

The Longitudinal Gluon Propagator in Lattice $SU(2)$ Gluodynamics at the Criticality

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- ▶ Phase transitions for pedestrains
- ▶ Phase transition in $SU(2)$
- ▶ Correlation between the **asymmetry** and the **Polyakov loop**
- ▶ Correlation between the **longitudinal propagator** and the **Polyakov loop**
- ▶ Regression analysis
- ▶ Evaluation of the critical exponents and amplitudes
- ▶ Conditional distributions of the longitudinal propagator
- ▶ Conclusions

Our main result:

$$D_L = C \cdot (T - T_c)^{0.326419(3)} \cdot (1 + \alpha(T))$$

- ▶ D_L - zero-momentum gluon propagator
- ▶ we consider $SU(2)$ lattice gauge theory in the Landau gauge
- ▶ $T_c \approx 297$ MeV
- ▶ $\lim_{T \rightarrow T_c} \alpha(T) = 0$

Ising model

$$\sigma_n = \pm 1$$

Finite-volume lattice: $\vec{n} = (n_1, \dots, n_D)$, $1 \leq n_\mu \leq L$, $n_\mu \in \mathbf{N}$

Infinite-volume lattice: $\vec{n} \in \mathbf{Z}^D$,

$$H = -J \sum_{|\vec{i}-\vec{j}|=1} \sigma_{\vec{i}} \sigma_{\vec{j}} - h \sum_{\vec{i} \in \mathbf{Z}^D} \sigma_{\vec{i}}$$

$$Z = \sum_{\sigma_n} e^{-H[\sigma]/T} = \exp\left(-\frac{F}{T}\right)$$

$$m = \frac{\partial F}{\partial h}, \quad S = -\frac{\partial F}{\partial T}, \quad \chi = \frac{\partial^2 F}{\partial h^2}, \quad c = -T \frac{\partial^2 F}{\partial T^2},$$

Phase transition: singular behavior of $F(T)$ at $T = T_c$

Critical exponents

$$\tau = \frac{T - T_c}{T_c}$$

$$m|_{h=0} \simeq C_\beta (-\tau)^\beta \quad (\tau < 0)$$

$$|m|_{\tau=0} \simeq C_\delta |h|^{1/\delta} \quad (\tau = 0)$$

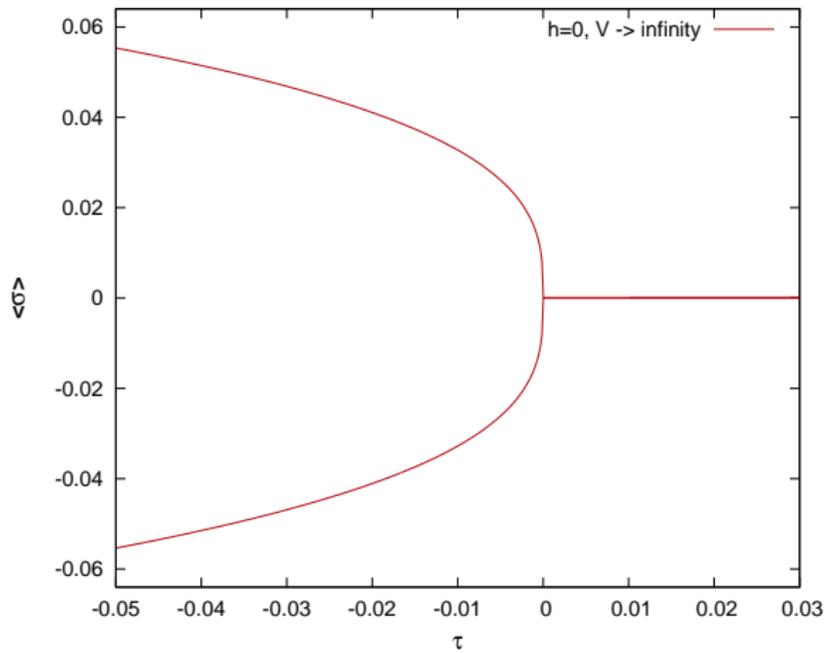
$$\chi = \frac{\partial m}{\partial h} \simeq C_\gamma |\tau|^{-\gamma}$$

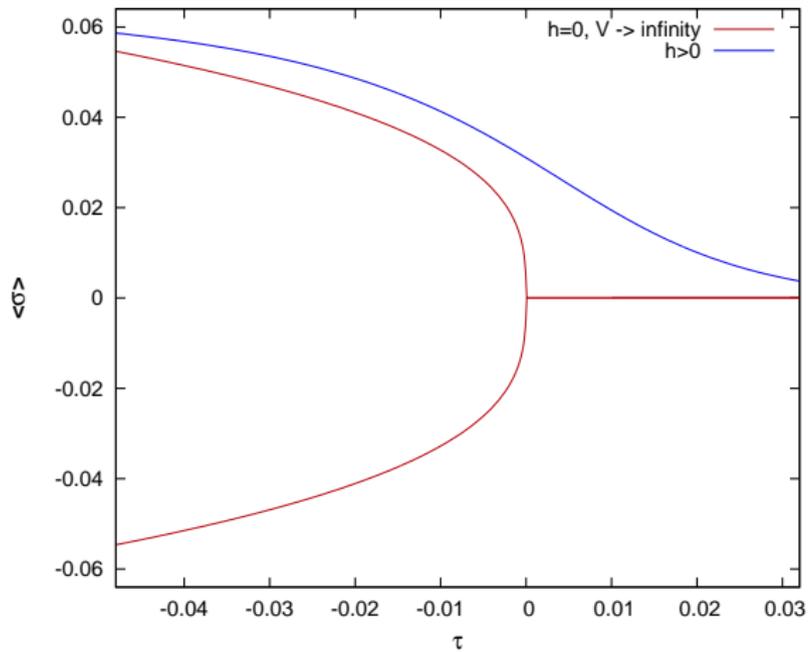
$$G(\vec{k})|_{\tau=0} = a^D \sum_{\vec{n} \in \mathbb{Z}^D} \langle \sigma_{a\vec{n}} \sigma_{\vec{0}} \rangle e^{i a \vec{n} \vec{k}} \simeq \frac{C_\eta}{|\vec{k}|^{2-\eta}}$$

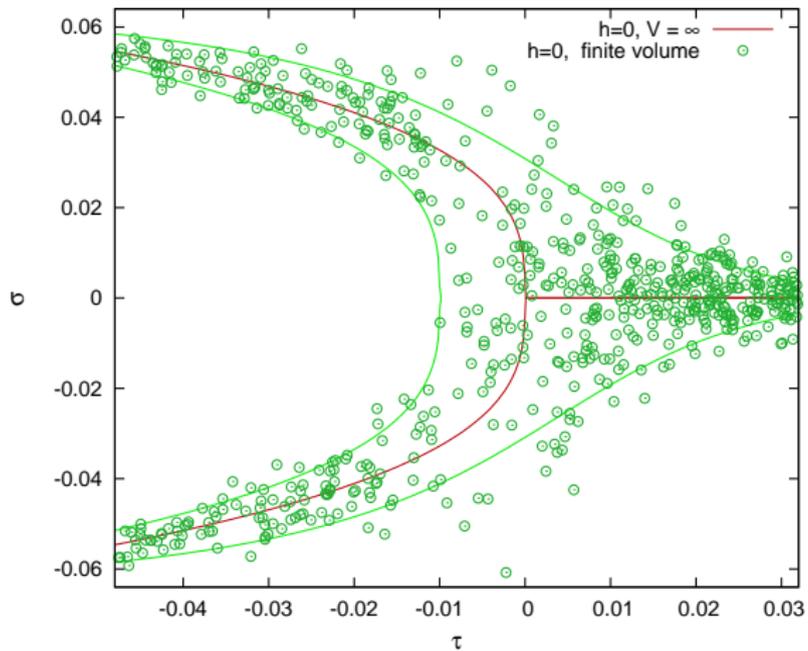
correlation length ξ : $\langle \sigma_{\vec{x}} \sigma_{\vec{0}} \rangle|_{\tau \neq 0} \sim e^{-|\vec{x}|/\xi}$

$$\xi \simeq C_\xi |\tau|^{-\nu}$$

heat capacity : $C_{h=0} \simeq C_C |\tau|^{-\alpha}$







We consider

Ising model \longrightarrow $SU(2)$ lattice gauge theory
magnetization \longrightarrow Polyakov loop

- ▶ Chromoelectric-chromomagnetic asymmetry
- ▶ Zero-momentum longitudinal gluon propagator

Lattices: $(\vec{x}, x_4) \in \Lambda(N_t \times N_s^3), \quad N_t = 8, \quad 32 \leq N_s \leq 88$

$$L(\vec{x}) = \frac{1}{2} \text{Tr} \prod_{x_4=1}^{N_t} U(\vec{x}, x_4; \mu = 4)$$

Polyakov loop: $\mathcal{P} = \frac{1}{N_s^3} \sum_{\vec{x}} L(\vec{x})$

$$\langle L(\vec{x}) L(\vec{0}) \rangle \simeq A \exp\left(-\frac{|\vec{x}|}{\xi}\right), \quad |\vec{x}| \rightarrow \infty$$

$$G(\vec{p}) = \frac{1}{N_s^3} \sum_{\vec{x}} \langle L(\vec{x}) L(\vec{0}) \rangle e^{i\vec{p}\vec{x}}$$

