Gravity in the High Energy Limit

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1. Working with Lev
2. Multi-Regge limit
3. Lipatov vertex in gravity
   1. From Feynman rules in Sudakov variables
   2. From Bern-Carrasco-Johansson (BCJ) Color-Kinematics duality
   3. From Cachazo-He-Yuan (CHY) in Sudakov representation
4. Double logarithms in gravity and supergravity
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In 1996 I heard about pomerons for the first time, from Donnachie. He told me about the work of a Russian scientist on the field.

A year later I studied BFKL with Forshaw, in Manchester. He used to say that to read Lev’s papers was like “hitting your head against the wall”

I first met Lev at a Low x Physics Workshop in 2001, in Krakow. He stood up after my talk and just stated “What you are saying is wrong”, with a very strong Russian accent. It felt like a big hole appeared under me and I was going down through it.

Since then we did not stop discussing about physics and everything else until August last year when we worked for a week in Cambridge. He had been in Madrid for a month in January.
We wrote four papers together. Funny enough, none of them on QCD:

2005 Reflexive Numbers & Berger Graphs from Calabi-Yau Spaces. with Velizhanin and Volkov
2008 BFKL Pomeron, Reggeized gluons & Bern-Dixon-Smirnov amplitudes
2008 N=4 SUSY amplitudes at high energies: the Regge cut piece
2012 Double-logarithms in Einstein-Hilbert gravity & supergravity. these last three with Bartels

Working with Lev you always had the feeling that something special was about to happen, that a great discovery was around the corner ...
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Hadron-hadron total cross section grows with energy:

\[ \sigma_{\text{tot}} \sim s^{\alpha(0)-1} = s^{0.1} \]  

(Donnachie-Landshoff)
Regge theory preludes QCD. Pomeron in terms of quarks & gluons?

Perturbation theory with large scale \( Q > \Lambda_{QCD} \rightarrow \alpha_s(Q) \ll 1 \).

\( s \gg t, Q^2 \rightarrow \alpha_s(Q) \log \left( \frac{s}{t} \right) \sim \mathcal{O}(1) \). Resummation needed.

\[
\sigma_{tot}(s=\epsilon^{y_A-y_B}) = \sum_{n=0}^{\infty} C_n^{LL} \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \ldots \int_{y_B}^{y_{n-1}} dy_n = \sum_{n=0}^{\infty} \frac{C_n^{LL}}{n!} \alpha_s^n (y_A - y_B)^n
\]
Multi-Regge linked to elastic amplitudes via optical theorem:

\[ \sigma_{\text{tot}}(s, e^{-y_0}) = \sum_{n=0}^{\infty} \left( \frac{1}{s} \right) \cdot \frac{1}{s} \sum_{n=0}^{\infty} = \frac{1}{s} \sum_{n=0}^{\infty} \right) \]

New degree of freedom = \( g_R \) ("Reggeized" gluon)

Pomeron = Bound state of 2 \( g_R \)

2-dimensional interaction Hamiltonian
Effective Feynman rules:

Gluon Regge trajectory: \( \omega(\vec{q}) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2} \)

Modified propagators in the \( t \)-channel:

\[
\left( \frac{s_i}{s_0} \right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}
\]

\[
\left( \frac{\alpha_s N_c}{\pi} \right)^2 \int d^2 \vec{k}_1 \frac{\theta \left( k_1^2 - \lambda^2 \right)}{\pi k_1^2} \int d^2 \vec{k}_2 \frac{\theta \left( k_2^2 - \lambda^2 \right)}{\pi k_2^2} \delta^{(2)} \left( \vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B \right) \times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega(\vec{k}_A)(Y - y_1)} e^{\omega(\vec{k}_A + \vec{k}_1)(y_1 - y_2)} e^{\omega(\vec{k}_A + \vec{k}_1 + \vec{k}_2)y_2}
\]

