Radiative corrections for a novel evaluation of the Leading Order Hadronic Contribution to the g-2 of the Muon

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on behalf of the proponents of the project
work based on:

C. M. Carloni Calame, M. Passera, L. Trentadue, and G. Venanzoni,

This talk is dedicated to Lev Lipatov 1940-2017

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Radiative Corrections for a Precision Determination of the Fine Structure Constant

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Abstract. We discuss the implications of a new proposed approach to determine $\alpha_{\mu}^{HLO}$ and $\alpha_{QED}$ by using space-like kinematics.

1 Introduction

This talk is dedicated to the memory of Lev Nikolaevich Lipatov (1940-2017).
is a new experiment to measure the Hadronic Leading Order (HLO) contribution to the muon g-2 by using

\[ \mu + e \rightarrow \mu + e \]

elastic scattering

Outlook:
- Physics Motivations
- Tools to perform the measurement
- The first testbeam (CERN)
- Future plans and developments
This talk is about how to measure the Vacuum (the hadronic contribution to)

Vacuum, since a long time (2500 years), constitutes an always present issue in Physics or, better, in Natural Sciences Philosophy

Parmenides, Democritos, Leucippos,…..Torricelli, von Guericke, Casimir, Schwinger, …..to mention only a few until nowadays

In Quantum Field Theory, in the perturbative phase, Vacuum is naturally represented by the vacuum polarization contribution
Vacuum Polarization makes $\alpha_{em}$ running assuming a well defined “effective” value at any scale. Vacuum polarization and the “effective charge” are defined by:

$$
e^2 \rightarrow e^2(q^2) = \frac{e^4}{1 + (\Pi(q^2) - \Pi(0))} \quad \alpha(q^2) = \frac{\alpha(0)}{1 - \Delta \alpha}; \quad \Delta \alpha = -\Re \left( \Pi(q^2) - \Pi(0) \right)$$

$\Delta \alpha$ takes contributions from leptonic and hadronic and gauge bosons elementary states.

Among these the non-perturbative $\Delta \alpha_{\text{had}}$

$$\Delta \alpha = \Delta \alpha_{\text{leptonic}} + \Delta \alpha_{\text{gb}} + \Delta \alpha_{\text{had}} + \Delta \alpha_{\text{top}}$$
The physics motivations

\[ a_\mu = \frac{g - 2}{2} \]

The muon g-2 has been measured with high precision

\[ a_\mu^{exp} = 116592089(63) \times 10^{-11} \]


The Standard Model prediction gives:

\[ a_\mu^{SM} = 116591783(35) \times 10^{-11} \]

F. Jegerlehner, MITP Workshop, 19-23 February 2018 Mainz

\[ \Delta a_\mu (exp - SM) = 306 \pm 72 \]

Systematics of the measurement ?
Systematics of the theoretical prediction ?
New Physics ?
Comparisons of the SM predictions with the measured g-2 value:

\[ a_{\mu}^{\text{EXP}} = 116592091 (63) \times 10^{-11} \]

E821 – Final Report: PRD73 (2006) 072 with latest value of \( \lambda = \mu_{\mu}/\mu_{\rho} \) from CODATA’10

<table>
<thead>
<tr>
<th>( a_{\mu}^{\text{SM}} \times 10^{11} )</th>
<th>( \Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>116591761 (57)</td>
<td>330 (85) \times 10^{-11}</td>
<td>3.9 [1]</td>
</tr>
<tr>
<td>116591818 (51)</td>
<td>273 (81) \times 10^{-11}</td>
<td>3.4 [2]</td>
</tr>
<tr>
<td>116591841 (58)</td>
<td>250 (86) \times 10^{-11}</td>
<td>2.9 [3]</td>
</tr>
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</table>

with the recent “conservative” hadronic light-by-light \( a_{\mu}^{\text{HNLO(lbl)}} = 102 (39) \times 10^{-11} \) of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

The muon g-2 - The Hadronic contribution

\[ K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \]

\[ a_{\mu}^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_e^2}^{\infty} ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_e^2}^{\infty} \frac{ds}{s} K(s)R(s) \]

\[ a_{\mu}^{\text{HLO}} = 6870 \ (42)_{\text{tot}} \times 10^{-11} \]

\[ = 6926 \ (33)_{\text{tot}} \times 10^{-11} \]

\[ = 6949 \ (37)_{\exp \ (21)_{\text{rad}}} \times 10^{-11} \]

F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2n)

M. Davier, arXiv:1612.02743

Hagiwara et al, JPG 38 (2011) 085003

from M. Passera
Comparison between the SM predictions and the experimental determinations

Theory parametrizations
DHMZ (M. Davier et al.), HLMNT (K. Hagiwara et al.)
SMXX is the average of the two previous values

BNL-E821 04 average is the current experimental value of $a_\mu$

New (g-2) exp. is the same central value with a fourfold improved precision of future g-2 experiments at Fermilab and J-PARC.

The physics motivations

will this possibly change in the next few years?