Critical exponents and amplitudes

$$\tau = \frac{T - T_c}{T_c}; \quad \tau > 0 \text{ -- deconfinement}$$

$$\langle \mathcal{P} \rangle \simeq B \tau^\beta$$

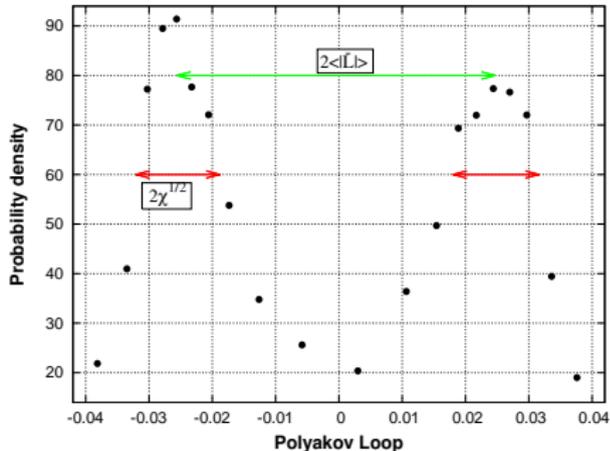
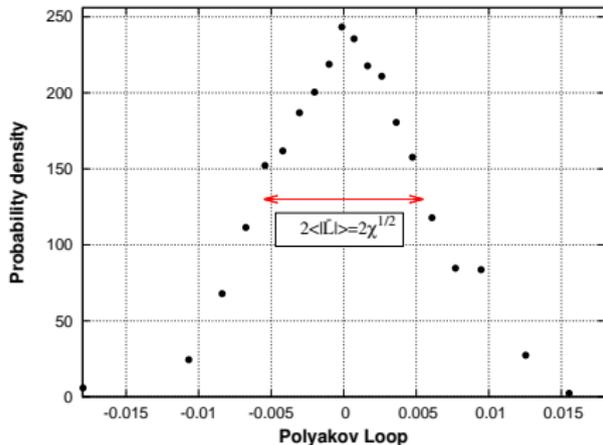
$$\xi \simeq \frac{f_\pm}{|\tau|^\nu}$$

$$\langle \mathcal{P}^2 \rangle - \langle \mathcal{P} \rangle^2 = G(\vec{0}) \simeq \frac{C_\pm}{N_s^3 |\tau|^\gamma}$$

$$\tau = 0 : \quad G(\vec{p}) \simeq \frac{H}{|\vec{p}|^{2-\eta}}$$

Universality hypothesis:

3D Ising model \Leftrightarrow SU(2) in 3+1



Binder 1981 – Ising model;
 Mitryuskin, Zadorozhny 1986 – Lattice $SU(2)$

Critical exponents

and amplitudes

3D Ising

F.Kos, D.Poland et al.
JHEP (2016)

$$\beta = 0.326419(3)$$

$$\gamma = 1.237075(10)$$

$$\eta = 0.036298(2)$$

$$\nu = 0.629971(4)$$

$SU(2)$, 4D

J.Engels, T.Schiedeler 1998

$$B = 0.825(1)$$

$$C_+ = 0.0587(8), C_- = 0.01243(12)$$

C_+/C_- is universal; for the 3D Ising universality class

$$C_+/C_- = 4.75(3) [1998]$$

Conformal bootstrap

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}}}$$

$$\nu = \frac{1}{3-2\Delta_{\epsilon}}, \quad \gamma = \frac{3-2\Delta_{\sigma}}{3-\Delta_{\epsilon}}$$

$$\langle A(x)B(y)C(z) \rangle = \frac{f_{ABC}}{|x-y|^{\Delta_A+\Delta_B-\Delta_C}|y-z|^{\Delta_B+\Delta_C-\Delta_A}|z-x|^{\Delta_C+\Delta_A-\Delta_B}}$$

$$\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4) \rangle = \sum_{\mathcal{O}} f_{\sigma\sigma\mathcal{O}}^2 C_{\Delta_{\mathcal{O}}\mathcal{O}}^{\Delta_{\sigma}}(x_1, x_2, x_3, x_4)$$

$$\sum_{\mathcal{O}} f_{\sigma\sigma\mathcal{O}}^2 \left(C_{\Delta_{\mathcal{O}}\mathcal{O}}^{\Delta_{\sigma}}(x_1, x_2, x_3, x_4) - C_{\Delta_{\mathcal{O}}\mathcal{O}}^{\Delta_{\sigma}}(x_3, x_2, x_1, x_4) \right) = 0$$

Poland, Rychkov et al., Nature 2016

The Chromo-Electric-Magnetic Asymmetry

$$\begin{aligned}\langle A_E^2 \rangle &= g^2 \langle A_4^a(x) A_4^a(x) \rangle, \\ \langle A_M^2 \rangle &= g^2 \langle A_i^a(x) A_i^a(x) \rangle.\end{aligned}\tag{1}$$

The quantity of particular interest is the (color) electric-magnetic asymmetry introduced by Chernodub and Ilgenfritz in 2008:

$$\langle \Delta_{A^2} \rangle \equiv \langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle.\tag{2}$$

Later we will use the dimensionless quantity

$$\Delta_{A^2} = \frac{\langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle}{T^2}.\tag{3}$$

We work in the Landau gauge $\partial_\mu A_\mu^a = 0$

Definition of the longitudinal (L) and transverse (T) propagators:

$$D_{\mu\nu}^{ab}(p) = \delta_{ab} \left(P_{\mu\nu}^T(p) D_T(p) + P_{\mu\nu}^L(p) D_L(p) \right),$$

where $P_{\mu\nu}^{T;L}(p)$ - orthogonal transverse (longitudinal) projectors

$$D_L(p) = \frac{1}{3} \sum_{a=1}^3 \langle A_0^a(p) A_0^a(-p) \rangle$$

$$D_T(p) = \begin{cases} \frac{1}{6} \sum_{a=1}^3 \sum_{i=1}^3 \langle A_i^a(p) A_i^a(-p) \rangle & p \neq 0 \\ \frac{1}{9} \sum_{a=1}^3 \sum_{i=1}^3 \langle A_i^a(p) A_i^a(-p) \rangle & p = 0 \end{cases}$$

We study critical behavior of the quantities

$$\mathcal{A} = \Delta_{A^2} - \Delta_{A^2}^C \quad (4)$$

and

$$\mathcal{D} = D_L(0) - D_L^C(0), \quad (5)$$

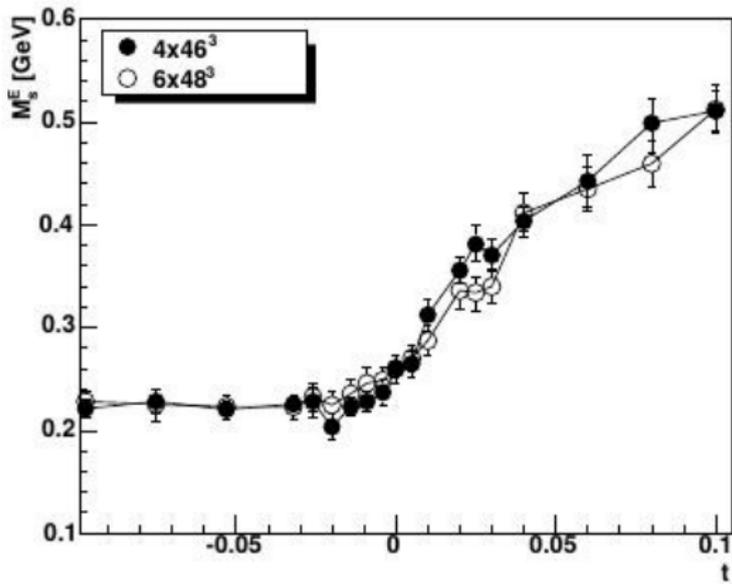
Asymptotic expansions of Δ_{A^2} and \mathcal{D}

in $\tau = \frac{T - T_c}{T_c}$ at $\tau \rightarrow 0_+$ have the form

$$\mathcal{A} \simeq B_A \tau^{\beta_A}, \quad (6)$$

$$\mathcal{D} \simeq B_D \tau^{\beta_D}, \quad (7)$$

We evaluate the critical exponents β_A and β_D and amplitudes B_A and B_D .



$$M^E = \frac{1}{\sqrt{D_L(0)}}$$

A.Maas, J.Pawlowski, L von Smekal, D.Spielmann, 2011

A.Maas *et al.*, 2011 (6×48^3):

$$M_E(\tau) = m_{gribov} + \theta(\tau)\mathcal{M}_+\tau^{\gamma_+/2} + \theta(-\tau)\mathcal{M}_-\tau^{\gamma_-/2} \quad (8)$$

$$m_{gribov} = 0.25_{-2}^{+3}; \quad \mathcal{M}_+ = 1.5_{-3}^{+1}, \quad \mathcal{M}_- = -0.07_{-17}^{+736}; \quad (9)$$

