Bayesian analysis for extracting properties of the nuclear equation of state from observational data

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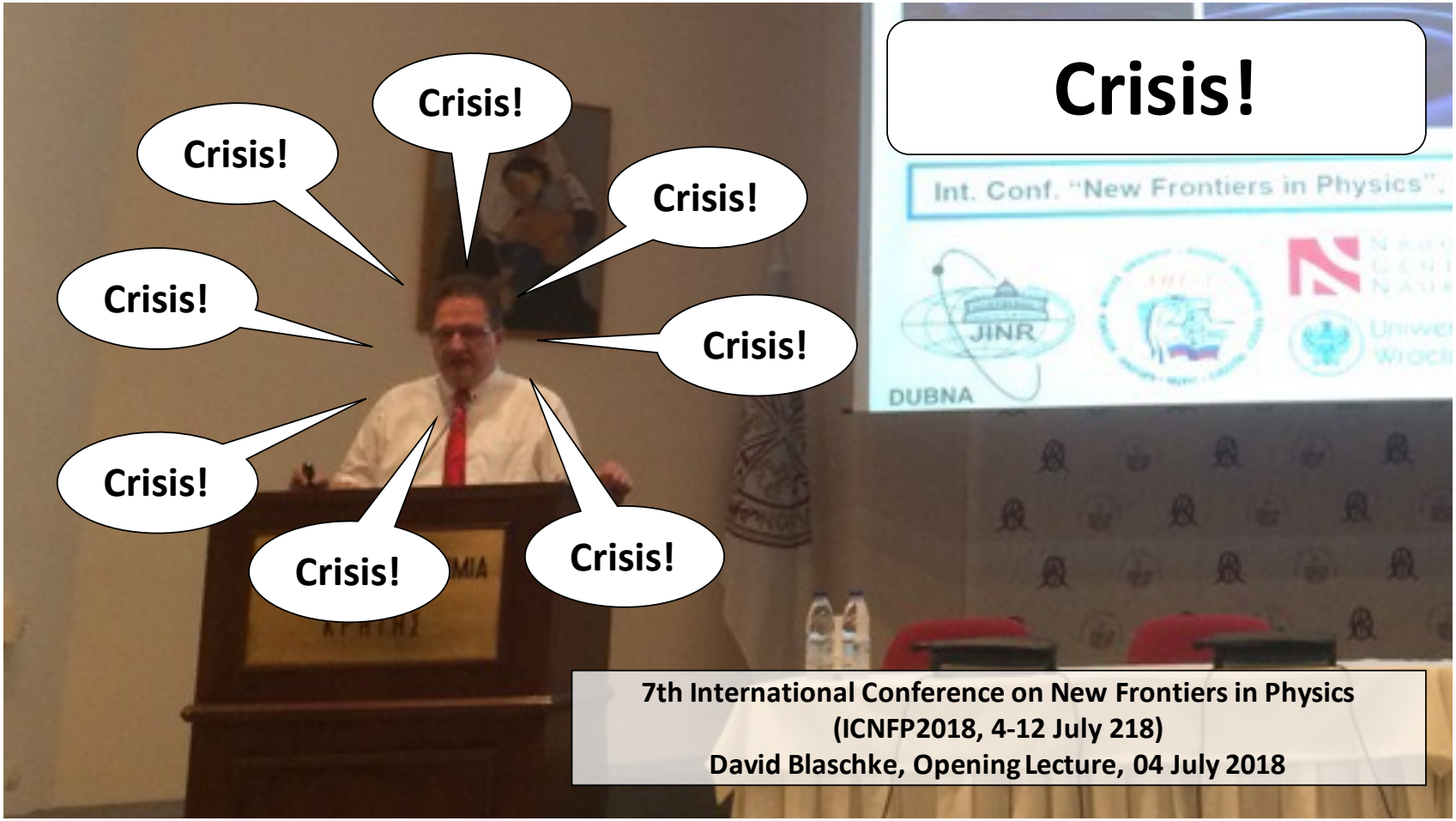
Crisis


Crisis!
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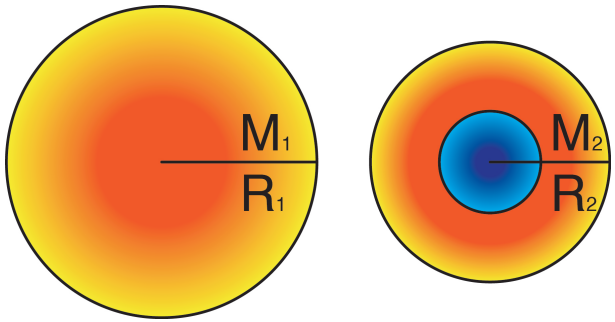
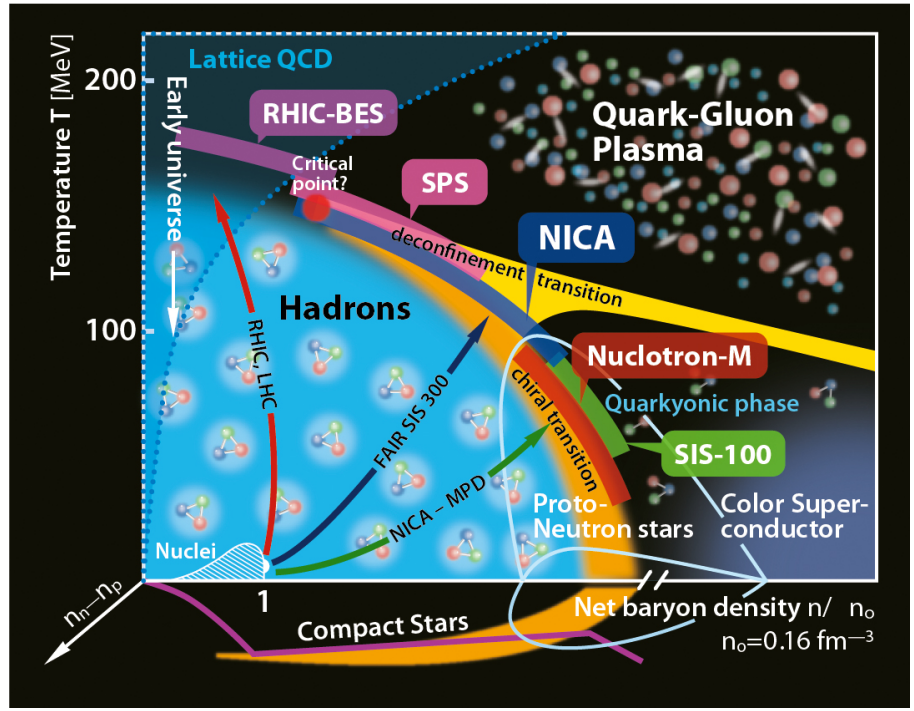
Int. Conf. "New Frontiers in Physics".
JINR
DUBNA
7th International Conference on New Frontiers in Physics
(ICNFP2018, 4-12 July 218)
David Blaschke, Opening Lecture, 04 July 2018

