Pak,PRD, (2015)

An extension of the direct $SU(2)_{CS} \times SU(N_F)$ product leads to a $SU(2N_F)$ group. This group contains the chiral symmetry of QCD $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup.

Its transformations are given by

$$\Psi \rightarrow \Psi' = \exp\left(i\frac{\epsilon^m T^m}{2}\right)\Psi,$$

where $m = 1, 2, ..., (2N_F)^2 - 1$ and the set of $(2N_F)^2 - 1$ generators is

$$\{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^n), (\tau^a \otimes \Sigma^n)\} \tag{3}$$

whith τ being the flavour generators with the flavour index a and $n = 1, 2, 3$ is the $SU(2)_{CS}$ index.

$SU(2N_F)$ is also a symmetry of the fermion charge, while not a symmetry of the Dirac eq.

Symmetries of the QCD action

Interaction of quarks with the gluon field in Minkowski space-time:

$$\overline{\Psi}\gamma^{\mu} D_{\mu}\Psi = \overline{\Psi}\gamma^{0} D_0\Psi + \overline{\Psi}\gamma^{i} D_i\Psi.$$

The first (temporal) term includes an interaction of the color-octet quark charge density

$$\bar{\Psi}(x)\gamma^{0}\frac{t^c}{2}\Psi(x) = \Psi(x)^{\dagger}\frac{t^c}{2}\Psi(x)$$

with the chromo-electric part of the gluonic field. It is invariant under $SU(2)_{CS}$ and $SU(2N_F)$. The spatial part contains a quark kinetic term and interaction with the chromo-magnetic field. It breaks $SU(2)_{CS}$ and $SU(2N_F)$.
The quark chemical potential term $\mu\Psi(x)^{\dagger}\Psi(x)$ in the QCD action

$$S = \int_0^{\beta} d\tau \int d^3x\overline{\Psi}[\gamma_{\mu} D_{\mu} + \mu\gamma_4 + m]\Psi,$$

is $SU(2)_{CS}$ and $SU(2N_F)$ invariant.

Low mode truncation

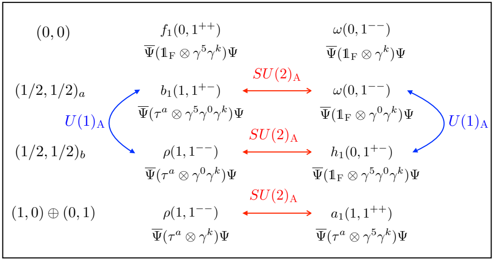
Banks-Casher:

$$< \bar{q}q >= -\pi\rho(0).$$

What we do:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

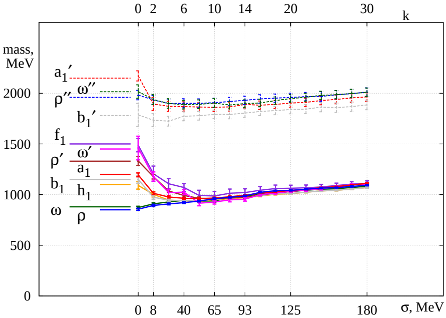
Chiral symmetry expectations for $J = 1$ mesons:



M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

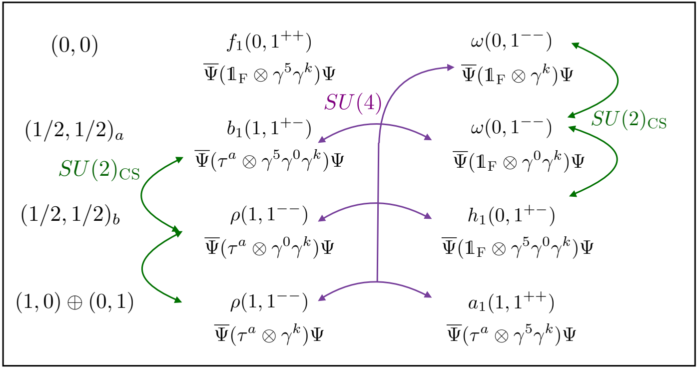
$$J = 1$$

We use $N_f = 2$ JLQCD overlap gauge configurations.



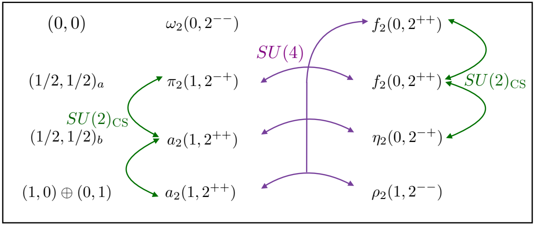
We clearly see a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian.

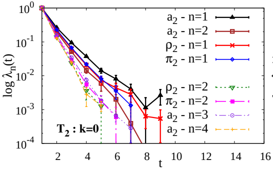
L.Ya.G., M. Pak, PRD 92(2015)016001

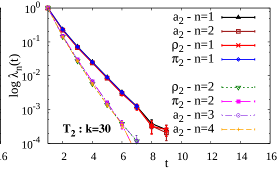


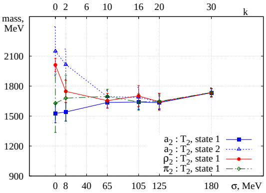
M. Denissenya, L.Ya.G, M.Pak, PRD 91(2015)114512

J=2 mesons.

