Towards understanding the origin of the microstates of large black holes

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References

Two groups of original papers by combinations of the authors:

The long string at the stretched horizon and the entropy of large non-extremal black holes arXiv:1505.04025
The thermal scalar and random walks in curved spacetime arXiv:1605.02785

Edge State Quantization: Vector Fields in Rindler arXiv:1801.09910
Edge Dynamics from the Path Integral: Maxwell and Yang-Mills arXiv:1804.07585

Some Background

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Beginning of the story was highlighted in the Opening Talk
Entropy for a classical solution is "naturally zero" (T=0)
If, however,

$$S_{BH} = \frac{(Area)_{horizon}}{4G_N}$$

then one is invited to look for microstates which make up the BH

Explicit solution was found for extremal BH, with $(Area)_{horizon} = 0$, in SUSY theory. Entropy of BH is given by quantum degeneracy of the lowest state.

Otherwise, it remains an open problem till now

Zero modes

Natural approach: look for zero modes which "connect" two neighboring solutions. Could be responsible for the entropy. However, the BH solution at large distances is fixed by conserving quantum numbers and conclude that there is no entropy enough, (basing on the knowledge of large distances.)

A way out could be zero modes confined to the distances "very close" to the horizon, or Soft hair
Seems, leading idea, with recent progress substantial.

Stretched horizon

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There is another problem: finiteness of S_{BH}.
In field theory, rather expect infinity.

$$ds^2 = -\left(1 - \frac{2G_N M}{r}\right)^{-1} dt^2 + \left(1 - \frac{2G_N M}{r}\right) dr^2 + d\mathbf{x}_\perp^2$$

Define $\rho \;=\; \sqrt{8G_N M(r - 2G_N M)}$

$$\beta_{local} \;=\; \beta_{infty}(4G_N M/\rho)$$

In field theory,

$$S_{matter\;\;falling} \;\sim\; \int d\rho T^3 \;\sim\; \frac{(Area)}{\epsilon^2}$$

Cannot allow $S_{matter\;\;falling} \;>\; S_{BH}$ ('t Hooft)
Need $\epsilon \;\sim\; (new - physics\;scale)$, or streched horizon

Conclusions to Introduction

Probably, need stretched horizon and edge states living there

Will consider two examples, stringy and field theoretic.
The stringy example is within "long-string" picture of Susskind

Field-theoretic example is electrostitic edge states living on the horizon, close to Strominger, Hawking+Perry+Strominger

Long string of L. Susskind

It takes long time for a string to reach horizon. Meantime string percolates in transverse directions (a la parton model) and becomes a "long string" near the horizon

Long string carries many degrees of freedom and might match S_{BH}. The idea is not immediately supported by equations.

Hagedorn phase transition

Even in flat space at high temperature strings become long and undergo a percolation transition at the Hagedorn temperature, β_H

$$\rho(E) \;\sim \exp\left(\beta_H E\right)$$

The two mechanisms are unified in theory of "thermal scalar" in Rindler space.

Thermal scalar

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In flat Euclidean space-time the Hagedorn transition manifested as vanishing of mass of once-wrapped-around-time mode:

$$M_{TS}^2 = \frac{\beta^2}{4\pi^2\alpha'^2} - \frac{4}{\alpha'} = \frac{1}{4\pi^2\alpha'^2}(\beta^2 - \beta_{Hagedorn}^2)$$

The $w = \pm 1$ modes form a complex scalar ϕ which is an order parameter for the percolation transition and becomes tachyonic above the Hagedorn temperature $T_{Hag} = \frac{1}{4\pi\sqrt{\alpha'}}$ (bosonic string).