Motivation

What if we have twins





- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?
- etc.

The picture of twin is made by Mark Kaltenborn

Bayesian analysis

What is Bayesian analysis?

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.
One of the most frequent case is analysis of probable values of model parameters.

Bayesian Analysis

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Bayes' theorem:

Likelihood Prior

$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Posterior Evidence

Prior: knowledge before experiment (logically)
Likelihood: Probability for data if the hypothesis was true
Posterior: Probability that the hypothesis is true given the data
Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

Bayesian Analysis

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

↓

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall\, i = 0..N-1$

↓

Calculating the coditional probabilities of the events:
$P(E\,|\vec{\pi}_i) = \prod_\alpha P(E_\alpha\,|\vec{\pi}_i)$,
where α is the index of the observational constraints.

↓

Calculating the *a posteriori* probabilities of the models:

$$P(\vec{\pi}_i\,|E) = \frac{P(E|\vec{\pi}_i)P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)P(\vec{\pi}_j)}$$

Neutron star mass and radius

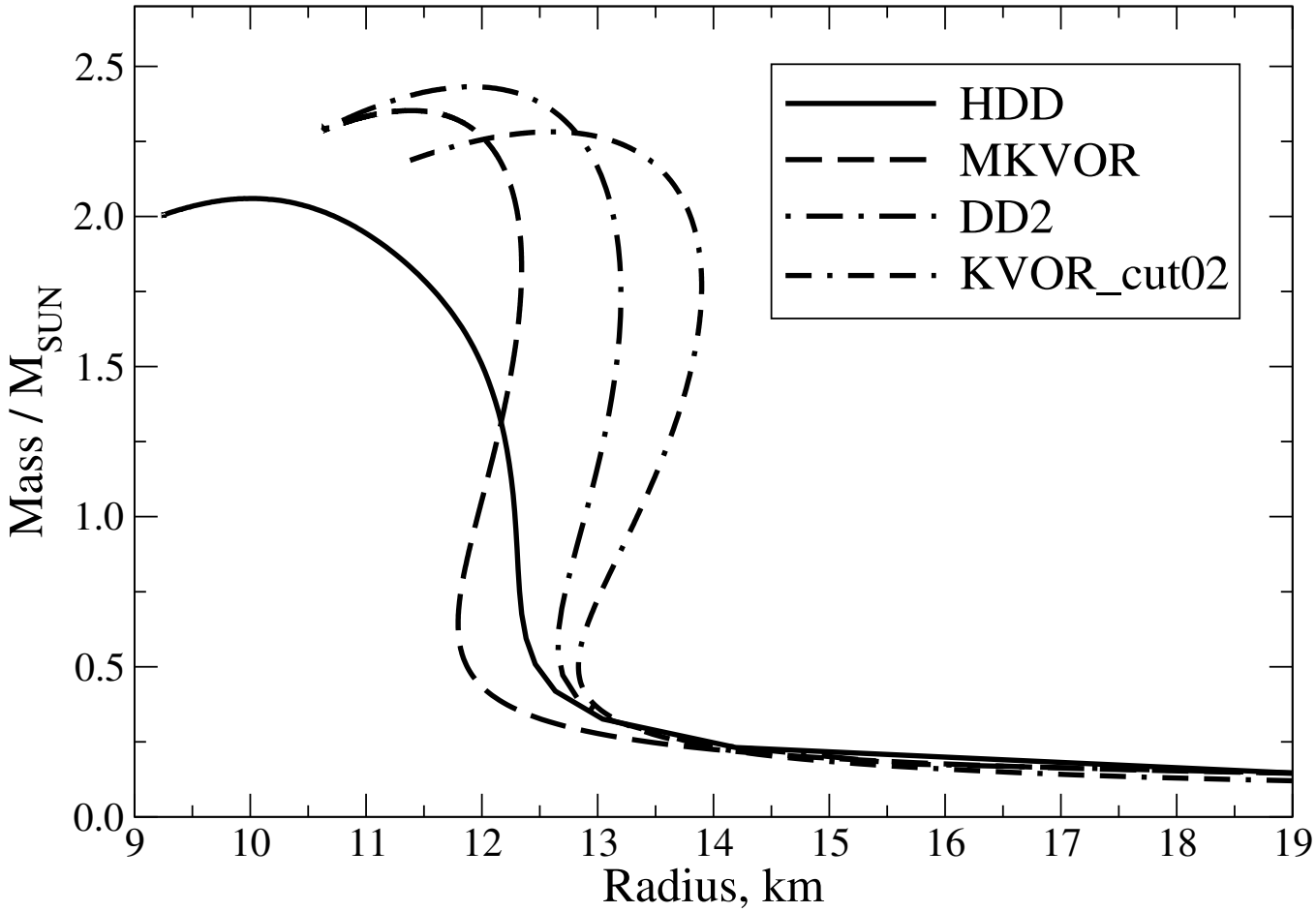
The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2,*]:

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2} \dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)}; \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r). \end{cases}$$

[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939).
[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

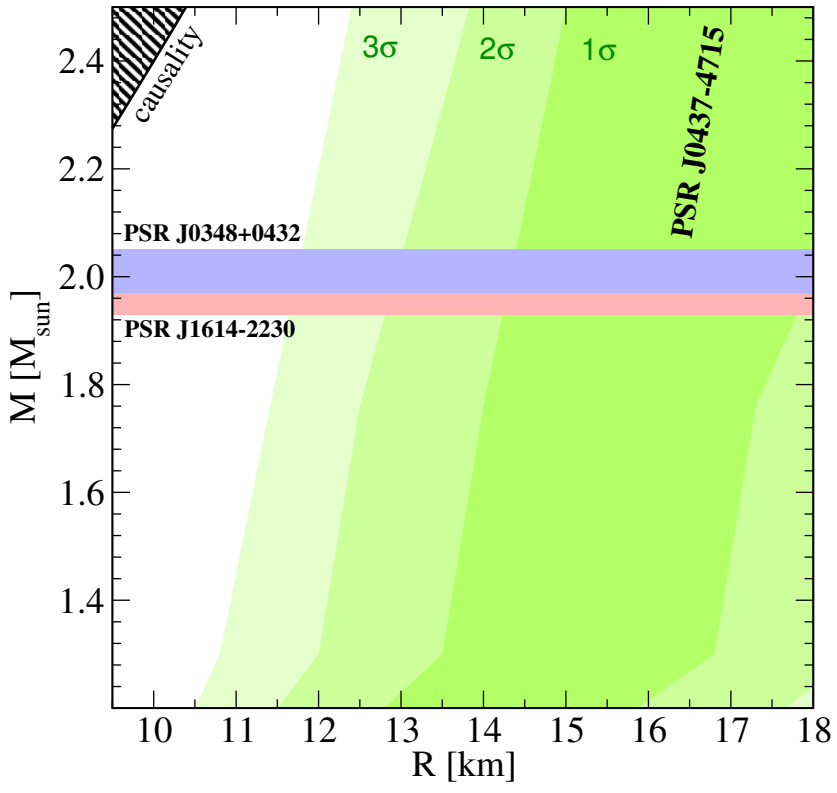
[*] Valid for static neutron stars.

Neutron star mass and radius relation



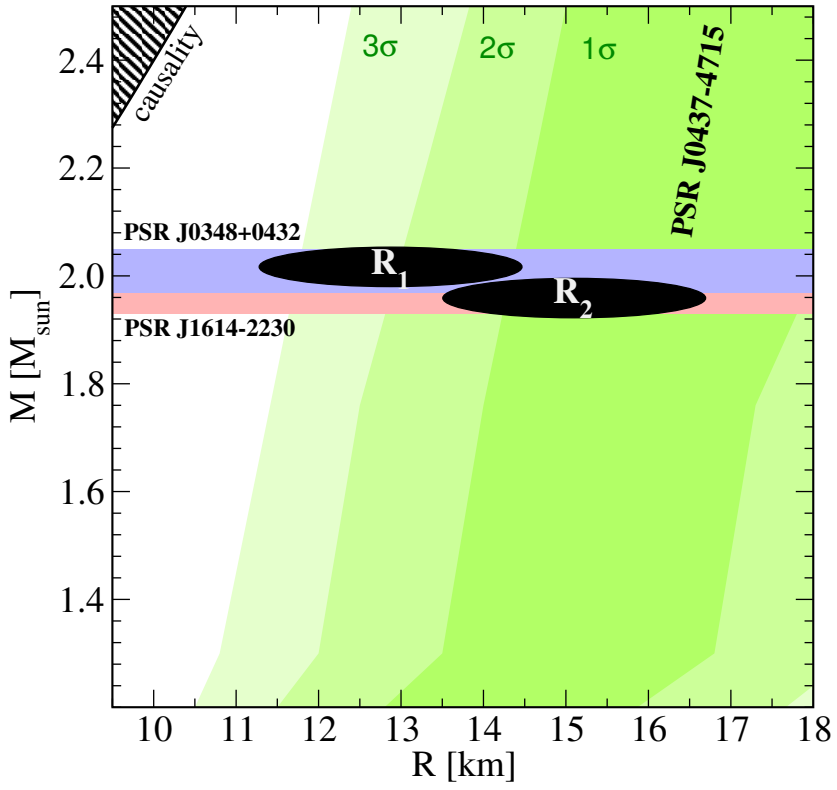
A. Ayriyan et. al. EPJ WoC 177, 07001 (2018)

Mass and Radius Constraints



Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. Ast. J. **762**, 96) and PSR J0348+0432 (Antoniadis *et al.* Sci. **340**, 6131) correspondingly.

Fictitious Radius Constraints



σ	R_1	R_2	R_1	R_2	R_1	R_2	R_1	R_2
0.5	11	13	11	15	13	15	15	11
1.0	11	13	11	15	13	15	15	11
1.5	11	13	11	15	13	15	15	11

We supposed that the inaccuracy of the fictitious measurements is normally distributed.

