To memory of

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On vacuum angle (in)dependence in Higgs regime

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Soon after introduction of the vacuum angle θ [Jackiw-Rebbi, Callan-Dashen-Gross, 1976] in QCD and Yang-Mills its UV origin was realized [Shifman-AV-Zakharov, 1980] Nonvanishing θ implies that CP is not conserved in strong interaction (modulus presence of massless fermions in the theory). To get rid of the problem the axion mechanism of screening θ was invented [Peccei-Quinn, 1977; Weinberg, 1978]. Another way out is a spontaneous breaking of CP invariance which provides a smallness of the effective θ [Nelson, Barr, 1984].

In the Standard Model based on the $\mathbf{SU(3)\times SU(2)\times U(1)}$ there is another vacuum angle $\theta_{\mathbf{EW}}$, associated with weak SU(2). Anselm and Johansen studied the problem in 1993.

Effects of θ_{EW} are exponentially suppressed, $\exp[-C/\alpha]$ — they involve instanton transitions. Moreover, Anselm and Johansen concluded that θ_{EW} is unobservable even at this level.

We'd like to address a similar issue of observability of vacuum angles in more general frameworks of Higgsed theories.

We then turn to our next question: θ observability in frameworks of grand unification theories.

$$G \to G_1 \times G_2 \times \ldots$$

Above the unification scale we have a single θ. Below this scale the vacuum angle for each group is calculable in terms of the original θ, it does not split into independent angles for each group. Different from Anselm-Johansen case! We will explain why.

Generalities: θ angle and its consequence

In any four-dimensional Yang-Mills theory based on the group G with nontrivial homotopy $\pi_3(G) = Z$ an additional parameter θ_G appears:

$$\Delta\mathcal{L}_\theta = \frac{\theta_G}{32\pi^2}\,\mathcal{F}^A_{\mu\nu}\widetilde{\mathcal{F}}^{\mu\nu,A}$$

This term is a total derivative so classically produces no effect. But nonperturbatively instanton transitions make it observable. Note that it works also in the Higgs regime at weak coupling.

In SM with the group $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ *a priori* we deal with two angles: θ_{QCD} associated with color SU(3) and θ_{EW} from the weak SU(2). No U(1) instantons!

While vacuum angles are certainly observable and bring in CP violation in purely bosonic theories introduction of fermion fields has a crucial impact.

$$\mathcal{L}_F = i\bar{\psi}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\psi^{\alpha} - \left[\psi^a_\alpha M_{ab}\psi^b_\beta\,\epsilon^{\alpha\beta} + \mathrm{H.c.}\right]$$

Extra U(1) symmetries:

$$\psi_P \longrightarrow \mathrm{e}^{\alpha_P}\psi_P$$

for each irreducible representation P. Due to chiral anomalies the fermion kinetic term is not invariant under this rotation and generate a shift of vacuum angle:

$$\theta_G \longrightarrow \theta_G - \sum_P 2T(P)\,\alpha_P \qquad\qquad \mathrm{Tr}\,(T^a T^b) = T(P)\delta^{ab}$$

For massless case it immediately shows that physics is θ-independent. Moreover, it is the case if there is a combination of phase rotations which keeps $\psi^a_\alpha M_{ab}\psi^{\bar b}_\beta\,\epsilon^{\alpha\beta}$ intact.

If no combination of phase rotation keeps mass term invariant then physics depends on the effective angle:

$$\bar{\theta}_G = \theta_G + \arg \mathrm{Det}\, M_{ab}$$

θ-independence without massless fermions and axions

The standard model case, SU(2)

The first generation of SM contains 4 SU(2) doublets:

$$\begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}, \quad \begin{pmatrix} u^i_\alpha \\ d^i_\alpha \end{pmatrix}, \quad i = 1, 2, 3$$

and 8 singets:

$$\nu^c_\alpha\,,\ e^c_\alpha\,,\ u^c_{\alpha,\,i}\,,\ d^c_{\alpha,\,i}$$

which include right-handed neutrino allowing for a Dirac mass to neutrino. Masses come from the Yukawa couplings:

$$\mathcal{L}_{\mathrm{Y}} = h_{AB}\, \bar{\phi}_k L^{k,A}_\alpha S^B_\beta \epsilon^{\alpha\beta} + \tilde{h}_{AB}\, \epsilon_{ik}\phi^i L^{k,A}_\alpha S^B_\beta \epsilon^{\alpha\beta} + \mathrm{H.c.}$$

The phase rotations of singlets is not anomalous and can be chosen to be opposite to the rotations of doublets. Keeping the Higgs field intact we have $\mathcal{L}_{\mathrm{Y}}$ invariant what proofs θ-independence.

