Gravitational Screens

Fil Simovic – July 10th 2018
ICNFP 2018 – Orthodox Academy of Crete

Dr. Laurent Freidel – Perimeter Institute for Theoretical Physics

Dr. Robert B. Mann – University of Waterloo

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

UNIVERSITY OF WATERLOO
CONCORDIA CUM VERITATE

Why Is Gravity Interesting?

Why Is Gravity Interesting?

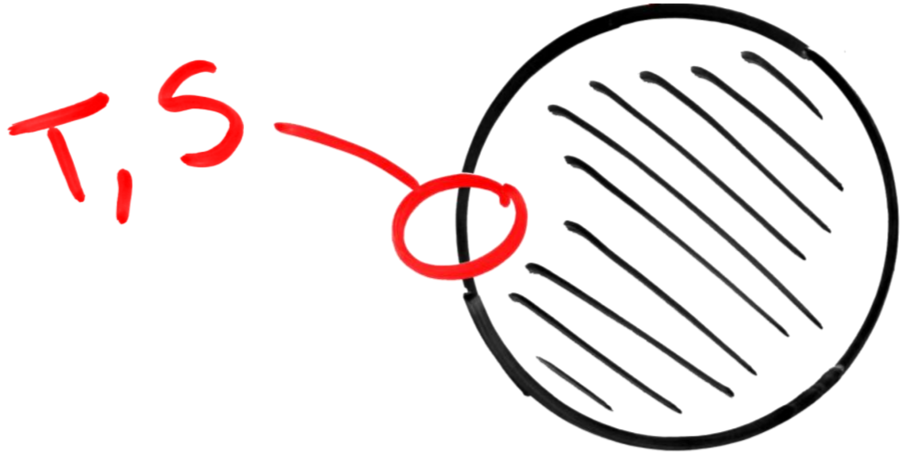
- Unusually weak: $G \sim 10^{-11} \;\longleftrightarrow\; k \sim 10^{10}$

Why Is Gravity Interesting?

- Unusually weak: $G \sim 10^{-11} \longleftrightarrow k \sim 10^{10}$
- Dominant force in large scale structure and evolution of the universe.

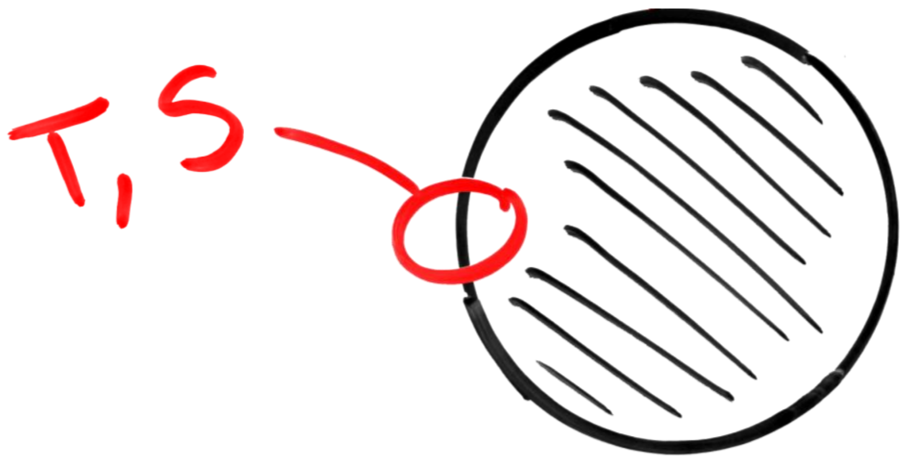
Why Is Gravity Interesting?

- Unusually weak: $G \sim 10^{-11} \quad \longleftrightarrow \quad k \sim 10^{10}$
- Dominant force in large scale structure and evolution of the universe.
- Appears to have thermodynamic properties:



Why Is Gravity Interesting?

- Unusually weak: $G \sim 10^{-11} \longleftrightarrow k \sim 10^{10}$
- Dominant force in large scale structure and evolution of the universe.
- Appears to have thermodynamic properties:



- **Can we understand the thermodynamic nature of gravity?**

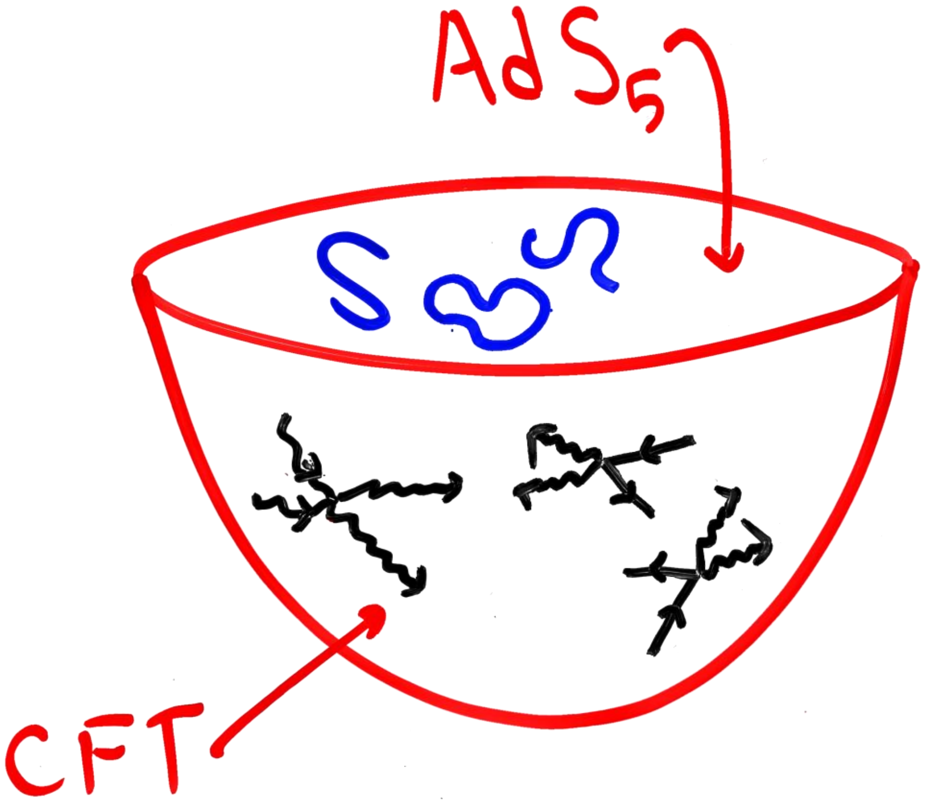
Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