Hadronic matter EoS

Excluded-volume DD2

The excluded volume correction is applied at supersaturation densities and has the effect of stiffening the EoS. The available volume fraction Φ_N for the motion of nucleons at given density n:

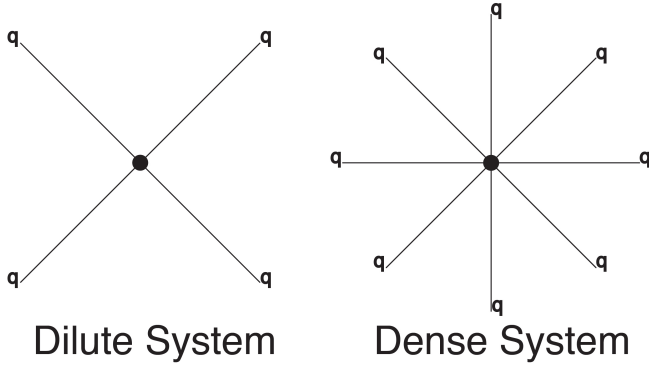
$$\Phi_N = \begin{cases} 1\ , & \text{if} \quad n \leq n_{\text{sat}} \\ \exp[-v|v|(n-n_{\text{sat}})^2/2]\ , & \text{if} \quad n > n_{\text{sat}}\ , \end{cases}$$

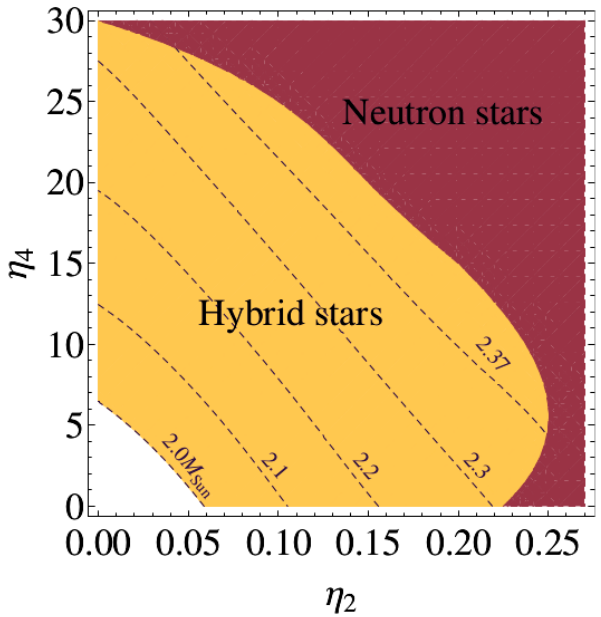
with $v = 16\pi r_N^3/3$ as the van-der-Waals excluded volume corresponding to a nucleon hard-core radius r_N, where $n_{\text{sat}} = 0.16\ \mathrm{fm}^{-3}$ is the density in the interior of atomic nuclei. We introduce the dimensionless parameter $p = 10 \times v[\mathrm{fm}^3]$, taking values between $p = 0$ and $p = 80$ [S. Typel, EPJA 52 (3) (2016)].

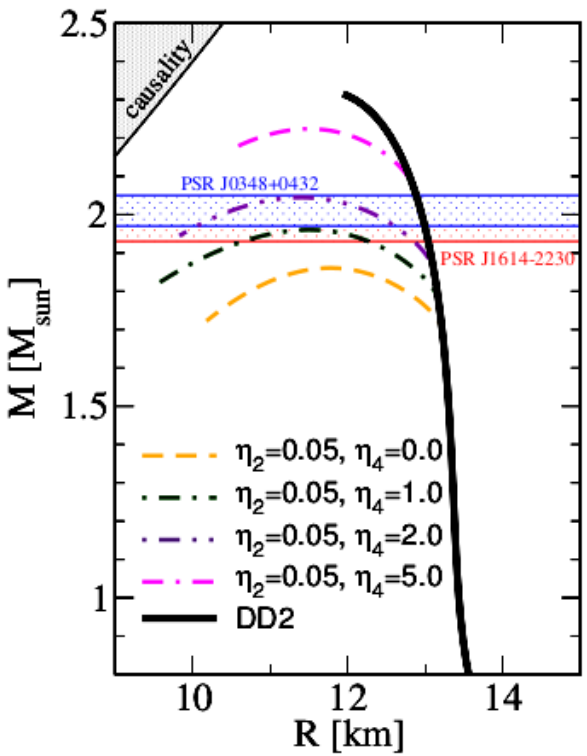
Quark matter EoS

Quark matter EoS with η_4 parameter

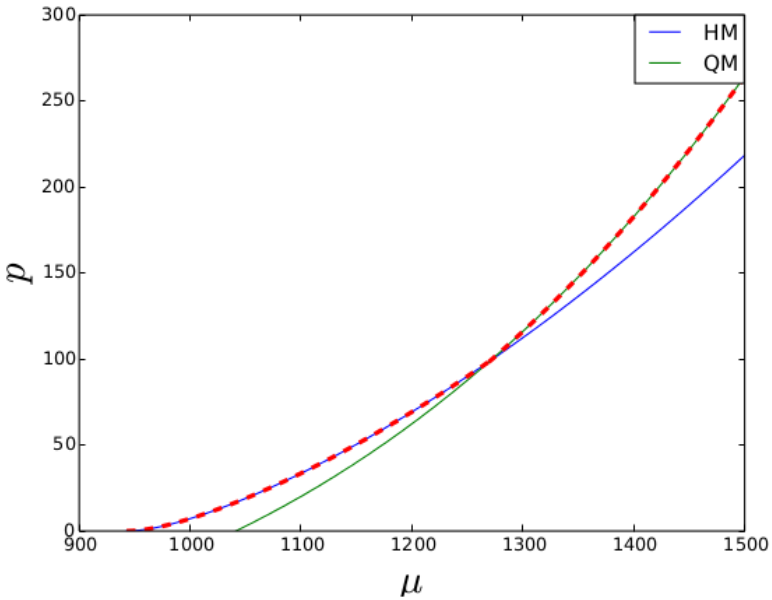
The quark matter was modelling by a two flavor Nambu-Jona- Lasinio (NJL) model with 8-quark interactions in the scalar and the vector channel [S. Benic, arxiv:1503.09145]. The η_2 was fixed [S. Benić, EPJA, 50:111 (2014)] and the η_4 – 8-quark vector NJL couplings parameter was variated from 0 to 30 with step 1 (so, $N_2 = 31$).