Although electroweak instantons are suppressed it is interesting a a matter of principle. Besides could be relevant for early cosmological time. The 't Hooft interaction generated by electroweak instantons is:

$$\Delta\dot{\mathcal{L}}_{\mathrm{inst}} \propto \prod_{A} L_{k}^{k,A}\, e^{-i\theta} \exp\Big(-\frac{8\pi^2}{g^2}\Big)$$

Clearly, one can eliminate $\exp(-i\theta)$ by redefining doublets fields. Thus, we see that in SM θ_{EW} is unobservable. The relevant U(1) is B+L which is anomalous, B-L is not. The above considerations are due by Anselm and Johansen.

Generalization to SU(N)

Consider SU(N) gauge theory with N with N complex Higgs fields ϕ^i_a, each in the gauge fundamental. Prior adding fermions the model Lagrangian is

$$\mathcal{L}_B = -\frac{1}{4g^2}\mathcal{F}^A_{\mu\nu}\mathcal{F}^{\mu\nu,A} + \mathcal{D}^\mu\bar{\phi}^a\,\mathcal{D}_\mu\phi_a - V(\phi)$$

$$A = 1, ..., N^2 - 1$$

$$V = \lambda_1 \sum_A \Big|\sum_a \bar{\phi}^a T^A \phi_a\Big|^2 + \lambda_2 \Big|\sum_a \bar{\phi}^a\phi_a - Nv^2\Big|^2$$

$$= \frac{\lambda_1}{2}\sum_{a,b}\Big[\bar{\phi}^a\phi_b\,\bar{\phi}^b\phi_a - \frac{1}{N}\bar{\phi}^a\phi_a\,\bar{\phi}^b\phi_b\Big] + \lambda_2\Big|\sum_a \bar{\phi}^a\phi_a - Nv^2\Big|^2$$

It is minimized by

$$\langle\phi^i_a\rangle = \mathrm{e}^{i\gamma}\,v\,\delta^i_a$$

N^2-1 gauge bosons are Higgsed with mass $M = gv$.

One of Higgs field s remains massless: U(1) symmetry

$$\phi \;\to\; \exp(i\alpha)\phi$$

One way to make it massive is to add

$$-\lambda_3\,\phi^{i_1}_{a_1}\,\phi^{i_2}_{a_2}\ldots\phi^{i_N}_{a_N}\,\varepsilon^{a_1 a_2\ldots a_N}\,\varepsilon_{i_1 i_2\ldots i_N}+\mathrm{H.c.}=-\mathrm{Re}\,\lambda_3\,\mathrm{Det}\,\phi$$

Then $\gamma = 0$. Both gauge and global flavor SU(N) are spontaneously broken, diagonal SU(N) survive.

Now let us add fermions: Dirac Ψ^i in gauge fundamental, i.e., two left-handed Weyl spinors

$$\Psi^i \longrightarrow \chi^i_\alpha\,,\quad \tilde\chi^\alpha_i, \qquad \alpha=1,2,\quad i=1,\ldots,N$$

together with two gauge singlets which are fundamental and anti fundamental of global SU(N)

$$\eta^\alpha_a\,,\ \tilde\eta^a_\alpha\,, \qquad \alpha=1,2,\quad a=1,\ldots,N$$

Next Yukawa terms:

$$\mathcal{L}_{\mathrm{Y}}=h_1\,\chi^i\bar\phi^a_i\,\eta_a+h_2\,\tilde\chi_i\phi^i_a\,\tilde\eta^a+\mathrm{H.c.}$$

With $\langle \phi^i_a \rangle = v\,\delta^i_a$ all 2N fermions become massive.

