QCD at high energies and Yangian symmetry

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My work with Lev N. Lipatov

first contact at LIYaF in 1978
start of joint work in 1981
some results

- QCD amplitudes in double-logs, 1983.
  "Double Logarithmic Asymptotics and Regge Singularities of Quark Amplitudes with Flavor Exchange"
- QCD Regge asymptotics, branch cuts and poles, 1989.
  "Bare Reggeons in Asymptotic Free Theories"
- QCD Regge asymptotics, effective action approach, 1993.
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Integrability in QCD discovered by L.N. Lipatov in 1993:
BFKL kernel is related to an integrable spin chain
higher symmetry (based on Yangians of sl_n type)

- in operator products, anomalous dimension

Braun, Derkachov, Manashov, 1998 ;

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- in Wilson loop expectation values

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Yangian symmetric correlators (YSC)

$$\Phi(\mathbf{x}_1, ..., \mathbf{x}_N), \mathbf{x}_i = (x_i^1, ..., x_i^n)$$

symmetry generated by $L_{ab} = x_a \partial_b, a, b, = 1, .., n \ (g\ell_n)$
L-operator $L(u)_{ab} = (u+1)\delta_{ab} + L_{ab}$.

at each point $\mathbf{x}_i$: $L_{i,ab} = x_{i,ab} \partial_{i,ab}, i = 1, ..., N$
scaling weights: $(\mathbf{x}_i \partial_i)\Phi = 2\ell_i \Phi$

YSC condition:

$$T_{ab}(\mathbf{u})\Phi = E(\mathbf{u})\delta_{ab}\Phi$$

monodromy matrix:

$$T(\mathbf{u}) = L_1(u_1, u_1^+)...L_N(u_N, u_N^+)$$

$$\mathbf{u} = \begin{pmatrix} u_1 & \dots & u_N \\ u_1^+ & \dots & u_N^+ \end{pmatrix}$$

work with D. Chicherin and S. Derkachov, 2013

Generic link integral forms (case $N = 4$)
in positions (Witten twistors)

$$\Phi(\mathbf{x}_1, ..., \mathbf{x}_4) = \int d^4 c \varphi(c) \delta^{(n)}(\mathbf{x}_1 - c_{13}\mathbf{x}_3 - c_{14}\mathbf{x}_4)\delta^{(n)}(\mathbf{x}_2 - c_{23}\mathbf{x}_3 - c_{24}\mathbf{x}_4)$$

in helicities $n = 2m$

$$\Phi(\bar{\lambda}_1, \lambda_1, ..., \bar{\lambda}_4, \lambda_4) = \int d^4 c \varphi(c)$$

$$\delta^{(m)}(\bar{\lambda}_1 - c_{13}\bar{\lambda}_3 - c_{14}\bar{\lambda}_4)\delta^{(m)}(\bar{\lambda}_2 - c_{23}\bar{\lambda}_3 - c_{24}\bar{\lambda}_4)$$
$$\delta^{(m)}(\lambda_3 + c_{13}\lambda_1 + c_{23}\lambda_2)\delta^{(m)}(\lambda_4 + c_{14}\lambda_1 + c_{24}\lambda_2)$$

Particular solutions used in the following
$N = 4$

$$(\varphi_X(c))^{-1} = c_{13}^{1+u_4-u_3}(c_{14}c_{23}-c_{13}c_{24})^{1+u_1^+-u_4}c_{24}^{1+u_2-u_1}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1^+ & 2^+ \end{pmatrix}$$

$$(\varphi_=(c))^{-1} = c_{13}c_{24}c_{23}^{u_2-u_1}(c_{14}c_{23}-c_{13}c_{24})^{1+u_1^+-u_3}c_{14}^{u_3-u_4}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2^+ & 1^+ \end{pmatrix}$$