The present experimental error as from the BNL E821 is

\[ \delta a_{\mu}^{Exp} \simeq 6.3 \cdot 10^{-10} [0.54 \text{ ppm}] \]

The new experiments in preparation at Fermilab and J-PARC are aiming to a precision of *

\[ \delta a_{\mu}^{Exp-FL/J-PARC} \simeq 1.6 \cdot 10^{-10} [0.14 \text{ ppm}] \]

(*assuming the same central value as today’s one)

The question is how to cope with such an improvement from the theoretical side
The physics motivations

\[ a_\mu = \frac{g - 2}{2} \]

\[ \Delta a_\mu (Exp - SM) \approx 28 \pm 8 \cdot 10^{-10} \]

Within the framework of low-energy high precision measurements the long-standing (~ 4\(\sigma\)) discrepancy between the experimental value of the muon anomalous magnetic moment and the Standard Model prediction is limited by strong interactions effects.

The present error on the leading order hadronic contribution to muon

\[ g - 2 \]

\[ \delta a_\mu^{HLO} \approx 4 \cdot 10^{-10} \]

It constitutes the main uncertainty of the SM predictions.
The physics motivations

In order to understand the discrepancy between the experimental measurement and the Standard Model prediction it is needed to reduce the theoretical uncertainty to have a more precise determination.

The largest contribution to the theoretical uncertainty comes from the term $\Delta \alpha_{had}$ which can be measured experimentally.

More theoretical work is necessary: Radiative corrections, Lattice evaluations, etc…

The Standard Dispersive Approach to the evaluation of the HLO contribution to the muon anomalous magnetic moment goes back to the ‘60s.
The Standard Dispersive Approach
\[ a^{HLO}_\mu = \left( \frac{\alpha}{\pi^2} \right) \int_0^\infty \frac{ds}{s} K(s) \text{Im} \Pi_{had}(s + i\epsilon) \quad \hat{K}(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x) \frac{s}{m_\mu^2}} \]

Optical Theorem

\[ \text{Im} \hat{\Pi}_{had}(s) \to \sigma_{tot}^{had}(s) \]

\[ a^{HLO}_\mu = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^\infty ds \frac{\hat{K}(s) R_{had}(s)}{s^2} \]

\[ R_{had}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \]
from F. Jegerlehner talk in Frascati March 23, 2016
Measurement of the running of $\alpha_{em}$

A direct measurement of $\alpha_{em}(s/t)$ in space/time-like regions can show the running of $\alpha_{em}(s/t)$

It can provide a test of "duality" (far away from resonances)

It has been done in past by few experiments at $e^+e^-$ colliders by comparing a "well-known" QED process with some reference (obtained from data or MC)

$$\left( \frac{\alpha(q^2)}{\alpha(q_0^2)} \right)^2 \sim \frac{N_{\text{signal}}(q^2)}{N_{\text{norm}}(q_0^2)}$$

$N_{\text{signal}}$ can be any QED process, muon pairs, etc…
$N_{\text{norm}}$ can be Bhabha process, pure QED as $\gamma\gamma$ pair production, a well as theory, or any other reference process.
We propose an alternative approach
The alternative approach of using a space-like formula for the vacuum polarization

\[
a_{\mu}^{HLO} = \frac{\alpha}{\pi} \int_0^1 dx \ (1 - x) \Pi_{\text{had}}(t(x)) = \frac{\alpha}{\pi} \int_0^1 dx \ (1 - x) \Delta \alpha_{\text{had}}(t(x))
\]

\[
a_{\mu}^{HLO} = \left( \frac{\alpha}{\pi} \right) \int_{-\infty}^0 \frac{dt}{\beta t} \left( \frac{1 - \beta}{1 + \beta} \right)^2 \Pi_{\text{had}}(t) = -\left( \frac{\alpha}{\pi} \right) \int_{-\infty}^0 \frac{dt}{\beta t} \left( \frac{1 - \beta}{1 + \beta} \right)^2 \Delta \alpha_{\text{had}}(t(x))
\]

\[
\beta = \sqrt{1 - \frac{4m_\mu^2}{t}}
\]

\[
t(x) = -\frac{x^2m_\mu^2}{1 - x}
\]

\[
\alpha(t) = \frac{\alpha(0)}{1 - \Delta \alpha(t)}
\]

\[
\Delta \alpha_{\text{had}}(t) \text{ is the hadronic contribution to the running of } \alpha
\]

\[
\Delta \alpha_{\text{had}}(t) = \Delta \alpha(t) - \Delta \alpha_{\text{lep}}(t)
\]

This may be obtained by using Bhabha scattering
\[ \Delta \alpha_i(t(x)) \]

\[ \Delta \alpha_{lep}(t) \]

\[ \Delta \alpha_{had}(t) \]

\[ i = \text{had} \]

\[ i = \text{lep} \]
The smooth integrand function \( \Delta \alpha_{\text{had}} \) is given by the total area under the curve

\[
(1 - x) \cdot \Delta \alpha_{\text{had}} \left( \frac{x^2 m^2}{x - 1} \right) \times 10^5
\]

for \( x \) in the range 0 to 1.

