Particle production at high energy: DGLAP, BFKL and beyond

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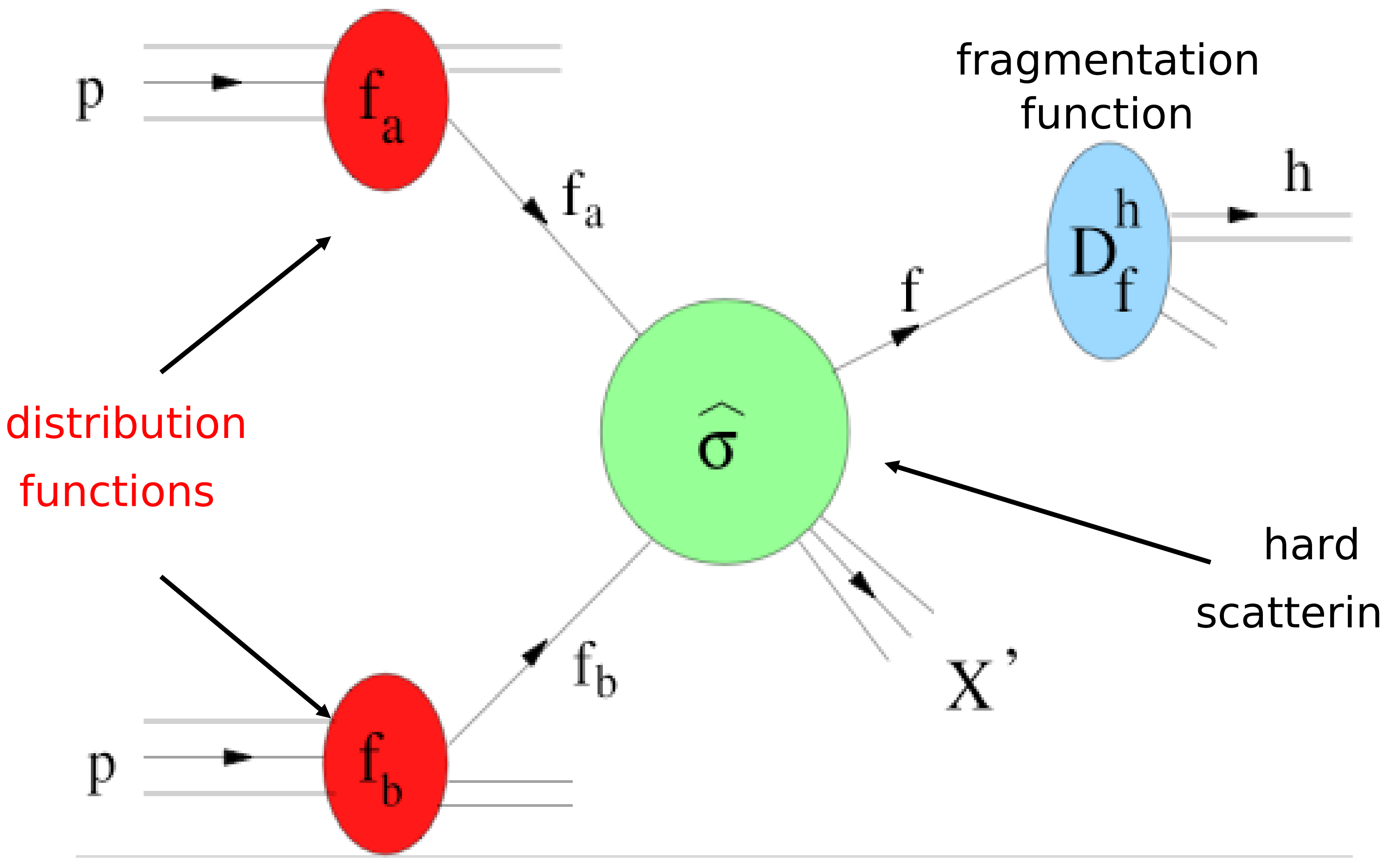
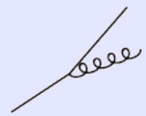
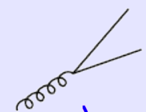
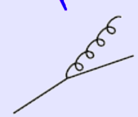
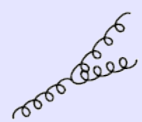
Particle production: pQCD