Thermal scalar in Rindler space

In type IIB superstring theory in Rindler space, we simply replace $\beta^2 \to \beta^2 G_{00}$ because of redshift:

$$M_{TS}^2 = \frac{\beta^2 G_{00}}{4\pi^2\alpha'^2} - \frac{2}{\alpha'} = \frac{\beta^2 a^2 \rho^2}{4\pi^2\alpha'^2} - \frac{2}{\alpha'}$$

where we used the Rindler metric:

$$ds^2 = a^2\rho^2 dt^2 - d\rho^2 - dX_\perp^2$$

Eigenmodes of the thermal scalar in Rindler space

$$\Big[-\partial_\rho^2 - \frac{1}{\rho}\partial_\rho + \frac{\beta^2 a^2 \rho^2}{4\pi^2\alpha'^2} - \frac{2}{\alpha'}\Big]\phi_n(\rho) = \lambda_n \phi_n(\rho)$$

$$\begin{aligned} \phi_n(\rho) &= \exp\Big(-\frac{a\beta\rho^2}{4\pi\alpha'}\Big) L_n\Big(\frac{a\beta\rho^2}{2\pi\alpha'}\Big) \\ \lambda_n &= \frac{1}{\pi\alpha'}\Big[a\beta(1+2n) - 2\pi\Big] \end{aligned}$$

Percolating strings in Rindler space: $T_{Hag} = T_{Unruh}$

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Thermal scalar contribution to 1 string partition function:

$$Z_{1TS} = A_H \int_0^\infty \frac{ds}{s} \sum_{n=0}^{\infty} e^{-s\lambda_n} \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}} e^{-sk_\perp^2}$$

Lowest mode dominates the infrared behavior:

$$\phi_0(\rho) = e^{-\frac{a\beta\rho^2}{4\pi\alpha'}} \qquad \lambda_0 = \frac{1}{\pi\alpha'}(a\beta - 2\pi)$$

so that

$$Z_{1TS} \approx A_H \int_0^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{D-2}{2}}} e^{-s\lambda_0} \approx A_H \int_0^\infty \frac{ds}{s} \frac{1}{(4\pi s)^{\frac{D-2}{2}}} e^{-\frac{s}{\pi\alpha'}[a\beta - 2\pi]}$$

which diverges when $\beta < \frac{2\pi}{a} = \beta_{Unruh}$ or $T_{Hag} = T_{Unruh}$

Hagedorn density of states in Rindler space?

Introducing the Rindler energy variable $E = \frac{sa}{\pi\alpha'}$, the 1 string partition function becomes:

$$Z_{1TS} \sim A_H \Big(\frac{a}{4\pi^2\alpha'^2}\Big)^{\frac{D-2}{2}} \int_0^\infty \frac{dE}{E^{\frac{D}{2}}}\ e^{\frac{2\pi}{a}E} e^{-\beta E} = \int_0^\infty dE\ \rho(E) e^{-\beta E}$$

or

$$\rho(E) = A_H \Big(\frac{a}{4\pi^2\alpha'^2}\Big)^{\frac{D-2}{2}} \frac{1}{E^{\frac{D}{2}}} e^{\frac{2\pi}{a}E} = A_H \Big(\frac{a}{4\pi^2\alpha'^2}\Big)^{\frac{D-2}{2}} \frac{1}{E^{\frac{D}{2}}} e^{\beta_{Unruh} E}$$

Hagedorn density of states for large black hole?

$$ds^2 \sim (\frac{\rho}{4GM})^2 dt^2 - d\rho^2 - dX_\perp^2 \rightarrow \beta_{Hag} = \beta_{Unruh} = 8\pi GM = \beta_{Hawking}$$

E = Rindler energy = energy at ∞

Bekenstein Hawking entropy and the stretched membrane

Let's build up large black hole in layers of δM:
The number of ways the layer can be added is:

$$\rho(\delta M) \sim e^{8\pi G M \delta M}$$

which represents a microscopic entropy of:

$$S(\delta M) = \delta S = 8\pi G M \delta M$$

and gives a total black hole entropy in accordance with the area law of Bekenstein-Hawking:

$$\begin{aligned} S(M) &= \int \delta S = \int 8\pi G M \delta M \\ &= 4\pi G M^2 = \frac{A}{4G} \end{aligned}$$