The peak values are: \( x_{\text{peak}} = 0.914 \) and \( t_{\text{peak}} = -0.108 \text{ GeV}^2 \).
2. Theoretical framework

The leading-order hadronic contribution to the muon $g-2$ is given by the well-known formula $[4,15]$

$$a_{\mu}^{HLO} = \frac{\alpha}{\pi^2} \int_{0}^{\infty} \frac{ds}{s} K(s) \operatorname{Im} \Pi_{\text{had}}(s + i\epsilon), \quad (1)$$

where $\Pi_{\text{had}}(s)$ is the hadronic part of the photon vacuum polarization, $\epsilon > 0$,

$$K(s) = \int_{0}^{1} dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_{\mu}^2)} \quad (2)$$

is a positive kernel function, and $m_{\mu}$ is the muon mass. As the total cross section for hadron production in low-energy $e^+e^-$ annihilations is related to the imaginary part of $\Pi_{\text{had}}(s)$ via the optical theorem, the dispersion integral in Eq. (1) is computed integrating experimental time-like ($s > 0$) data up to a certain value of $s$ $[2,18,19]$. The high-energy tail of the integral is calculated using perturbative QCD $[20]$.

Alternatively, if we exchange the $x$ and $s$ integrations in Eq. (1) we obtain $[21]$

$$a_{\mu}^{HLO} = \frac{\alpha}{\pi} \int_{0}^{1} dx (x-1) \overline{\Pi}_{\text{had}}[t(x)], \quad (3)$$

where $\overline{\Pi}_{\text{had}}(t) = \Pi_{\text{had}}(t) - \Pi_{\text{had}}(0)$ and
The space-like kinematics allows a direct comparison with the lattice evaluations.

$$t(x) = \frac{x^2 m^2}{x - 1} < 0$$

is a space-like squared four-momentum. If we invert Eq. (4), we get 
$$x = (1 - \beta) (t/2m^2),$$
with
$$\beta = (1 - 4m^2/t)^{1/2},$$
and from Eq. (3) we obtain

$$a^\text{HLO}_\mu = \frac{\alpha}{\pi} \int_{-\infty}^{0} \bar{\Pi}_{\text{had}}(t) \left( \frac{\beta - 1}{\beta + 1} \right)^2 \frac{dt}{t\beta}. \quad (5)$$

Eq. (5) has been used for lattice QCD calculations of $a^\text{HLO}_\mu$ [22]; while the results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [23].


To summarize

\[
a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_{0}^{1} (1 - x) \Pi_{\text{had}} \left( -\frac{x^2}{1 - x} m_{\mu}^2 \right) dx
\]

\[
t = \frac{x^2 m_{\mu}^2}{x - 1} \quad 0 \leq -t < +\infty
\]

\[
x = \frac{t}{2 m_{\mu}^2} \left( 1 - \sqrt{1 - \frac{4 m_{\mu}^2}{t}} \right); \quad 0 \leq x < 1;
\]

\[
t = -s \sin^2 \left( \frac{\theta}{2} \right)
\]

\[
\Delta \alpha_{\text{had}}(t) = -\Pi_{\text{had}}(t) \quad \text{for } t < 0
\]

with the “t” kernel

\[
a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_{0}^{1} (1 - x) \Delta \alpha_{\text{had}} \left( -\frac{x^2}{1 - x} m_{\mu}^2 \right) dx
\]
AN EXAMPLE OF A
SPACE-LIKE APPROACH

BASED ON THE PER-MILLE ACCURACY
 EVALUATION OF THE **BHABHA SCATTERING**
 CROSS-SECTION
Abstract

A method to determine the running of $\alpha$ from a measurement of small-angle Bhabha scattering is proposed and worked out. The method is suited to high statistics experiments at $e^+e^-$ colliders, which are equipped with luminometers in the appropriate angular region. A new simulation code predicting small-angle Bhabha scattering is also presented.
The method to measure the running of $\alpha$ exploits the fact that the cross section for the process $e^+e^- \rightarrow e^+e^-$ can be conveniently decomposed into three factors:

$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt} \left( \frac{\alpha(t)}{\alpha(0)} \right)^2 (1 + \Delta r(t))$$

(3)

each one of them known with an accuracy of at least 0.1%

1st factor

$$\frac{d\sigma^0}{dt} = \frac{d\sigma^B}{dt} \left( \frac{\alpha(0)}{\alpha(t)} \right)^2 .$$

The Born cross section contains all the soft and virtual corrections

Bhabha is a pure QED process
Quarks enter only in loops
2nd factor

\[ \left( \frac{\alpha(t)}{\alpha(0)} \right)^2 \]

Vacuum polarization effects gives the running of alpha

3rd factor

\[ (1 + \Delta r(t)) \]

with all the real and virtual effects not incorporated in the running of alpha
\[ \alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)}, \]

\[ \alpha(0) \] is the Sommerfeld fine structure constant measured with a precision of \( O(10^{-9}) \).

\[ \Delta\alpha(q^2) \] from loop contributions to the photon propagator
Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

OPAL Collaboration
Abstract

Using the OPAL detector at LEP, the running of the effective QED coupling $\alpha(t)$ is measured for space-like momentum transfer from the angular distribution of small-angle Bhabha scattering. In an almost ideal QED framework, with very favourable experimental conditions, we obtain:

$$\Delta \alpha(-6.07 \text{ GeV}^2) - \Delta \alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},$$