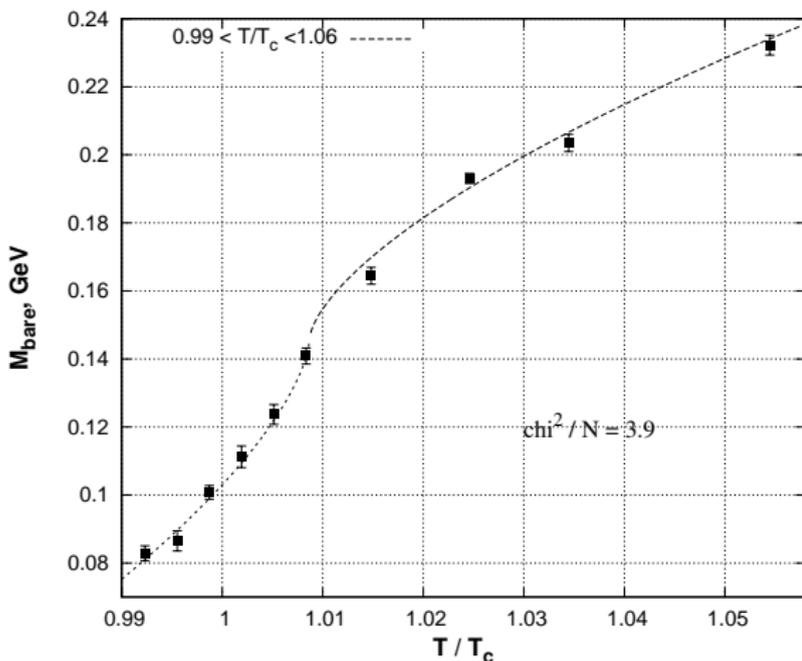
$$\gamma_+ = 1.54_{0.05}^{-12}, \quad \gamma_- = 0.6_{+5}^{+45}$$

Our **previous** result 2015,

($N_t = 8$, extrapolation to the infinite-volume limit):

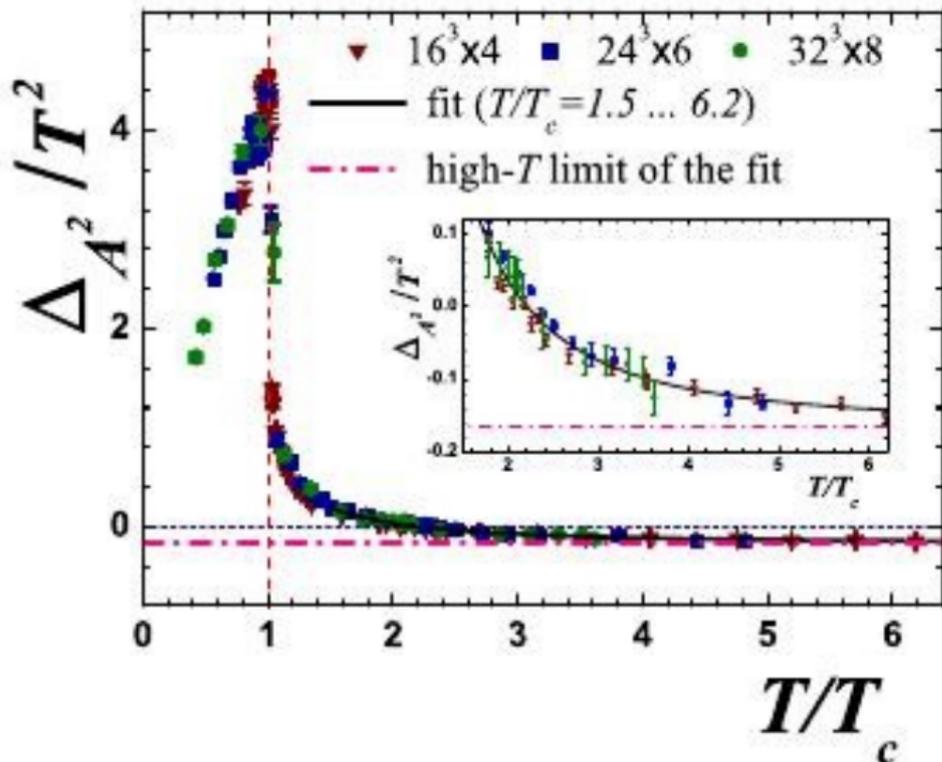
$$m_{gribov} = 0.217(3); \quad \mathcal{M}_+ = 0.93(11), \quad \mathcal{M}_- = -1.23(19); \quad (10)$$

$$\gamma_+ = \gamma_- = 0.63(3)$$

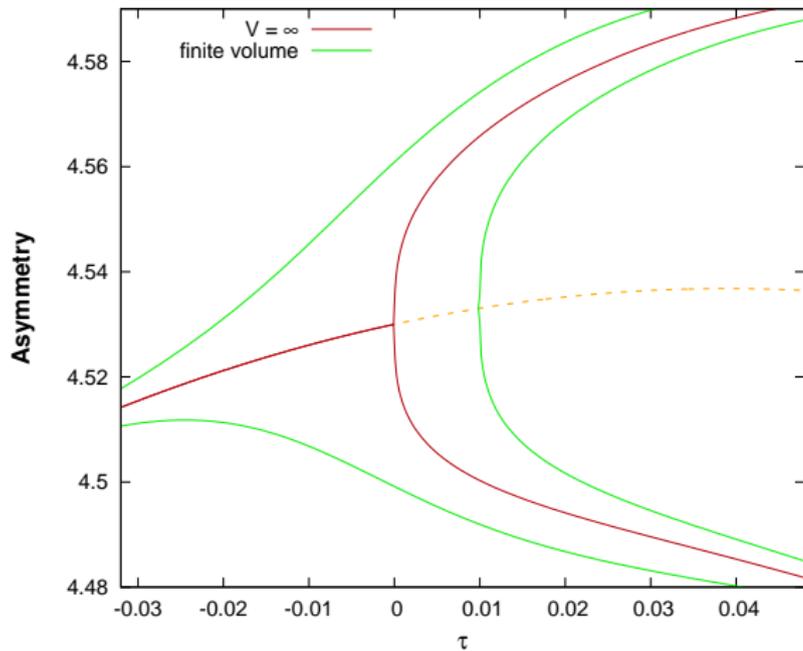


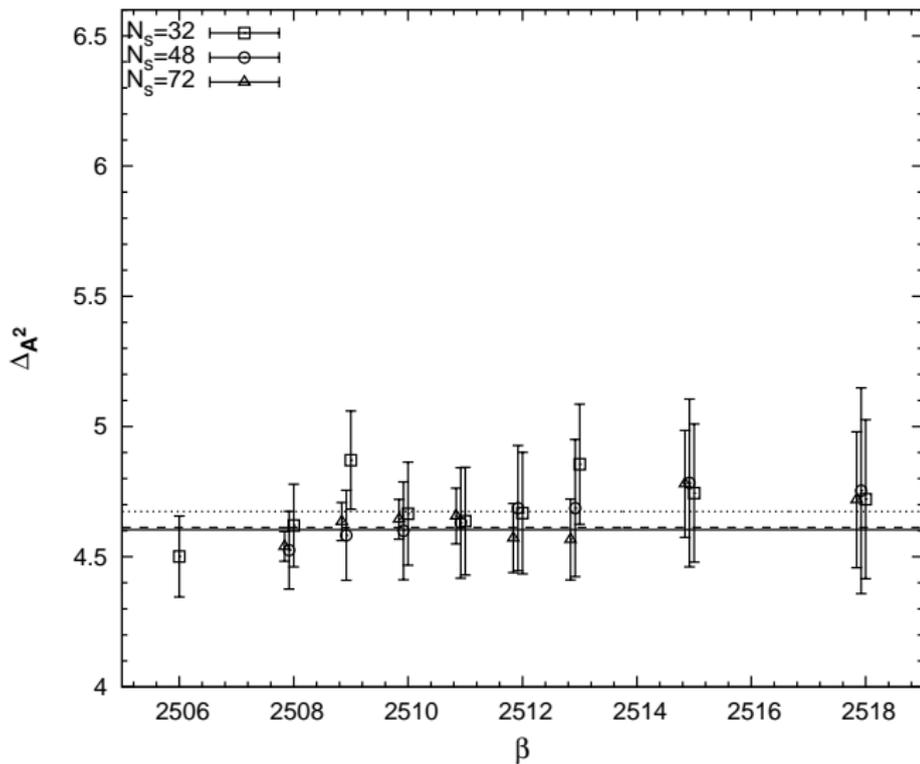
Unjustified assumptions:

- ▶ **Maas' et al.:** Correlator $\langle A_0(\vec{x})A_0(\vec{0}) \rangle$ is associated with the same critical exponent (γ) as that of Polyakov loops
- ▶ **Our :** Negative Polyakov-loop sector can be safely ignored