collinear factorization: separation of long and short distances



DGLAP evolution of
parton distribution functions

$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

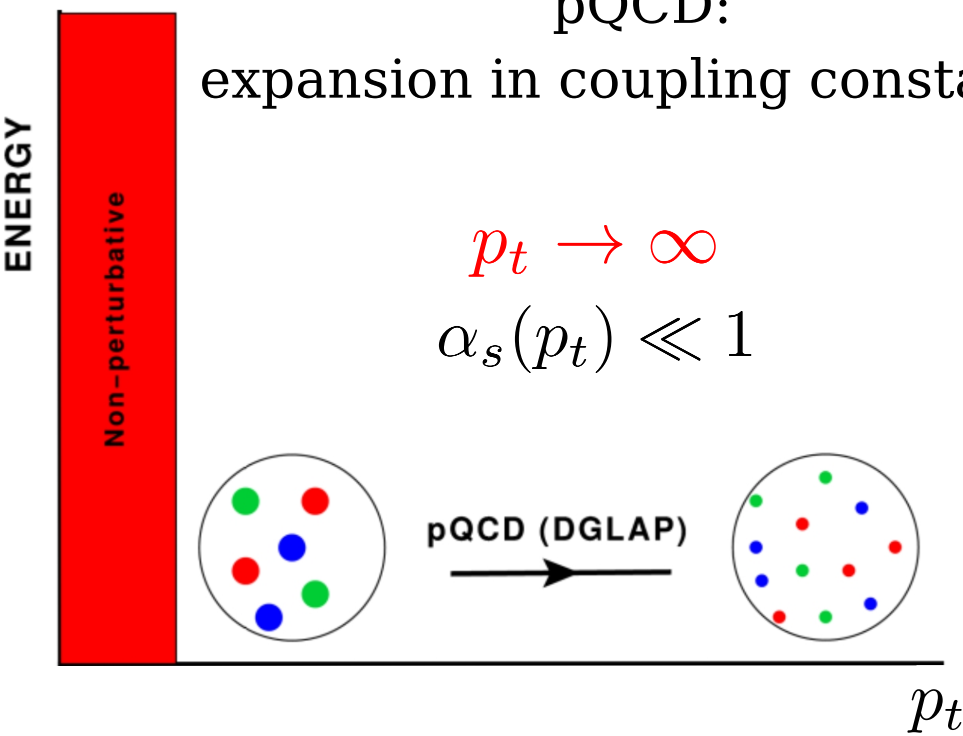





pQCD: DGLAP evolution is essential

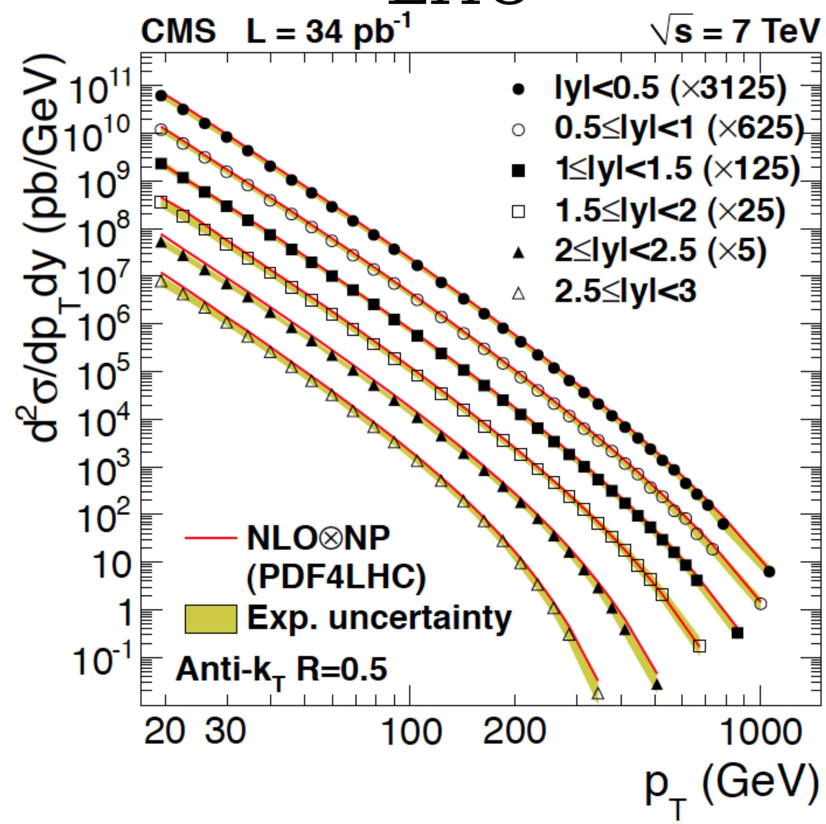
pQCD:
expansion in coupling constant

$$p_t \to \infty$$

$$\alpha_s(p_t) \ll 1$$



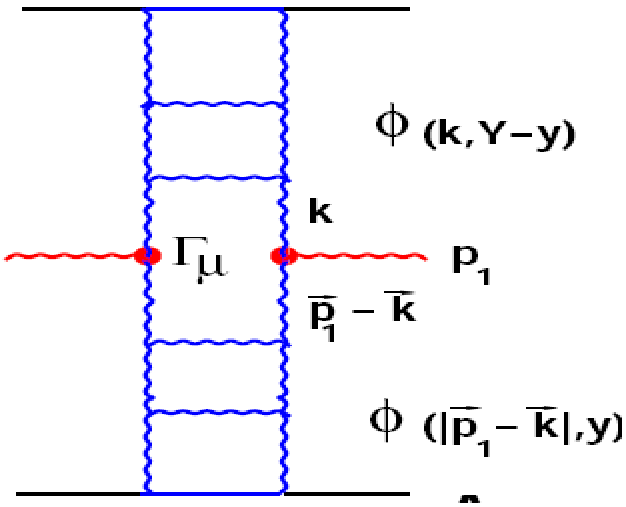
LHC



but bulk of QCD phenomena happens at low p_t

particle production in Regge limit

$$\sqrt{S} \to \infty \qquad \begin{array}{l} p_t \sim \text{fixed} \\ \alpha_s(p_t) \ll 1 \end{array}$$



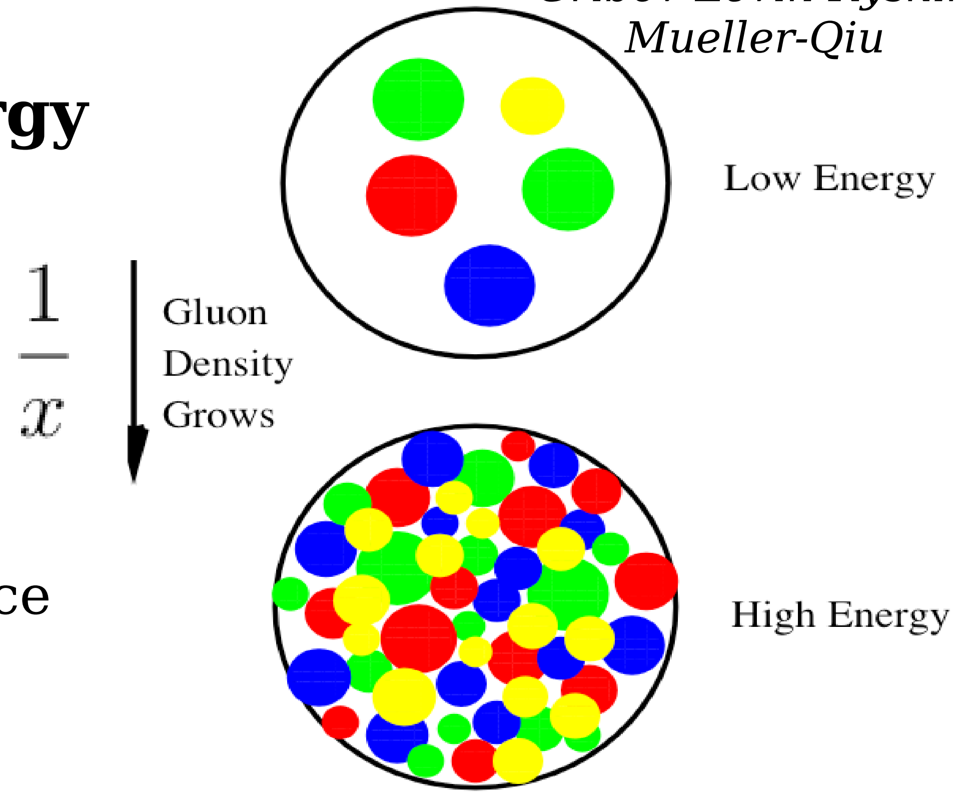
$$\frac{d\sigma}{d^2p_t\,dy} \sim \int d^2k_t\,\phi(k_t, y)\,\phi(p_t - k_t, y)$$

Φ : intrinsic gluon distribution satisfies the BFKL evolution equation

linear equation: issues with unitarity, diffusion,....