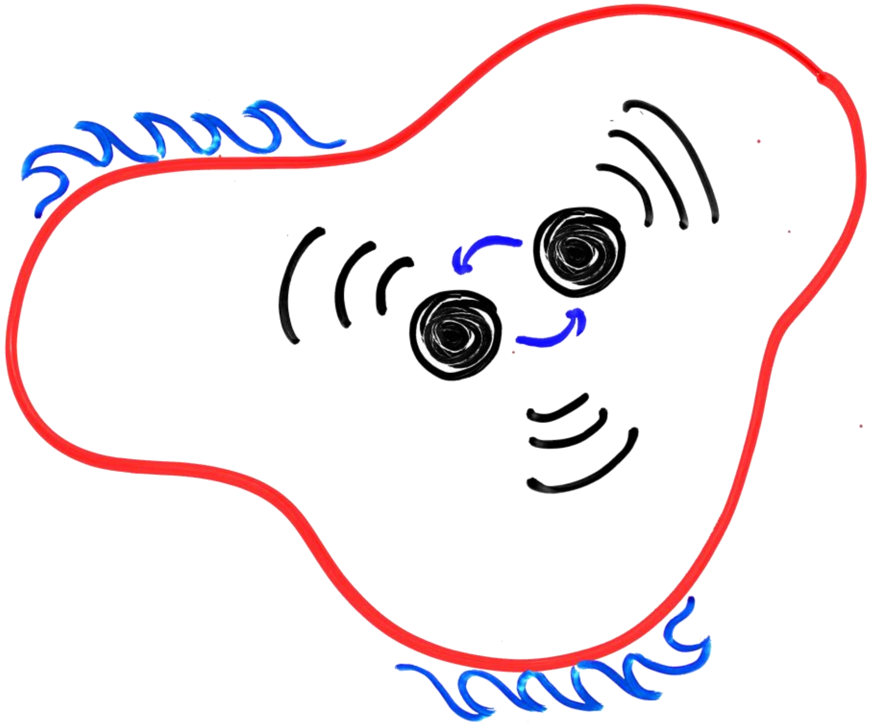
  - AdS/CFT Correspondence



Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

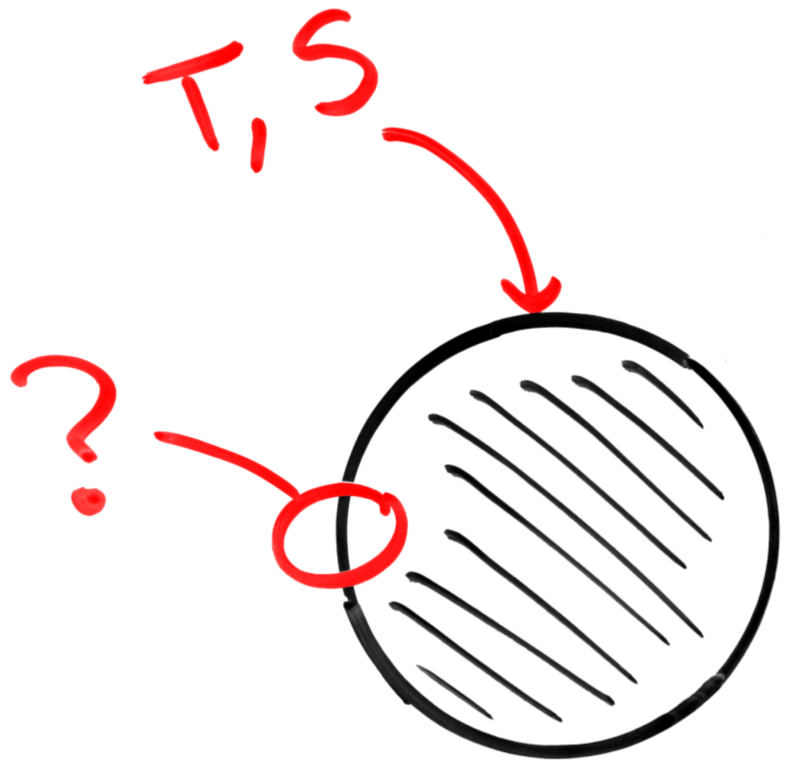
  - AdS/CFT Correspondence
  - Fluid/Gravity Correspondence



Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

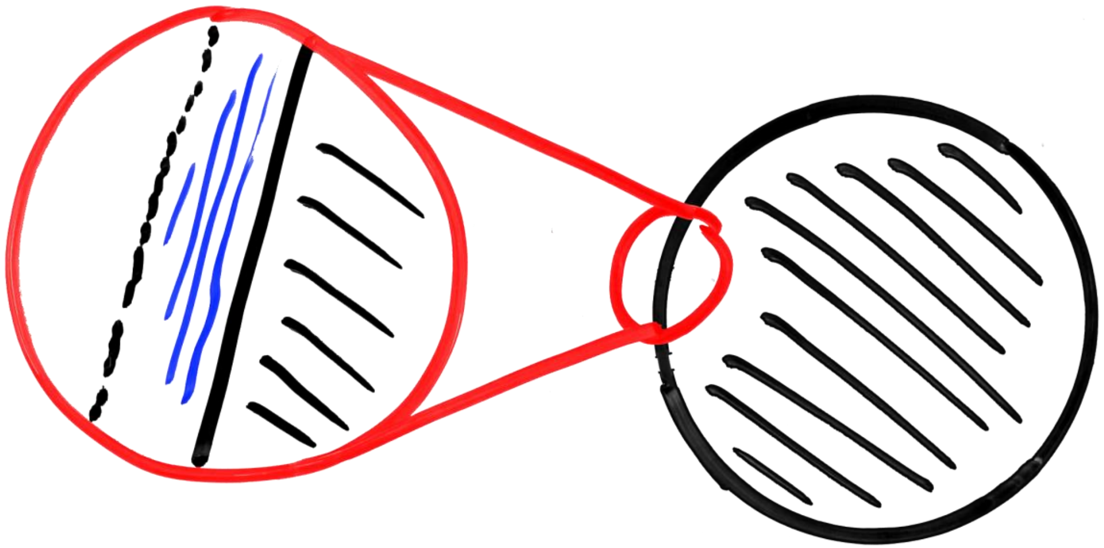
  - AdS/CFT Correspondence
  - Fluid/Gravity Correspondence
  - Black Hole Thermodynamics



Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

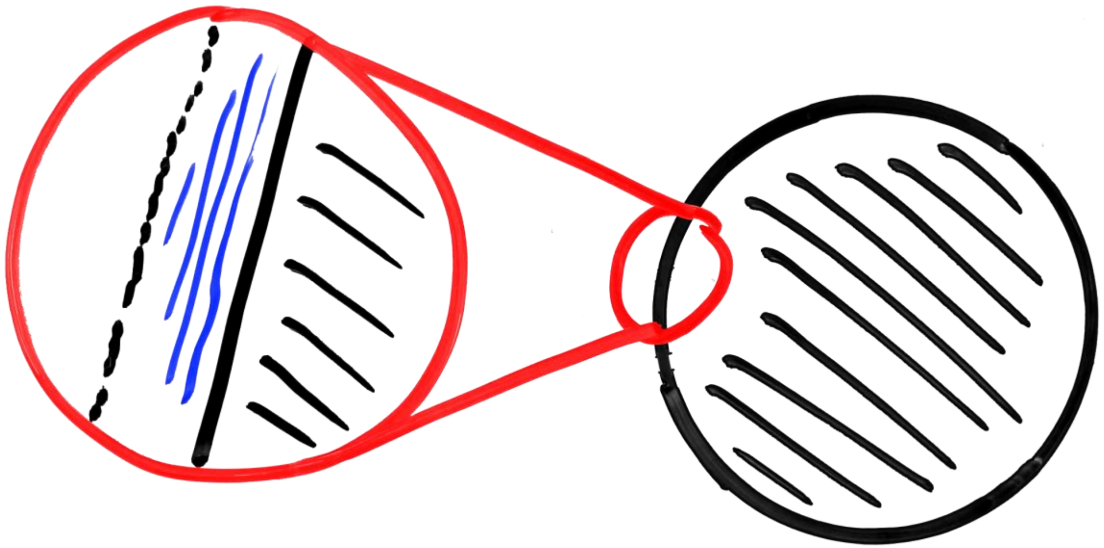
  - AdS/CFT Correspondence
  - Fluid/Gravity Correspondence
  - Black Hole Thermodynamics
  - Membrane Paradigm



Holography

- We can often describe N-dimensional systems using information on an N-1 dimensional boundary.

  - AdS/CFT Correspondence
  - Fluid/Gravity Correspondence
  - Black Hole Thermodynamics
  - Membrane Paradigm



- How can we overcome these issues?

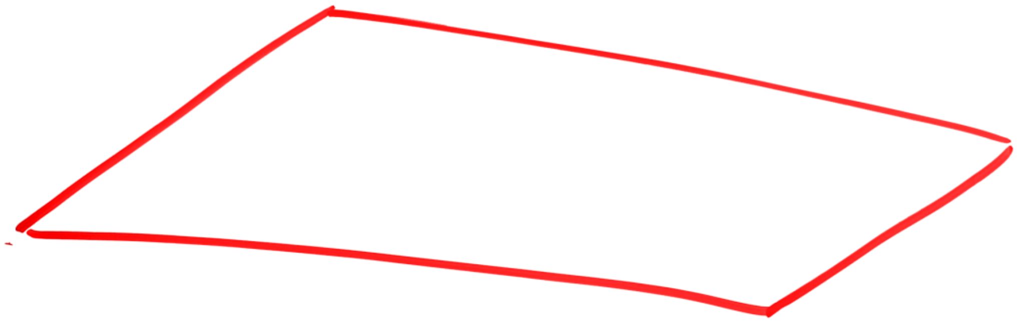
L. Freidel, Y. Yokokura - Classical and Quantum Gravity, 32 (2015) - arXiv:1405.4881

Hypersurfaces

- Consider a region of space bounded by a 2-sphere $\mathcal{S}$:

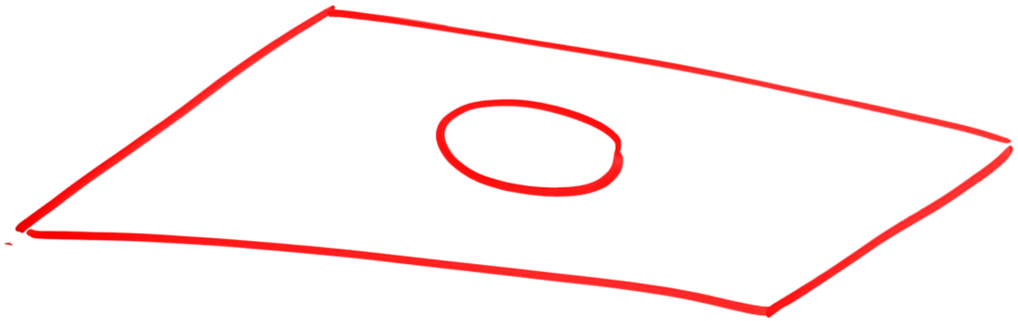
Hypersurfaces

- Consider a region of space bounded by a 2-sphere $\mathcal{S}$:



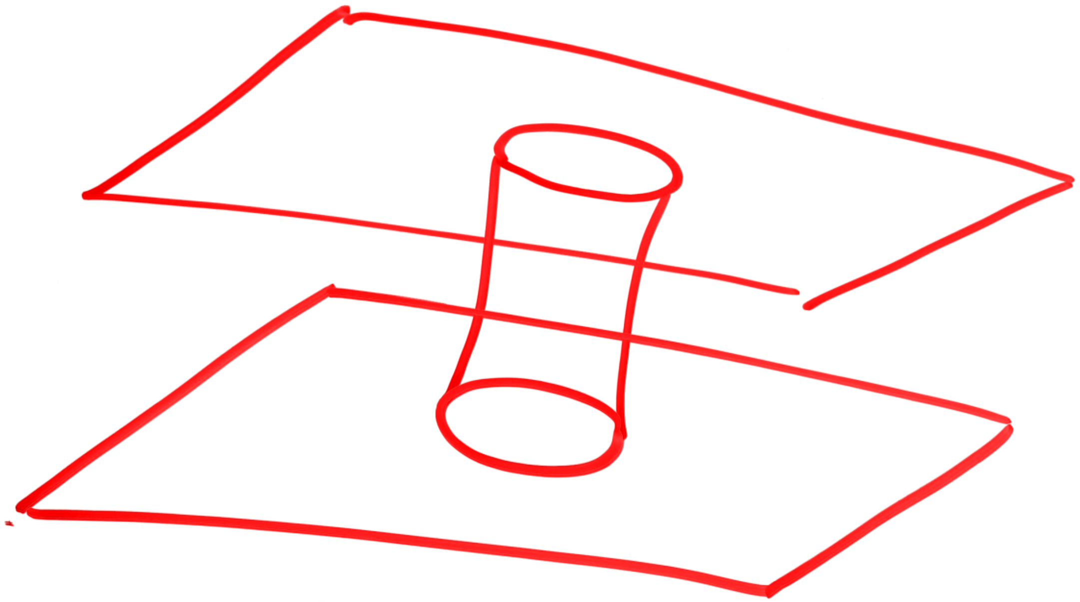
Hypersurfaces

- Consider a region of space bounded by a 2-sphere $\mathcal{S}$:



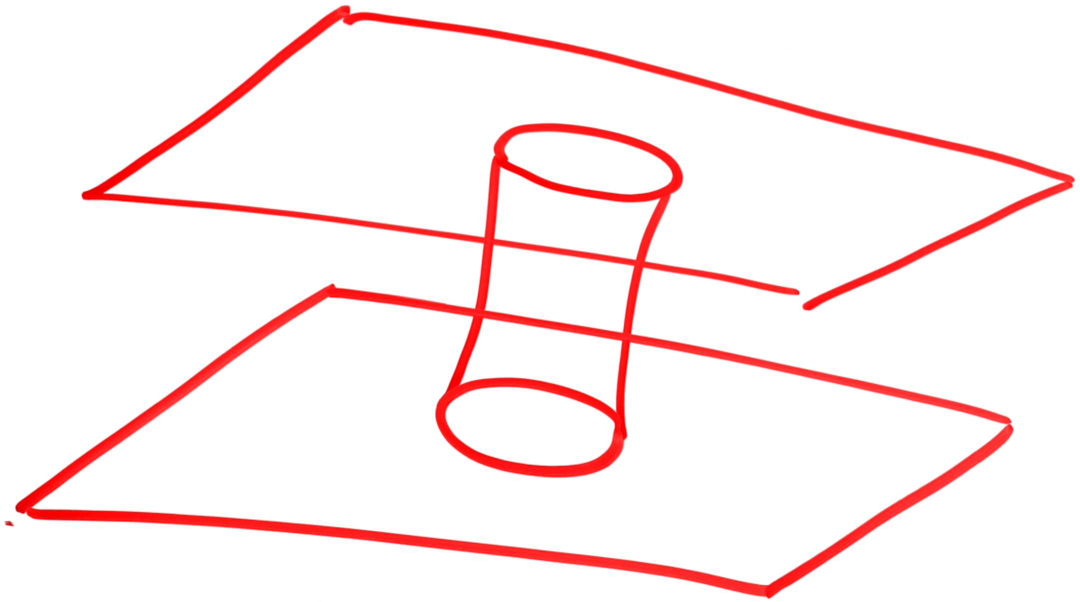
Hypersurfaces

- Consider a region of space bounded by a 2-sphere $\mathcal{S}$:



Hypersurfaces

- Consider a region of space bounded by a 2-sphere $\mathcal{S}$:



- The time evolution of $\mathcal{S}$ defines a time-like hypersurface Σ

Hypersurfaces

- By construction we have that:

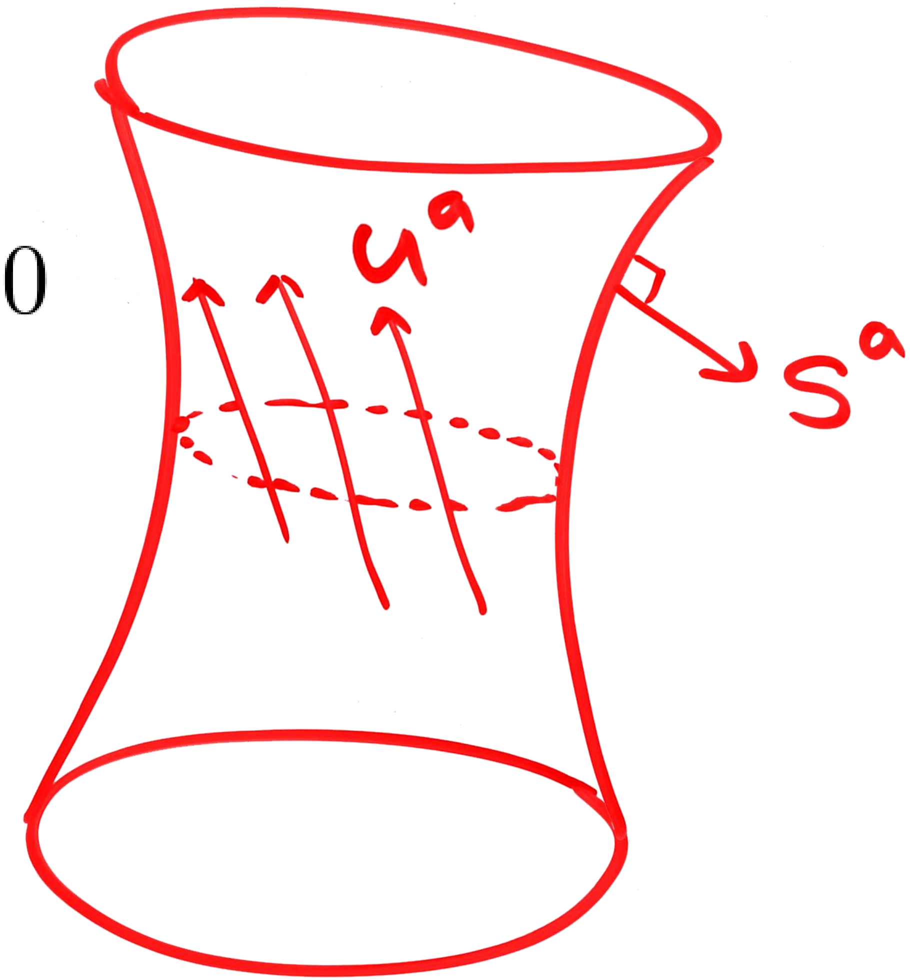
$$s^a s_a = 1, \quad u^a u_a = -1, \quad s^a u_a = 0$$