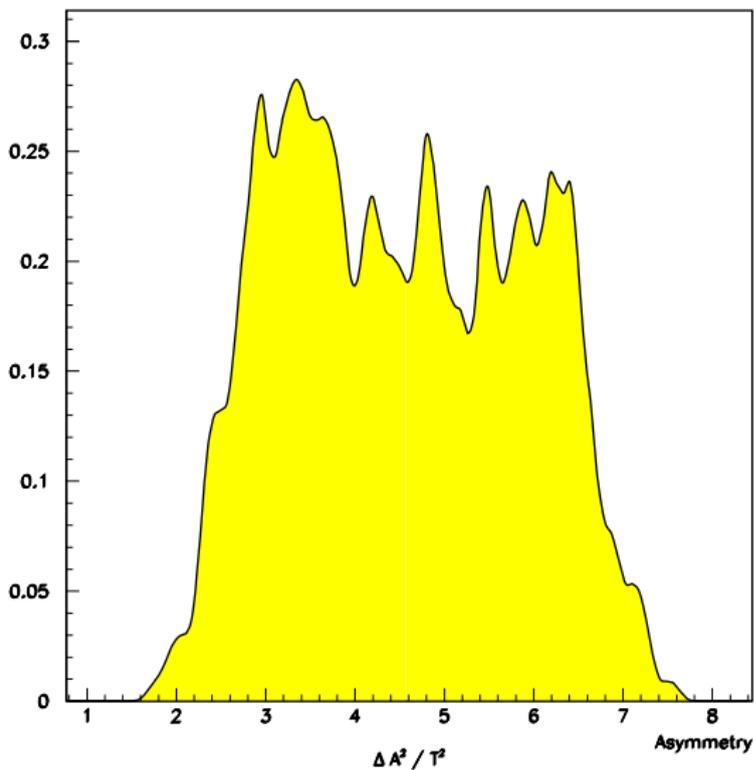


Chernodub, Ilgenfritz 2008

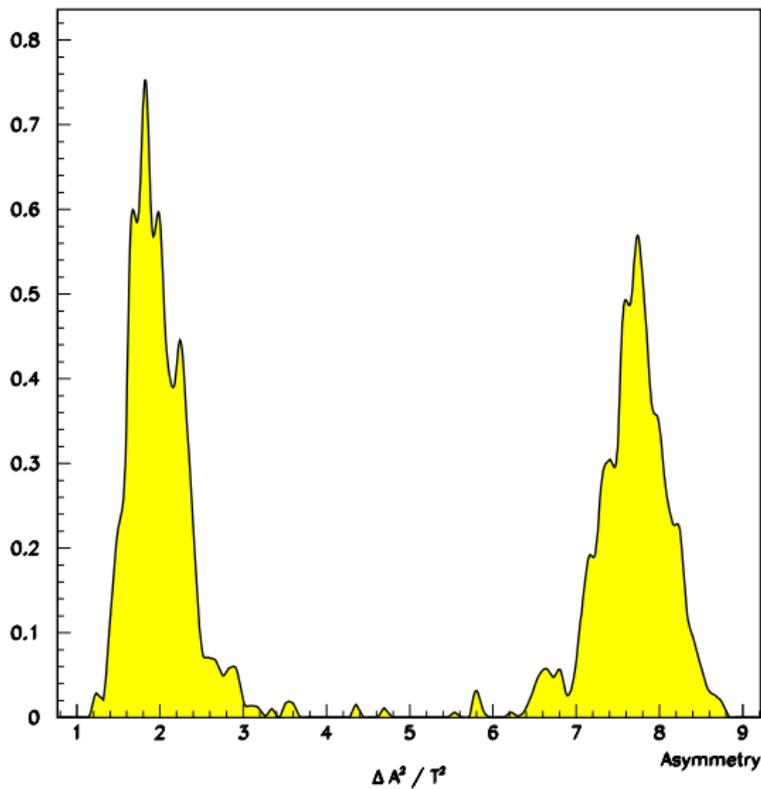




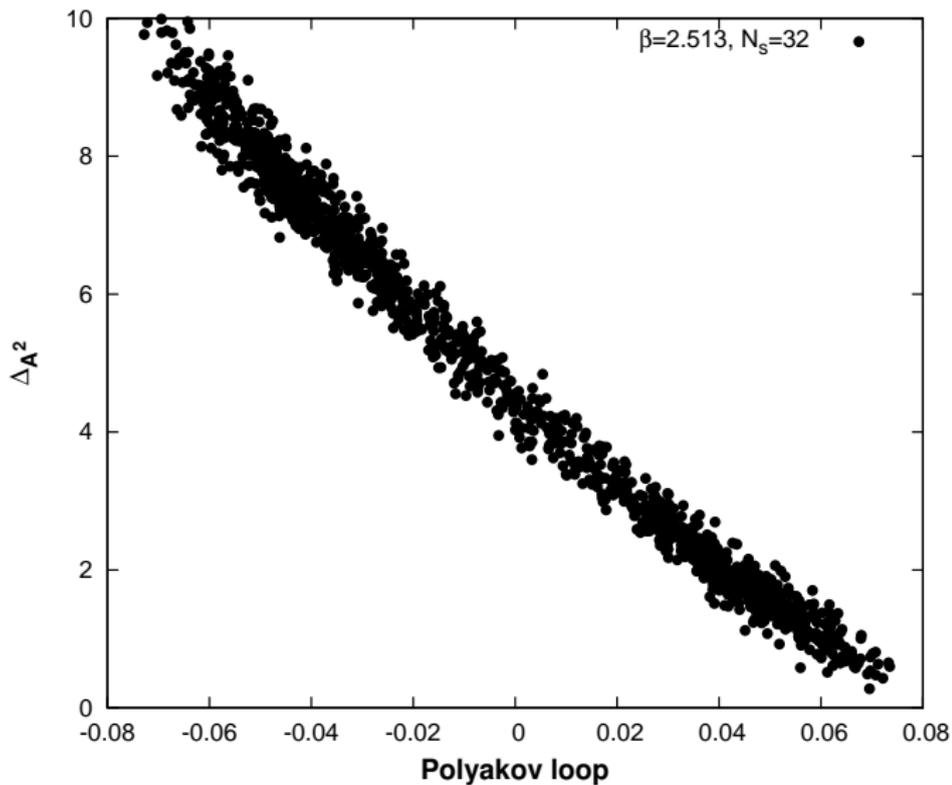
Average value of Δ_{A^2} versus T , $T_c \rightarrow \beta = 2.5104(2)$



$$T/T_c = 0.9925; L = 6.0 \text{ fm}; 72^3 \times 8$$

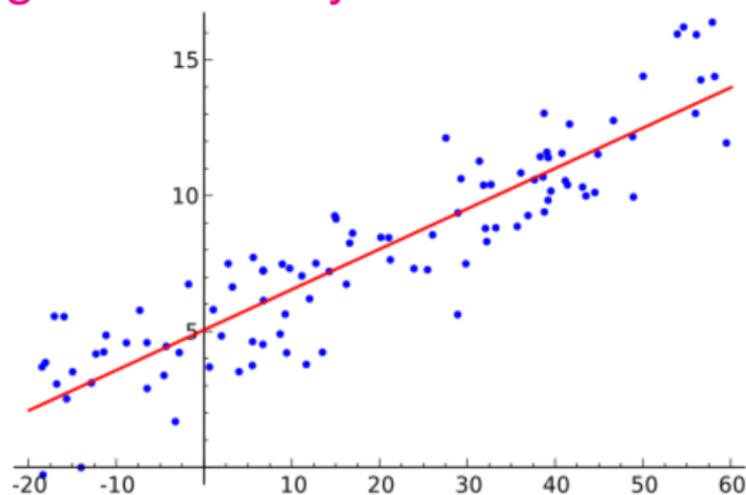


$T/T_c = 1.0024$; $L = 5.8$ fm; $72^3 \times 8$



Scatter plot: asymmetry versus Polyakov loop; $\tau = 0.008$, $L = 5.9$ fm

Regression analysis



Y (regressand)
depends on
X (regressor)

The problem: to find the conditional expectation value of Y

as a function of X:

$$E(Y|X) = f(X, \theta),$$

here $f(X, \theta) = \theta_0 + \theta_1 X$

Quantities under consideration

- ▶ The conditional cumulative distribution function (CDF)

$$F(\Delta|\mathcal{P}) \quad (\text{on this page } \Delta \equiv \Delta_{A^2})$$

describes the distribution of gauge-field configurations in the asymmetry for a fixed value \mathcal{P} of the Polyakov loop.