Conclusions to thermal scalar in Rindler space

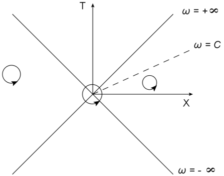
At least formally, the BH entropy is reproduced with no free parameter. Physical picture is also quite transparent:

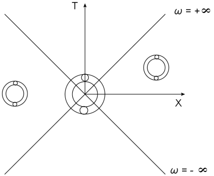
Where do the black hole microstates live?
As the layer with mass δM drops to the horizon, it forms a long string gas with energy and pressure distribution determined by the thermal scalar zeromode:

$$\phi_0(\rho) = e^{-\frac{a\beta_{Unruh}\rho^2}{4\pi\alpha'}} = e^{-\frac{\rho^2}{2\alpha'}}$$

The radial pressure is proportional to ϕ_0^2 and sustains a long string atmosphere of width $\approx \sqrt{\alpha'} = l_s$ which constitutes a stringy version of the stretched membrane.

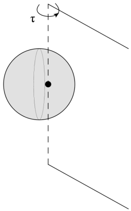
Black hole entropy , entanglement and edge states in string theory

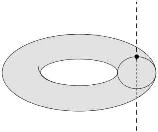




A fiducial observer outside the large black hole sees the vacuum as a thermal density matrix so that for large black holes (Rindler space) thermal entropy is entanglement entropy. This entanglement entropy is a one loop quantity described by paths encircling the origin. In string theory they are genus 1 torus graphs encircling the origin. This entanglement entropy has a clear state counting interpretation.

Black hole entropy , entanglement and edge states in string theory





Genus zero (sphere) string graphs induce the classical Einstein-Hilbert action (Tseytlin). Sphere graphs that contain the origin describe open strings frozen at the horizon. They are edge states that arise because the horizon cuts the closed string.

Genus one (torus) string graphs containing the origin describe open strings at the horizon that emit closed strings. They describe entanglement between edge states and have no clear state counting interpretation (negative contribution). They arise because of non- factorisation of Hilbert space.

Entanglement of edge states in gauge theories

Because of Gauss' law, the physical Hilbert space in gauge theories does not factorize. A Wilson line cut by a surface S is not invariant under large gauge transformations that do not vanish at S. To restore gauge invariance, one introduces charges at the cut ends, the edge states:

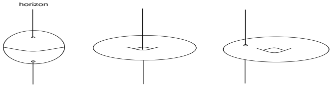
$$\mathcal{H}_L = \mathcal{H}_{\text{edge},L} \otimes \mathcal{H}_{\text{bulk},L}, \qquad \mathcal{H}_R = \mathcal{H}_{\text{edge},R} \otimes \mathcal{H}_{\text{bulk},R}.$$

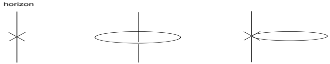
The physical Hilbert space is then embedded in the factorisable extended Hilbert space:

$$\mathcal{H}_{phys} \subset \mathcal{H}_{\text{ext}} = \mathcal{H}_L \otimes \mathcal{H}_R.$$

Gauss' law entangles the edge states by matching the charges:

$$\mathcal{H}_{phys} = \bigoplus_{q \in \mathcal{H}_{\text{edge}}} (q_L \otimes \mathcal{H}_{\text{bulk},L}) \otimes (q_R \otimes \mathcal{H}_{\text{bulk},R}).$$





$$\beta F = \int d^d x \sqrt{g} \mathcal{L}_{\text{eff}}$$

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \int_{\epsilon^2}^{\infty} \frac{ds}{(4\pi s)^{d/2}} e^{-sm^2} \left(\frac{c_0}{s} + c_1 R + \mathcal{O}(s) \right)$$

$$c_1 = \begin{cases} 1/6 & \text{minimally coupled scalar} \\ 1/12 \quad 2^{[d/2]} & \text{Dirac fermion (with } 2^{[d/2]} \text{ components)} \\ (d-2)/6 - 1 & \text{abelian gauge field (negative contact term)} \end{cases}$$