Gribov-Levin-Ryskin
Mueller-Qiu

A hadron at high energy



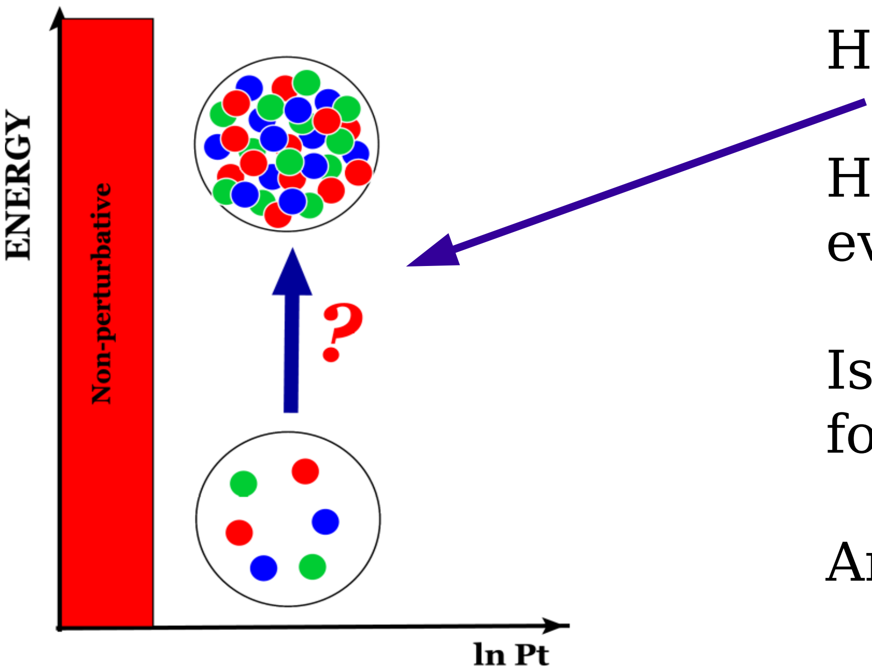
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron becomes a dense system of gluons:
<u>gluon saturation</u> (Color Glass Condensate)

differs from pQCD in two aspects:

multiple scattering (Eikonal approximation)
large logs of 1/x

Dynamics of *universal gluonic matter:*
gluon saturation



How does this happen ?

How do correlation functions evolve ?

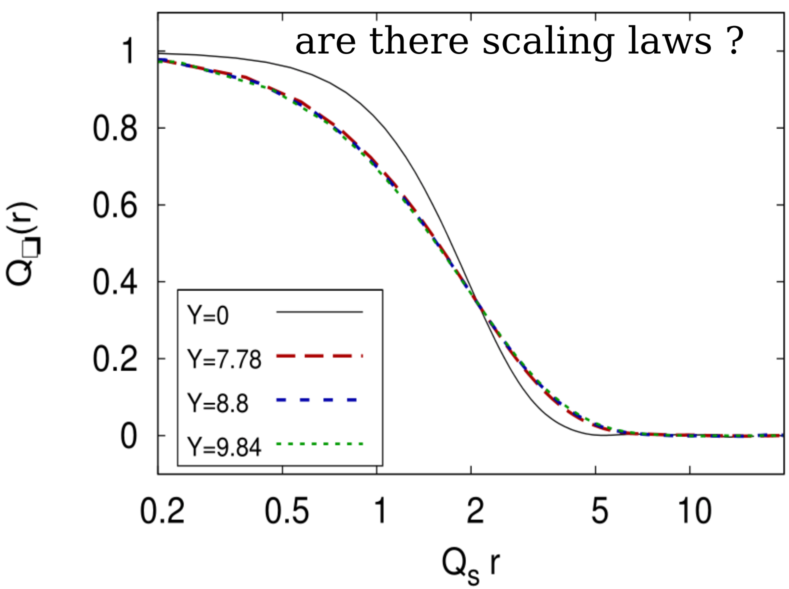
Is there a universal fixed point for the RG evolution of d.o.f

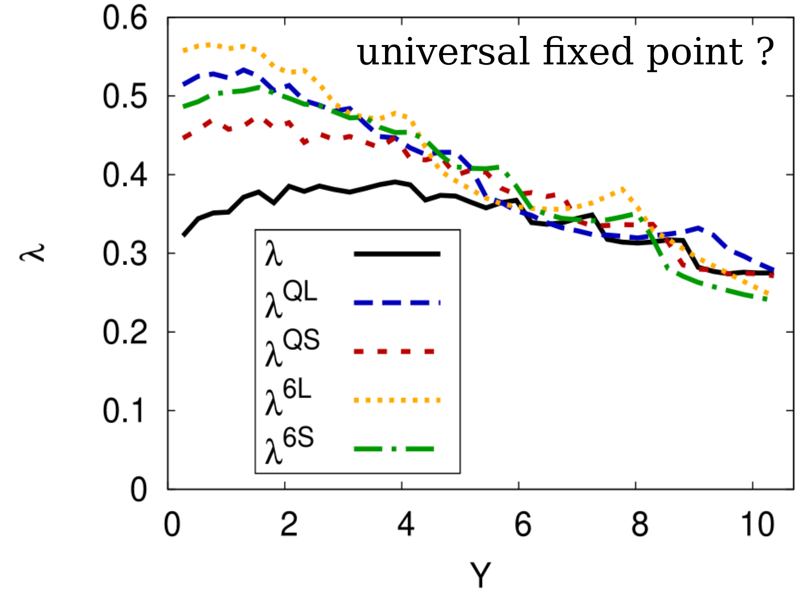
Are there scaling laws ?