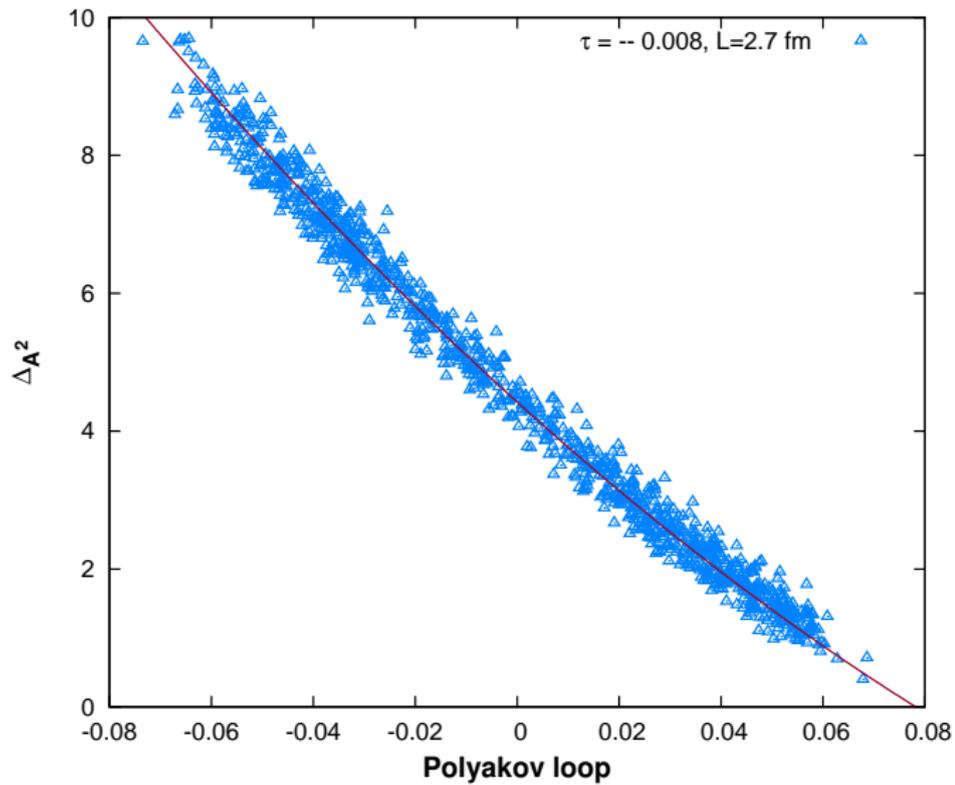
- ▶ The conditional expectation

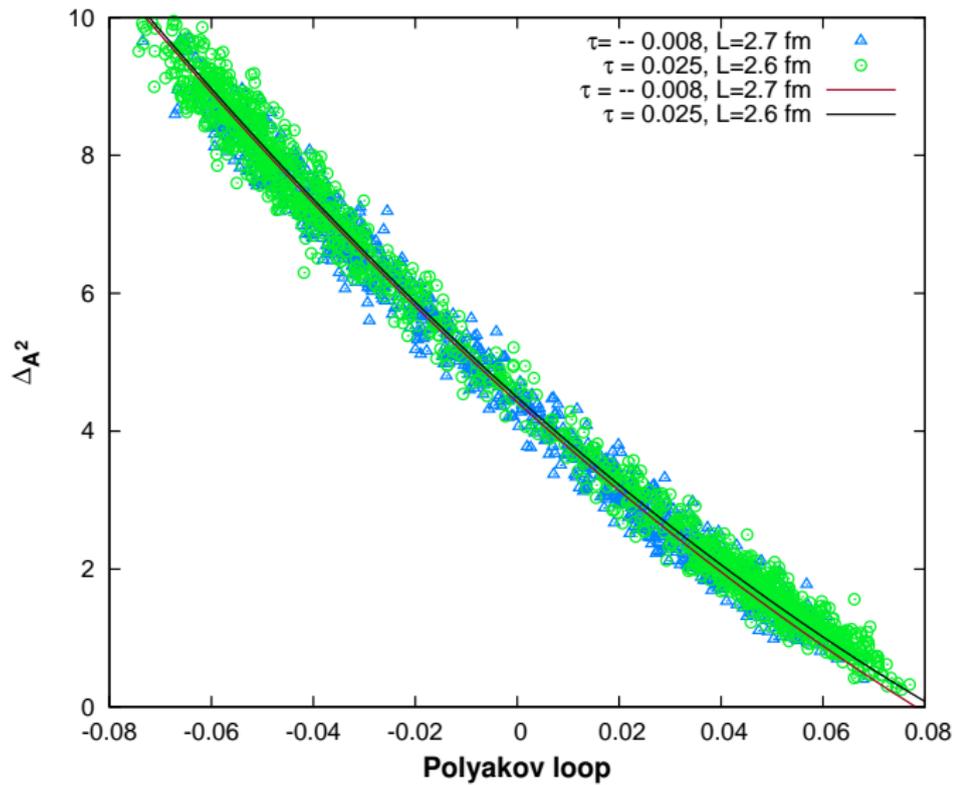
$$\langle \Delta \rangle_{\mathcal{P}} = E(\Delta|\mathcal{P}) = \int \frac{dF(\Delta|\mathcal{P})}{d\Delta} \Delta d\Delta.$$

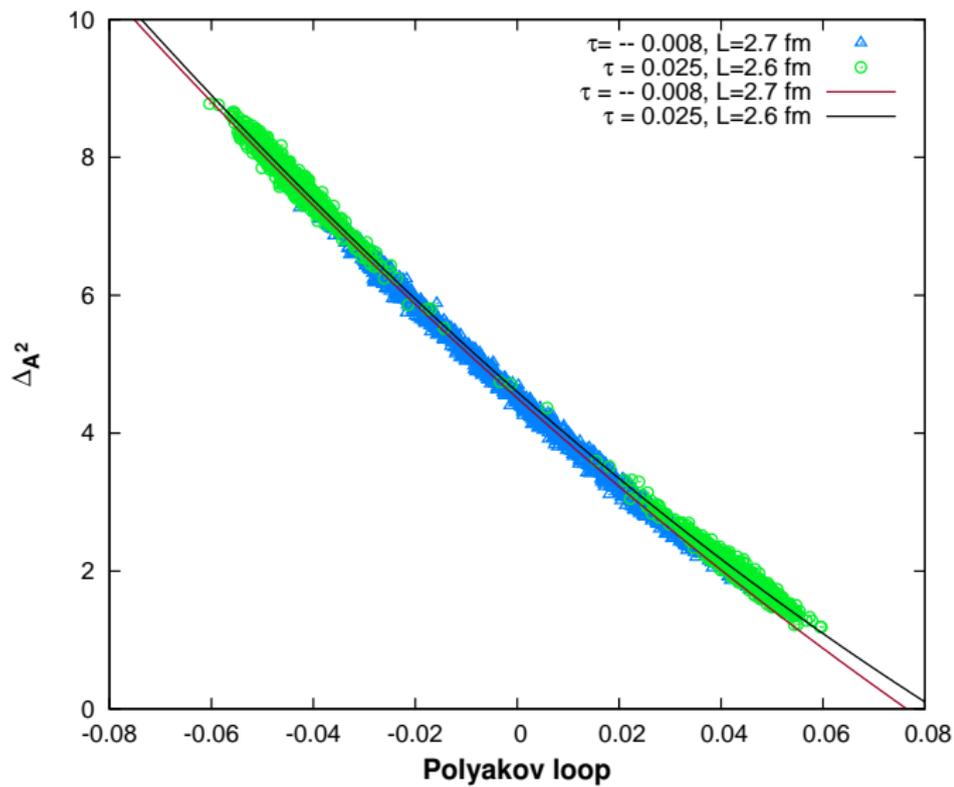
- ▶ As $T \rightarrow T_{c+}$ (that is, at $\mathcal{P} \sim 0$) it can be fitted to a polynomial:

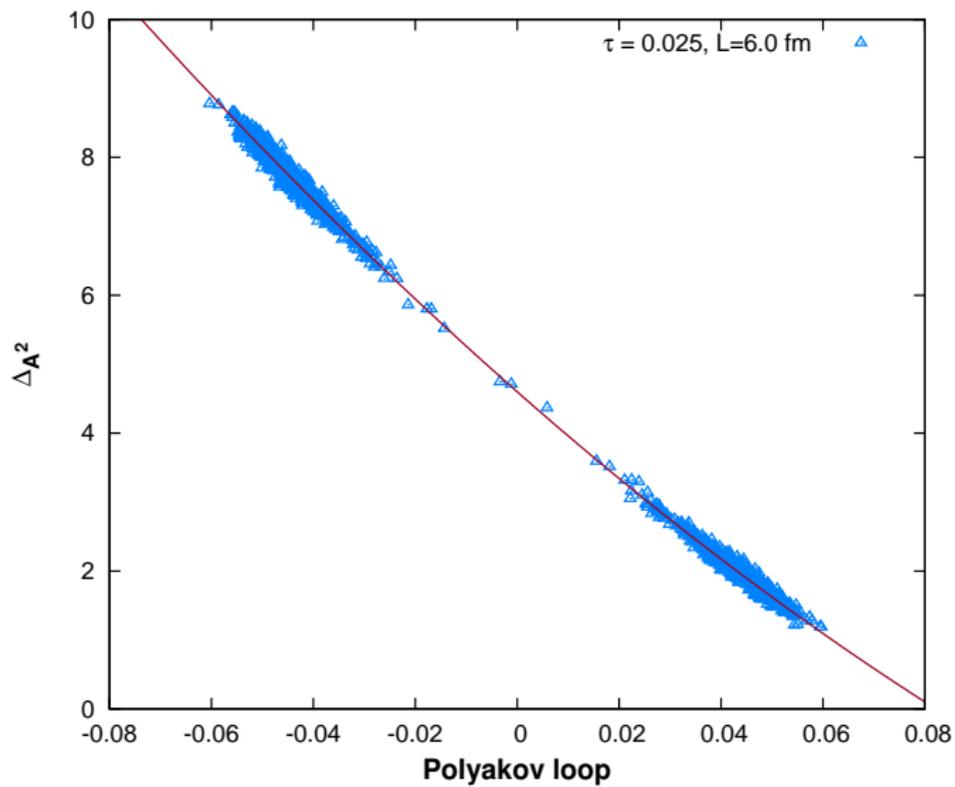
$$E(\Delta|\mathcal{P}) \simeq \Delta^C + \sum_{j=1}^n A_j \mathcal{P}^j.$$

We employ the method of least squares to determine \mathcal{A}_C and A_j

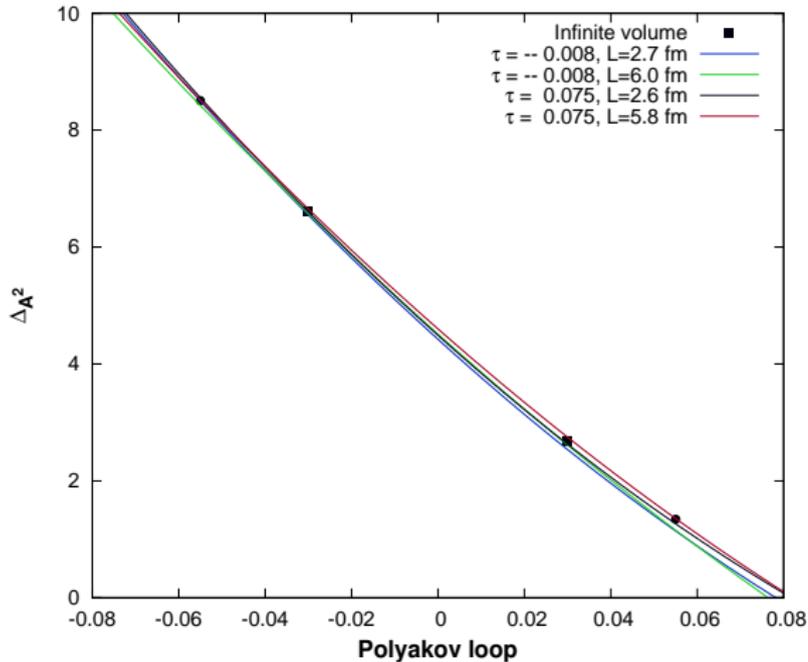








Infinite-volume limit



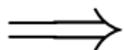
distributions tend to dots on the regression curve