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty. This agrees with current evaluations of $\alpha(t)$. The null hypothesis that $\alpha$ remains constant within the above interval of $-t$ is excluded with a significance above 5$\sigma$. Similarly, our results are inconsistent at the level of 3$\sigma$ with the hypothesis that only leptonic loops contribute to the running. This is currently the most significant direct measurement where the running $\alpha(t)$ is probed differentially within the measured $t$ range.
The method used follows the above parametrization/factorization of the Bhabha cross-section

\[
\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt} \left( \frac{\alpha(t)}{\alpha_0} \right)^2 (1 + \epsilon) (1 + \delta_\gamma) + \delta_Z
\]

\[
\frac{d\sigma^{(0)}}{dt} = \frac{4\pi\alpha_0^2}{t^2}
\]

We determined the effective slope of the Bhabha momentum transfer distribution which is simply related to the average derivative of \(\Delta \alpha\) as a function of \(\ln t\) in the range \(2 \text{ GeV}^2 \leq -t \leq 6 \text{ GeV}^2\). The observed \(t\)-spectrum is in good agreement with Standard Model predictions. We find:

\[\Delta \alpha(-6.07 \text{ GeV}^2) - \Delta \alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},\]

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty.

This measurement is one of only a very few experimental tests of the running of \(\alpha(t)\) in the space-like region, where \(\Delta \alpha\) has a smooth behaviour. We obtain the strongest direct evidence for the running of the QED coupling ever achieved differentially in a single experiment, with a significance above 5\(\sigma\). Moreover we report clear experimental evidence for the hadronic contribution to the running in the space-like region, with a significance of 3\(\sigma\).
Such an approach was possible with a per-mille accuracy of the Bhabha cross-section 1992-1997....
all started in 1992 with a preprint:

**Small angles Bhabha scattering: Two loop approximation**

Victor S. Fadin (Novosibirsk, IYF), E.A. Kuraev (Dubna, JINR), L.N. Lipatov (St. Petersburg, INP), N.P. Merenkov (Kharkov, KIPT), L. Trentadue (CERN)

Dec 1992 - 20 pages

JINR-E2-92-577

then the general program:

**Generalized eikonal representation of the small angle e+ e- scattering amplitude at high-energy**

Victor S. Fadin, E.A. Kuraev, L. Trentadue (Dubna, JINR), L.N. Lipatov (St. Petersburg, INP), N.P. Merenkov (Kharkov, KIPT)

1993

Small Angle Bhabha Scattering for LEP

A. Arbuzov a V. Fadin b E. Kuraev a
L. Lipatov c N. Merenkov d L. Trentadue e

We present the results of our calculations to a one, two, and three loop approximation of the e⁺e⁻→e⁺e⁻ Bhabha scattering cross-section at small angles. All terms contributing to the radiatively corrected cross-section, within an accuracy of δσ/σ = 0.1%, are explicitly evaluated and presented in an analytic form. O(α) and O(α²) contributions are kept up to next-to-leading logarithmic accuracy, and O(α³) terms are taken into account to the leading logarithmic approximation. We define an experimentally measurable cross-section by integrating the calculated distributions over a given range of final-state energies and angles. The cross-sections for exclusive channels as well as for the totally integrated distributions are also given.
and a few months later:

Small-angle electron–positron scattering with a per mille accuracy


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\textsuperscript{b} Budker Institute for Nuclear Physics, Novosibirsk State University, 630090, Novosibirsk, Russia
\textsuperscript{c} St. Petersburg Institute of Nuclear Physics, Gatchina, Leningrad region, 188350, Russia
\textsuperscript{d} Institute of Physics and Technology, Kharkov, 310108, Ukraine
\textsuperscript{e} Theoretical Physics Division, CERN, CH-1211 Geneva 23, Switzerland

Received 22 December 1995; revised 25 June 1996; accepted 6 September 1996
The goal of the analytical result was aiming at a precision

\[ \left| \frac{\delta \sigma}{\sigma} \right| < 0.001 \]

since the accuracy reached at the time was still inadequate. According to the evaluations the theoretical estimates were still incomplete, moreover, are in disagreement with each other up to 0.5%, far from the required theoretical and experimental accuracy.

\[ \theta_1 < \theta_- = \vec{p}_1 \vec{p}_1' \equiv \theta < \theta_3 \quad , \quad \theta_2 < \theta_+ = \vec{p}_2 \vec{p}_2' < \theta_4 \quad , \quad 0.01 \lesssim \theta_i \lesssim 0.1 \text{ rad} \quad , \quad (2) \]

where \( \vec{p}_1, \vec{p}_1' \) (\( \vec{p}_2, \vec{p}_2' \)) represent the momenta of the initial and of the scattered electron (positron) in the center-of-mass frame.
At small angles the main contribution comes from one photon exchanged in the t-channel
(due to the eikonal approximation logarithmic terms from multiple-photon exchange diagrams do cancel)

\[
\frac{d\sigma}{d\theta^2} \sim \theta^{-4} .
\]

Let us now estimate the correction of order $\theta^2$ to this contribution. If

\[
\frac{d\sigma}{d\theta^2} \sim \theta^{-4}(1 + c_1 \theta^2) ,
\]

then, after integration over $\theta^2$ in the angular range as Eq. (2), one obtains:

\[
\int_{\theta^2_{\text{min}}}^{\theta^2_{\text{max}}} \frac{d\sigma}{d\theta^2} d\theta^2 \sim \theta^{-2}(1 + c_1 \theta^2_{\text{min}} \ln \frac{\theta^2_{\text{max}}}{\theta^2_{\text{min}}}).
\]