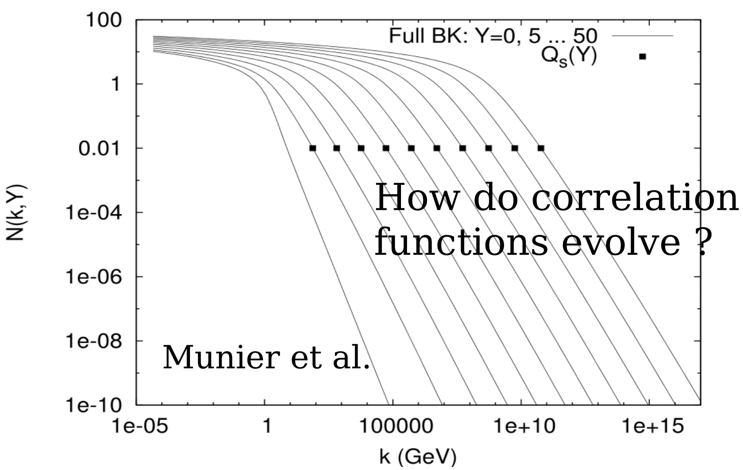
dense system of gluons
growth with energy: <u>JIMWLK</u> eq.

$$\frac{\partial \mathbf{W_E}[\mathbf{A}]}{\partial \mathbf{E}} = \mathcal{H}\, \mathbf{W_E}[\mathbf{A}]$$