Is there a similar picture in gravity?
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We reviewed Lev’s calculation (SV-Serna-VazquezMozo)\textsuperscript{2012}

where the amplitude is the sum of two gauge invariant parts
\[ t = k_1^2, \quad t' = k_2^2 \]

\[ k_1 = \alpha_1 p + \beta_1 q + k_1^\perp \quad k_2 = \alpha_2 p + \beta_2 q + k_2^\perp. \]

Sudakov expansion

Multi-Regge kinematics (MRK):

\[ 1 \gg |\alpha_1| \gg |\alpha_2| = -\frac{t'}{s} \quad 1 \gg |\beta_2| \gg |\beta_1| = -\frac{t}{s} \]

Universal Reggeized g - Reggeized g - g Effective Vertex (Lipatov) in MRK:

\[ = i g \eta_{\mu\nu} \left\{ \left( \alpha_1 - \frac{2t}{s\beta_2} \right) p^\nu + \left( \beta_2 - \frac{2t'}{s\alpha_1} \right) q^\nu - \left( k_1^\perp + k_2^\perp \right)^2 \right\} \]
The Closest Calculation in Einstein-Hilbert Gravity:
Nice Trick is Still Nice but More Tricky:

\[
\begin{align*}
\text{Diagram 1} & + \text{Diagram 2} + \frac{t}{t - t'} \left\{ \text{Diagram 3} + \text{Diagram 4} \right\} = \text{Diagram 5} \\
\text{Diagram 6} & + \frac{t'}{t' - t} \left\{ \text{Diagram 7} + \text{Diagram 8} \right\} = \text{Diagram 9} \\
\frac{t'}{t' - t} \text{Diagram 10} & + \frac{t}{t - t'} \text{Diagram 11} = 0
\end{align*}
\]

Exact Amplitude is the Sum of Two Gauge Invariant Sub-Amplitudes:
Using the same Sudakov expansion and Multi-Regge kinematics:

Universal Reggeized G - Reggeized G - G Effective Vertex (Lipatov):

Subtraction Term to Fullfil Steinman Relations (no simultaneous singularities in overlapping channels).
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This can be calculated also using the Color-Kinematics Duality (Johansson-SV-Serna-VazquezMozo)\textsuperscript{2013}. With scalars and one gluon in general we have
\[ A_5 = g^3 \sum_{i=1}^{15} \frac{c_i n_i}{d_i} \]

\( c_i \) are color factors (\( c_1 = f^{a_5 a_3 b} f^{b a_4 c} f^{c a_2 a_1} \))

\( d_i = \prod_{\alpha_i} s_{\alpha_i} \) are products of kinematical invariants

\( n_i \) are functions of momenta and polarizations from Feynman rules

\( c_i \) satisfy Jacobi identities (\( j_1 \equiv c_{12} - c_9 + c_{15} = 0 \))

These relations are not satisfied if \( c_i \rightarrow n_i \)

Perform the transformation \( A_5 = \sum_{i=1}^{15} \frac{c_i n_i}{d_i} + \sum_{\alpha=1}^{9} \gamma_{\alpha j_{\alpha}} = \sum_{i=1}^{15} \frac{c_i n_i'}{d_i} \), such that the Jacobi identities with \( c_i \rightarrow n'_i \) are satisfied.

The gravitational amplitude from BCJ double-copy:

\[ -i \mathcal{M} = \left( \frac{\kappa}{2} \right)^3 \sum_{i=1}^{15} \frac{n'_i \tilde{n}_i'}{d_i} \]

has the correct Lipatov’s Regge limit. Test of double copy prescription.
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Gravity in the High Energy Limit

Recent idea for amplitudes without Feynman diags. (Cachazo-He-Yuan):

$$A_n = i g^{n-2} \int \frac{d^n \sigma}{\text{Vol}[\text{SL}(2, \mathbb{C})]} \sigma_{kl} \sigma_{lm} \sigma_{mk} \prod_{i \neq k, l, m} \delta \left( \sum_{j \neq i}^{n} \frac{2 \mathbf{p}_i \cdot \mathbf{p}_j}{\sigma_{ij}} \right) I_L I_R$$

$I_L$ carries the color traces $I_L = \sum_{\beta \in S_n/\mathbb{Z}_n} \frac{\text{Tr}(T^a \beta(1) T^a \beta(2) \cdots T^a \beta(n))}{\sigma_{\beta(1)\beta(2)} \sigma_{\beta(2)\beta(3)} \cdots \sigma_{\beta(n)\beta(1)}}$.