Also terms of the type:

\[
\frac{m^2}{Q^2} \quad m = m_e, m_\mu
\]

if \[Q^2 \simeq 1 \text{GeV}^2\] may be omitted
2. Born cross section and one-loop virtual and soft corrections

The Born cross section for Bhabha scattering within the Standard Model is well known [8]:

\[
\frac{d\sigma^B}{d\Omega} = \frac{\alpha^2}{8s} \{4B_1 + (1 - c)^2B_2 + (1 + c)^2B_3\},
\]

(5)

where

\[
B_1 = \left(\frac{s}{t}\right)^2 \left|1 + (g_v^2 - g_a^2)\xi\right|^2, \quad B_2 = \left|1 + (g_v^2 - g_a^2)\chi\right|^2,
\]

\[
B_3 = \frac{1}{2} \left|1 + \frac{s}{t} + (g_v + g_a)^2\left(\frac{s}{t}\xi + \chi\right)\right|^2 + \frac{1}{2} \left|1 + \frac{s}{t} + (g_v - g_a)^2\left(\frac{s}{t}\xi + \chi\right)\right|^2,
\]

\[
\chi = \frac{\Lambda s}{s - m_z^2 + \text{i}M_z\Gamma_z}, \quad \xi = \frac{\Lambda t}{t - M_z^2},
\]

\[
\Lambda = \frac{G_FM_z^2}{2\sqrt{2}\pi\alpha} = (\sin 2\theta_w)^{-2}, \quad g_a = -\frac{1}{2}, \quad g_v = -\frac{1}{2}(1 - 4\sin^2 \theta_w),
\]

\[
s = (p_1 + p_2)^2 = 4\varepsilon^2, \quad t = -Q^2 = (p_1 - q_1)^2 = -\frac{1}{2} s(1 - c),
\]

\[
c = \cos \theta, \quad \theta = \frac{\vec{p}_1 \cdot \vec{q}_1}{s}.
\]

Here \(\theta_w\) is the Weinberg angle. In the small-angle limit \((c = 1 - \theta^2/2 + \theta^4/24 + \ldots)\), expanding formula (5) leads to
the weak interaction contribution

\[
\frac{d\sigma^B}{d\theta} = \frac{8\pi\alpha^2}{\varepsilon^2\theta^4} \left( 1 - \frac{\theta^2}{2} + \frac{9}{40}\theta^4 + \delta_{\text{weak}} \right),
\]

where \( \varepsilon = \sqrt{s}/2 \) is the electron or positron initial energy and the weak correction term \( \delta_{\text{weak}} \), connected with diagrams with \( Z^0 \)-boson exchange, is given by the expression

\[
\delta_{\text{weak}} = 2g_v^2\xi - \frac{\theta^2}{4} (g_v^2 + g_a^2) \text{Re} \chi + \frac{\theta^4}{32} (g_v^4 + g_a^4 + 6g_v^2g_a^2)|\chi|^2.
\]

One can see from Eq. (7) that the contribution \( c_1^w \) of the weak correction \( \delta_{\text{weak}} \) into the coefficient \( c_1 \) introduced in Eq. (3)

\[
c_1^w \lesssim 2g_v^2 + \frac{(g_v^2 + g_a^2)}{4} \frac{M_Z}{\Gamma_Z} + \theta_{\text{max}}^2 \frac{(g_v^4 + g_a^4 + 6g_v^2g_a^2)}{32} \frac{M_Z^2}{\Gamma_Z^2} \simeq 1.
\]
virtual + soft photon contribution

\[ \frac{d\sigma^{(1)}_{\text{QED}}}{dc} = \frac{d\sigma_{\text{QED}}^B}{dc} \left( 1 + \delta_{\text{virt}} + \delta_{\text{soft}} \right), \quad (9) \]

where \( d\sigma_{\text{QED}}^B \) is the Born cross section in the pure QED case (it is equal to \( d\sigma^B \) with \( g_a = g_v = 0 \)) and

\[ \delta_{\text{virt}} + \delta_{\text{soft}} = 2 \frac{\alpha}{\pi} \left[ 2 \left( 1 - \ln \frac{4\varepsilon^2}{m^2} + 2 \ln \left( \cot \frac{\theta}{2} \right) \right) \ln \frac{\varepsilon}{\Delta \varepsilon} + \int_{\cos^2(\theta/2)}^{\sin^2(\theta/2)} \frac{dx}{x} \ln(1 - x) \right. \]

\[ - \frac{23}{9} + \frac{11}{6} \ln \frac{4\varepsilon^2}{m^2} \right] + \frac{\alpha}{\pi} \frac{1}{(3 + c^2)^2} \left[ \frac{\pi^2}{3} (2c^4 - 3c^3 - 15c) \right. \]

\[ + 2(2c^4 - 3c^3 + 9c^2 + 3c + 21) \ln^2 \left( \sin \frac{\theta}{2} \right) \]

\[ - 4(c^4 + c^2 - 2c) \ln^2 \cos \frac{\theta}{2} - 4(c^3 + 4c^2 + 5c + 6) \ln^2 \left( \tan \frac{\theta}{2} \right) \]

