Particle production at high energy: DGLAP, BFKL and beyond

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Particle production: pQCD

collinear factorization: separation of long and short distances

(fragmentation function)

(distribution functions)

(hard scattering)
DG\textbf{LAP} evolution of parton distribution functions

\[
\frac{d}{d \ln \mu} \begin{pmatrix} q(x, \mu) \\ g(x, \mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} (z, \alpha_s) \begin{pmatrix} q(x/z, \mu) \\ g(x/z, \mu) \end{pmatrix}
\]
pQCD: DGLAP evolution is essential

pQCD: expansion in coupling constant

\[ p_t \rightarrow \infty \]

\[ \alpha_s(p_t) \ll 1 \]

but bulk of QCD phenomena happens at low \( p_t \)

\( \text{LHC} \)

\( \sqrt{s} = 7 \text{ TeV} \)

CMS \( L = 34 \text{ pb}^{-1} \)

\( \delta \sigma/dp_T dy \) (pb/GeV)

- \( |y| < 0.5 \) (\( \times 3125 \))
- \( 0.5 \leq |y| < 1 \) (\( \times 625 \))
- \( 1 \leq |y| < 1.5 \) (\( \times 125 \))
- \( 1.5 \leq |y| < 2 \) (\( \times 25 \))
- \( 2 \leq |y| < 2.5 \) (\( \times 5 \))
- \( 2.5 \leq |y| < 3 \)

Anti-\( k_T \) R=0.5

NLO@NP (PDF4LHC)

Exp. uncertainty
particle production in Regge limit

\[ \sqrt{S} \to \infty \]

\[ p_t \sim \text{fixed} \]

\[ \alpha_s(p_t) \ll 1 \]

\[ \frac{d\sigma}{d^2p_t \ dy} \sim \int d^2k_t \ \phi(k_t, y) \ \phi(p_t - k_t, y) \]

\( \Phi \): intrinsic gluon distribution satisfies the BFKL evolution equation

linear equation: issues with unitarity, diffusion,....
radiated gluons have the same size \((1/Q^2)\) - the number of partons increase due to the increased longitudinal phase space.

A hadron at high energy becomes a dense system of gluons: *gluon saturation* (Color Glass Condensate)

differs from pQCD in two aspects:
- multiple scattering (Eikonal approximation)
- large logs of \(1/x\)
Dynamics of universal gluonic matter: *gluon saturation*

How does this happen?

How do correlation functions evolve?

Is there a universal fixed point for the RG evolution of d.o.f?

Are there scaling laws?

dense system of gluons
growth with energy: JIMWLK eq.

\[
\frac{\partial W_E[A]}{\partial E} = \mathcal{H} W_E[A]
\]
JIMWLK: scaling properties

How do correlation functions evolve?

connection to statistical physics

universal fixed point?

are there scaling laws?

HERA: Stasto et al.
QCD phase space

connecting JIMWLK/BFKL with DGLAP?

Jet physics
Energy loss in Quark-Gluon Plasma
Interactions of UHE neutrinos, ...
dense target (proton/nucleus) as a background color field

\[ J^\mu_a \sim \delta^{\mu-} \rho_a \]

\[ D_\mu J^\mu = D_- J^- = 0 \]
\[ \partial_- J^- = 0 \quad \text{(in A+ = 0 gauge)} \]

solution to classical EOM:

\[ A^-(x^+, x_t) \equiv n^- S(x^+, x_t) \]

scattering of a quark from background color field

\[ A^-(x^+, x_t) \]
\[ i\mathcal{M}_1 = (ig) \int d^4x_1 \, e^{i(q-p)\cdot x_1} \tilde{u}(q) \left[ \not\! \! \p \, S(x_1) \right] u(p) \]
\[ = (ig)(2\pi)\delta(p^+ - q^+) \int d^2x_{1t} \, dx_1^+ \, e^{i(q^- - p^-)x_1^+} \, e^{-i(q_t - p_t)x_{1t}} \tilde{u}(q) \left[ \not\! \! \p \, S(x_1^+, x_{1t}) \right] u(p) \]

\[ i\mathcal{M}_2 = (ig)^2 \int d^4x_1 \, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1 - p)\cdot x_1} \, e^{i(q - p_1)\cdot x_2} \tilde{u}(q) \left[ \not\! \! \p \, S(x_2) \, \frac{ip_1}{p_1^2 + i\epsilon} \not\! \! \p \, S(x_1) \right] u(p) \]

\[ \int \frac{dp_1^-}{(2\pi)} \, e^{ip_1^-(x_2^+ - x_2^+)} = \frac{-i}{2p^+} \theta(x_2^+ - x_1^+) \, e^{ip_1^+ (x_1^+ - x_2^+)} \]

Contour integration over the pole leads to time ordering of scattering.