Our main assumption

$$\Delta_{A^2} = \Delta_{A^2}^C + A_1 \mathcal{P} + \bar{o}(\mathcal{P})$$

where

$$\lim_{\mathcal{P} \rightarrow 0} \frac{\bar{o}(\mathcal{P})}{\mathcal{P}} = 0$$



$$\begin{aligned}\beta_{\mathcal{A}} &= \beta = 0.326419(3), \\ B_{\mathcal{A}} &= A_1 B = -54.02(24)\end{aligned}$$

$$\Delta_{A^2} = \Delta_{A^2}^C + A_1 \mathcal{P} + A_2 \mathcal{P}^2 + \dots$$

From this expansion it follows that

$$\mathcal{A} = \Delta_{A^2} - \Delta_{A^2}^C \simeq A_1 \mathcal{P} \simeq A_1 B \tau^\beta, \quad (11)$$

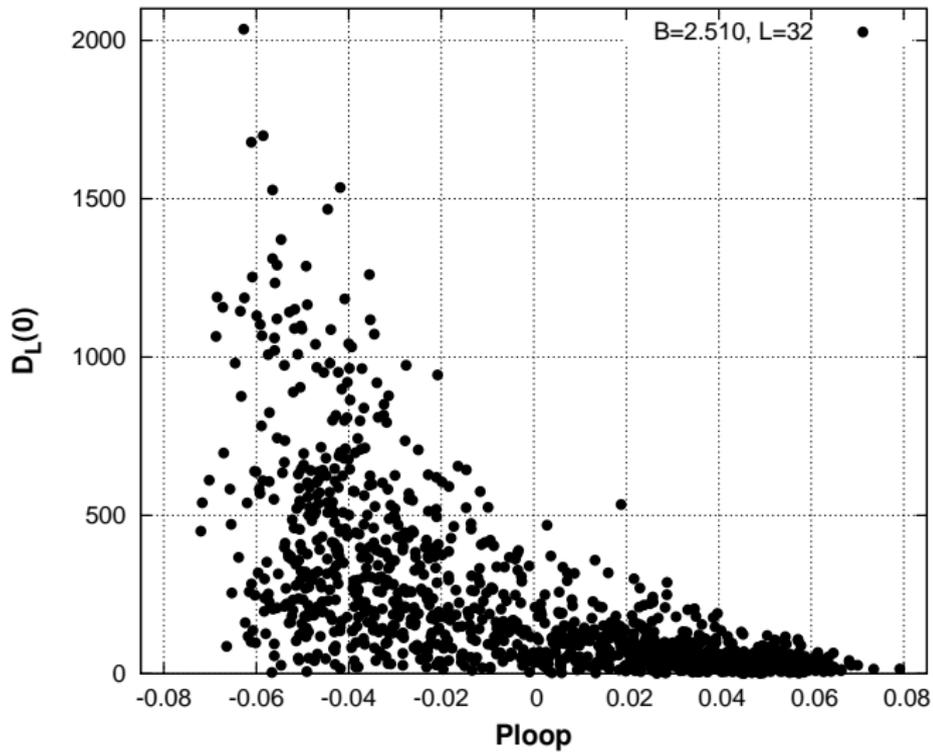
whereas, by definition,

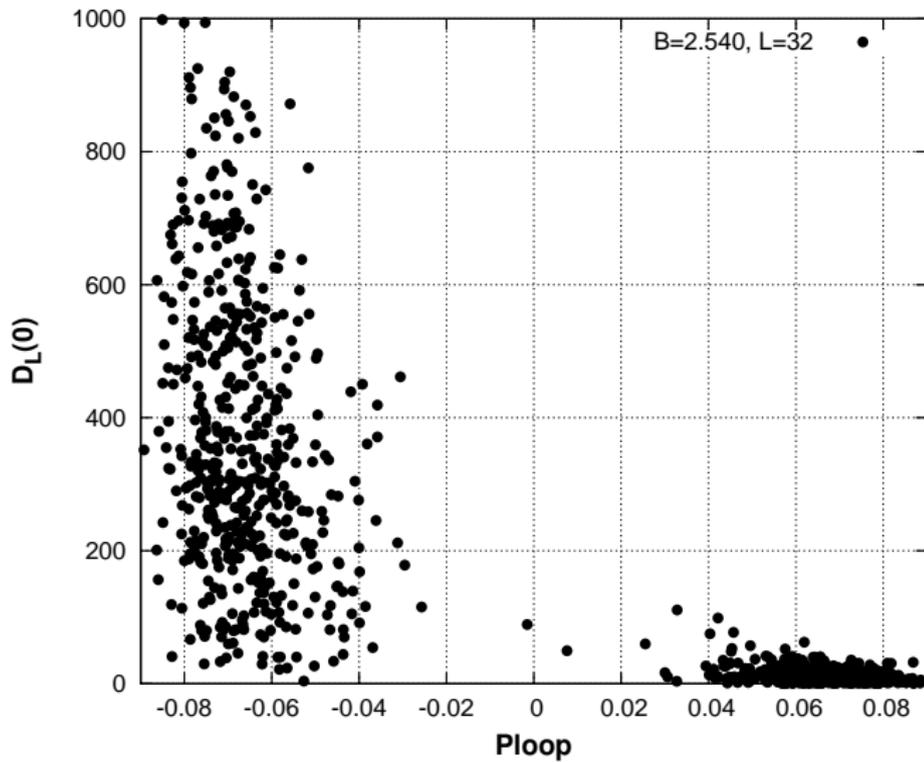
$$\mathcal{A} \simeq B_{\mathcal{A}} \tau^{\beta_{\mathcal{A}}}. \quad (12)$$

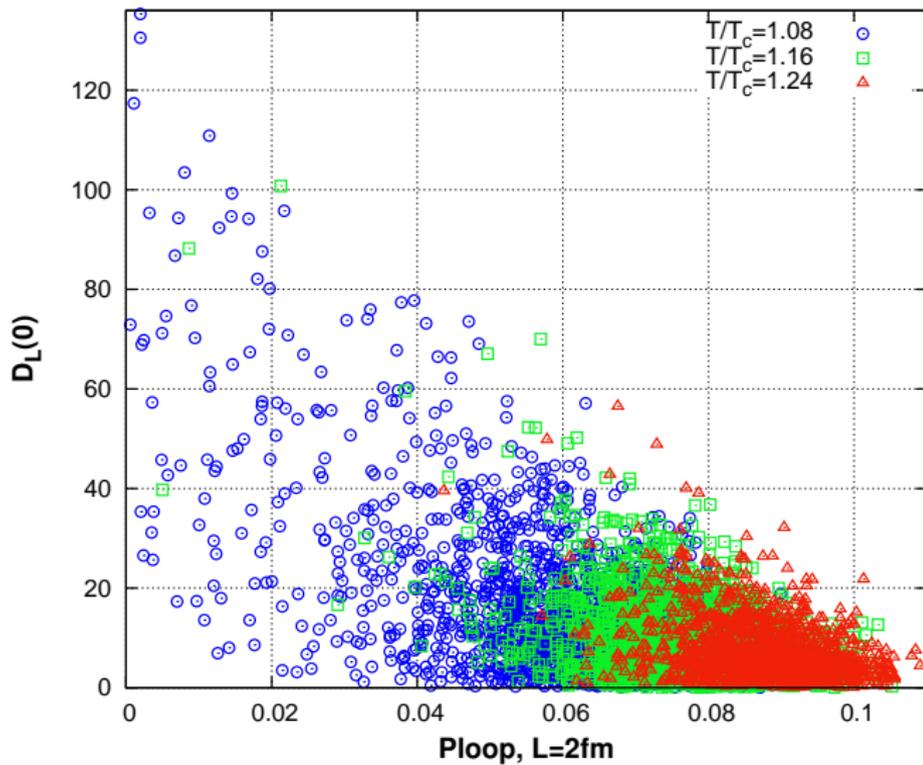
Therefore,

$$\begin{aligned} \beta_{\mathcal{A}} &= \beta = 0.326419(3), \\ B_{\mathcal{A}} &= A_1 B = -54.02(24) \end{aligned}$$

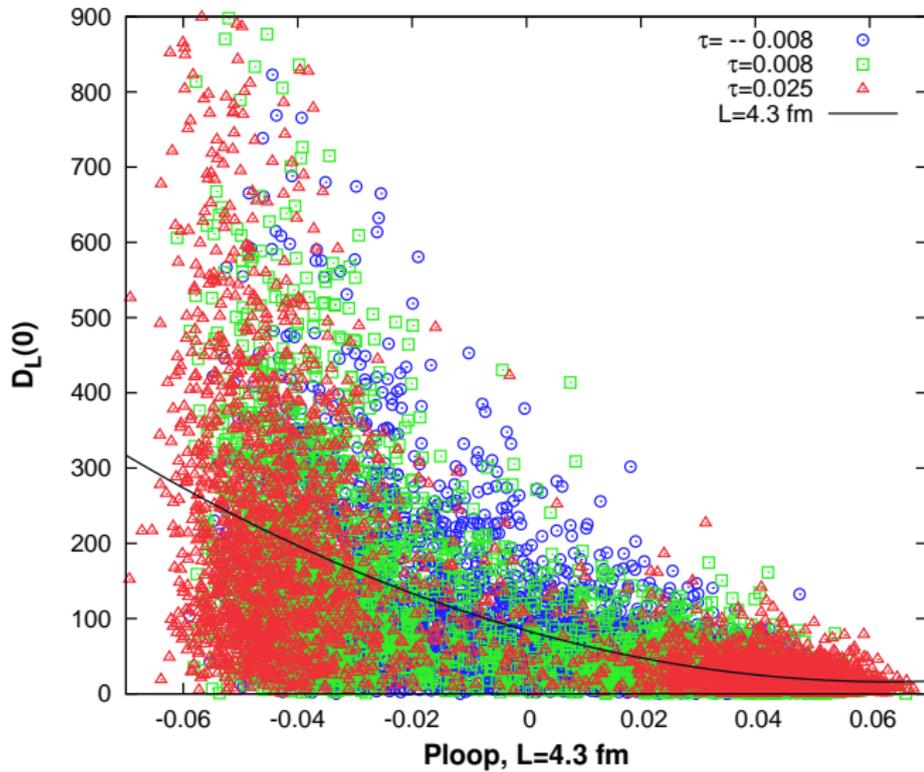
At $\tau < 0$ $\mathcal{A} \approx 0$ is a smooth function