$I_R = \text{Pf}' M_n$ is the reduced Pfaffian of $M_n = \begin{pmatrix} M_A & -M_C^T \\ M_C & M_B \end{pmatrix}$ where

$$M_{ij}^A = \begin{cases} \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{\sigma_{ij}}, & i \neq j \\ 0, & i = j \end{cases}, \quad M_{ij}^B = \begin{cases} \frac{\mathbf{e}_i \cdot \mathbf{e}_j}{\sigma_{ij}}, & i \neq j \\ 0, & i = j \end{cases}, \quad M_{ij}^C = \begin{cases} \frac{\mathbf{e}_i \cdot \mathbf{p}_j}{\sigma_{ij}}, & i \neq j \\ -\sum_{k \neq i} \frac{\mathbf{e}_i \cdot \mathbf{p}_k}{\sigma_{ik}}, & i = j \end{cases}$$

$\mathbf{e}_i$: $i$-th gauge boson polarization. $(\text{Pf } A)^2 = \det(A)$.

$\text{Pf}' M_n$ removes $k$-th row, $\ell$-th column, times $(-1)^{k+\ell} \sigma_{k\ell}^{-1}$.

Gravity: $I_L = \text{Pf}' M_n$, $I_R = \text{Pf}' M_n$. 

Agustín Sabio Vera (UAM, IFT)
Gravity in the High Energy Limit

\[ s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j, \quad \sigma_{ij} = \sigma_i - \sigma_j \]

\[ \mathcal{A}_n = i \, g^{n-2} \int \frac{d^n\sigma}{\text{Vol}[\text{SL}(2, \mathbb{C})]} \sigma_{kl}\sigma_{lm}\sigma_{mk} \prod_{i \neq k,l,m} \delta (S_i(\sigma)) \, I_L I_R \]

Support of the integral on Scattering Equations: \( S_i(\sigma) \equiv \sum_{j \neq i} \frac{s_{ij}}{\sigma_{ij}} = 0 \)

\((n - 3)!\) solutions \( (\sigma_1^{(i)}, \ldots, \sigma_n^{(i)}) \) as \( n \)-punctured spheres.
To find all of them is a very complicated algebraic problem.

\textbf{Sudakov variables} simplify the finding of solutions greatly.
(Chachamis-Medrano-SV-VázquezMozo)\textsuperscript{2018}

Parametrize on-shell momenta with energy & \( S^2 \) stereographic coordinates:

\[ p_j = \omega_j \left( 1, \frac{\zeta_j + \bar{\zeta}_j}{1 + \zeta_j \bar{\zeta}_j}, i \frac{\bar{\zeta}_j - \zeta_j}{1 + \zeta_j \bar{\zeta}_j}, \frac{\zeta_j \bar{\zeta}_j - 1}{1 + \zeta_j \bar{\zeta}_j} \right) \]

In dimension four the solution \( \sigma_j = \zeta_j = e^{Y_j + i \phi_j} \) by Fairlie always exists.
Geometric interpretation rapidity $Y_j$ & azimuthal angle $\phi_j$

$$\sigma_j = \zeta_j = e^{Y_j + i\phi_j} \text{ on punctured sphere.}$$
Take incoming momenta as \( p = \frac{\sqrt{s}}{2} (1, 0, 0, 1), \quad q = \frac{\sqrt{s}}{2} (1, 0, 0, -1) \)

Then \( \sigma_p = \frac{e^{i\phi}}{\epsilon} \longrightarrow \infty, \quad \sigma_q = -e^{Y_q + i\phi} = -\epsilon e^{i\phi} \longrightarrow 0 \)

Four-point amplitude with \( p + q \rightarrow p' + q' \).