Yukawa terms are invariant under

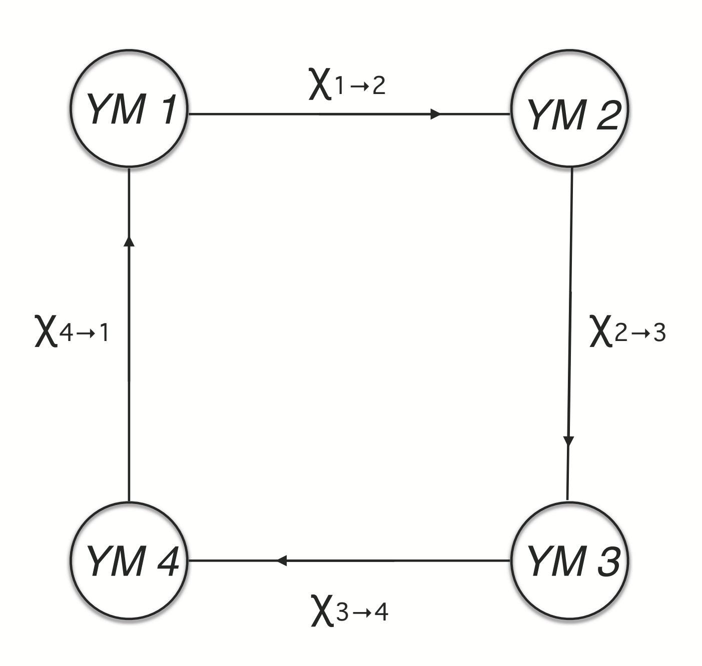
$$\chi^i_\alpha \to e^{i\alpha}\chi^i_\alpha\,, \quad \tilde{\chi}^\alpha_i \to e^{i\tilde{\alpha}}\tilde{\chi}^\alpha_i\,, \qquad \eta^\alpha_a \to e^{-i\alpha}\eta^\alpha_a\,, \quad \tilde{\eta}^a_\alpha \to e^{-i\tilde{\alpha}}\tilde{\eta}^a_\alpha$$

Anomaly for nonsinglets then gives .

$$\theta \longrightarrow \theta - N(\alpha + \tilde{\alpha})$$

what shows that θ is unobservable.

Quiver generalizations



$$\mathrm{SU}(N)\times\mathrm{SU}(N)\times\mathrm{SU}(N)\times\mathrm{SU}(N)$$

Grand unification and θ-dependence

SU(5) unification. Fermions of one generation are in one decuplet $X_\alpha^{[ab]}$ and one antiquintet $\overline{V}_{\alpha,a}$, $a = 1, 2, ..., 5$

SU(5) is broken to $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ at large UV scale v_{G} by an adjoint Higgs field. 12 superheavy bosons with masses $\sim g v_{\mathrm{G}}$. Then another Higgs field φ^a in the fundamental rep gives $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)} \to \mathrm{SU(3)} \times \mathrm{U(1)}$ at the electroweak scale v_{EW}.

Two Yukawa terms are responsible for fermion masses

$$\lambda_1 \epsilon^{\alpha\beta} X_\alpha^{[ab]} \, \overline{V}_{\beta,a} \bar{\varphi}_b \, , \quad \lambda_2 \epsilon^{\alpha\beta} X_\alpha^{[ab]} X_\beta^{[cd]} \varphi^f \varepsilon_{abcdf} \, .$$

At the scales above v_{G} we have an unique θ - term. With complex couplings it is $\bar{\theta} = \theta + \arg(\lambda_1 \lambda_2)$. There is no phase rotations which leave Yukawa terms intact and change θ.

Thus, $\bar{\theta}$ is in principle observable in SU(5). [Rubakov had a similar conclusion in 2016]

What is different with SM in SU(2) corner? In SU(5) there is a perturbative amplitude with $\Delta B = \Delta L = \pm 1$ due to Higgs exchange. It is proportional to $\lambda_1 \lambda_2$ and interferes with instanton induced amplitude,

$$\left(\boldsymbol{X}^{[ab]} \boldsymbol{X}^{[cd]} \boldsymbol{X}^{[fg]} \boldsymbol{V}_g \, \varepsilon_{abcdf} \right) \, e^{-i\theta} \, \exp\left(- \frac{8\pi^2}{g^2} \right)$$

Interference contains the combination

$$\mathrm{Re}\left(\lambda_1 \lambda_2 \mathrm{e}^{i\theta} \right) = |\lambda_1||\lambda_2| \mathrm{e}^{i\bar{\theta}}$$

For scales μ below v_{G} we deal with different running in SU(3) and SU(2) corners.

Conclusions

The θ angle implications crucially depend on the UV structure of the theory. Depending on details, θ independence can emerge without massless fermions and/ or axions.

We generalize Anselm-Johansen example to a broad class of theories.

Unification at high scale results in case of SU(5) in observable θ- related effects in the SU(2) corner. Under spontantenous breaking $G \to G_1 \times G_2 \times ...$ subgroups inherit one and the same θ .