Homoscedasticity is severely broken



Critical behavior of the bare longitudinal propagator

$$D_L(0) = D_L^C(0) + D_1 \mathcal{P} + D_2 \mathcal{P}^2 + \dots$$

From this expansion it follows that

$$\mathcal{D} = D_L(0) - D_L^C(0) \simeq D_1 \mathcal{P} \simeq D_1 B \tau^\beta, \quad (13)$$

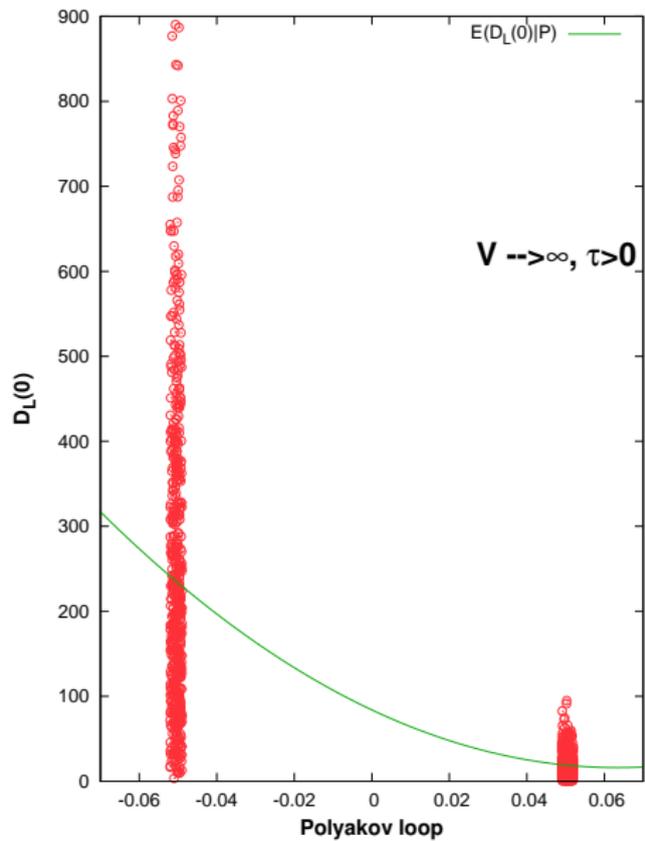
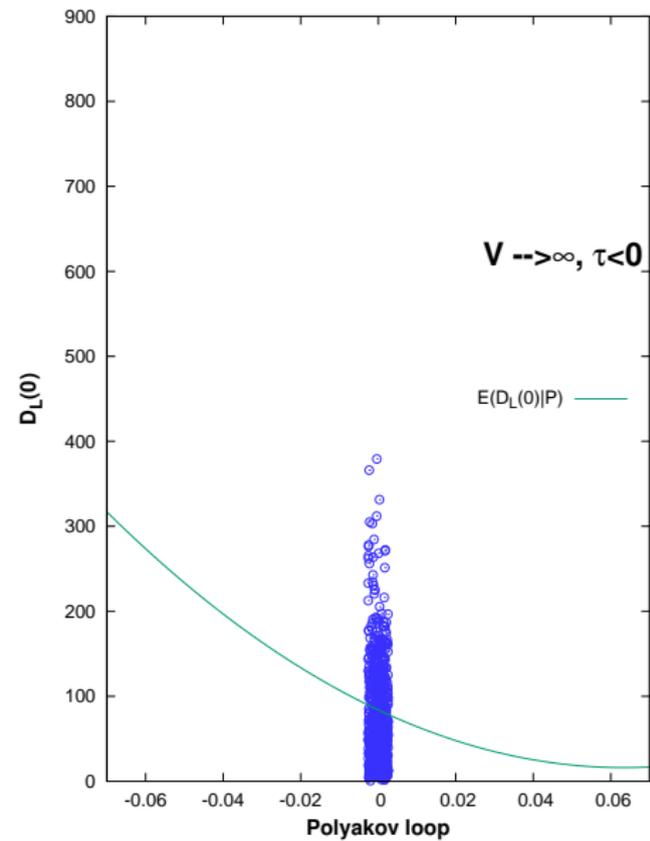
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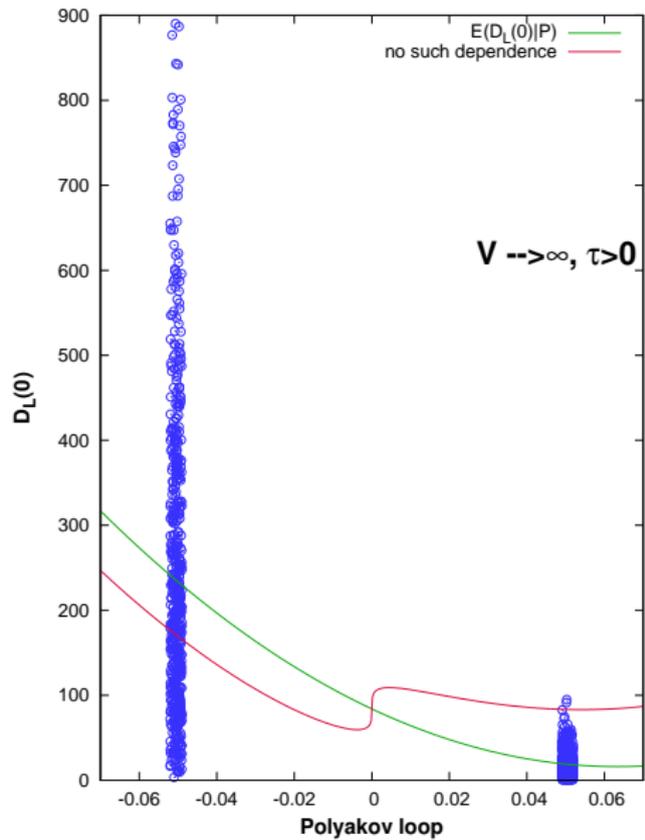
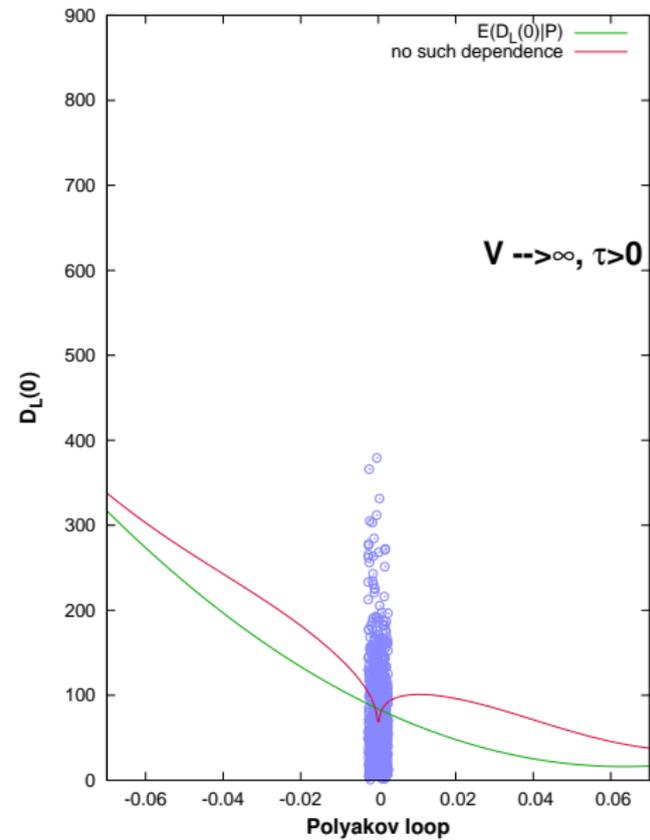
$$\mathcal{D} \simeq \mathcal{D} \tau^{\beta_{\mathcal{D}}}. \quad (14)$$