- The metrics on Σ and $\mathcal{S}$ are:

$$h_{ab} = g_{ab} - s_a s_b$$
$$q_{ab} = h_{ab} + u_a u_b$$

- The extrinsic curvature is:

$$H_{ab} = h_a{}^c h_b{}^d \nabla_c s_d$$



Hypersurfaces

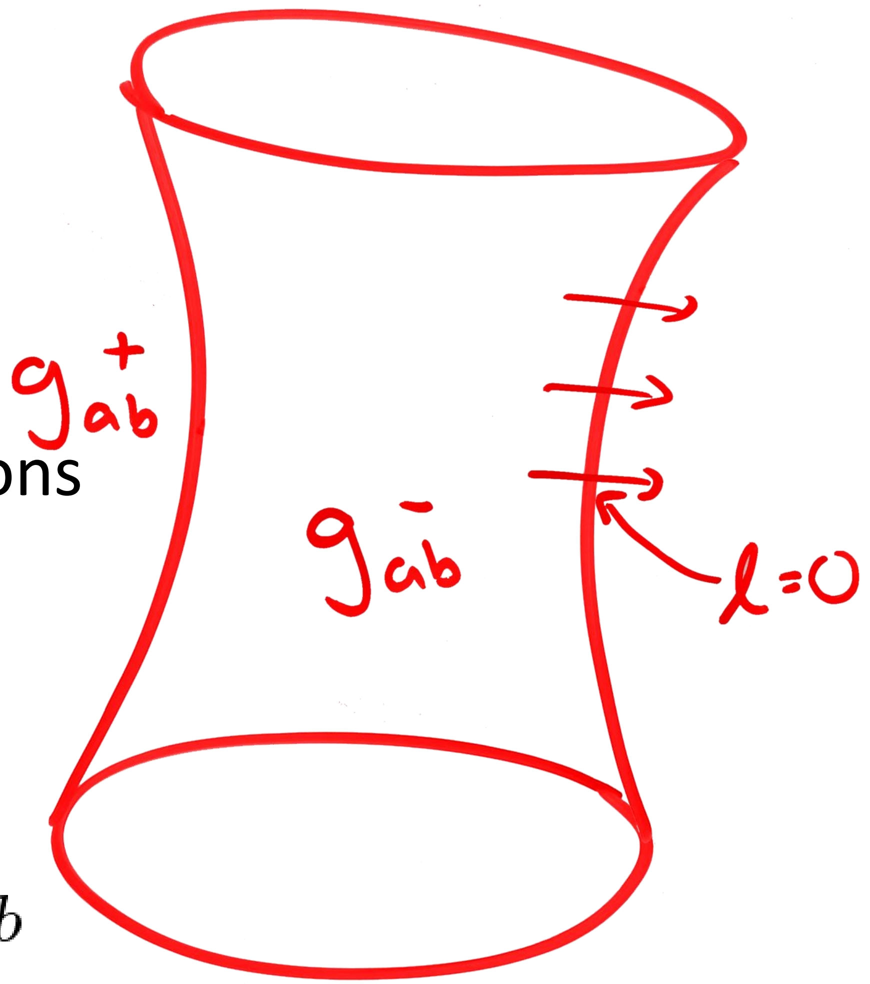
- Discontinuity in extrinsic curvature across the boundary implies presence of singular stress energy on the boundary.

- Comes from requirement that Einstein equations are well-defined in terms of distributions:

$$g_{ab} = \Theta(l) g^{+}_{ab} + \Theta(-l) g^{-}_{ab}$$

$$T_{ab} = \Theta(l) T^{+}_{ab} + \Theta(-l) T^{-}_{ab} + \delta(l) S_{ab}$$

$$S_{ab} = [H] h_{ab} - [H_{ab}]$$



Hypersurfaces

- Consider replacing spacetime within the surface with stress tensor supported entirely on the surface:

$$S_{ab} = Hh_{ab} - H_{ab}$$

Hypersurfaces

- Consider replacing spacetime within the surface with stress tensor supported entirely on the surface:

$$
\begin{aligned}
S_{ab} &= H h_{ab} - H_{ab} \\
&= (h_c{}^e h_d{}^f \nabla_e s_f) h^{cd} h_{ab} - h_a{}^c h_b{}^d \nabla_c s_d \\
&= -\theta_s u_a u_b - \omega_a u_b - u_a \omega_b + (\gamma_u + \tfrac{1}{2}\theta_s) q_{ab} - \tilde{\Theta}_{sab}
\end{aligned}
$$

- We have defined: $\theta_s \equiv q^{ab} \nabla_a s_b$ $\qquad$ $\gamma_u \equiv s \cdot \nabla_u u$

EXPANSION $\qquad$ **NORMAL ACCELERATION**

Hypersurfaces

What is this stress tensor?

$$S_{ab} = -\theta_s u_a u_b - \omega_a u_b - u_a \omega_b + (\gamma_u + \tfrac{1}{2}\theta_s) q_{ab} - \tilde{\Theta}_{sab}$$

Hypersurfaces

What is this stress tensor?

$$S_{ab} = -\theta_s u_a u_b - \omega_a u_b - u_a \omega_b + (\gamma_u + \tfrac{1}{2}\theta_s) q_{ab} - \tilde{\Theta}_{sab}$$

Let’s consider a fluid stress tensor decomposition:

$$S_{ab} = e u_a u_b + \pi_a u_b + \pi_b u_a + p q_{ab} + \Pi_{ab}$$

Hypersurfaces

What is this stress tensor?

$$S_{ab} = -\theta_s u_a u_b - \omega_a u_b - u_a \omega_b + (\gamma_u + \tfrac{1}{2}\theta_s) q_{ab} - \tilde{\Theta}_{sab}$$

Let’s consider a fluid stress tensor decomposition:

$$S_{ab} = e u_a u_b + \pi_a u_b + \pi_b u_a + p q_{ab} + \Pi_{ab}$$

The stress tensor on the boundary can be interpreted as that of a relativistic fluid with:

$$e = -\frac{\theta_s}{8\pi G}, \quad \pi_a = -\frac{\omega_a}{8\pi G}, \quad p = \frac{\gamma_u + \frac{1}{2}\theta_s}{8\pi G}, \quad \Pi_{ab} = -\frac{\tilde{\Theta}_{sab}}{8\pi G}$$

Projected Einstein Equations

- The Einstein equations with zero cosmological constant are:

$$G_{ab} \equiv R_{ab} - Rg_{ab} = T_{ab}$$

Projected Einstein Equations

- The Einstein equations with zero cosmological constant are:

$$G_{ab} \equiv R_{ab} - Rg_{ab} = T_{ab}$$

- Their projection onto Σ is given by the Gauss-Codazzi equations:

$$D_b S^{ba} = T_{cb} s^b h^{ca}, \qquad D_a V_b \equiv h_a{}^c h_b{}^d \nabla_c V_d$$

Projected Einstein Equations

- The Einstein equations with zero cosmological constant are:

$$G_{ab} \equiv R_{ab} - Rg_{ab} = T_{ab}$$

- Their projection onto Σ is given by the Gauss-Codazzi equations:

$$D_b S^{ba} = T_{cb} s^b h^{ca}, \qquad D_a V_b \equiv h_a{}^c h_b{}^d \nabla_c V_d$$

- Projected into directions along u_a and orthogonal to u_a gives conservation of momentum and energy:

$$\boxed{(D_b S^{ba}) q_{ad} = (T_{cb} s^b h^{ca}) q_{ad},} \quad \boxed{(D_b S^{ba}) u_a = (T_{cb} s^b h^{ca}) u_a}$$

MOMENTUM ENERGY

Projected Einstein Equations

- Conservation of momentum on Σ:

$$\boxed{0 = \theta_s \mathrm{a}_c - D_c(\gamma_u + \tfrac{1}{2}\theta_s) + D_b \tilde{\Theta}^b_{sc} - D_u \omega_c + (8\pi G) T_{sc} - (\tilde{\Theta}_{ubc} + \epsilon_{bc} + \tfrac{3}{2}\theta_u q_{bc})\omega^b}$$

- Conservation of energy on Σ:

$$\boxed{0 = D_u \theta_s + \theta_s \theta_u - (\gamma_u + \tfrac{1}{2}\theta_s)\theta_u - \tilde{\Theta}_s^{ab}\Theta_{uab} + (\mathrm{d}_a + 2\mathrm{a}_{uc})\omega^a + (8\pi G) T_{su}}$$

- Dots denote derivatives along the fluid velocity: $\dot{e} \equiv u^a \nabla_a e$

$$T_{su} \equiv T_{ab} s^a u^b = \text{ energy flux through the surface}$$

$$T_{sa} \equiv T_{ab} s^b = \text{ momentum flux through surface}$$

Relativistic Hydrodynamics

- The evolution of systems with large numbers of particles cannot be solved by tracking each particle individually.

Relativistic Hydrodynamics

- The evolution of systems with large numbers of particles cannot be solved by tracking each particle individually.

- When the length scale of a system of particles is extremely large compared to the average particle separation, we use a *fluid* description.

Relativistic Hydrodynamics

- The evolution of systems with large numbers of particles cannot be solved by tracking each particle individually.

- When the length scale of a system of particles is extremely large compared to the average particle separation, we use a *fluid* description.