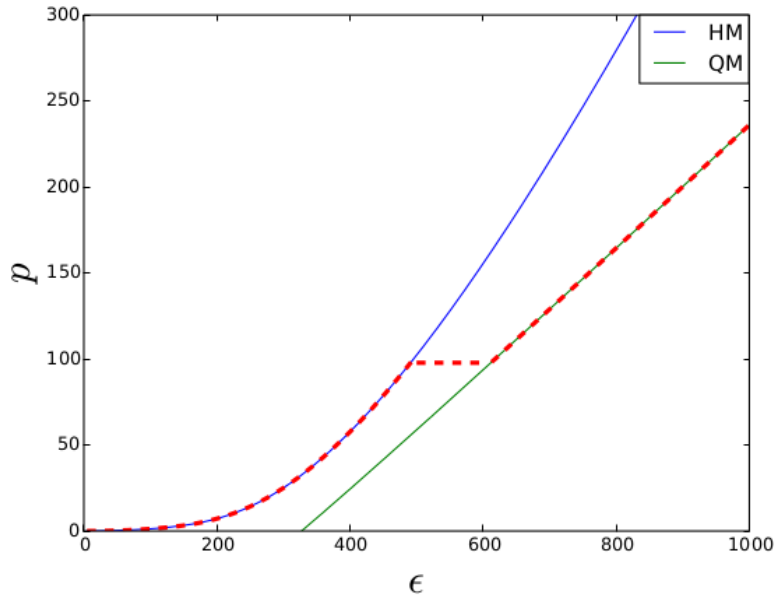




Hybrid EoS



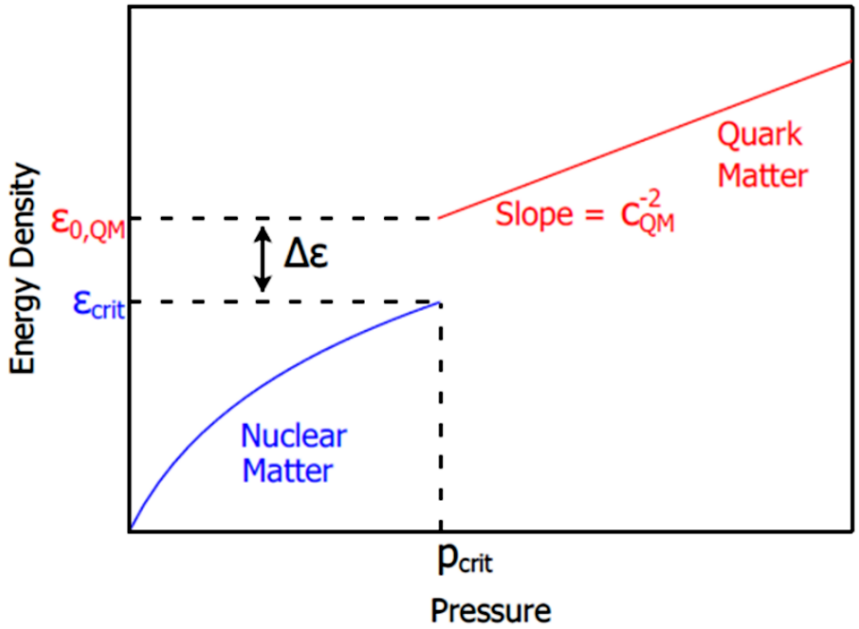
(A) $p(\mu)$ functions.

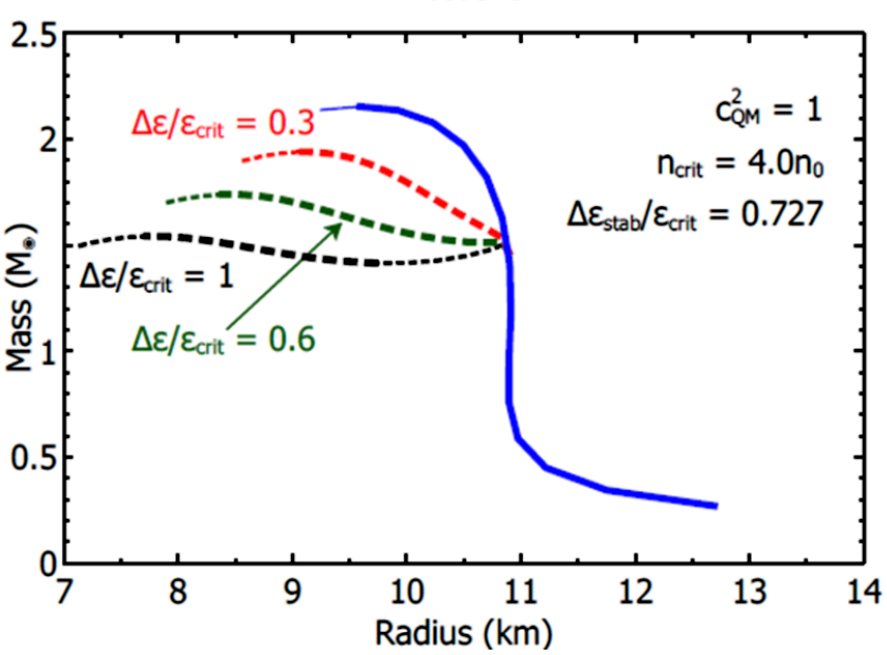


(B) $p(\epsilon)$ functions.