Ignore all terms: \( O\left(\frac{p_t}{p^+}, \frac{q_t}{q^+}\right) \)

and use \( \not\! \! \p \, \frac{p_1}{2p^+} \not\! \! \p = \not\! \! \p \)

\[ i\mathcal{M}_2 = (ig)^2 (-i)(i) \, 2\pi \delta(p^+ - q^+) \int dx_1^+ \, dx_2^+ \, \theta(x_2^+ - x_1^+) \int d^2x_{1t} \, e^{-i(q_t - p_t)\cdot x_{1t}} \tilde{u}(q) \left[ S(x_2^+, x_{1t}) \not\! \! \p \, S(x_1^+, x_{1t}) \right] u(p) \]
\[ i\mathcal{M}_n = 2\pi \delta(p^+ - q^+) \bar{u}(q) \not\! \not p \int d^2x_t \ e^{-i(q_t - p_t) \cdot x_t} \]

\[ \left\{ (ig)^n (-i)^n (i)^n \int dx_1^+ dx_2^+ \cdots dx_n^+ \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \right\} u(p) \]

sum over all scatterings gives (Eikonal approximation)

\[ i\mathcal{M}(p, q) = 2\pi \delta(p^+ - q^+) \bar{u}(q) \not\! \not p \int d^2x_t \ e^{-i(q_t - p_t) \cdot x_t} \ [V(x_t) - 1] \ u(p) \]

with \[ V(x_t) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ n^- S_a(x^+, x_t) t_a \right\} \]

including propagation “backward” gives \[ V^\dagger \]
quark propagator in the background color field

\[ S_F(q, p) \equiv (2\pi)^4 \delta^4(p - q) \underbrace{S^0_F(p)}_{\text{no interaction}} + \underbrace{S^0_F(q) \tau_f(q, p) S^0_F(p)}_{\text{interaction}} \]

with \( S^0_F(p) = \frac{i}{p + i\epsilon} \)

\[ \tau_f(q, p) \equiv (2\pi)\delta(p^+ - q^+) \hat{\nu} \int d^2x_t \ e^{i(q_t - p_t) \cdot x_t} \{ \theta(p^+) [V(x_t) - 1] - \theta(-p^+) [V^\dagger(x_t) - 1] \} \]
DIS total cross section

\[ \sigma_{\text{DIS}}^{\text{total}} = 2 \int_0^1 dz \int d^2x_t d^2y_t \left| \Psi(k^\pm, k_t | z, x_t, y_t) \right|^2 T(x_t, y_t) \]

can be written in closed form in terms of Bessel functions \( K_0, K_1 \)

\[ T(x_t, y_t) \equiv \frac{1}{N_c} \text{Tr} \left( 1 - V(x_t) V^\dagger(y_t) \right) \]

\[ V(x_t) \equiv \cdots \equiv \cdots \sim 1 + O(gA) + O(g^2A^2) \]

**total cross section** = probability of photon decaying into a quark anti-quark pair \( \text{QED} \)

probability of the quark anti-quark "dipole" scattering on the target \( \text{QCD} \)
Evolution (energy dependence) of dipoles

\[
\frac{d}{dy} T(x_t - y_t) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2} \times \\
[T(x_t - z_t) + T(z_t - y_t) - T(x_t - y_t) - T(x_t - z_t)T(z_t - y_t)]
\]

\text{BFKL}

\[\tilde{T}(p_t) \rightarrow \log \left[ \frac{Q_s^2}{p_t^2} \right] \text{ saturation region}\]

\[\tilde{T}(p_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right]^\gamma \text{ extended scaling region}\]

\[\tilde{T}(p_t) \rightarrow \frac{1}{p_t^2} \left[ \frac{Q_s^2}{p_t^2} \right] \text{ pQCD region}\]