$N = 3$

$$\varphi^{-++} = \left(c_{12}^{1+u_3-u_2} c_{13}^{1+u_1^+-u_3} \right)^{-1}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1^+ \end{pmatrix}$$

$$\varphi^{--+} = \left(c_{23}^{1+u_2^+-u_1^+} c_{13}^{1+u_1^+-u_3} \right)^{-1}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1^+ & 2^+ \end{pmatrix}$$

YSC - relations to QCD

- $n = 4, \bar{\lambda}_{i,\dot{\alpha}}, \lambda_{i,\alpha}$ $\ell_i + 1 =$ helicity $\rightarrow$ QCD tree amplitudes
- $n = 2, k_i = \bar{\lambda}_i \lambda_i, l_i + \frac{1}{2} =$ parton helicity $\rightarrow$ DGLAP kernels
- $n = 2, x_i = \frac{x_{i,1}}{x_{i,2}} \rightarrow$ ERBL/DGLAP kernels for GPD
- $n = 2, x_i = \frac{x_{i,1}}{x_{i,2}}, \ell_i \rightarrow 0 \rightarrow$ BFKL kernel
- convolution construction of YSC $\rightarrow$ Gell-Mann-Low coeff. in $\alpha_S(Q^2)$

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QCD amplitudes

$$\Phi_{hel} = \varphi(c^*, h_1, h_2, \varepsilon)\delta(\sum k_i)$$

$$k_{i,\dot{\alpha}\alpha} = \bar{\lambda}_{i,\dot{\alpha}}\lambda_{i,\alpha}, 2\ell_i = u_i^+ - u_i, 2h_i = 2\ell_i + 2$$

$$< ij >= \lambda_{i,1}\lambda_{j,2} - \lambda_{i,2}\lambda_{j,1}, \quad [ij] = \lambda_{i,1}\lambda_{j,2} - \lambda_{i,2}\lambda_{j,1}$$

$$c_{13}^* = \frac{[14]}{[34]} = -\frac{< 32 >}{< 12 >}, c_{14}^* = \frac{[13]}{[43]} = -\frac{< 42 >}{< 12 >},$$

$$c_{23}^* = \frac{[24]}{[34]} = -\frac{< 31 >}{< 21 >}, c_{24}^* = \frac{[23]}{[43]} = -\frac{< 41 >}{< 21 >}.$$

$$c_{14}^* c_{23}^* - c_{13}^* c_{24}^* = -\frac{[12]}{[34]} = \frac{< 43 >}{< 12 >}$$

$$\varphi_X(c^*; h_1, h_2, \varepsilon) = \left(\frac{[14][23]}{[12][34]}\right)^{\varepsilon} \frac{[12]^{1+2h_1}[34]^{1-2h_2}}{[14][23]^{1+2h_1-2h_2}}$$

$$\varphi_=(c^*; h_1, h_2, \varepsilon) = \left(\frac{[13][24]}{[12][34]}\right)^{\varepsilon} \frac{[12]^{1+2h_1}[34]^{1-2h_2}}{[14][23][24]^{2h_1-2h_2}}$$

gluon-gluon

$$M(1,1,-1,-1) = \varphi_X(c^*,1,1,0),\ \ M(-1,-1,1,1) = \varphi_X(c^*,-1,-1,0),$$
$$M(1,-1,-1,1) = \varphi_X(c^*,1,-1,4), M(-1,1,1,-1) = \varphi_X(c^*,-1,1,0),$$

$$M(1,1,-1,-1) = \varphi_=(c^*,1,1,0),\ \ M(-1,-1,1,1) = \varphi_=(c^*,-1,-1,0),$$
$$M(1,-1,1,-1) = \varphi_=(c^*,1,-1,4), M(-1,1,-1,1) = \varphi_=(c^*,-1,1,0).$$

quark - quark

$$M(\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2}) = \varphi_X(c^*,\tfrac{1}{2},\tfrac{1}{2},0),$$

$$M(\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2}) = \varphi_X(c^*,\tfrac{1}{2},-\tfrac{1}{2},3),\;\; M(-\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}) = \varphi_X(c^*,-\tfrac{1}{2},\tfrac{1}{2},1)$$