Introduce Sudakov representation

\[
q_1 \equiv p - p' = \alpha (p - q) + q_1, \quad q_1 = q_1^\perp (0, \cos \theta_1, \sin \theta_1, 0).
\]
There is only one, \((n-3)!\), solution to the Scattering Equations:

\[
\sigma_p = \infty, \quad \sigma_q = 0, \quad \sigma_{p'} = -\frac{Q_1}{\alpha}, \quad \sigma_{q'} = \frac{Q_1}{1-\alpha}
\]

\[
Q_j = \frac{q_j}{\sqrt{s}} e^{i\theta_j}, \quad |Q_1|^2 = \alpha(1 - \alpha)
\]

The four-point scalar amplitude is easy to calculate:

\[
\mathcal{A}_4^{\varphi^3} = \int dz_{p'} \frac{z_{pq}^2 z_{qq'}^2 z_{q'}^2 p}{(z_{pq} z_{qq'} z_{q'} p z_{p'})^2} \delta(S_{p'}) = \int \frac{dz_{p'}}{(z_{p'} - \sigma_{q'})^2} \delta \left( \frac{s_{p' q}}{z_{p'}} - \frac{s_{p' q'}}{z_{p'} - \sigma_{q'}} \right)
\]

The Jacobian is very important:

\[
\mathcal{A}_4^{\varphi^3} = \int dz_{p'} \left[ z_{p'} - \frac{Q_1}{1-\alpha} \right]^{-2} \frac{Q_1^2}{s\alpha^3(\alpha - 1)} \delta \left( z_{p'} + \frac{Q_1}{\alpha} \right)
\]

\[
= \left[ \frac{\alpha^2(1 - \alpha)^2}{Q_1^2} \right] \left[ \frac{Q_1^2}{s\alpha^3(\alpha - 1)} \right] = \frac{(\alpha - 1)}{s\alpha} = \frac{1}{s} + \frac{1}{t}
\]
Five-point amplitude $p + q \rightarrow p' + k + q'$. Sudakov representation:

$$q_1 = p - p' = \alpha_1 p + \beta_1 q + q_1,$$
$$q_2 = q' - q = \alpha_2 p + \beta_2 q + q_2,$$
$$k = q_1 - q_2 = (\alpha_1 - \alpha_2) p + (\beta_1 - \beta_2) q + q_1 - q_2,$$

Scattering Equations with $(n-3)! = 2$ complex conjugated solutions:

$$\sigma_{p'}^{(+)} = \sigma_{p'}^{(-)*} = \frac{Q_1 e^{-i\theta_2}}{\beta_1},$$
$$\sigma_{q'}^{(+)} = \sigma_{q'}^{(-)*} = \frac{Q_2 e^{-i\theta_2}}{1 + \beta_2},$$
$$\sigma_{k}^{(+)} = \sigma_{k}^{(-)*} = \frac{(Q_1 - Q_2) e^{-i\theta_2}}{\beta_1 - \beta_2}.$$

In Sudakov space the CHY approach is much simpler.
Punctures on the Riemann sphere for the five-particle amplitude
\[ A_{5}^{\varphi^3} = \int dz_{p'} dz_{q'} \, \delta (S_{p'}) \, \delta (S_{q'}) \, \frac{z_{pq}^2 z_{qk}^2 z_{kp}^2}{(z_{pq} z_{qq'} z_{q'k} z_{kp} z_{p'p})^2} \]

\[ = \int dz_{p'} dz_{q'} \, J^{-1} \delta (z_{p'} - \sigma_{p'}) \, \delta (z_{q'} - \sigma_{q'}) \, \frac{z_{k}^2}{z_{q'}^2 z_{q'k}^2 z_{kp}^2} + \text{c.c.} \]