- The fluid is composed of fluid elements, large enough so that microscopic effects are averaged out, but small enough to maintain local thermodynamic equilibrium within the element.

Relativistic Hydrodynamics

- The evolution of systems with large numbers of particles cannot be solved by tracking each particle individually.

- When the length scale of a system of particles is extremely large compared to the average particle separation, we use a *fluid* description.

- The fluid is composed of fluid elements, large enough so that microscopic effects are averaged out, but small enough to maintain local thermodynamic equilibrium within the element.

- We work in the *Eckart frame* which is aligned with the fluid velocity u^a.

Relativistic Hydrodynamics

- The fundamental equations representing relativistic fluids are conservation of mass and conservation of stress-energy:

$$\nabla_a J^a = 0, \qquad \nabla_a T^{ab} = 0$$

Relativistic Hydrodynamics

- The fundamental equations representing relativistic fluids are conservation of mass and conservation of stress-energy:

$$\nabla_a J^a = 0, \qquad \nabla_a T^{ab} = 0$$

- J^a is the mass density current and can be written $J^a = \rho u^a$ where u^a is the fluid velocity in it’s rest frame and ρ is the mass density.

Relativistic Hydrodynamics

- The fundamental equations representing relativistic fluids are conservation of mass and conservation of stress-energy:

$$\nabla_a J^a = 0, \qquad \nabla_a T^{ab} = 0$$

- J^a is the mass density current and can be written $J^a = \rho u^a$ where u^a is the fluid velocity in it's rest frame and ρ is the mass density.

- A perfect fluid is one with no viscosity, shear stress, or heat flux:

$$T^{ab} = (e + p) u^a u^b + p g^{ab}$$

Relativistic Hydrodynamics

- The fundamental equations representing relativistic fluids are conservation of mass and conservation of stress-energy:

$$\nabla_a J^a = 0, \qquad \nabla_a T^{ab} = 0$$

- J^a is the mass density current and can be written $J^a = \rho u^a$ where u^a is the fluid velocity in it's rest frame and ρ is the mass density.

- A general fluid can have viscosity, shear stress, and heat flux:

$$T^{ab} = e u^a u^b + (p + \pi) q^{ab} + u^a q^b + u^b q^a + \Pi^{ab}$$

Relativistic Hydrodynamics

- The conservation equations are:

$$\boxed{0 = -\dot{e} - (e + p + \pi)\theta - (\mathrm{d}_a + 2\mathrm{a}_a)q^a + \Pi^{ab}\sigma_{ab}}$$

$$\boxed{\begin{aligned} 0 = {} & \mathrm{a}_a(e + p + \pi) + \mathrm{d}_a(p + \pi) + (\mathrm{d}_c + \mathrm{a}_c)\Pi^c{}_a \\ & + q_{ab}\dot{q}^b + (\omega_{ca} + \sigma_{ca} + \tfrac{3}{2}\theta\, q_{ca})q^c \end{aligned}}$$

$\omega_{ab} = \text{twist},$ $\theta = \text{expansion}$ $\sigma_{ab} = \text{shear}$

(antisymmetric) (trace) (symmetric trace-free)

The Dictionary

- Compare conservation of energy from projected Einstein equations to conservation of energy for the relativistic fluid:

$$\boxed{\begin{aligned} 0 &= \theta_s \mathrm{a}_c - D_c(\gamma_u + \tfrac{1}{2}\theta_s) + D_b \tilde{\Theta}^b_{sc} - D_u \omega_c + (8\pi G) T_{sc} - (\tilde{\Theta}_{ubc} + \epsilon_{bc} + \tfrac{3}{2}\theta_u q_{bc})\omega^b \\ 0 &= -e a_c - (\mathrm{d}_c + a_c)(p + \pi) - (\mathrm{d}_a + a_a)\Pi^a{}_c - q_{cb}\dot{q}^b + (\sigma_{ac} + \omega_{ac} + \tfrac{3}{2}\theta q_{ac}) q^a \end{aligned}}$$

The Dictionary

- Compare conservation of energy from projected Einstein equations to conservation of energy for the relativistic fluid:

$$0 = \theta_s \mathrm{a}_c - D_c(\gamma_u + \tfrac{1}{2}\theta_s) + D_b \tilde{\Theta}^b_{sc} - D_u \omega_c + (8\pi G) T_{sc} - (\tilde{\Theta}_{ubc} + \epsilon_{bc} + \tfrac{3}{2}\theta_u q_{bc})\omega^b$$
$$0 = -e a_c - (\mathrm{d}_c + a_c)(p + \pi) - (\mathrm{d}_a + a_a)\Pi^a{}_c - q_{cb}\dot{q}^b + (\sigma_{ac} + \omega_{ac} + \tfrac{3}{2}\theta q_{ac})q^a$$

- Similarly for conservation of momentum:

$$0 = D_u \theta_s + \theta_s \theta_u - (\gamma_u + \tfrac{1}{2}\theta_s)\theta_u - \tilde{\Theta}^{ab}_s \Theta_{uab} + (\mathrm{d}_a + 2\mathrm{a}_{uc})\omega^a + (8\pi G) T_{su}$$
$$0 = -\dot{e} - e\theta - (p + \pi)\theta + \Pi^{ab}\sigma_{ab} - (\mathrm{d}_a + 2\mathrm{a}_a)q^a$$

- We have a direct correspondence between geometric variables describing screen evolution and thermodynamic variables describing a relativistic fluid!

The Dictionary

$$e = -\frac{\theta_s}{8\pi G}$$ energy density = radial expansion

$$p + \pi = \frac{\gamma_u + \frac{1}{2}\theta_s}{8\pi G}$$ pressure = normal accel. + radial exp.

$$\theta = \theta_t$$ expansion = temporal expansion

$$\Pi_{ab} = -\frac{\tilde{\Theta}_{sab}}{8\pi G}$$ viscous stress = radial extrisic curvature

$$\sigma_{ab} = \Theta_{uab}$$ shear = temporal extrinsic curvature

$$q_a = -\frac{\omega_a}{8\pi G}$$ heat flow = normal one-form

Constitutive Relations

- Need a ‘constitutive relation’ or ‘equation of state’ to close the system, a relationship between stress and strain or energy and pressure/density:

$$\Pi(\sigma) \quad e(p,\rho) \quad e(p) \quad s(e)$$

- From the dictionary, a relationship between stress and strain is fixed by radial and temporal extrinsic curvature:

$$\Pi(\sigma) \quad \longleftrightarrow \quad \tilde{\Theta}_s(\Theta_u)$$

- If we know screen evolution, we know the constitutive relation, since $\tilde{\Theta}_s$ and Θ_u are determined by s^a and u^a.

Entropy and Temperature

- Entropy production in the fluid can come from viscous dissipation or heat fluxes:

$$\dot{s} = \beta\, \Pi_{ab}\, \sigma^{ab} + q^a d_a \beta$$

- The viscous dissipation term $\Pi_{ab}\sigma^{ab}$ maps to $\tilde{\Theta}_{sab}\Theta_u^{ab}$ which is related to the Weyl tensor, so entropy production in the fluid is related to gravitational wave propagation in the bulk.

Entropy and Temperature

- Entropy production in the fluid can come from viscous dissipation or heat fluxes:

$$\dot{s} = \beta\, \Pi_{ab}\, \sigma^{ab} + q^a d_a \beta$$

- The viscous dissipation term $\Pi_{ab}\sigma^{ab}$ maps to $\tilde{\Theta}_{sab}\Theta_u^{ab}$ which is related to the Weyl tensor, so entropy production in the fluid is related to gravitational wave propagation in the bulk.