$$M(\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2}) = \varphi_=(c^*,\tfrac{1}{2},\tfrac{1}{2},0),$$

$$M(\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}) = \varphi_=(c^*,\tfrac{1}{2},-\tfrac{1}{2},3),\;\; M(-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2}) = \varphi_=(c^*,-\tfrac{1}{2},\tfrac{1}{2},1)$$

quark-gluon

$$M(1,\tfrac{1}{2},-1,-\tfrac{1}{2}) = \varphi_X(c^*,1,\tfrac{1}{2},1), \quad M(1,-\tfrac{1}{2},-1,\tfrac{1}{2}) = \varphi_X(c^*,1,-\tfrac{1}{2},3).$$

$$M(1,\tfrac{1}{2},-\tfrac{1}{2},-1) = \varphi_=(c^*,1,\tfrac{1}{2},1), \quad M(1,-\tfrac{1}{2},\tfrac{1}{2},-1) = \varphi_=(c^*,1,-\tfrac{1}{2},3).$$

selection of YSC by asymptotics or analytic structure

Parton kernels

$$\Phi_{5,2} = \delta^{(4)}(\sum_{1}^{5} \lambda_{k,\alpha}\bar{\lambda}_{k,\dot{\alpha}}) \prod_{1}^{5} < i-1, i >^{-1+2h_{i-2}+2h_{i+1}}$$

$\sum_1^5 h_i = 1.$

work with G. Savvidy, 2016

collinear limit

in $M_{5,2}$ $<i,i+1>\to 0$

$$k_i \to zp,\;\; k_{i+1} \to (1-z)p,\;\; \lambda_i \to \sqrt{z}\lambda_p,\;\; \lambda_{i+1} \to \sqrt{1-z}\lambda_p,$$

.

$$<i-1,i>\to<i-1,p>\sqrt{z},\;\; <i+1,i+2>\to<p,i+2>\sqrt{1-z}$$

$$M_{5,2} \to \tilde{M}_{4,2} <i,i+1>^{-1} Split(h_p,h_i,h_{i+1})$$

$$Split(h_p,h_i,h_{i+1}) = \frac{\sqrt{z(1-z)}}{z^{\eta h_i}(1-z)^{\eta h_{i+1}}}$$

$\eta = h_i + h_{i+1} + h_p = \pm 1$

Parton kernels from split factors

$$P^{h_C}_{h_B h_A}(z) = (Split(h_B, h_C, h_A))^2 = z^{1-2\eta h_B}(1-z)^{1-2\eta h_C}, A \to B + C$$

$$w_h^{-h'}(z) = \sum_{h''}(Split(h', h'', h))^2$$

gluon-gluon:

$$w_1^1 = z(1-z)[z^{-1}(1-z)^{-1}]^2 + z(1-z)[z^{+1}(1-z)^{+1}]^2 = \frac{1+z^4}{z(1-z)}$$

$$w_1^{-1} = z(1-z)[z^{-1}(1-z)^{+1}]^2 = \frac{(1-z)^3}{z}$$

quark-quark:

$$w_{\frac{1}{2}}^{\frac{1}{2}} = z(1-z)[z^{-\frac{1}{2}}(1-z)^{-\frac{1}{2}}]^2 + z(1-z)[z^{+\frac{1}{2}}(1-z)^{+\frac{1}{2}}]^2 = \frac{1+z^2}{1-z}$$

gluon-quark:

$$w_1^{\frac{1}{2}} = z(1-z)[z^{\frac{1}{2}}(1-z)^{-\frac{1}{2}}]^2 = z^2$$

$$w_1^{-\frac{1}{2}} = z(1-z)[z^{-\frac{1}{2}}(1-z)^{+\frac{1}{2}}]^2 = (1-z)^2$$

quark-gluon:

$$w_{\frac{1}{2}}^{1} = z(1-z)[z^{-1}(1-z)^{-\frac{1}{2}}]^2 = z^{-1}$$

$$w_{\frac{1}{2}}^{-1} = z(1-z)[z^{-1}(1-z)^{+\frac{1}{2}}]^2 = \frac{(1-z)^2}{z}$$

Kernels for hypothetical partons of other helicities are obtained too.