\[ = \frac{2}{s^2} \text{Re} \left[ \left( \frac{\sigma_{p'}}{\sigma_{q'}} \right) \frac{1}{LL - RR} \right] \]

\[ = \frac{1}{s^2} \left[ \frac{1}{\alpha_1 + \beta_1} - \frac{1}{\alpha_2 + \beta_2} + \frac{1}{(\alpha_1 + \beta_1)\beta_1} - \frac{1}{\beta_1\alpha_2} + \frac{1}{\alpha_2(\alpha_2 + \beta_2)} \right] \]

where

\[ L = \frac{\sigma_{p'k}}{\sigma_{p'q'}} \left[ (\alpha_1 - 1) \frac{\sigma_{q'}}{\sigma_{p'}} + \beta_1 \frac{\sigma_{p'}}{\sigma_{q'}} \right], \quad R = \left( \frac{\sigma_{p'} \sigma_{p'k}}{\sigma_{p'q'}} \right) \frac{(1-\alpha_1+\alpha_2-\beta_1+\beta_2)(\alpha_1+\beta_1)}{(\alpha_1-\alpha_2+\beta_1)\sigma_{p'}-(1+\beta_2)\sigma_{q'}} \]

\[ \tilde{O} \left( \alpha_1, \alpha_2, \beta_1, \beta_2, \theta_1 - \theta_2 \right) = O \left( 1 - \alpha_2, 1 - \alpha_1, -1 - \beta_2, -1 - \beta_1, \theta_2 - \theta_1 \right) \]

Factorization channels are encoded in the Jacobian ...

Geometrical meaning of subtraction to double copy found by Lev?
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Four-graviton scattering in $N$-SUGRA

At 1-loop there are three contributions ($\alpha = \text{Newton's constant}/\pi$):

\[
\mathcal{M}_{4,(N=8)}^{(1)} = \alpha t \ln \left( \frac{-s}{-t} \right) \ln \left( \frac{-u}{-t} \right) \\
\text{Double Logs} + \alpha \frac{t}{2} \ln \left( \frac{-t}{\lambda^2} \right) \left( \ln \left( \frac{-s}{-t} \right) + \ln \left( \frac{-u}{-t} \right) \right) \\
\text{Trajectory} - \alpha \frac{(s-u)}{2} \ln \left( \frac{-t}{\lambda^2} \right) \ln \left( \frac{-s}{-u} \right) \\
\text{Eikonal}
\]
In the Regge limit \( u \simeq -s \)

\[
\mathcal{M}_{4,(N=8)}^{(1)} \simeq (\alpha t) \ln^2 \left( \frac{s}{-t} \right) + (\alpha t) \ln \left( \frac{-t}{\lambda^2} \right) \ln \left( \frac{s}{-t} \right) + i \pi (\alpha s) \ln \left( \frac{-t}{\lambda^2} \right)
\]

Can we calculate the Double Logs to all orders? We can use

\[
\mathcal{A}_{4,(N)} = \mathcal{A}_{4}^{\text{Born}} \left( \frac{s}{-t} \right)^{\alpha t \ln \left( \frac{-t}{\lambda^2} \right)} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{-t} \right)^{\omega} f_{\omega}^{(N)}
\]
The diagrammatic origin of the Double Logs is

\[ = \begin{array}{c}
\text{virtual gravitons with lowest energy} \\
\text{a pair of t-channel gravitons/gravitinos with lowest energy}
\end{array} + 2 \]