- We can use the Gibbs relation and the Euler relation to determine the screen fluid's entropy and temperature:

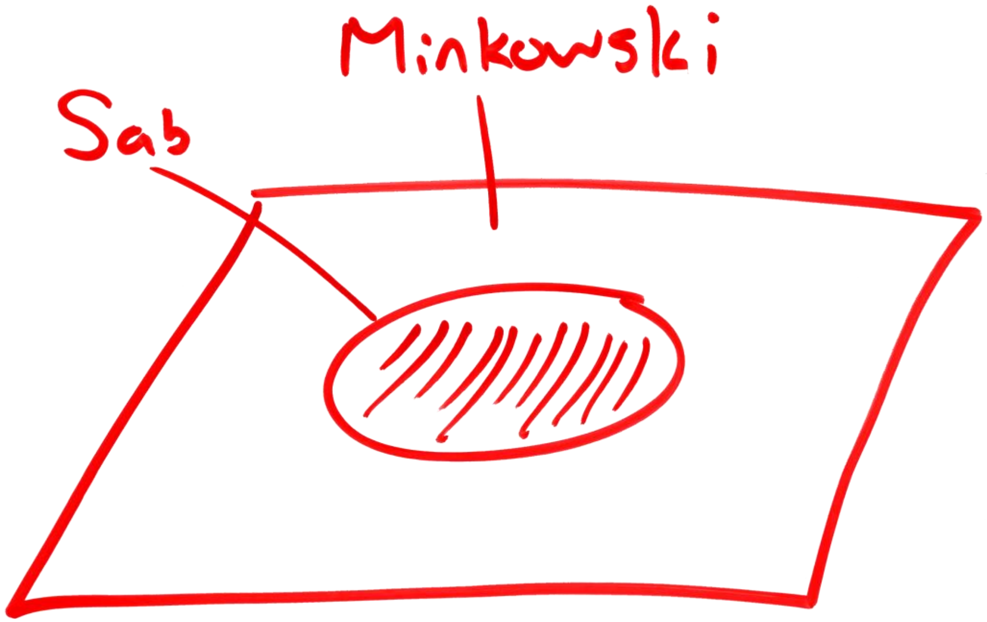
$$de = T ds \qquad e + p = Ts + \mu n$$

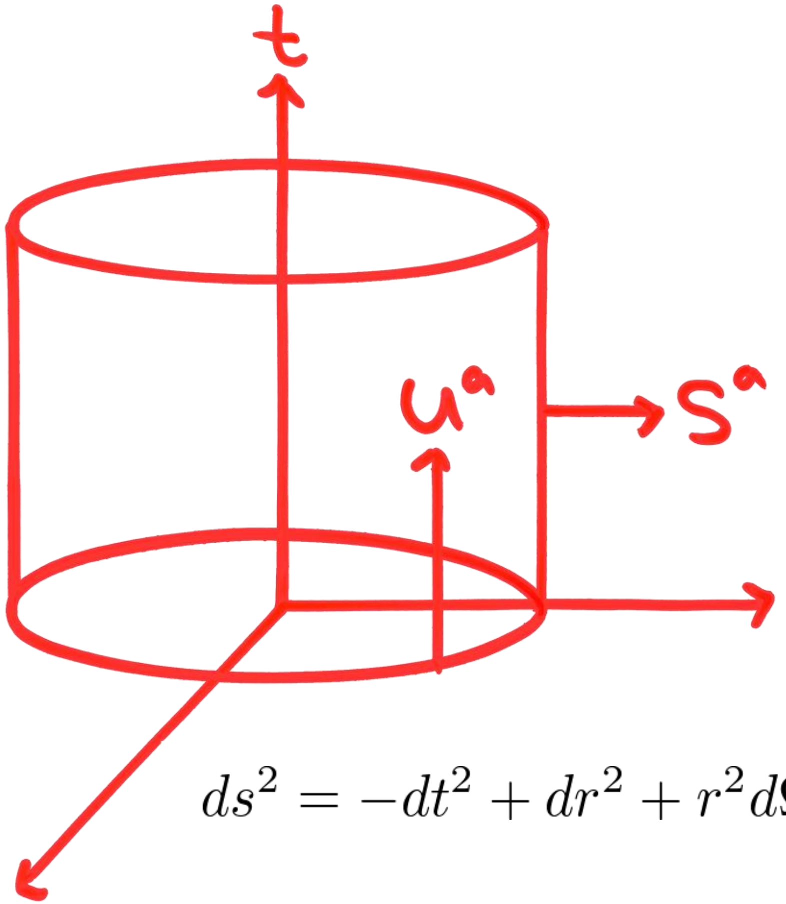
Static Screens

- Consider a static screen in flat space:

Static Screens

- Consider a static screen in flat space:

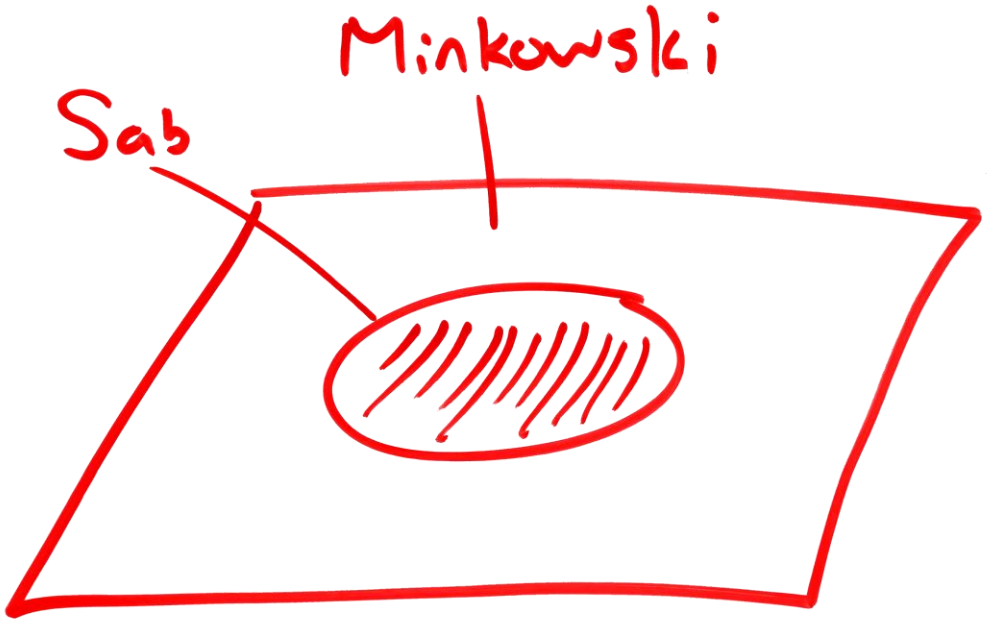


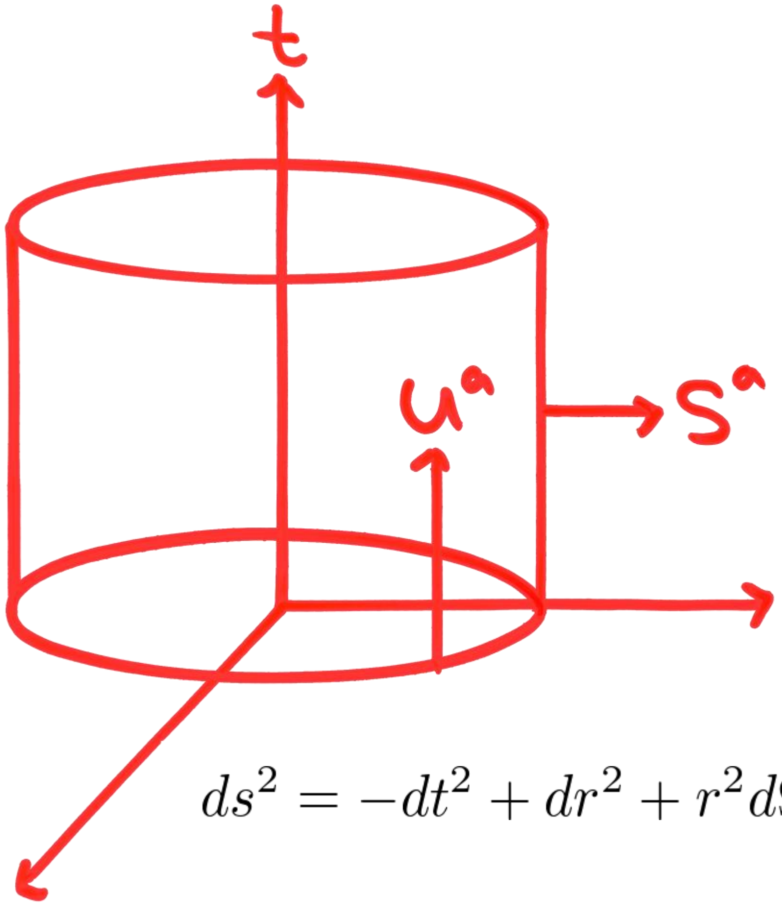


$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

Static Screens

- Consider a static screen in flat space:





$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

- The geometric variables are:

$$\theta_s = 2/r \qquad \theta_u = 0$$

$$\gamma_s = 0 \qquad \gamma_u = 0$$

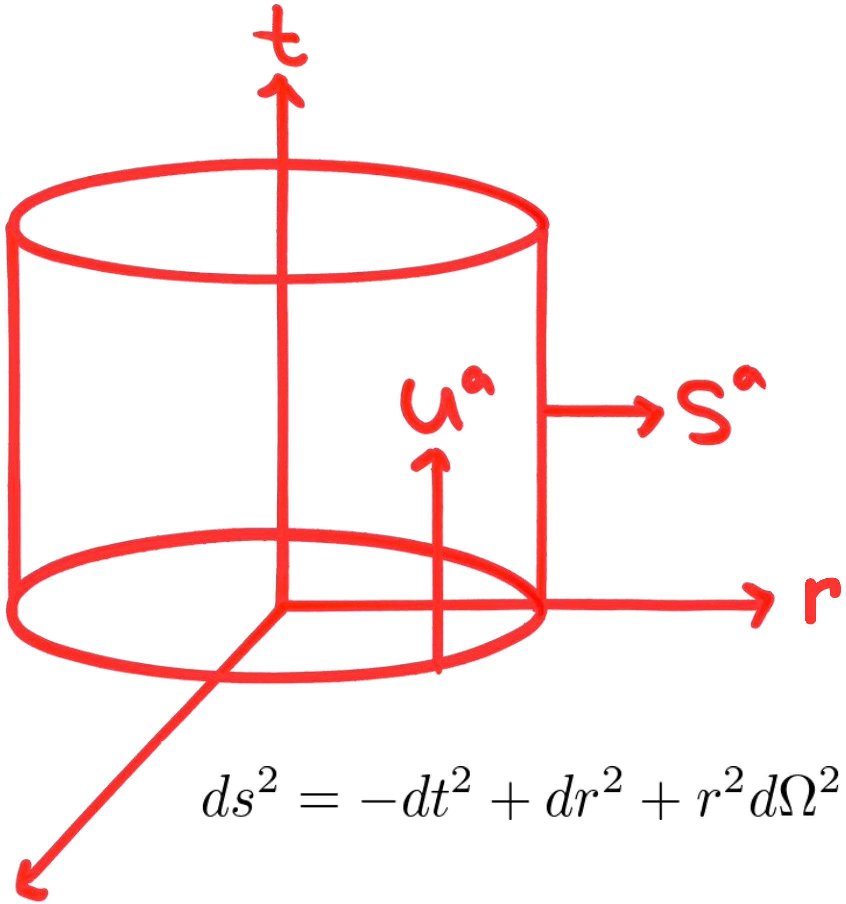
Static Screens

- The equation of state is:

$$e(p) = -2p \quad \longleftrightarrow \quad s(e) = Ce^2$$

- The temperature is therefore:

$$\frac{1}{T} = \frac{ds}{de} \quad \rightarrow \quad T = \frac{1}{2Ce}$$

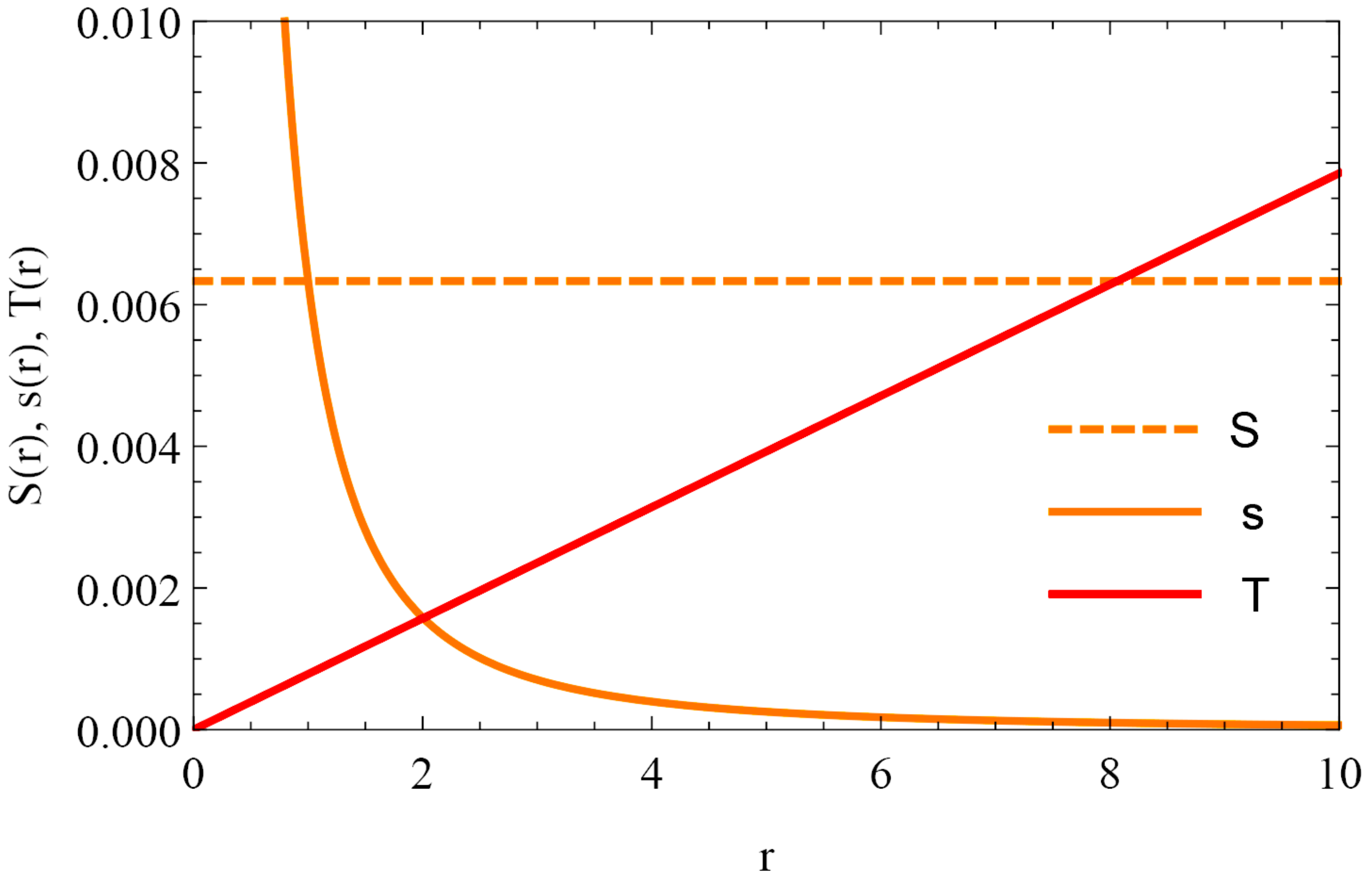


$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

Static Screens

- In terms of screen radius:

$$T(r) = \frac{r}{4C} \qquad s(r) = \frac{4C}{r^2}$$



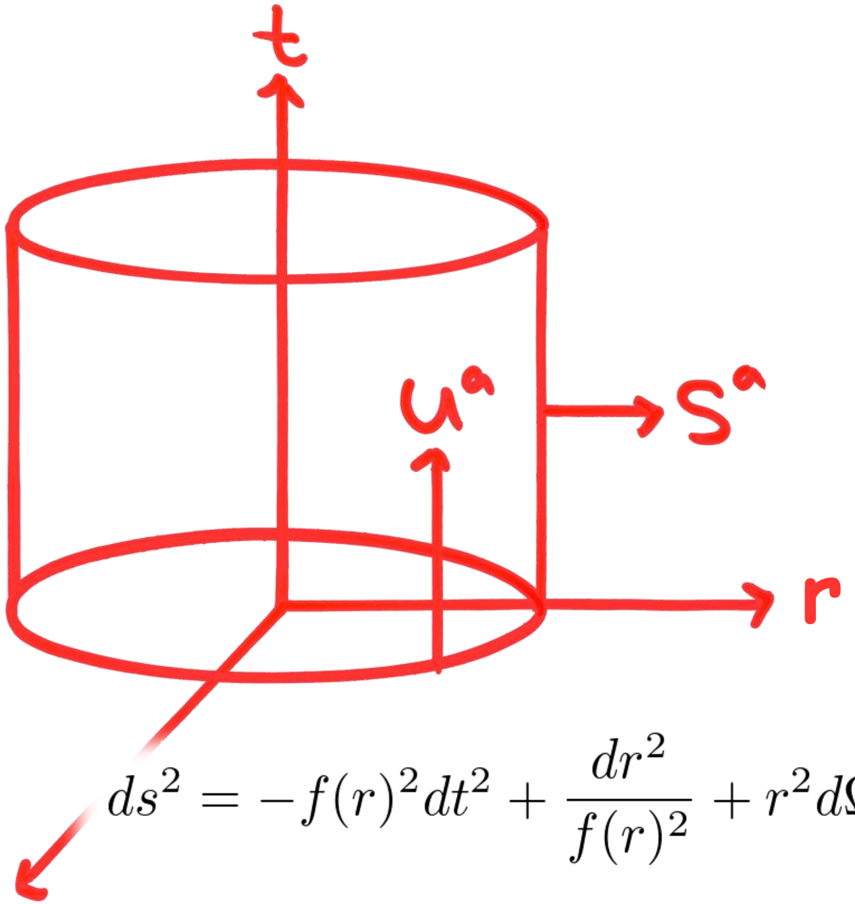
$$S = A \times s(r) = \text{constant}$$

Screen does not attribute entropy to flat space. ✓

Static Screens

- Consider now a static screen in a Schwarzschild spacetime.