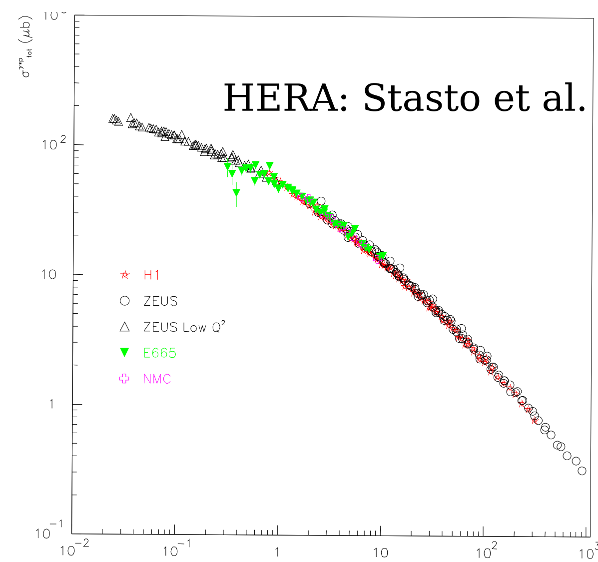
JIMWLK: scaling properties



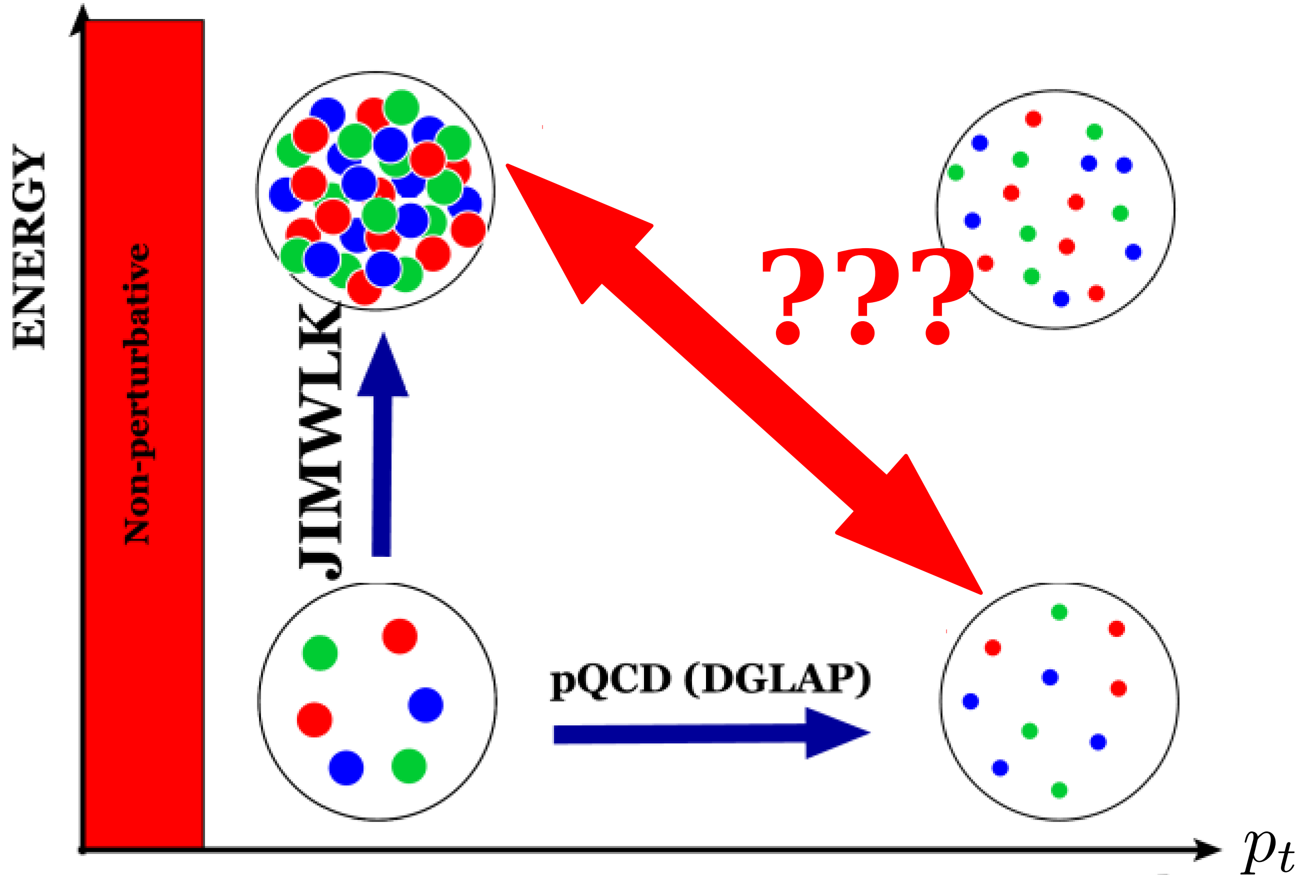




connection to statistical physics



QCD phase space



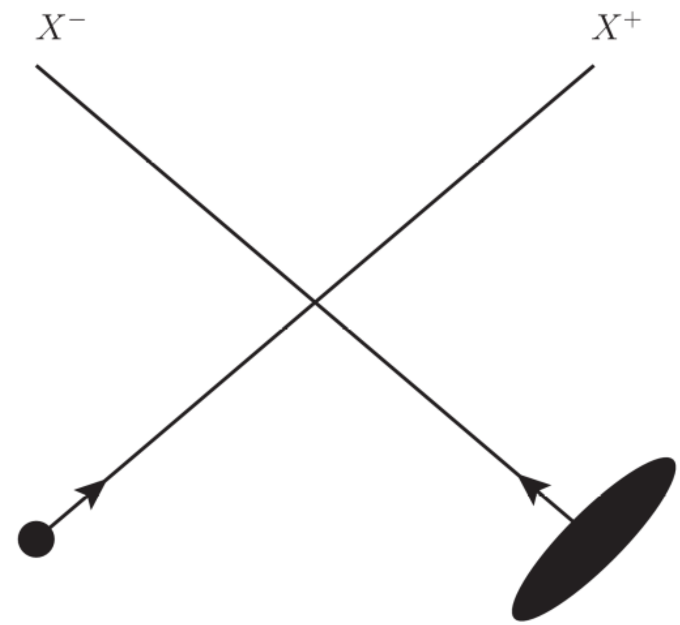
connecting JIMWLK/BFKL with DGLAP?

Jet physics
Energy loss in Quark-Gluon Plasma
Interactions of UHE neutrinos, ...

dense target (proton/nucleus) as a background color field

$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$

$$\begin{aligned} D_\mu J^\mu &= D_- J^- = 0 \\ \partial_- J^- &= 0 \end{aligned}$$ (in A+ = 0 gauge)



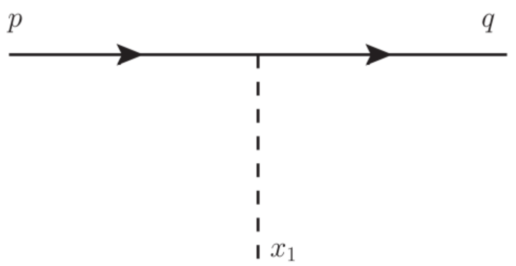
solution to classical EOM: $$A^-(x^+, x_t) \equiv n^- S(x^+, x_t)$$

scattering of a quark from background color field $A^-(x^+, x_t)$

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\ \left[\not n\, S(x_1)\right]\ u(p)$$

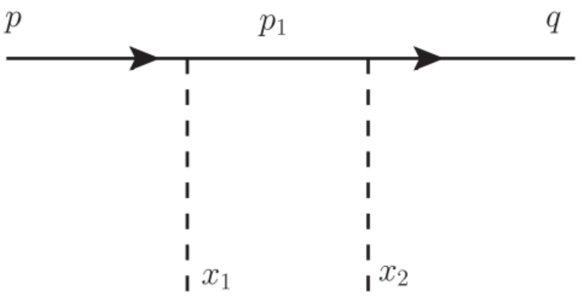
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}}$$

$$\bar{u}(q)\left[\not n\, S(x_1^+, x_{1t})\right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2\int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2}$$

$$\bar{u}(q)\ \left[\not n\, S(x_2)\, \frac{i\not p_1}{p_1^2 + i\epsilon}\, \not n\, S(x_1)\right]\ u(p)$$



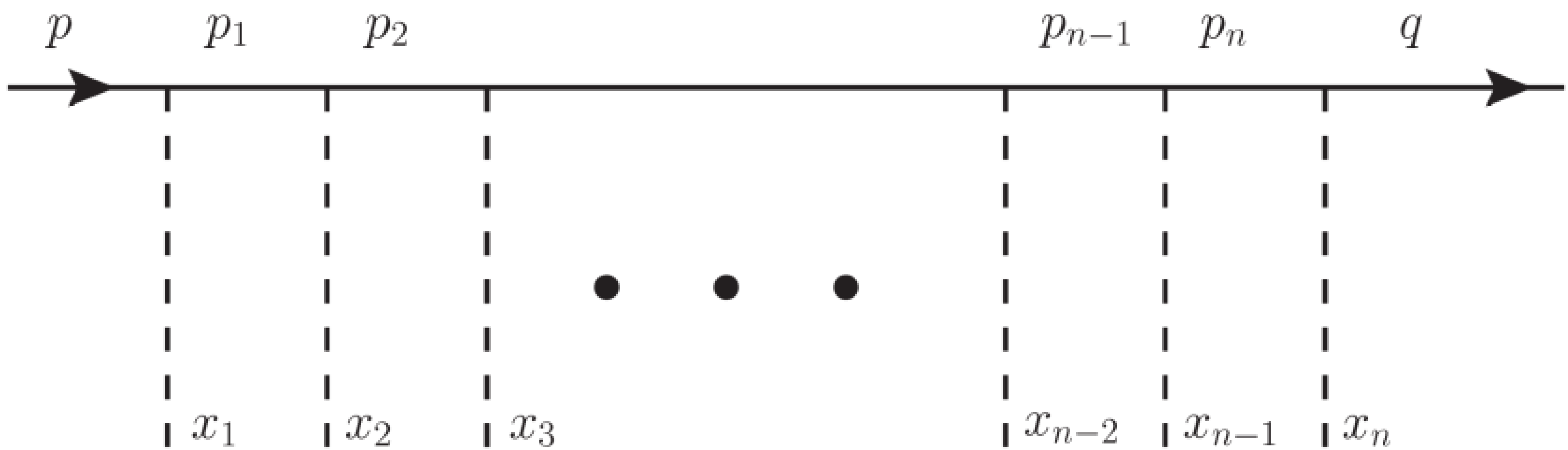
$$\int \frac{dp_1^-}{(2\pi)}\frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\,\theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

contour integration over the pole leads to time ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not n\,\frac{\not p_1}{2p^+}\,\not n = \not n$

$$i\mathcal{M}_2 = (ig)^2\,(-i)(i)\,2\pi\delta(p^+ - q^+)\int dx_1^+\, dx_2^+\,\theta(x_2^+ - x_1^+)\int d^2x_{1t}\, e^{-i(q_t-p_t)\cdot x_{1t}}$$

$$\bar{u}(q)\ \left[S(x_2^+, x_{1t})\,\not n\, S(x_1^+, x_{1t})\right]\ u(p)$$



$$
\begin{aligned}
i\mathcal{M}_n \;\; = \;\; & 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\
& \Big\{ (ig)^n\,(-i)^n (i)^n \int dx_1^+\, dx_2^+ \,\cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+) \,\cdots \theta(x_2^+ - x_1^+) \\
& \big[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t)\,\cdots S(x_2^+, x_t) S(x_1^+, x_t)\big] \Big\}\, u(p)
\end{aligned}
$$

sum over all scatterings gives (Eikonal approximation)

$$
i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\; [V(x_t) - 1]\; u(p)
$$

with $\quad V(x_t) \equiv \hat{P}\, \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a \right\}$