Split amplitude and 3-point YSC at $n = 2$

with the monodromy characterized by

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1^{+} \end{pmatrix}$$

in helicity form is ($k_i = \lambda_i \bar{\lambda}_i$)

$$\Phi_3^{-++,\lambda} = \int \mathrm{d}^2 c\, \varphi^{-++}\, \delta(\bar{\lambda}_1 - c_{12}\bar{\lambda}_2 - c_{13}\bar{\lambda}_3)\delta(\lambda_2 + c_{12}\lambda_1)\delta(\lambda_3 + c_{13}\lambda_1) =$$

$$= \left(\frac{\bar{\lambda}_1}{\lambda_1}\right)^{a_1} \left(\frac{\bar{\lambda}_2}{\lambda_2}\right)^{a_2} \left(\frac{\bar{\lambda}_3}{\lambda_3}\right)^{a_3} k_1^{-a_1} k_2^{-a_2} k_3^{-a_3}\ \delta(k_1 + k_2 + k_3).$$

Unlike the 4-dimensional case: $2a = 2\ell + 1$ and $a_1 + a_2 + a_3 = \eta\frac{1}{2}$:

The case $\eta = -1$ comes with the other 3-pont YSC

$$\Phi_3^{--+} \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1^+ & 2^+ \end{pmatrix}$$

$$(Split(h_B, h_C, h_A))^2 \sim \Phi_3^{--+} \otimes \Phi_3^{+--} = \Phi_4^{--++}$$

$$\Phi^{-++,k}(a_1, a_2, a_3; k_1, k_2, k_3) \sim (k_1 k_2 k_3)^{\frac{1}{2}}\ k_1^{-\eta a_1} k_2^{-\eta a_2} k_3^{-\eta a_3}$$

$$Split(h_1, h_2, h_3; z) = \Phi^{-++,k}(h_1, h_2, h_3 - \tfrac{1}{2}\eta; -z, z-1, 1)$$

The relation arising from symmetry in subscripts $1, 2, 3$ match the crossing rules for parton evolution kernels:

$$P_{h_A h_C}^{h_B}(z) = \pm z P_{h_C h_A}^{h_B}(\frac{1}{z})$$

$$w_{h'}^{-h}(z) \;=\; z\, w_{h}^{-h'}(\frac{1}{z})$$

Parton kernels and 4-point YSC at $n = 2$

with J. Fuksa 2016

$$\Phi^{--++} = \int dc_{24} \varphi_4(c^{(0)}(c_{24})) \; \frac{1}{\lambda_1 \bar{\lambda}_3} \delta(\lambda_1 \bar{\lambda}_1 + \lambda_2 \bar{\lambda}_2 + \lambda_3 \bar{\lambda}_3 + \lambda_4 \bar{\lambda}_4)$$

our case: with φ_X, the $n = 2$ analogon of the YSC for tree amplitudes.
$k = \bar{\lambda}\lambda$

$$\lambda_1^{u_1 - u_3 - 1} \lambda_2^{u_2 - u_4 - 1} \bar{\lambda}_3^{u_1 - u_3 - 1} \bar{\lambda}_4^{u_2 - u_4 - 1} \; \delta(k_1 + k_2 + k_3 + k_4) \times$$

$$\int dc c^{u_1 - u_2 - 1} \; (k_1 + k_4 + c)^{u_3 - u_4 - 1} \; (k_2 k_4 + c(k_1 + k_2))^{u_4 - u_1 + 1}$$

application to kernels in the limit $k_1 + k_2 = 0$
$1, 2, 3, 4, \to 1', 2', 1, 2$

$$\sim \quad (k_1 k_{1'})^{-h_1 - h_2 + v} (k_{1'} - k_1)^{-1-2v}$$

reproduces parton splitting functions $z = \frac{k_{1'}}{k_1}$
$g \to g, \quad h_1 = h_2 = -1, v = 0, \; -1, \; -2$

$$z w_1^1(z) = \frac{1 + z^4}{1 - z} = 2\frac{z^2}{1 - z} + 4z(1 - z) + (1 - z)^3$$

more interesting: It reproduces the kernels of GPD scale evolution.