$$e = \frac{r-2m}{4\pi r^2}, \quad p = \frac{m-r}{8\pi r^2}$$



$$ds^2 = -f(r)^2 dt^2 + \frac{dr^2}{f(r)^2} + r^2 d\Omega^2$$

$$f(r) = \sqrt{1 - \frac{2m}{r}}$$

Static Screens

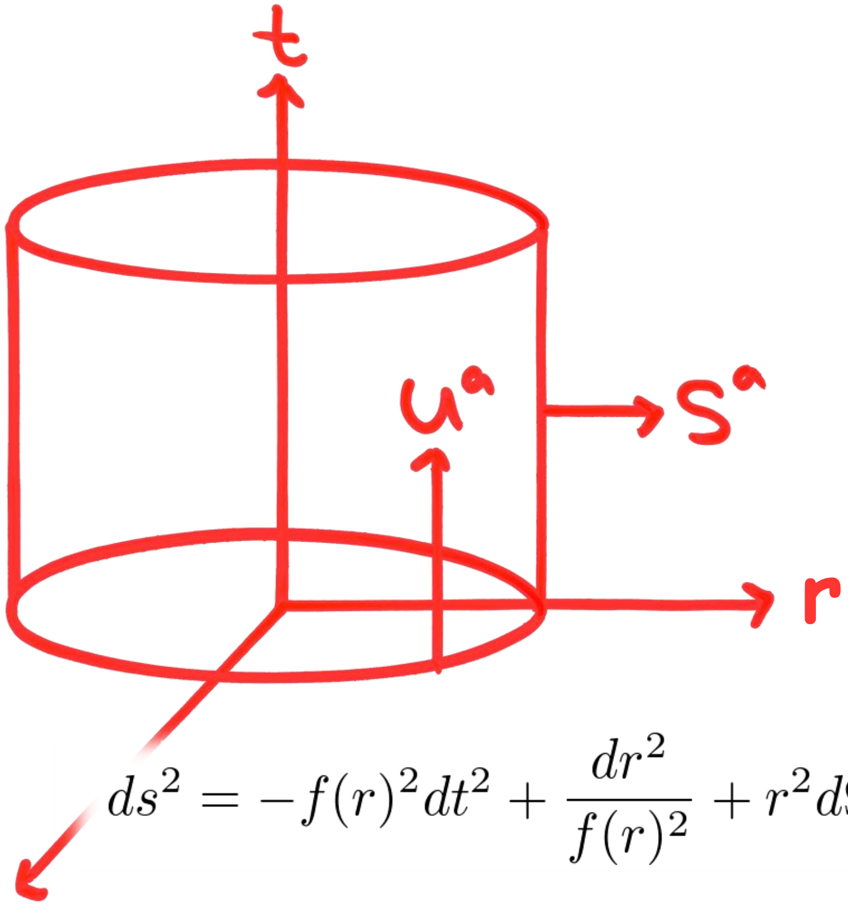
- Consider now a static screen in a Schwarzschild spacetime.

$$e = \frac{r - 2m}{4\pi r^2}, \quad p = \frac{m - r}{8\pi r^2}$$

- The equation of state is:

$$e(p) = \frac{4p\left(\sqrt{1 - 32\pi mp} + 32\pi mp - 1\right)}{\left(1 + \sqrt{1 - 32\pi mp}\right)^2}$$

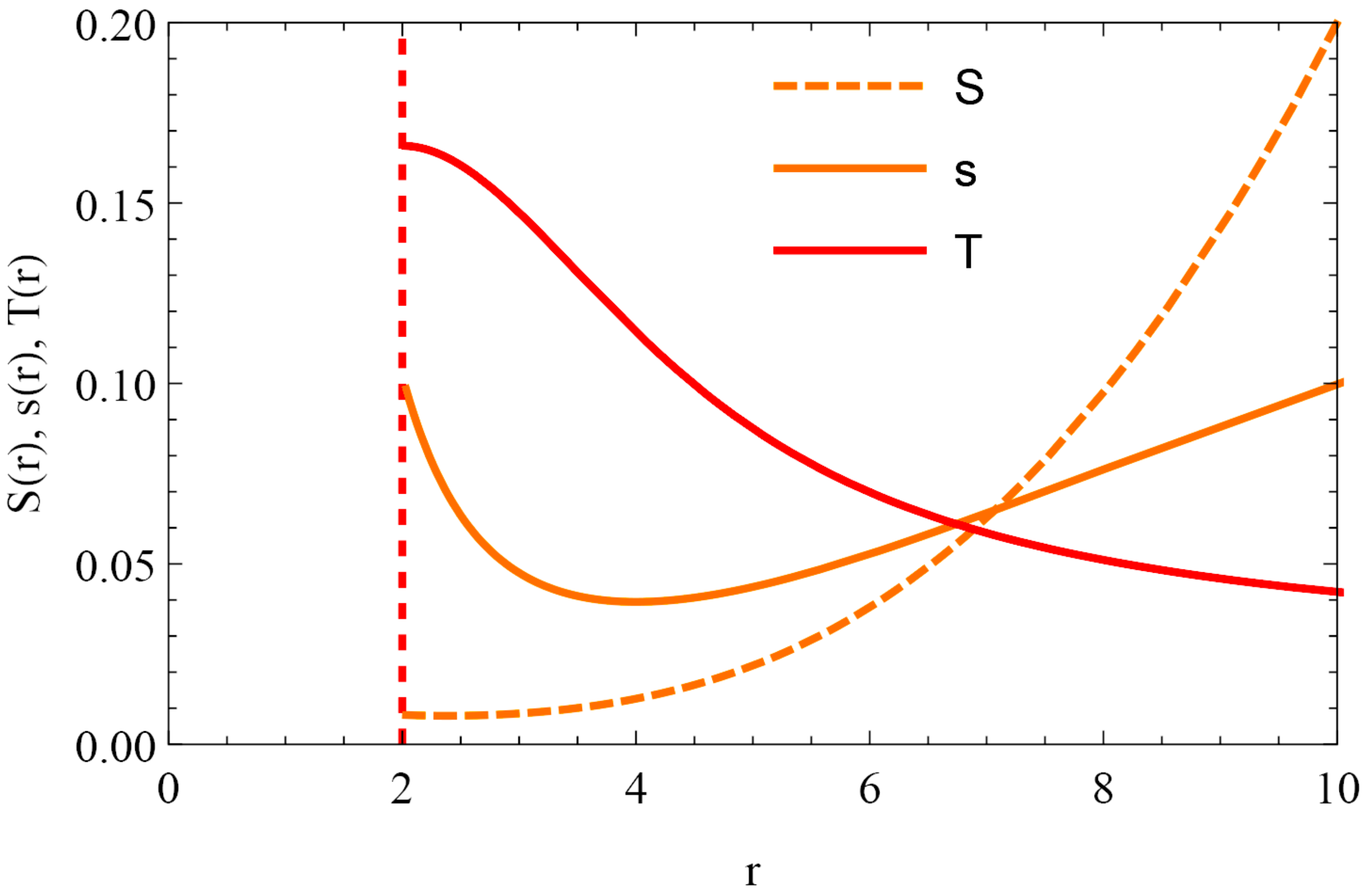
$$\approx -2p + 16\pi p^2 m + \mathcal{O}(m^2)$$



$$ds^2 = -f(r)^2 dt^2 + \frac{dr^2}{f(r)^2} + r^2 d\Omega^2$$

$$f(r) = \sqrt{1 - \frac{2m}{r}}$$

Static Screens



- Entropy density falls off *slower* than $1/r^2$.
- Total fluid entropy *increases* when more space is enclosed.

✓

Future Work

- Classify which types of screens and evolutions have holographic fluid descriptions that are *physical* (satisfy the laws of thermodynamics).

- When is a perfect fluid insufficient? When do we have to resort to a non-equilibrium description?

- Can the fluid dynamics capture all of the gravitational physics present inside the screen (eg. gravitational waves)?

Conclusions

- Einstein's equations projected onto a time-like hypersurface are equivalent to relativistic hydrodynamic conservation laws.

- This allows a holographic description of gravitational physics in a bulk region of spacetime in terms of a fluid on the boundary of that region.

- Provides a framework where the thermodynamic properties of gravity can be understood without restriction to spatial boundaries or null surfaces.

Questions?