The corresponding equation is

\[
f^{(N)}_{\omega} = 1 - (\alpha t) \frac{d}{d\omega} \left( \frac{f^{(N)}_{\omega}}{\omega} \right) + (\alpha t) \left( \frac{N - 6}{2} \right) \left( \frac{f^{(N)}_{\omega}}{\omega} \right)^2
\]

with perturbative solution \((N = \text{number of gravitinos})\):

\[
f^{(N)}_{\omega} = 1 + (\alpha t) \left( \frac{N - 4}{2\omega^2} \right) + (\alpha t)^2 \left( \frac{N - 4}{2\omega^4} \right) \left( \frac{N - 3}{2} \right)
\]

\[
- (\alpha t)^3 \frac{(N - 4) \left( 5N^2 - 26N + 36 \right)}{8\omega^6} + \ldots
\]
In agreement with the two-loop results for $N = 4, \ldots, 8$ SUGRA obtained using the conjecture that gravity is a double copy of gauge theories (Dixon, Bern, Carrasco, Johansson).

We have predictions to all orders e.g. $N = 8$:

$$\mathcal{A}_{4,(N=8)} = \mathcal{A}_{4}^{\text{Born}} \left( \frac{-t}{\lambda^2} \right)^{\alpha t \left( \ln \left( \frac{s}{-t} \right) + i\pi \left( \frac{s}{t} \right) \right)}$$

$$\times \left\{ 1 + 2 \left( \frac{\alpha t}{2} \right) \ln^2 \left( \frac{s}{-t} \right) + \frac{5}{3} \left( \frac{\alpha t}{2} \right)^2 \ln^4 \left( \frac{s}{-t} \right) + \frac{37}{45} \left( \frac{\alpha t}{2} \right)^3 \ln^6 \left( \frac{s}{-t} \right) + \frac{353}{1260} \left( \frac{\alpha t}{2} \right)^4 \ln^8 \left( \frac{s}{-t} \right) + \frac{583}{8100} \left( \frac{\alpha t}{2} \right)^5 \ln^{10} \left( \frac{s}{-t} \right) + \ldots \right\}$$

Which the BCJ conjecture should reproduce for different $N$. 
We can resum all terms (Bartels-Lipatov-ASV):
Solution of a Schrödinger equation in terms of parabolic cylinder functions

\[ N = 4 \text{ SUGRA is a critical theory among} \]
Finite \( (N > 4) \)
Non-finite \( (N < 4) \) theories at high energies
Eikonal phase is modified. Example, in $N = 8$ SUGRA:

$$\delta^{(8)}_{DL}(\rho, s, \lambda) = -\frac{s\kappa^2}{4\pi} \ln(\lambda\rho) - \frac{s\kappa^2}{8\pi} \frac{\beta_1}{\sqrt{\beta_1^2 + \beta_2^2}} \frac{\rho_c(s)}{\rho}$$

$$-\frac{s\kappa^2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)(2(n-1))!}{(2n+1)n!(n-1)!} \cos \left( (2n+1) \arctan \frac{\beta_2}{\beta_1} \right) \left( \frac{\rho_c(s)}{4\rho} \right)^{2n+1}$$

Eikonal phases in Born and DL approximations as a function of $\rho$
The associated Einstein deflection angle is

\[
\frac{\partial \delta_{DL}^{(8)}}{\partial \rho} = -\frac{s\kappa^2}{4\pi \rho} + \frac{s\kappa^2}{8\pi} \frac{\beta_1}{\sqrt{\beta_1^2 + \beta_2^2}} \frac{\rho_c(s)}{\rho^2} + \frac{4s\kappa^2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n(2n - 1)(2(n - 1))!}{n!(n - 1)!\rho_c(s)} \cos \left( (2n + 1) \arctan \frac{\beta_2}{\beta_1} \right) \left( \frac{\rho_c(s)}{4\rho} \right)^{2n+2}
\]

Critical lines in the impact parameter vs \(s\) plot: \(\rho_c(s) = 2\sqrt{\beta_1^2 + \beta_2^2 \kappa \ln s}\)
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Let us see how far we are able to push the limits of MRK, now without the help of a friend and a true giant in physics ...

Lev N. Lipatov, Southampton, August 2017