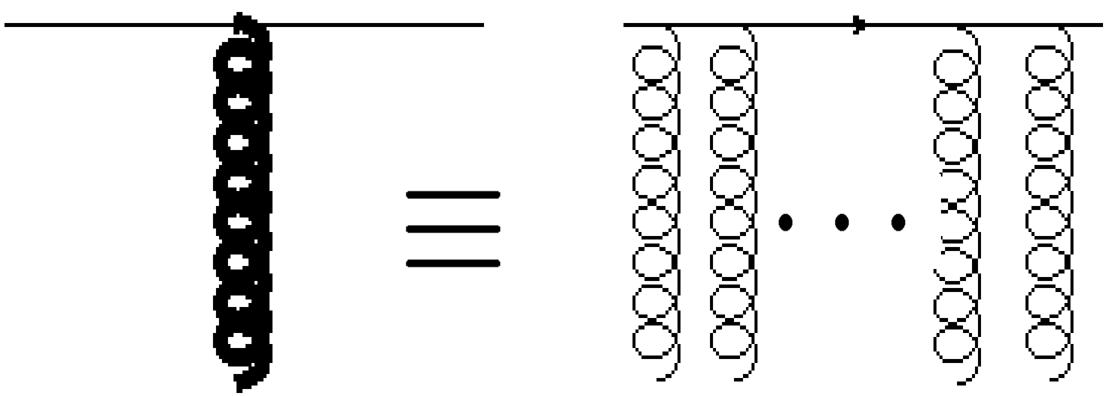
including
propagation
“backward” gives $V^\dagger$

quark propagator in the background color field

$$\mathrm{S}_F(q,p) \equiv (2\pi)^4 \underbrace{\delta^4(p-q)\, S_F^0(p)}_{\text{no interaction}} + S_F^0(q) \underbrace{\tau_f(q,p)}_{\text{interaction}} S_F^0(p)$$

with $\quad S_F^0(p) = \dfrac{i}{\not{p} + i\epsilon}$

$$\begin{aligned}\tau_f(q,p) \quad &\equiv \quad (2\pi)\delta(p^+ - q^+)\, \not{n} \int d^2x_t\, e^{i(q_t - p_t)\cdot x_t} \\ &\quad \left\{\theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1]\right\}\end{aligned}$$

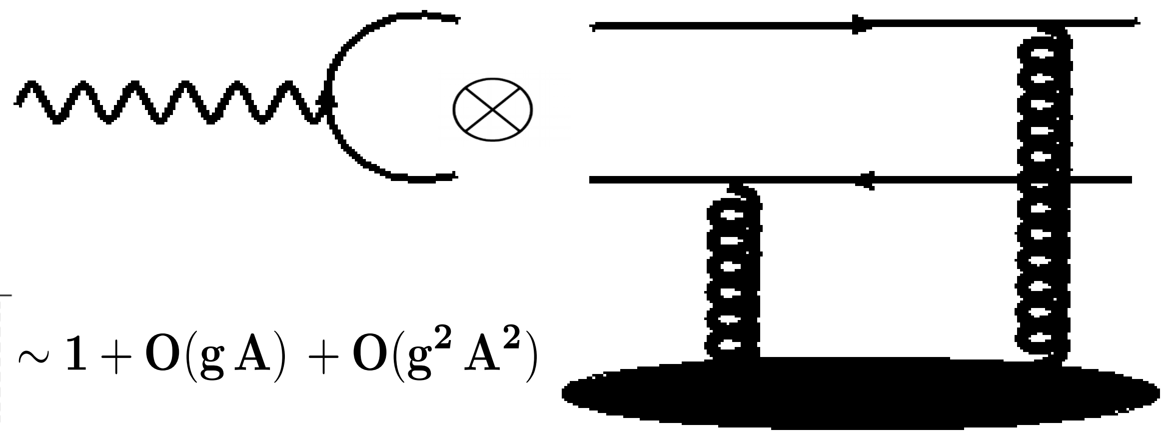


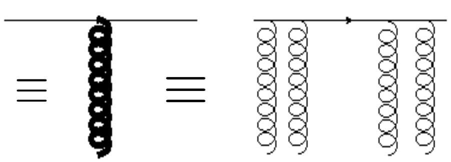
DIS total cross section

$$\sigma^{\mathrm{total}}_{\mathrm{DIS}} = 2\int_0^1 dz \int d^2x_t\, d^2y_t \left|\Psi(k^{\pm}, k_t | z, x_t, y_t)\right|^2 T(x_t, y_t)$$

can be written in closed form in terms of Bessel functions K_0, K_1

$$T(x_t, y_t) \equiv \frac{1}{N_c} \mathrm{Tr}\left\langle 1 - V(x_t) V^{\dagger}(y_t) \right\rangle$$



$V(x_t) \equiv$  $\sim 1 + O(g\,A) + O(g^2\,A^2)$

total cross section =

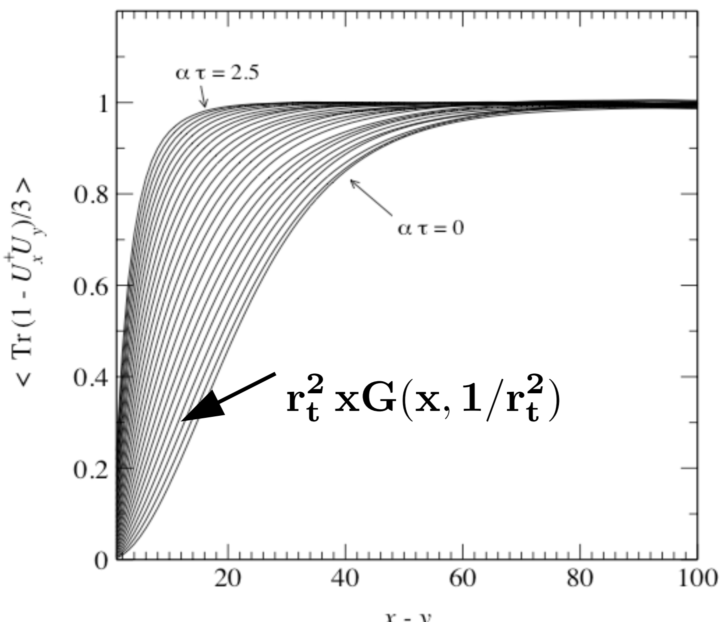
probability of photon decaying into a quark anti-quark pair $\otimes$ *probability of the* ***quark anti-quark "dipole"*** *scattering on the target*

