On Quantum Fields at High Temperature

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A $C^\star$-algebra analysis (*History*)

From $C^\star$-algebras (N.P. Landsman 1988) The necessity of doubling the number of degrees of freedom as soon as $T, \mu > 0$, avoiding ill-defined $\left[\delta(P^2 - m^2)\right]^N$ of perturbative expansions based on previous 1974 Dolan-Jackiw real-time propagator,

$$D(P) = \frac{i}{P^2 - m^2 + i\varepsilon} + 2\pi n_B(p_0)\,\delta(P^2 - m^2)$$

with,

$$n_B(p_0) = \frac{1}{e^{|p_0|/T} - 1}$$

However, nothing in the Hilbert space representation obtained through the *GNS* construction allows one to define any reliable Perturbation Theory .. in the standard $T = 0$ sense!. Apart maybe from intractable exception(s)

On the same time, though .. (*History*)

Braaten-Pisarski/Frenkel-Taylor/Taylor-Wong constructed the *Resummation* Program of *Hard Thermal Loops*, *HTL*, the effective $P.T$. ruling the leading order thermal fluctuations at momentum scale eT (QED), gT (QCD).

Necessary because for $\mathcal{O}(k) = eT$ one-loop corrected field functions are on the same order of magnitude as bare ones, *e.g.*, ($K^2 = k_0^2 - k^2$),

$$g\varphi_6^3, \quad Re\Sigma_{RR}^{HTL}(k_0,k) \simeq C^{st}\,[\frac{g^2T^2}{k^2}]\,K^2\,\frac{k_0}{k}\ln\frac{k_0+k}{k_0+k} = \mathcal{O}(K^2)$$

Propagators must be 're-summed'. In R/A real-time formalism, for example,

$$^*D_{RR}(K) = \frac{i}{K^2 - \Sigma_{RR}^{HTL}(k_0,k) + i\varepsilon k_0}$$

A successful Resummation Program of 'Hard Thermal Loops' ..(*History*)

- 'Hard Thermal Loops' are 1-loop (order e^2T^2, g^2T^2) gauge invariant quantities .. even in QCD
- Solving the gluon damping rate puzzle.
- Obey Ward identities .. even in QCD

$HTL - QCD$ effective actions are *Lie-algebra valuations* of $HTL - QED$ effective actions : $A_\mu \rightarrow \sum A_\mu^a T^a$. Obvious in fermionic sector. In bosonic sector, a lagrangian density like

$$\frac{{m_i}^2}{2}\int_0^{\infty} d\lambda \int_{-\infty}^{+\infty} \frac{d\sigma}{2\sqrt{\pi}}\, e^{\frac{-\sigma^2}{4}} \left\langle \widehat{K}^\alpha \widehat{K}^\beta Tr\left\{ \left[D_\mu, D_\alpha\right] e^{-\sigma\sqrt{\lambda}\,\widehat{K}\cdot D} \left[D^\mu, D_\beta\right] e^{+ig\sigma\sqrt{\lambda}\,\widehat{K}\cdot A} \right\} \right\rangle . \quad {m_\gamma}^2 = \frac{e^2T^2}{6}, {m_g}^2 = C_A\frac{g^2T^2}{6} + C_F\frac{g^2T}{12}$$

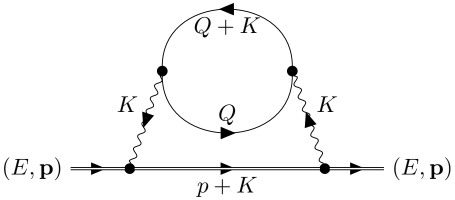
encompasses $\mathcal{L}_\gamma^{QED}$ and $\mathcal{L}_g^{QCD}$, differing only the Lie-algebra valuation of A_μ (*T.G., 2010*).

High-T Quantum Fields: *IR* problems .. or something else?

Besides a (previous) series of IR singularities which revealed to be erroneous, admittedly, the RP meets serious *IR* obstructions.

1 The *IR* logarithmic divergence of a rapid ($v \to 1$) massive fermion damping rate through a plasma by the fermion's mass shell at high T $(1 + n_B(k_0) \simeq \frac{T}{k_0} + \frac{1}{2} + ..)$

$$\gamma(E,p) \simeq \lim_{v=1} \frac{e^2 T}{2\pi} \int_{E|1-v|/2}^{k^\star} \frac{\mathrm{d}k}{k} + reg.$$



IR problems .. or something else?