Rindler quantisation of Maxwell and entanglement entropy of edge states

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Maxwell in Lorentz gauge in Rindler space:

$$S_M = \int_{\mathcal{M}} d^{d+1}x\sqrt{-g}\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\nabla_\mu A^\mu \nabla_\nu A^\nu - \partial^\mu \bar{c}\partial_\mu c\right),$$

Variation of action results in boundary conditions ($\partial\mathcal{M} = \partial\Sigma$time):

$$n_\mu \frac{\partial \mathcal{L}}{\partial\partial_\mu A^R_\nu}\delta A^R_\nu\Big|_{\partial\Sigma}(\mathbf{x}) = 0.$$

This is satisfied by PEC or PMC boundary conditions. PMC in Rindler space turn out to be most natural and determine the bulk Hilbert space:

$$\mathbf{E}_\perp|_{\partial\mathcal{M}}(\mathbf{x}) = \mathbf{0}, \quad \rho\mathbf{B}_{||}|_{\partial\mathcal{M}}(\mathbf{x}) = \mathbf{0}, \quad A^\rho|_{\partial\mathcal{M}}(\mathbf{x}) = 0$$

Rindler quantisation of Maxwell and entanglement entropy of edge states

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How ensure that Wilson line algebra is extended to horizon?

$$\boxed{\Phi_\Omega \mathcal{W}_\mathcal{C} = \mathcal{W}_\mathcal{C}\theta(\Omega \cap \mathcal{C}), \quad \forall \Omega, \mathcal{C}},$$

or for $\mathcal{C} \perp$ horizon and $\Omega \subset$ horizon:

$$\int_\mathcal{C} A\Phi_\Omega = i\theta(\mathcal{C} \cap \Omega),$$

On the horizon, only the $\omega = 0$ mode $A_{\mathbf{k}}^{(1)}$ has non zero radial electric flux and the large gauge $A_{\mathbf{k}}^{(G)}$ a non zero radial Wilson line, so that:

$$A_{edge} = -\sum_{\mathbf{k}} \left(\frac{1}{k} q_{\mathbf{k}} A_{\mathbf{k}}^{(1)} + k a_{\mathbf{k}} A_{\mathbf{k}}^{(G)} \right),$$

The relevant components for the commutator are:

$$A_\rho = -\sum_{\mathbf{k}} a_{\mathbf{k}} \partial_\rho \phi_{\mathbf{k}}, \quad E^\rho = \sum_{\mathbf{k}} q_{\mathbf{k}} \phi_{\mathbf{k}}$$

Rindler quantisation of Maxwell and entanglement entropy of edge state

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With the probability distribution $p(E_\perp) = e^{-2\pi E(E_\perp)}/Z_{\text{edge}}$, the edge entanglement entropy $S_{Edge} = -\text{Tr}\rho_R^{Edge} \log \rho_R^{Edge}$can be written as

$$S_{\text{edge}} = -\int [\mathcal{D}E_\perp(\mathbf{x})]\, p(E_\perp) \ln p(E_\perp).$$

and reproduces the Kabat contact term contribution. It has a state counting interpretation as it counts the number of classical horizon edge states. In the $\epsilon \to 0$ limit, the Rindler energy $E(E_\perp)$ becomes zero and the edge states are zero energy static coherent photon states. The Rindler vacuum is infinitely degenerate and the edge states provide soft Maxwell hair to the horizon. This degeneracy suggests a natural interpretation of the edge states as black hole microstates.

Conclusions

There is progress in understanding edge states which might be responsible for the BH entropy

On the other hand, inclusion of matter edge states brings well known problems as (how many fields one should take into account)

Edge states have also other applcations, (more mathematical in nature)