QED *QCD*

Evolution (energy dependence) of dipoles

$$\frac{d}{dy}\mathbf{T}(\mathbf{x_t}-\mathbf{y_t}) = \frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{z_t}\,\frac{(\mathbf{x_t}-\mathbf{y_t})^2}{(\mathbf{x_t}-\mathbf{z_t})^2(\mathbf{y_t}-\mathbf{z_t})^2}\times$$

$$\Big[\underbrace{\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})+\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})-\mathbf{T}(\mathbf{x_t}-\mathbf{y_t})}_{\text{BFKL}}-\mathbf{T}(\mathbf{x_t}-\mathbf{z_t})\mathbf{T}(\mathbf{z_t}-\mathbf{y_t})\Big]$$



Rummukainen-Weigert, NPA739 (2004) 183
NLO: *Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)*

$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \log\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$ saturation region

$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{1}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]^{\gamma}$ extended scaling region

$\tilde{\mathbf{T}}(\mathbf{p_t}) \to \frac{1}{\mathbf{p_t^2}}\left[\frac{\mathbf{Q_s^2}}{\mathbf{p_t^2}}\right]$ pQCD region

rich phenomenology:
ep, eA, pp, pA, AA

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field S involves only small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

allow hard scattering by including one all x field during which there is large momenta exchanged and quark can get deflected by a large angle.

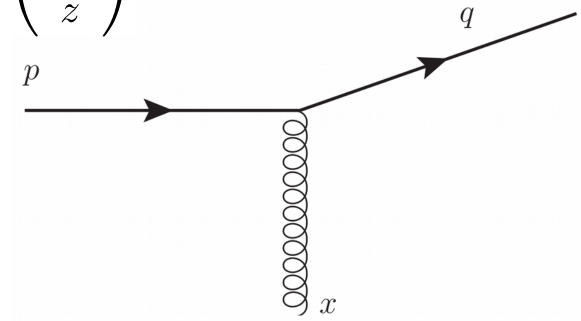
$$A^\mu_a(x^+, x^-, x_t)$$

include Eikonal multiple scattering before and after (along a different direction) the hard scattering

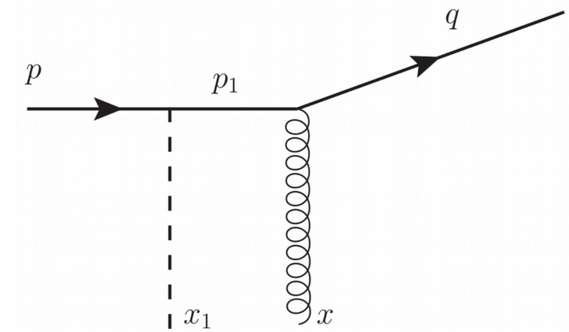
hard scattering: large deflection
scattered quark travels in the new "z" direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

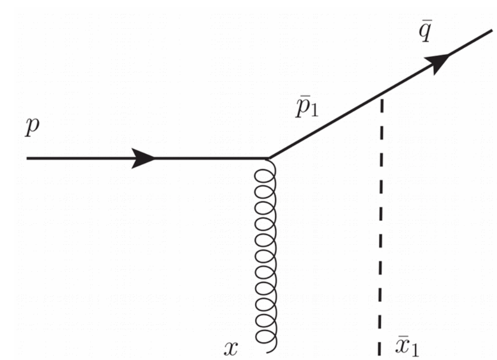
$$i\mathcal{M}_1 = (ig) \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q})\, \left[\not{A}(x)\right]\, u(p)$$



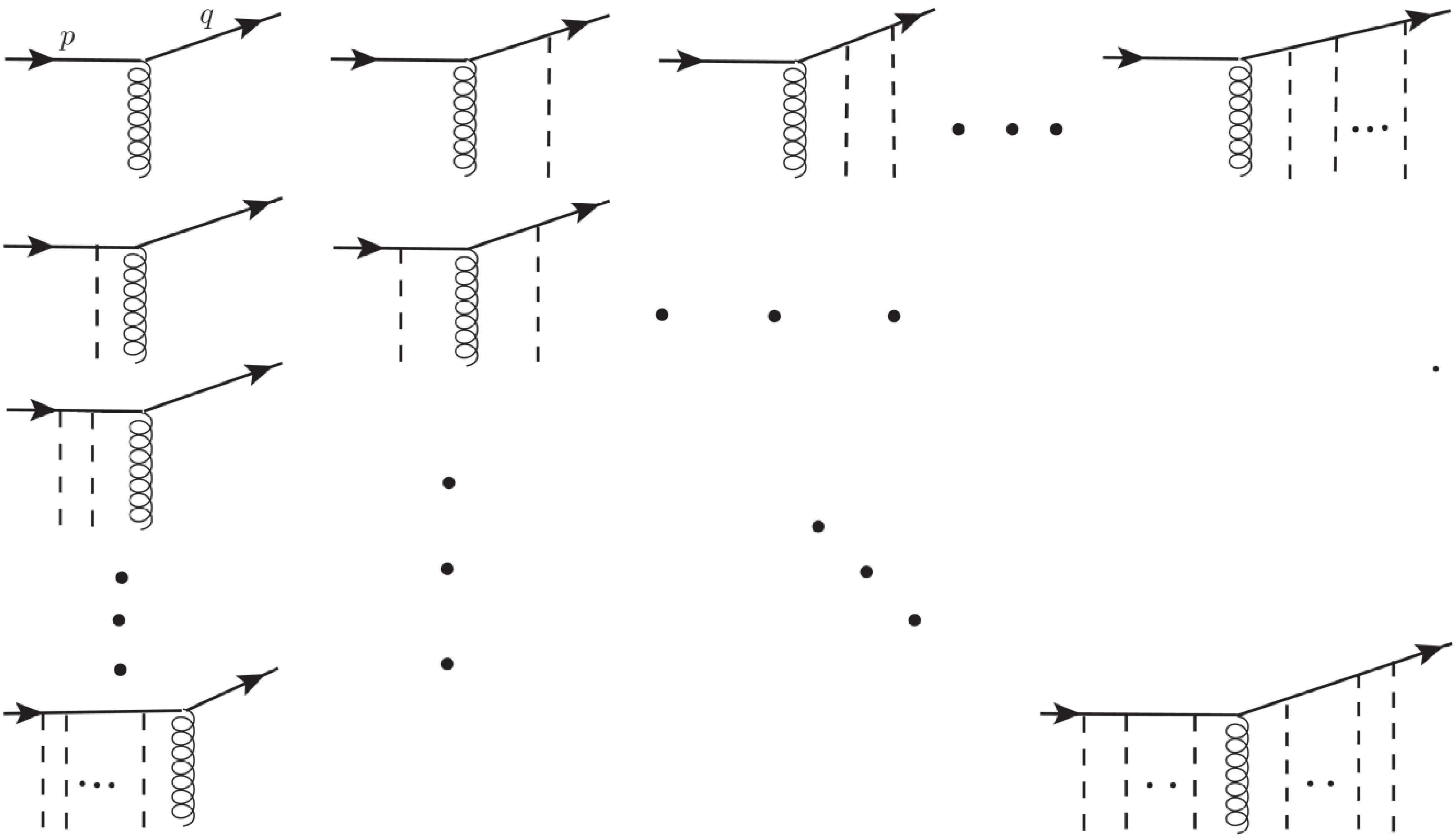
$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x\, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(\bar{q}-p_1)x} \\ &\qquad \bar{u}(\bar{q})\, \left[\not{A}(x)\, \frac{i\not{p}_1}{p_1^2+i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p) \end{aligned}$$