\[ + \frac{2}{3} (11c^3 + 33c^2 + 21c + 111) \ln \left( \sin \frac{\theta}{2} \right) \]

\[ + 2(c^3 - 3c^2 + 7c - 5) \ln \left( \cos \frac{\theta}{2} \right) \]

\[ + 2(c^3 + 3c^2 + 3c + 9) \delta_t - 2(c^3 + 3c)(1 - c) \delta_s \right]. \]
Let us define $\Sigma_0^0$ to be equal to $\Sigma_0|_{\Pi=0}$ (see Eq. (21)), which corresponds to the Born cross section obtained by switching off the vacuum polarization contribution $\Pi(t)$. For the experimentally observable cross section we obtain

$$\sigma = \frac{4\pi\alpha^2}{Q_1^2} \Sigma_0^0 (1 + \delta_0 + \delta^\gamma + \delta^{2\gamma} + \delta^{e^+e^-} + \delta^{3\gamma} + \delta^{e^+e^-\gamma}),$$

(94)

where

$$\Sigma_0^0 = \Sigma_0|_{\Pi=0} = 1 - \rho^{-2} + \Sigma_W + \Sigma_\theta|_{\Pi=0}$$

(95)

and

$$\delta_0 = \frac{\Sigma_0 - \Sigma_0^0}{\Sigma_0^0}, \quad \delta^\gamma = \frac{\Sigma^\gamma}{\Sigma_0^0}, \quad \delta^{2\gamma} = \frac{\Sigma^{2\gamma}}{\Sigma_0^0}, \ldots$$

(96)

The numerical results are presented in Table 1.
Table 1
The values of $\delta^i$ in per cent for $\sqrt{s} = 91.161$ GeV, $\theta_1 = 1.61^\circ$, $\theta_2 = 2.8^\circ$, $\sin^2 \theta_W = 0.2283$, $l^* = 2.4857$ GeV.

<table>
<thead>
<tr>
<th>$x_c$</th>
<th>$\delta_0$</th>
<th>$\delta^\gamma$</th>
<th>$\delta_{\text{leading}}^{2\gamma}$</th>
<th>$\delta_{\text{non-leading}}^{2\gamma}$</th>
<th>$\delta^{e^+e^-}$</th>
<th>$\delta^{e^+\gamma^e}$</th>
<th>$\delta^{3\gamma}$</th>
<th>$\sum \delta^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>4.120</td>
<td>-8.918</td>
<td>0.657</td>
<td>0.162</td>
<td>-0.016</td>
<td>-0.017</td>
<td>-0.019</td>
<td>-4.031±0.006</td>
</tr>
<tr>
<td>0.2</td>
<td>4.120</td>
<td>-9.226</td>
<td>0.636</td>
<td>0.156</td>
<td>-0.027</td>
<td>-0.011</td>
<td>-0.016</td>
<td>-4.368±0.006</td>
</tr>
<tr>
<td>0.3</td>
<td>4.120</td>
<td>-9.626</td>
<td>0.615</td>
<td>0.148</td>
<td>-0.033</td>
<td>-0.008</td>
<td>-0.013</td>
<td>-4.797±0.006</td>
</tr>
<tr>
<td>0.4</td>
<td>4.120</td>
<td>-10.147</td>
<td>0.586</td>
<td>0.139</td>
<td>-0.039</td>
<td>-0.005</td>
<td>-0.010</td>
<td>-5.356±0.006</td>
</tr>
<tr>
<td>0.5</td>
<td>4.120</td>
<td>-10.850</td>
<td>0.539</td>
<td>0.129</td>
<td>-0.044</td>
<td>-0.003</td>
<td>-0.006</td>
<td>-6.115±0.006</td>
</tr>
<tr>
<td>0.6</td>
<td>4.120</td>
<td>-11.866</td>
<td>0.437</td>
<td>0.132</td>
<td>-0.049</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-7.229±0.006</td>
</tr>
<tr>
<td>0.7</td>
<td>4.120</td>
<td>-13.770</td>
<td>0.379</td>
<td>0.130</td>
<td>-0.057</td>
<td>-0.001</td>
<td>0.005</td>
<td>-9.194±0.006</td>
</tr>
<tr>
<td>0.8</td>
<td>4.120</td>
<td>-17.423</td>
<td>0.608</td>
<td>0.089</td>
<td>-0.069</td>
<td>0.001</td>
<td>0.013</td>
<td>-12.661±0.006</td>
</tr>
<tr>
<td>0.9</td>
<td>4.120</td>
<td>-25.269</td>
<td>1.952</td>
<td>-0.085</td>
<td>-0.085</td>
<td>0.005</td>
<td>0.017</td>
<td>-19.379±0.006</td>
</tr>
</tbody>
</table>

$x_c \left( \frac{\alpha L}{\pi} \right)^3 \leq 0.3 \times 10^{-4}$ (for $x_c = 0.5$).

Regarding all the uncertainties (a)-(g) and (82) as independent ones we conclude the total theoretical uncertainty of our results to be ±0.006%.