2 The collinear singularity of the soft photon emission rate out of a quark-gluon plasma: $Im\,\Pi_R(Q)$ proportional to $(D = 4 + 2\varepsilon)$

$$\frac{C^{st}}{\varepsilon} \int \frac{d^4P}{(2\pi)^4}\, \delta(\widehat{Q}\cdot P)\, (1 - 2n_F(p_0)) \sum_{s=\pm 1,\, V=P,P'} \pi(1 - s\frac{v_0}{v})\beta_s(V)$$

However ..

.. while the latter does not even exist (*K. Bouakaz, T.G. 2013*), the former does not exist either, as being an ill-posed problem (see below).

IR problems or .. something else?

Under a disguise of *IR* problems, what shows up instead is a structural obstruction to perturbative attempts.

- Not perceived *as such* by the $C^\star$-algebra analysis ..
- .. as being revealed in a specific way through diagrammatic higher number of loop calculations

Proceeding with *History* ..

There is another (semi-classical) Resummation Program .. (terminal?)

In *QCD*, a similar effective $P.T.$ rules softer order $g^2 T$ thermal fluctuations. (*1998-99: Blaizot-Iancu/ Bödeker*). Enjoys the same properties as the *HTL*-Resummation Program:

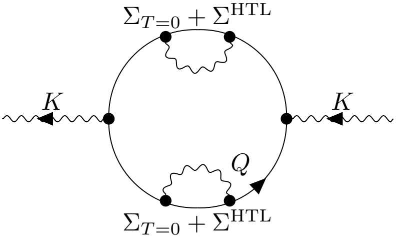
- For external momenta $k_0 \leq k \sim g^2 T$ ultra-soft amplitudes are as large as the corresponding *HTL* and tree-level ones.
- They are gauge fixing independent (covariant and Coulomb-like gauges..)
- Satisfy simple Ward identities

DIFFERENCES ARE ..

- No abelian counterpart (QED has no *HTL*-proper-vertices other than N-photon-2-electrons)
- *Utra-soft* amplitudes receive contributions from an infinite series of multi-loop diagrams (*ladder diagrams*)

Back to *diagrammatics* ..(no more semi-classical guides)

3 loop corrections to the photon polarization tensor $\Pi(K)$ do not seem to define any *invariant* fluctuation at a momentum scale other than the *ultrasoft* $e^2 T$ (*QED*), and $g^2 T$ (*QCD*) scales ..



At $k/T << 1$, leading order terms are of form,

$$\delta\Pi_L^{(3)}(k_0, k) \simeq K^2 \, [\frac{e^2 T}{k}]^3 \, \delta F_L^{(3)}(k_0/k, k/T)$$

$$\mathcal{O}(\delta F_L^{(3)}) = \mathcal{O}(Q_0(k_0/k), Q_1(k_0/k)) = \mathcal{O}(1)$$

Back to *diagrammatics* ..

The 1-loop renormalized (*i.e.* $T=0$) *e.m.* vertex (external lines on mass shell $Q^2=(Q+K)^2=m^2$) reads (*QED*),

$$\Gamma^{\mu}_{T=0}(K)=\gamma^{\mu}\,F_1^{ren.}(K^2)+\frac{i}{2m}\,\sigma^{\mu\nu}k_{\nu}\,F_2^{ren.}(K^2)$$

$F_1^{ren.}$ (and $F_2^{ren.}$) is (are) gauge-invariant at any order of *P.T.*

At $\sinh^2(\theta/2)=-K^2/4m_e^2$, and $C^{st.}=1+\ln\frac{\mu}{m_e}$

$$F_1(K^2)=-\frac{\alpha}{\pi}\left[C^{st}(1-\theta\coth\theta)+2\coth\theta\int_0^{\theta/2}\mathrm{d}\varphi\,\varphi\tanh\varphi+\frac{\theta}{4}\tanh\frac{\theta}{2}\right]$$

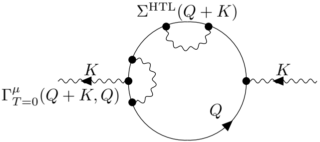
whose small θ-limit reads

$$F_1(K^2)\simeq\frac{\alpha}{3\pi}\frac{K^2}{m_e^2}\left(-\frac{3}{8}\right).$$

F_2: No role here.

.. so that ..