$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x\, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4}\, e^{i(\bar{p}_1-p)x}\, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \\ &\qquad \bar{u}(\bar{q})\, \left[\not{\bar{n}}\, \bar{S}(\bar{x}_1)\, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2+i\epsilon}\not{A}(x)\right]\, u(p) \end{aligned}$$



with $\quad \bar{S}^{\mu} = \Lambda^{\mu}_{\ \nu}\, S^{\nu}$


p
q

$$S_F(p,\bar{q}) = (2\pi)^4\delta^4(p-\bar{q})\,S_F^0(p) + S_F^0(p)\,\tau_{hard}(p,\bar{q})\,S_F^0(\bar{q})$$

$$\begin{aligned}
\tau_{hard}(p,\bar{q}) \;\; \equiv \;\; & (ig)\int d^4x \int \frac{d^2k_t}{(2\pi)^2}\,\frac{d^2\bar{k}_t}{(2\pi)^2}\,d^2z_t\,d^2\bar{z}_t\,e^{i(\bar{k}-k)x}\,e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\,e^{-i(k_t-p_t)\cdot z_t} \\
& \Big\{\theta(p^+)\,\theta(\bar{q}^+)\,V(z_t,x^+)\,\not{n}\,\frac{\not{k}}{2k^+}\not{A}(x)\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\not{\bar{n}}\,\overline{V}(x^+,\bar{z}_t) - \\
& \theta(-p^+)\,\theta(-\bar{q}^+)\,V^\dagger(z_t,x^+)\,\not{n}\,\frac{\not{k}}{2k^+}\not{A}(x)\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\not{\bar{n}}\,\overline{V}^\dagger(x^+,\bar{z}_t)\Big\}
\end{aligned}$$

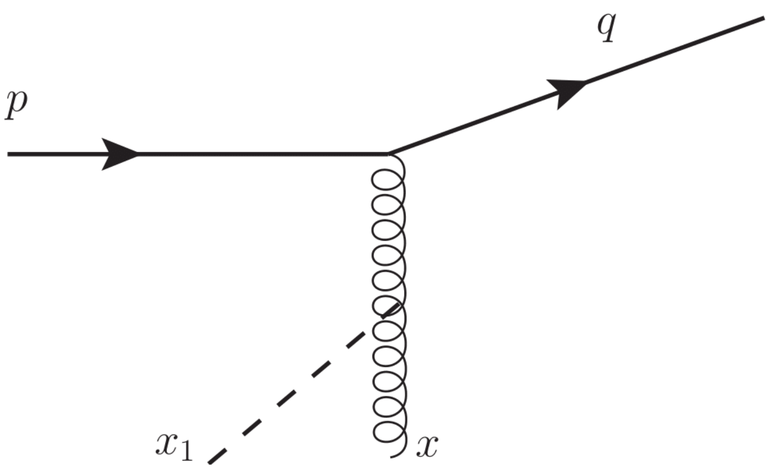
with

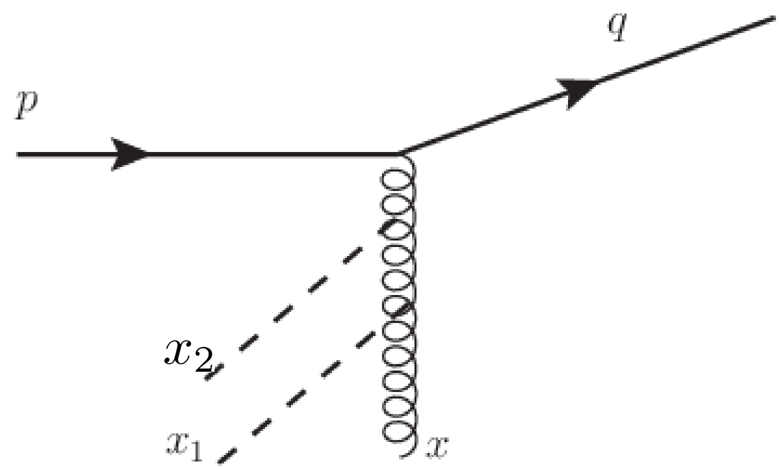
$$\overline{V}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\,\bar{S}_a^-(\bar{z}_t,\bar{z}^+)\,t_a\right\}$$

all “bar-ed” quantities are in a rotated frame where quark’s new direction of propagation (after a hard scattering) is $\bar{z}$

this quark propagator is the building block for DIS structure functions, single inclusive particle production in pA,....

but there is more to do: interactions of large and small x modes





in progress

SUMMARY

Collinear factorization with DGLAP evolution at high p_t

CGC with JIMWLK evolution at high energy

- ***Leading Log + rc works well***
  - ***it has been used to fit a wealth of data; ep, eA, pp, pA, AA***
  - ***need to eliminate/minimize late time/hadronization effects***
- ***NLO corrections are becoming available***

CGC breaks down at large x (high p_t)

Toward a unified formalism: CGC + DGLAP