\text{rich phenomenology: } ep, eA, pp, pA, AA

Rummukainen-Weigert, NPA739 (2004) 183

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $S$ involves only small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s}, \ p^- = 0, p_t = 0)$$

$$S = S(p^+ \sim 0, p^- / P^- \ll 1, p_t)$$

allow hard scattering by including one all x field during which there is large momenta exchanged and quark can get deflected by a large angle.

$$A^\mu_a(x^+, x^-, x_t)$$

include Eikonal multiple scattering before and after (along a different direction) the hard scattering
hard scattering: large deflection
scattered quark travels in the new “z” direction:  

\[ (\bar{x}, \bar{y}, \bar{z}) = \mathcal{O} (x, y, z) \]

\[
iM_1 = (ig) \int d^4x \ e^{i(q-p)x} \bar{u}(q) \left[ \hat{A}(x) \right] u(p)
\]

\[
iM_2 = (ig)^2 \int d^4x \ d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} e^{i(p_1-p)x_1} e^{i(q-p_1)x} \bar{u}(q) \left[ \hat{A}(x) \frac{i\phi_1}{p_1^2 + i\epsilon} \hat{S}(x_1) \right] u(p)
\]

\[
iM_2 = (ig)^2 \int d^4x \ d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} e^{i(\bar{p}_1-p)x} e^{i(q-\bar{p}_1)x_1} \bar{u}(q) \left[ \hat{\bar{S}}(\bar{x}_1) \frac{i\phi_1}{\bar{p}_1^2 + i\epsilon} \hat{A}(x) \right] u(p)
\]

\[
\text{with } \quad \bar{S}^{\mu} = \Lambda^\mu_\nu S^\nu
\]
\[ S_F(p, \bar{q}) = (2\pi)^4 \delta^4(p - \bar{q}) \ S_F^0(p) + S_F^0(p) \ \tau_{\text{hard}}(p, \bar{q}) \ S_F^0(\bar{q}) \]

\[ \tau_{\text{hard}}(p, \bar{q}) \equiv (ig) \int d^4x \int \frac{d^2k_t}{(2\pi)^2} \frac{d^2\bar{k}_t}{(2\pi)^2} \ d^2z_t \ d^2\bar{z}_t \ e^{i(k - \bar{k})x} e^{-i(\bar{q}_t - k_t) \cdot \bar{z}_t} e^{-i(k_t - p_t) \cdot z_t} \]

\[
\begin{align*}
\left\{ \theta(p^+) \ \theta(\bar{q}^+) \ V(z_t, x^+) \ \slashed{\bar{\eta}} \ \frac{k}{2k^+} \ A(x) \ \frac{\bar{k}}{2\bar{k}^+} \ \slashed{\eta} \ \overline{V}(x^+, \bar{z}_t) - \\
\theta(-p^+) \ \theta(-\bar{q}^+) \ V^\dagger(z_t, x^+) \ \slashed{\bar{\eta}} \ \frac{k}{2k^+} \ A(x) \ \frac{\bar{k}}{2\bar{k}^+} \ \slashed{\eta} \ \overline{V}^\dagger(x^+, \bar{z}_t) \right\}
\end{align*}
\]

with

\[ \overline{V}(x^+, \bar{z}_t) \equiv \hat{P} \ \exp \left\{ ig \int_{x^+}^{+\infty} d\bar{z}^+ \ S^-_{\alpha}(\bar{z}_t, \bar{z}^+) \ t_{\alpha} \right\} \]

all "bar-ed" quantities are in a rotated frame where quark’s new direction of propagation (after a hard scattering) is \( \bar{z} \)

this quark propagator is the building block for DIS structure functions, single inclusive particle production in pA,....
but there is more to do: interactions of large and small x modes

in progress
SUMMARY

Collinear factorization with DGLAP evolution at high $p_t$

CGC with JIMWLK evolution at high energy

Leading Log + rc works well

- it has been used to fit a wealth of data; ep, eA, pp, pA, AA
- need to eliminate/minimize late time/hadronization effects

NLO corrections are becoming available

CGC breaks down at large $x$ (high $p_t$)

Toward a unified formalism: CGC + DGLAP