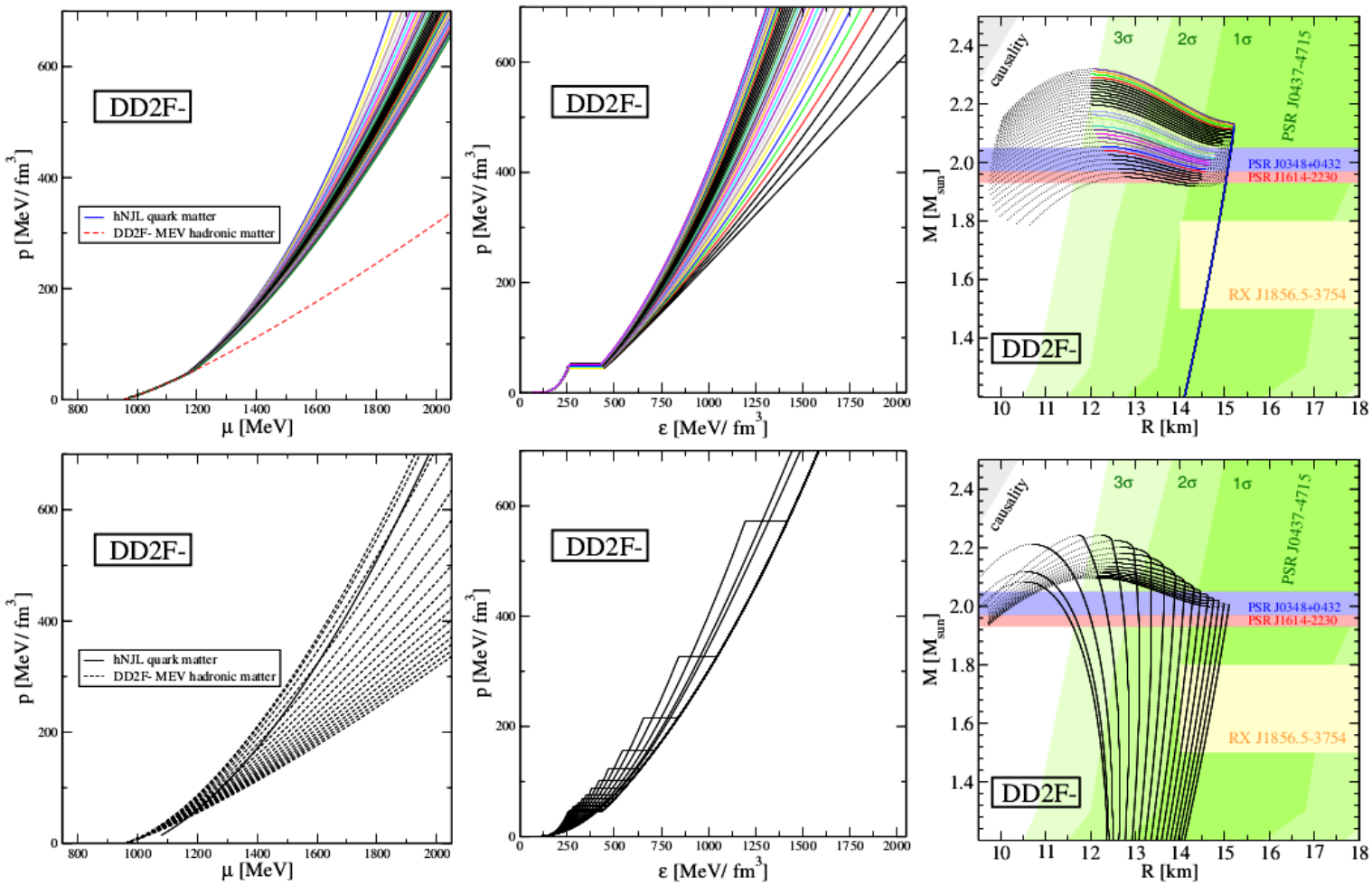
$$p(\mu) = \max\left[p_H(\mu), p_Q(\mu)\right]$$

NS families


Energy Density
$\varepsilon_{0,QM}$
$\Delta\varepsilon$
ε_{crit}
Quark Matter
Slope = c_{QM}^{-2}
Nuclear Matter
p_{crit}
Pressure


Pressure
Mass ($M_\odot$)
Radius (km)
$\Delta\varepsilon/\varepsilon_{crit}$ = 0.3
$\Delta\varepsilon/\varepsilon_{crit}$ = 1
$\Delta\varepsilon/\varepsilon_{crit}$ = 0.6
c_{QM}^2 = 1
n_{crit} = 4.0n_0
$\Delta\varepsilon_{stab}/\varepsilon_{crit}$ = 0.727

Hybrid matter EoS



Variations of the hybrid EoS for the DD2F$^-$ model. *Upper row*. The hadronic EoS is kept fixed while the quark EoS is allowed to vary for the parameters $\eta_4 = 0, 1, 2..30$.

Vector of Parameters

Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values $\overrightarrow{\pi} = \{p, \eta_4\}$:

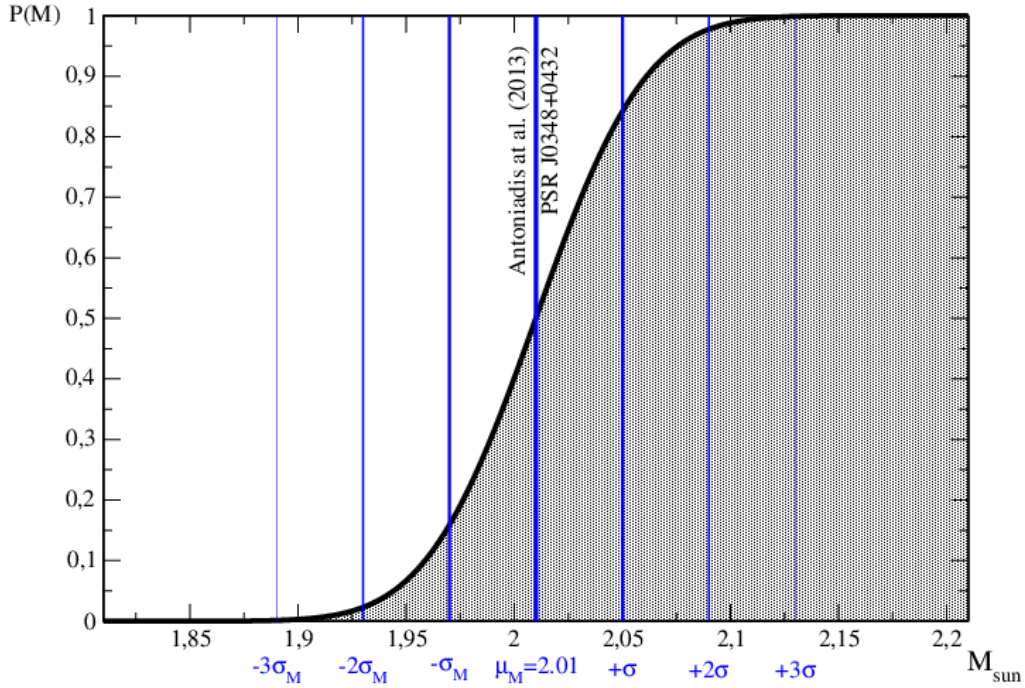
$$\overrightarrow{\pi}_i = \left\{p_{(k)}, \eta_{4(l)}\right\},$$

$i = 1 \ldots N$ (here $N = N_1 \times N_2$), $i = N_2 \times k + l$ and $k = 0 \ldots N_1 - 1$, $l = 0 \ldots N_2 - 1$

Likelihood

Likelihood for Mass Constraint for given π_i

$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [Antoniadis et al. Science 340].



Likelihood

Likelihood for fictitious radius constraints

The Likelihood is corresponding the area where the configurations have values of radii predicted from the model when the masses are in the interval of the observational mass range $M_{G_j} \pm \Delta M_j$:

$$\begin{aligned} P\left(E_{F_\alpha} | \overrightarrow{\pi}_i\right) &= \Phi\left(R_\alpha(M_{G_\alpha} + \Delta M_\alpha; \overrightarrow{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \\ &- \Phi\left(R_\alpha(M_{G_\alpha} - \Delta M_\alpha; \overrightarrow{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \end{aligned}$$

Because in some cases (mainly for the hybrid stars) the $R_\alpha(M_{G_\alpha}; \overrightarrow{\pi}_i)$ can be not uniquely defined functions, we have excluded possible overlaps of the boxes of the radius probability regions on the $M_{G_\alpha} \otimes R_\alpha$ plane to avoid the double counting.

Likelihood

Likelihood of all constraints for π_i

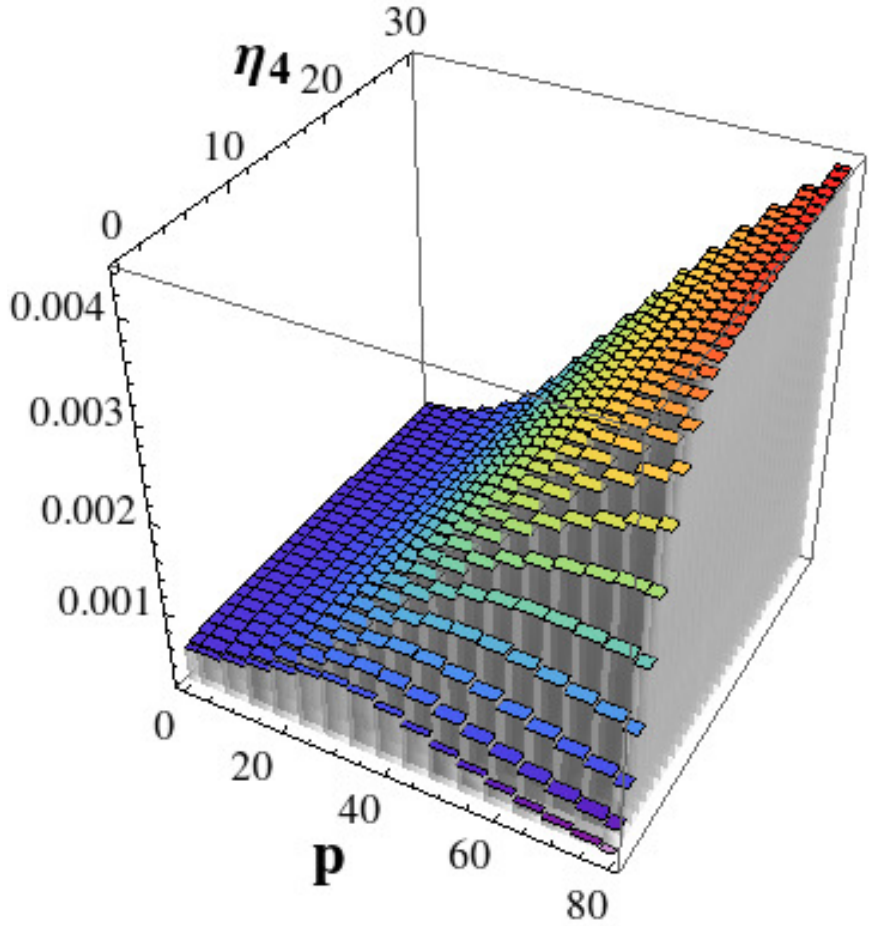
Taking to the account assumption that these measurements are independent on each other we can calculate the Likelihood:

$$P\left(E\,|\overrightarrow{\pi}_i\right) = \prod_{\alpha} P\left(E_\alpha\,|\overrightarrow{\pi}_i\right).$$

Posterior

$$P\left(\overrightarrow{\pi}_i\,|E\right) = \frac{P\left(E\,|\overrightarrow{\pi}_i\right) P\left(\overrightarrow{\pi}_i\right)}{\sum_{j=0}^{N-1} P\left(E\,|\overrightarrow{\pi}_j\right) P\left(\overrightarrow{\pi}_j\right)}.$$