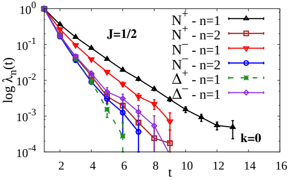


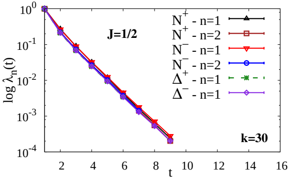


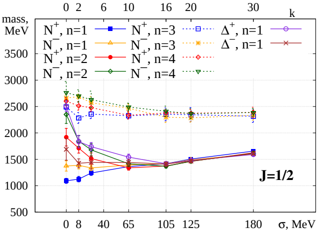




$J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, PRD 92 (2015) 074508







(Near) zero modes of Euclidean QCD and $SU(2)_{CS}$-$SU(4)$ breaking.

The Euclidean $SU(2)_{CS}$ generators:

$$\boldsymbol{\Sigma} = \{\gamma^k, -i\gamma^5\gamma^k, \gamma^5\} \ . \tag{4}$$

The $SU(2)_{CS}$ generators do not commute with the Dirac operator.

What is the intrinsic dynamical reason for $SU(2)_{CS}$ breaking: the zero modes of the Dirac operator?

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \tag{5}$$

The zero mode is chiral, L or R, depending on the topological charge $Q \neq 0$.
Atiyah-Singer:

$$Q = n_L - n_R$$

.

At $Q \neq 0$, there is an asymmetry between L and R.
Conclusion: The zero modes break explicitly $SU(2)_{CS}$ and $SU(2N_F)$.

Conclusions to part I and prediction for high T

Observed on the lattice $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of hadrons upon elimination of the near-zero modes are symmetries of confinement in QCD that is due to chromo-electric charge-charge interaction.

The chromo-magnetic interaction in QCD contributes only to the near-zero modes. It breaks explicitly both $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of confinement.

The hadron spectra observed in a real world can be viewed as a result of splitting of the primary energy levels with the $SU(4)$ symmetry by means of dynamics associated with the near-zero modes.

Above Tc $SU(2)_L \times SU(2)_R$ gets restored and the near-zero modes of the Dirac operator are suppressed. Then we expect emergence of $SU(2)_{CS}$ and $SU(4)$ - no deconfinement.

Spatial correlators at high T. Full QCD, no truncation.

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L.Ya.G., S. Hashimoto, C. B. Lang, S. Prelovsek, PRD 96 (2017) 094501

$N_f = 2$ QCD with the chirally symmetric Dirac operator.

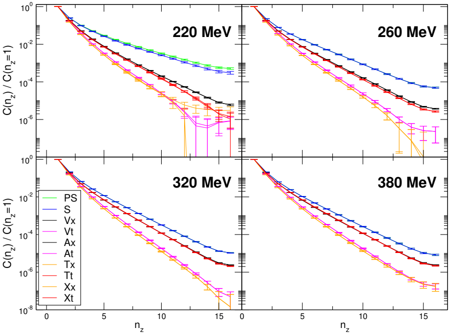
$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(\mathbf{0}, 0)^\dagger \rangle \,.$$

where $\mathcal{O}_\Gamma(x) = \bar{q}(x)\Gamma \frac{\vec{\tau}}{2} q(x)$ are all possible $J = 0$ and $J = 1$ local operators:

Name	Dirac structure	Abbreviation	
Pseudoscalar	γ_5	PS	] $U(1)_A$
Scalar	$\mathbb{1}$	S	
Axial-vector	$\gamma_k\gamma_5$	**A**	] $SU(2)_A$
Vector	γ_k	**V**	
Tensor-vector	$\gamma_k\gamma_3$	**T**	] $U(1)_A$
Axial-tensor-vector	$\gamma_k\gamma_3\gamma_5$	**X**	

Table : Operators considered in this work and their transformation properties. The open vector index k denotes the components 1, 2, 4, *i.e.* x, y, t.

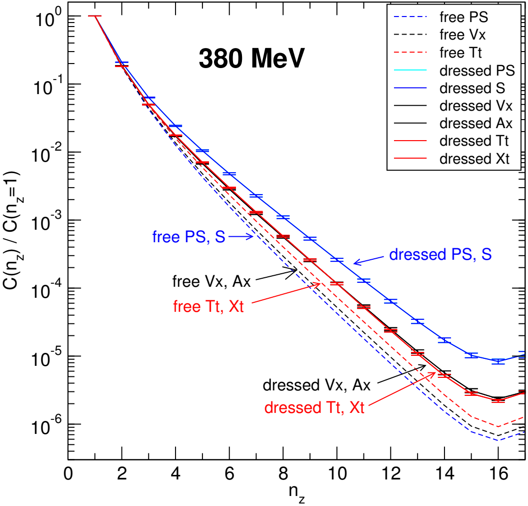
Spatial correlators at high T. Full QCD, no truncation.