Bukhvostov, Frolov, Kuraev, Lipatov, 1995.

R.K, S. Derkachov, 2001

Operators with YSC as kernels

$$[R \cdot \psi](x_1, x_2) = \int dx_1' dx_2' \psi(x_1', x_2') \Phi(x_1', x_2', x_1, x_2|\mathbf{u})$$

For complex $x \to x, \bar{x}$, positions in transverse subspace,

$$[\mathbf{R}\psi](x_1, x_2) = \int dx_1' dx_2' d\bar{x}_1' d\bar{x}_2' \psi(x_1', x_2', \bar{x}_1', \bar{x}_2') \Phi(x_1', x_2', x_1, x_2|\mathbf{u}) \Phi(\bar{x}_1', \bar{x}_2', \bar{x}$$

The YSC condition implies Yang-Baxter RLL relations.

$$\mathbf{R}_{12}(u) = P_{12} + u\mathbf{H_{12}} + ...$$

L.N. Lipatov, 1993

Use the solution φ_X at $n = 2$ in normal coordinates

$$\Phi_X(x'_1, x'_2, x_1, x_2|\mathbf{u}) = x_{12}^{-2}\varphi_X(c^*)$$

$$c^*_{13} = \frac{x_{14}}{x_{34}}, c^*_{14} = -\frac{x_{13}}{x_{34}}, c^*_{23} = \frac{x_{24}}{x_{34}}, c^*_{24} = -\frac{x_{23}}{x_{34}}$$

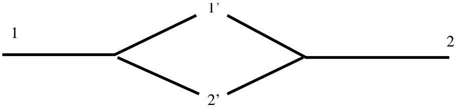
At $\ell_1 = \ell_2 = 0$ the decomposition in $u = \varepsilon$ leads to the BFKL kernel in the dipole form

$$\int \frac{|x_{12}|^2 d^2 x_3}{|x_{13}|^2 |x_{23}|^2} \left(\delta^{(2)}(x_{21'})\delta(x_{2'3}) + \delta^{(2)}(x_{12'})\delta(x_{1'3}) - \delta^{(2)}(x_{12'})\delta(x_{21'})\right)$$

Nikolaev, Zakharov, 1994, Mueller, 1994
Balitsky; Kovchegov 1999

Bartels, Lipatov, Vacca, 2004

Asymptotic freedom



$$\int d\mathbf{x}_{1'} d\mathbf{x}_{2'} \Phi_3^{-++}(1,2',1')\, \Phi_3^{--+}(1',2',2) = B(-2\ell_{1'}, -2\ell_{2'})\Phi_2^{-+}(1,2)$$

$2\ell + 2 = 2h = 2\bar{h} + 2\varepsilon$, $2\bar{h}$ integer,

$$B(2h+2, -2h+2) =$$

$$\frac{\pi}{6\sin(\pi(2-2h))}\frac{\Gamma(2h+2)}{\Gamma(2h-1)} = -\frac{1}{12\varepsilon}(2\bar{h}+1)2\bar{h}(2\bar{h}-1) +$$

$$(-1)^{2\bar{h}}\frac{-12\bar{h}^2+1}{6}$$

Relation to classical papers on parton evolution:
momentum sum rule

$$\int_0^1 dz P^{h}_{-h,1}(z) = B(2+2h, 2-2h)$$

L.N. Lipatov, 1994; Altatrelli, Parisi, 1994

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