Each of these contributions to $\sigma$ has a sign that can change because of the interplay between real and virtual corrections. The cross section corresponding to the Born diagrams for producing a real particle is always positive, whereas the sign of the radiative corrections depends on the order of perturbation theory. For the virtual corrections at odd orders it is negative, and at even orders it is positive. When the aperture of the counters is small the compensation between real and virtual corrections is not complete. In the limiting case of small aperture ($\rho \to 1, x_c \to 1$) the virtual contributions dominate.
Running of $\alpha_{\text{em}}$

$$\Delta \alpha = \frac{\alpha M_Z^2}{3\pi} \Re \int_4^{\infty} ds \frac{R(s)}{(s - M_Z^2 - i\epsilon)}$$

$\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$

- **time-like**
- **space-like**
A new possibility
via
\[ \mu e \rightarrow \mu e \]
scattering
$\mu e \rightarrow \mu e \quad 1.3 \times 10^7 \mu/s$

- High intensity muon beam available in the CERN North Area $E = 150$ GeV
- pure t-channel process
  \[
  \frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha(0)} \right|^2
  \]
  \[
  s \simeq 0.16 \text{ GeV}^2 \quad -0.14 \leq t \leq 0 \text{ GeV}^2 \quad 0 \leq x \leq 0.93
  \]

- the $2 \rightarrow 2$ kinematics reads
  \[
  t = 2m_e^2 - 2m_e E_e, \quad s = m_\mu^2 + m_e^2 + 2m_e E_\mu^2
  \]
  \[
  E_e = m_e \frac{1 + r^2 c_e^2}{1 - r^2 c_e^2}, \quad \theta_e = \arccos \left( \frac{1}{r} \sqrt{\frac{E_e - m_e}{E_e + m_e}} \right)
  \]
  \[
  r = \frac{\sqrt{(E_\mu^2 - m_\mu^2)}}{E_\mu + m_e}, \quad c_e = \cos \theta_e
  \]
  \[
  0 < \theta_e < 31.85 \text{ mrad} \quad \leftrightarrow \quad 139.8 > E_e > 1 \text{ GeV} \quad \leftrightarrow \quad -0.143 < t < -10^{-3} \text{ GeV}^2
  \]

- Same process can be used for signal and normalization
  - differential cross-section at LO (including vacuum polarization) as a function of $\theta_e$
  - effect due to $\Delta \alpha_{\text{had}}(t)$
  - for instance the region $\theta_e > 20 \div 25 \text{ mrad}$ can be used as normalization
$10^{-3}$

LO cross section

$\Delta \alpha_{\text{had}} \times 10^3$

Signal

$\sim 10^{-5} < \Delta \alpha_{\text{had}} < 10^{-3}$

$\theta_e$ (mrad)

Normalization

$\Delta \alpha_{\text{had}} < \sim 10^{-5}$

$N_{\text{data}}(t_i)$

$N_{\text{norm data}}^\text{norm}$

$\theta_e$ [mrad]
Muon and electron scattering angles are correlated. This very important constraint may be used to select elastic events, reject background from radiative events and minimize systematics.

Muon beam momentum = 150 GeV

Electron scattering angle (mrad)

Muon scattering angle (mrad)

- $x = 0.0$, $E_e = 0.2$ GeV
- $x = 0.1$, $E_e = 0.5$ GeV
- $x = 0.2$, $E_e = 0.9$ GeV
- $x = 0.3$, $E_e = 1.4$ GeV
- $x = 0.4$, $E_e = 2.9$ GeV
- $x = 0.5$, $E_e = 5.5$ GeV
- $x = 0.6$, $E_e = 9.8$ GeV
- $x = 0.7$, $E_e = 17.8$ GeV
- $x = 0.8$, $E_e = 35.0$ GeV
- $x = 0.9$, $E_e = 88.5$ GeV
- $x = 0.928$, $E_e = 130.7$ GeV
- $x = 0.932$, $E_e = 139.5$ GeV
Detector design/optimization

- Electromagnetic calorimeter needed to:
  - Perform the PID: muon/electron discrimination.
    - PID capabilities also reconstructing the electromagnetic shower in the tracking system.
  - Triggering: (muon in) AND (ECAL $E > E_{\text{th}}$)
    - There is an alternative trigger condition: (muon in) AND (2 prongs into a given module)
- Establish how to measure $E_e$ in order to get rid of events with electron energy below 1 GeV

U. Marconi at the CSN1 May 2017
The Detector

i) Initial muons have to be tagged with their direction and momentum

ii) 60 Be (C) layers interfaced with Si planes spaced by 1m air gap modularly spaced

iii) The use of a low Z material in order to reduce multiple scattering and background

iv) A final EM calorimeter to discriminate e/mu at small angles (2-3 mrad)
Statistics

\[ \mu \text{ beam} \quad 1.3 \times 10^7 \mu/s \quad \text{for} \quad 2 \times 10^7 \text{s/yr} \]
\[ 2 \times 10^{12} \text{ events/yr} \quad \text{statistical precision} \quad 0.3\% \text{ in 2 yrs running} \]

Systematics

many effects have to be under control:
efficiencies (uniformity, acceptance, tracking, trigger, PID)
alignment of the Si planes, uncertainties in vertex location, incoming muon momentum, effect of multiple scattering (different in “control” and “signal” regions) .........................(many others, can be studied with data themselves).