The 3-loop fluctuation of the internal photonic K-line,



is *invariant*

$$K^{\mu}\,\Pi^{(3;ren.)}_{\mu\nu}(k_0,k)=0$$

and yields,

$$\delta\Pi^{(3;ren.)}_{L}(k_0,k)=-K^2\,\frac{C^{norm.}}{24\pi^2}\left[\frac{e^3\,T}{k}\right]^2\delta\mathcal{F}^{(3;ren.)}_{L}(k_0,k)\,,$$

$T=0$- renormalized, $\delta\Pi^{(3;ren.)}_{L}(k_0,k)$ identifies $e^3\,T$ a new momentum scale of gauge invariant *vacuum and statistical mixed* fluctuations to be re-summed along the K-line.

L-peculiarities preserved by T-contributions

At $k/T << 1$, transverse degrees yield $\delta\Pi_T^{(3;ren.)}(k_0,k)$ a leading order part

$$-\frac{\left[e^3 T\right]^2}{24\pi} C^{norm.} \left[\frac{T}{k}\right]^2 \left(\int_0^\infty \frac{\mathrm{d}x}{x} \int_0^x \mathrm{d}y\, y \tanh\frac{y}{2}\right)^{ren.}$$

leading to

$$\mathcal{O}\left({}^{(3)}\rho_T(k_0,k)\right) = \mathcal{O}\left(\frac{k_0^2 - k^2}{T^2}\right) \times \mathcal{O}\left({}^{(3)}\rho_L(k_0,k)\right)$$

T-degrees preserve the L-peculiarity and need no re-summation

As a result for $n = 1, 2, 3, \ldots$

$$\delta_n \gamma(E,p) \simeq \frac{e^2 T}{2\pi} \int_{e^{[n+\frac{1}{2}]}T}^{e^{[n-\frac{1}{2}]}T} k\mathrm{d}k \int_{-k}^{+k} \frac{\mathrm{d}k_0}{k_0} \left\{ {}^{(n)}\rho_L(k_0,k) + {}^{(n)} Transv. \right\}$$

With

$$^{(n)}\rho_L(k_0,k) = \frac{1}{k^2 + C^{(n;ren.)} \left[e^n T \right]^2 \delta\mathcal{F}_L^{(n;ren.)}(k_0,k)}$$

HTL-case at $n = 1$

$$^{(1)}\rho_L \equiv {}^{\star}\rho_L\,, \quad \delta\mathcal{F}_L^{(1)}(k_0,k) = Q_1(k_0,k)\,, \quad C^{(1)} = \frac{1}{3}$$

As a result for $n = 1, 2, 3, ..$

$$\delta_n \gamma(E,p) = \frac{e^2 T}{2\pi} \int_{e^{[n+\frac{1}{2}]}T}^{e^{[n-\frac{1}{2}]}T} k\,\mathrm{d}k \int_{-k}^{+k} \frac{\mathrm{d}k_0}{k_0} \{^{(n)}\rho_L(k_0,k) + {}^{(n)}Transv.\}$$

and

$$\delta_n \gamma(E,p) = \frac{e^2 T}{2\pi} \mathcal{O}_n(1)\,, \quad n = 1, 2, 3, ..$$

Whatever the somewhat arbitrarily defined ranges

$$e^{[n+\frac{1}{2}]}T \leq k \leq e^{[n-\frac{1}{2}]}T$$

Therefore

- The ultra-soft fluctuations at momentum scale $e^2 T$ are not *terminal*. Softer fluctuations, such as $e^3 T$, emerge out of specific higher number of loop diagrams.
- By construction these mixed fluctuations are gauge invariant.
- They contribute the zeroth order approximation to $\gamma(E,p)$ .. let aside sub-leading ones
- The old *IR* divergence plaguing the rapid fermion damping rate by the mass shell was clearly an ill-posed problem

$$\gamma(E,p) \simeq \lim_{v=1} \frac{e^2 T}{2\pi} \int_{E|1-v|/2}^{k^\star} \frac{\mathrm{d}k}{k}, \qquad k^\star = e^{[1-\frac{1}{2}]} T$$

The $HTL - P.T.$ shouldn't have been extended down to 0 but only to some $k_{min} = e^{[1+\frac{1}{2}]} T$

Moreover

- Besides a number of *invariant fluctuations* to be controlled in order to get the leading part only of a given observable,

- the very principle of their separation, *i.e.*, the clear-cut separation of momentum scales T, $g(T)T$, $g^2(T)T$, $g^3(T)T$, is deprived of any physical realization even at very high T.