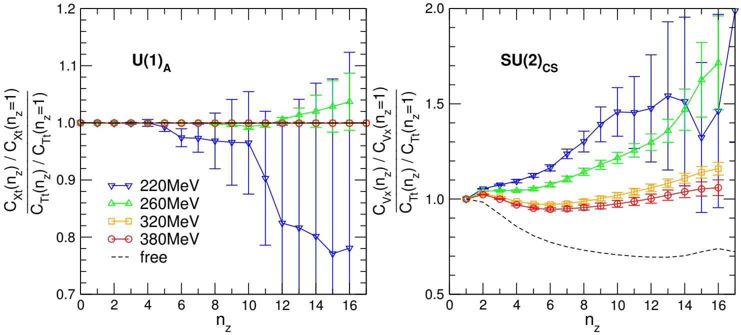
In total we observe three different multiplets:

$$E_1(U(1)_A): \quad PS \leftrightarrow S$$
$$E_2(SU(4)): \quad V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x$$
$$E_3(SU(4)): \quad V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t$$

Spatial correlators at high T. Full QCD, no truncation.



Spatial correlators at high T. Full QCD, no truncation.

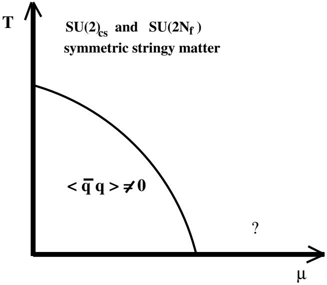


At T=380 MeV we observe approximate $SU(2)_{CS}$ and $SU(4)$ symmetries at the level of 5%.

Will a non-zero chemical potential break this symmetry?

$$S = \int_0^\beta d\tau \int d^3x \overline{\Psi}[\gamma_\mu D_\mu + \mu\gamma_4 + m]\Psi, \quad (6)$$

The quark chemical potential term $\mu\Psi^\dagger\Psi$ is invariant under $SU(2)_{CS}$ and $SU(2N_F)$.



Parity doublets and chiralspin symmetry, L.Ya.G., M. Catillo, arXiv:1804.0717

Parity doublets: B. W. Lee, Chiral dynamics, 1972

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}, \qquad (7)$$

$$\psi_R = \frac{1}{\sqrt{2}}(\Psi_+ + \Psi_-); \quad \psi_L = \frac{1}{\sqrt{2}}(\Psi_+ - \Psi_-), \qquad (8)$$

The $(0,1/2) \oplus (1/2,0)$ representation of $SU(2)_L \times SU(2)_R$ is

$$\Psi \rightarrow \exp\left(\imath \frac{\theta_V^a \tau^a}{2} \otimes \mathbb{1}\right) \Psi; \quad \Psi \rightarrow \exp\left(\imath \frac{\theta_A^a \tau^a}{2} \otimes \sigma_1\right) \Psi. \qquad (9)$$

$$\begin{aligned} \mathcal{L} &= i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - m\bar{\Psi}\Psi \\ &= i\bar{\Psi}_+\gamma^\mu \partial_\mu \Psi_+ + i\bar{\Psi}_-\gamma^\mu \partial_\mu \Psi_- - m\bar{\Psi}_+\Psi_+ - m\bar{\Psi}_-\Psi_- \qquad (10) \\ &= i\bar{\Psi}_L\gamma^\mu \partial_\mu \Psi_L + i\bar{\Psi}_R\gamma^\mu \partial_\mu \Psi_R - m\bar{\Psi}_L\Psi_L - m\bar{\Psi}_R\Psi_R \end{aligned}$$

Parity doublets and chiralspin symmetry, L.Ya.G., M. Catillo, arXiv:1804.0717

The parity doublet can be unitary transformed into a doublet

$$\tilde{\Psi} = \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix}. \quad (11)$$

$$\begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix} \to \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right) \begin{pmatrix} \Psi_R \\ \Psi_L \end{pmatrix}. \quad (12)$$

It then follows that the parity doublet Lagrangian is $SU(2)_{CS}$- and $SU(2N_F)$-invariant with the generators of $SU(2N_F)$ being

$$\{(\tau^a \otimes \mathbb{1}), (\mathbb{1} \otimes \sigma^n), (\tau^a \otimes \sigma^n)\}. \quad (13)$$

At high T baryon-like objects are parity doublets.
The chemical potential term in the QCD action is chiralspin symmetric and does NOT match with a single quark excitation. It matches with the parity doublet excitation. Condensations of parity doublets at large μ ?

Conclusions to part II

We observe emergence of approximate $SU(2)_{CS}$ and $SU(4)$ symmetries with increasing temperature.

Emergence of $SU(2)_{CS}$ and $SU(4)$ symmetries indicates that the chromo-magnetic interaction is suppressed while confining chromo-electric interaction is still active.

These symmetries are incompatible with the scenario of a plasma of asymptotically free, deconfined quarks and gluons.

Elementary objects: chiral quarks connected by chromo-electric field. Strings?