Theory

Electroweak radiative corrections (including subleading logarithmic and mass contributions) have to be under control at the NNLO accuracy
This idea has been proposed in 2016 at the “Physics Beyond Collider Workshop” at CERN

- The idea has been presented in 2016 to the “Physics Beyond Collider Study Group”

- C. Matteuzzi and G. Venanzoni are members of the board as the experiment representatives.

- Physics Beyond Collider Study Group will select in fall 2018 experiments aiming to:
  
  - Enrich and diversify the CERN scientific program:
    - Exploit the unique opportunities offered by CERN’s accelerator complex and scientific infrastructure
    - Complement the laboratory’s collider programme (LHC, HL-LHC and possible future colliders).
    - The scientific findings will be collected in a report to be delivered by the end of 2018.

This document will also serve as input to the next update of the European Strategy for Particle Physics.

Also proposed to the INFN NSCI in 2017 and 2018
Theory

Our final TH goal: a running MC for the ratio of the SM cross sections in the signal and normalization regions below, at the level, of 10 ppm

Muon-electron scattering: theory progress

- NLO QED corrections known & checked. MC @ NLO ready and tailored to the fixed target kinematics.

- NNLO: Missing MI for the planar 2-loop box diagrams computed.

- NNLO amplitudes: virtual 2-loop, real-virtual, double real, automation, subtractions...

- NNLO hadronic contributions

- Fixed-order NNLO + Resummation

- Towards a MC at NNLO

- Interplay with lattice calculations

M. Passera  CERN - PBC  March 2 2018
from U. Marconi
Master integrals for the NNLO virtual corrections to $\mu e$ scattering in QED: the non-planar graphs

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ABSTRACT: We evaluate the master integrals for the two-loop non-planar box-diagrams contributing to the elastic scattering of muons and electrons at next-to-next-to-leading order in QED. We adopt the method of differential equations and the Magnus exponential to determine a canonical set of integrals, finally expressed as a Taylor series around four space-time dimensions, with coefficients written as combination of generalised polylogarithms. The electron is treated as massless, while we retain full dependence on the muon mass. The considered integrals are also relevant for crossing-related processes, such as di-muon production at $e^+e^-$ colliders, as well as for the QCD corrections to top-pair production at hadron colliders. In particular our results, together with the planar master integrals recently computed, represent the complete set of functions needed for the evaluation of the two-loop virtual next-to-next-to-leading order QED corrections to $e\mu \to e\mu$ and $e^+e^- \to \mu^+\mu^-$.
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TEST Beams 2017/2018
at CERN
2017 Test Beam Goals

• Measure the effect of low Z materials on high energy electrons and check GEANT4 simulations.
• Carbon targets of various thickness: 4 to 20 mm
• Electron beams of 12 GeV and 20 GeV
• Detector response measured directly (by taking data removing the target)
• Detector alignment with high-energy pions and muons
• Run with muon beam to possibly detect elastic scattering events.
Test Beam 2017: the setup

- We used the UA9 tracking system to record scattering data
- Alignment, tracking, pattern recognition done by ourselves starting from scratch
- Geant4 simulation is an evolving subject: again developed from scratch
2017 Test Beam setup and target

Thanks to the UA9 Collaboration
(particularly M. Garattini, R. Iaconageli,
M. Pesaresi), J. Bernhard

from U. Marconi
2017 Test Beam Result

Evidence of the elastic scattering

160 GeV muon beam, 8 mm C target
Golden selection: single track in and two tracks out

from U. Marconi
Diffusion angle [-2,2] mrad

Diffusion angle [-2,2] mrad

Diffusion angle [-0.5,0.5] mrad

20mm, 12 GeV

from U. Marconi
from U. Marconi

8mm, 20 GeV

Diffusion angle [-2,2] mrad

Diffusion angle [-0.5,0.5] mrad
Test Beam 2018

- COMPASS uses hadron beams. Muon beams are used for alignment and calibration once per 1 - 2 weeks.
- We can use both muons beams and muons from pion decays.
  - This implies being able to run from April to October.

- Main objectives
  - Select elastic events
  - Effects of the muon beam momentum mean and resolution
  - Measure the tracking efficiency uniformity versus the scattering angle.
  - Measure the relation angle-energy of the scattered electrons.
Setup located in the North Area behind COMPASS detector

Test Beam 2018

EHN2 Test Beams 2018

- MUonE: Measure $\mu e$ scattering on 2 target modules with Silicon instrumentation + 1 EM calorimeter. Total length ~ 3m.
- Compass TPC: Measure $\mu p$ scattering in high pressure TPC + Silicon telescope

from U. Marconi
Tentative timeline of the project

- Studies with Geant4 (underway)
- Detector geometry, number of planes, thickness, calorimeter for pid,…
- Test beam 2018 (muons) 2019 (electrons)
- Assemble the detector -> 2020
- Start to collect data 2021
Conclusions

• We propose to measure muon electron scattering by using the muon beam in the CERN North Area (150 GeV) to extract the space-like quark vacuum polarization.

• This measurement will allow to obtain the leading hadronic contribution to the g-2 in a new independent way and will constitute a crosscheck with previous time-like determinations and with the lattice results.

• The goal is to determine the origin of the presently observed discrepancy between experiments and Standard Model predictions of the g-2 and the origin if within SM or if it could be attributed to BSM physics.
The End