  Checked up to $10^{25-30}\,T_c$ in pure *YM* (*O. Akerlund and P. de Forcrand, 2013*. A condition necessary to a consistent implementation of the renormalization group *à la* Wilson fails to be met, *J.P. Blaizot, 1999*)

Proposal

The $C^\star$-algebraic analysis is pertinent :

@ high T Quantum Fields call for non-perturbative methods.

- What Lattices have long been doing .. their ways ..

- What has been undertaken partly (*Blaizot and Iancu 1998, H.M. Fried, T.G., Y.-M. Sheu, 2005, 2008*) by means of functional calculations of 2-pt field functions

Results exhibit a rich variety of possible behaviors in terms of couplings, masses, Temperature and external momentum .. as can be guessed on the bases of perturbative attempts !

To wit

$$S'(\omega, z_0) = i\frac{2m}{\omega}\left\{\frac{1}{2}\,e^{-i\omega z_0 - \frac{A^2}{4\omega^2}z_0^2}\right.$$
$$\left. - e^{-\frac{\omega}{T} - \frac{A^2}{4\omega^2}(z_0^2 - \frac{1}{T^2})}\cos\left(\left[\omega - 2T(\frac{A}{2\omega T})^2\right] z_0\right)\right\}$$

where,

$$A^2 = \frac{4\alpha}{3\pi}(\vec{p}^{\,2})^2\left(1 + (\frac{2\pi T}{p})^2\right) - \frac{4\alpha^2}{3\pi}(\vec{p}^{\,2})^2 \ln\left(\frac{\vec{p}^{\,2}}{m^2}\right)$$

for a *BN*- approximated propagator of an energetic fermion entering a thermalized *QED* plasma in thermal equilibrium @T.

$$z_0 = x_0 - y_0,\ \vec{p} = \vec{p}(z_0) = \vec{p}(0)\,e^{-\Gamma z_0},\ \Gamma = \frac{2}{\sqrt{3\pi}}\frac{\alpha c}{\lambda_c}\left(\frac{k_B T}{mc^2}\right)^2$$

H.M. Fried, T.G., Y.-M. Sheu, 2008.

The End

Expressions ..

$$\delta\mathcal{F}_L^{(3;ren.)}(k_0,k) = \frac{1}{\pi}\left[\frac{k_0}{k}Q_0(\frac{k_0}{k}) + Q_1(\frac{k_o}{k})\right]\left(\int_0^\infty \frac{\mathrm{d}x}{x}\frac{\mathrm{d}}{\mathrm{d}x}\, x\tanh\frac{x}{2}\right)^{ren.}$$
$$-\frac{1}{4\pi^2}Q_1(\frac{k_o}{k})\left(\int_0^\infty \frac{\mathrm{d}x}{x}\tanh x\right)^{ren.}$$
$$+\frac{1}{8\pi^2}\left[\frac{k}{k_0}\int_0^\infty \mathrm{d}x\int_0^x \frac{\mathrm{d}x_0}{x_0-x}\left[\frac{\tanh\frac{x_0}{2}}{x_0}\ln\frac{k_0x_0+kx}{k_0x_0-kx} - \frac{\tanh\frac{x}{2}}{x}\ln\frac{k_0+k}{k_0-k}\right]\right]^{ren.}$$

$$C^{(3;ren.)} = \frac{C^{norm.}}{24\pi^2}\left[-\frac{1}{\pi}\left(\int_0^\infty \frac{\mathrm{d}x}{x}\frac{\mathrm{d}}{\mathrm{d}x}\, x\tanh\frac{x}{2}\right)^{ren.}\right.$$
$$\left. +\frac{1}{4\pi^2}\left(\int_0^\infty \mathrm{d}x\int_0^x \mathrm{d}x_0\,\frac{\tanh\frac{x_0}{2} - \tanh\frac{x}{2}}{x_0-x}\right)^{ren.}\right]$$

Expressions ..

$$\mathcal{O}_n(1) = \frac{1}{2}\ln C^{(n;ren.)} + \frac{1}{2}\ln\frac{1}{e} + \mathcal{O}(\sqrt{\alpha})$$

Or,

$$\mathcal{O}_3(1) = \frac{1}{1 - C^{(3)}[e^3 T/m_e]^2}\ln\frac{1}{e}$$

if

$$F_1(K^2) \simeq \frac{\alpha}{3\pi}\frac{K^2}{m_e^2}(-\frac{3}{8})$$

is used.