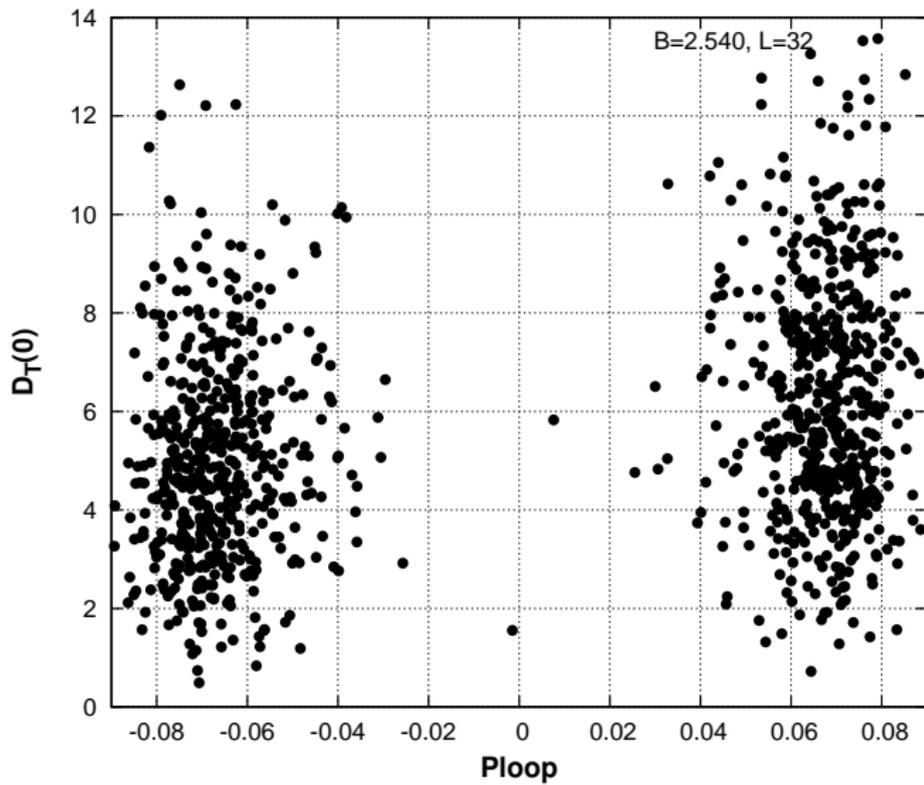
Therefore,

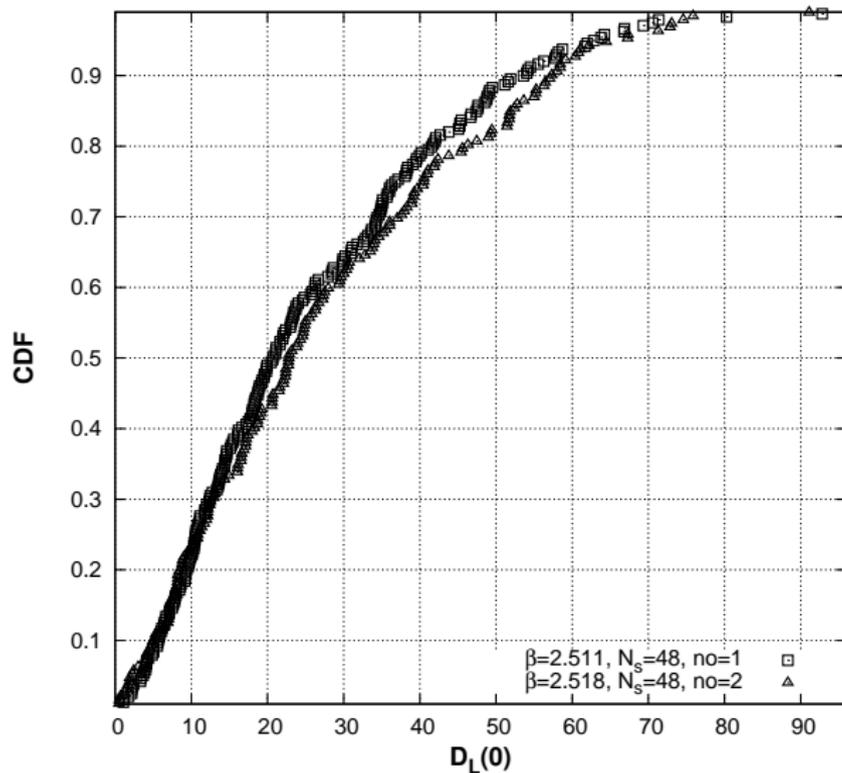
$$\begin{aligned} \beta_{\mathcal{D}} &= \beta = 0.326419(3), \\ B_{\mathcal{D}} &= D_1 B = -1832(60) \text{ GeV}^{-2} \end{aligned}$$

$$a^{-1} \sim 2.5 \text{ GeV}$$

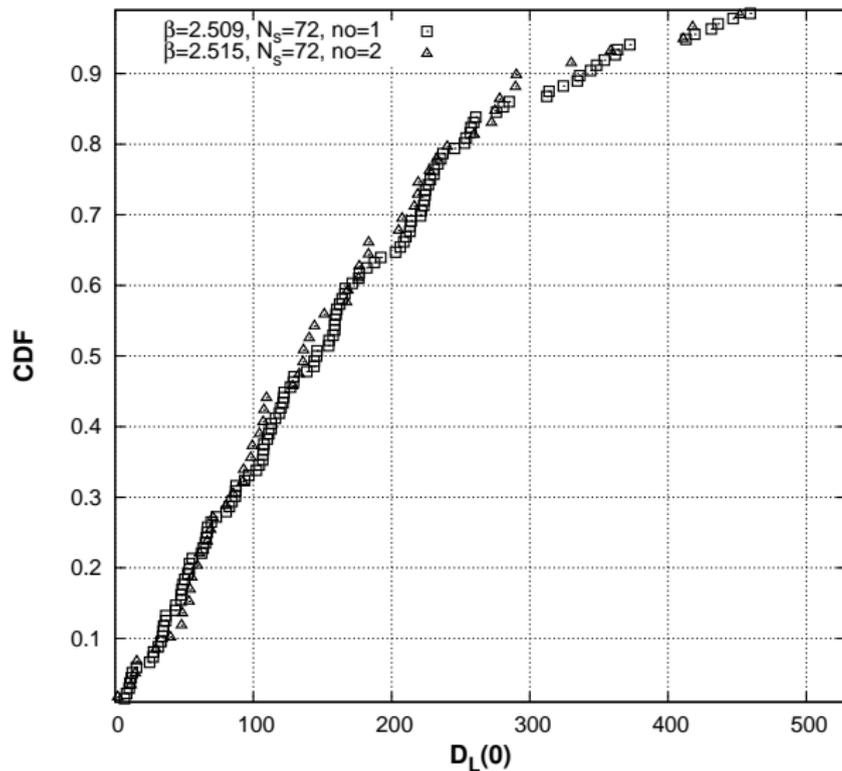




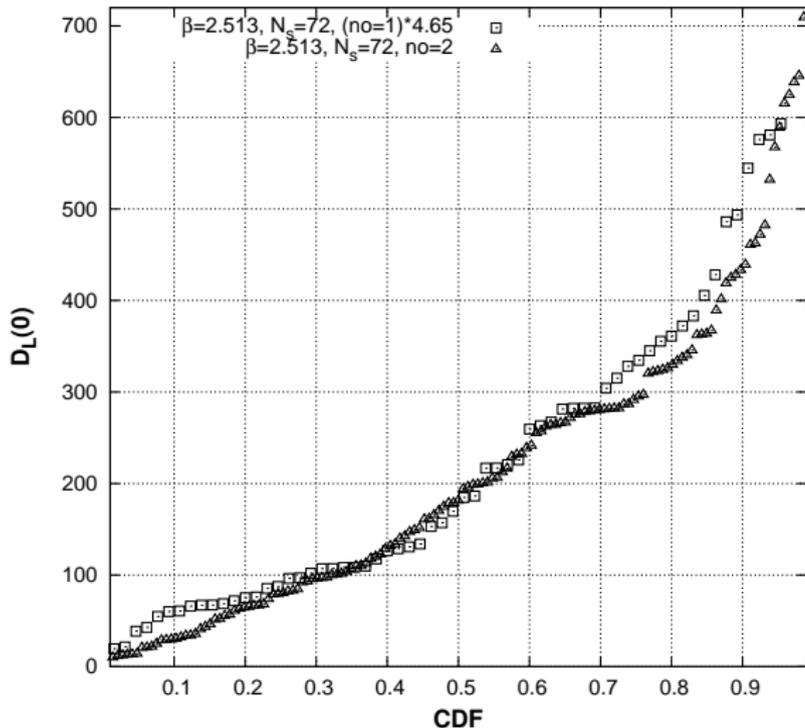




$0.035 < \mathcal{P} < 0.040$; $L = 4 \text{ fm}$; $1 \rightarrow \tau = 0.002$; $2 \rightarrow \tau = 0.025$



$-0.030 < \mathcal{P} < -0.025$; $L = 6$ fm; $1 \rightarrow \tau = -0.0045$; $2 \rightarrow \tau = 0.0148$



$L = 6 \text{ fm};$
 $\tau = 0.0048;$

$1 \rightarrow 0.035 < \mathcal{P} < 0.040;$ $2 \rightarrow -0.030 < \mathcal{P} < -0.025;$

Conclusions

- ▶ Both the asymmetry and the longitudinal propagator have a significant correlation with the Polyakov loop.
- ▶ Regression analysis reveals the dependence of each of these quantities on the Polyakov loop \mathcal{P} as follows:

$$D \simeq D_0 + D_1\mathcal{P} + D_2\mathcal{P}^2$$

- ▶ Such dependence implies that in the infinite-volume limit both Δ_{A^2} and $D_L(0)$

$$\beta_{\mathcal{A}} = \beta_{\mathcal{D}} = \beta = 0.326419(3)$$

- ▶ $B_{\mathcal{A}} = -54.02(24)$, $B_{\mathcal{D}} = -1832(60)$ GeV⁻² (bare quantities)
- ▶ Scaling in the conditional distribution of $D_L(0)$ is observed