BA results with M-R data



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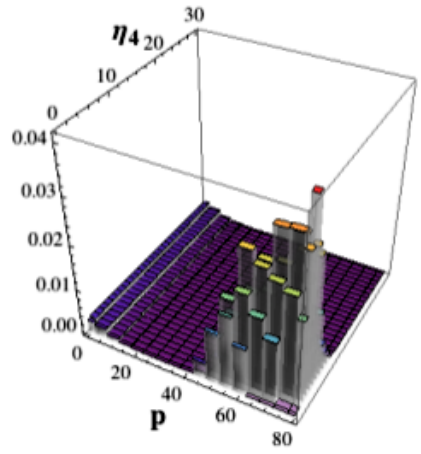
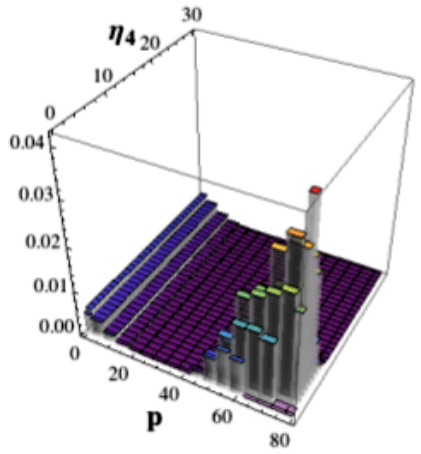
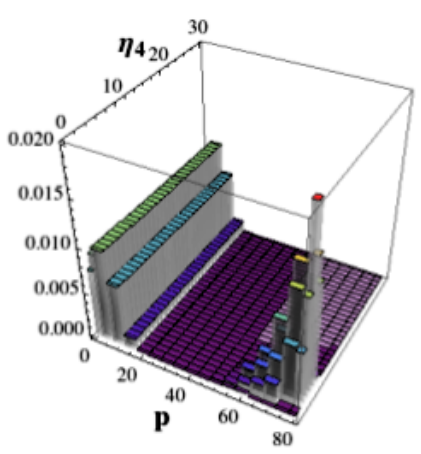
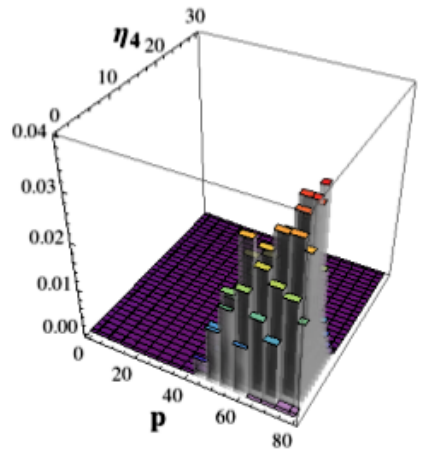
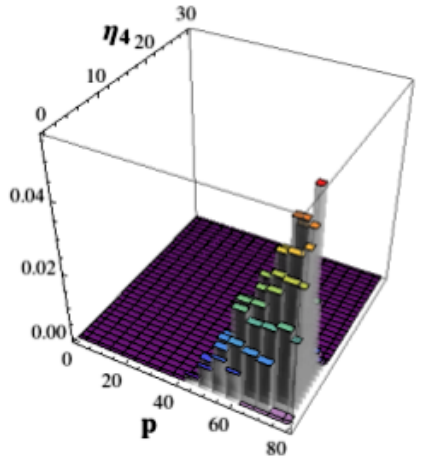
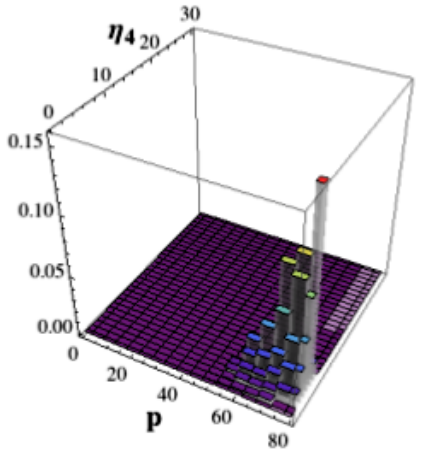
BA results with fictitious radius measuremets

Radius error $\sigma \rightarrow$ R_A, R_D	1.5 km	1.0 km	0.5 km
11 km, 13 km			
11 km, 15 km			

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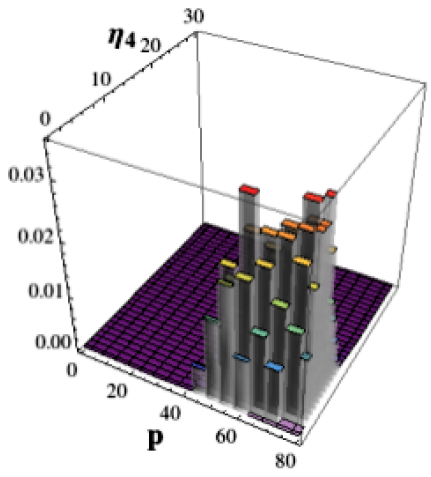
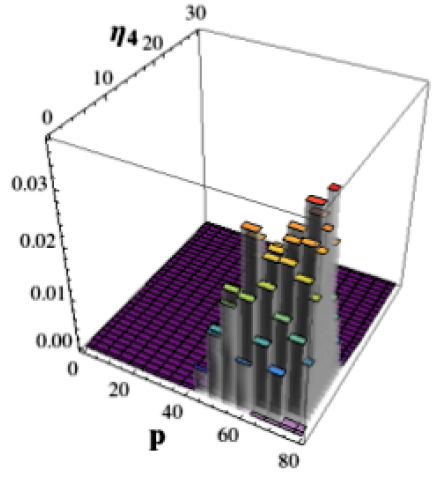
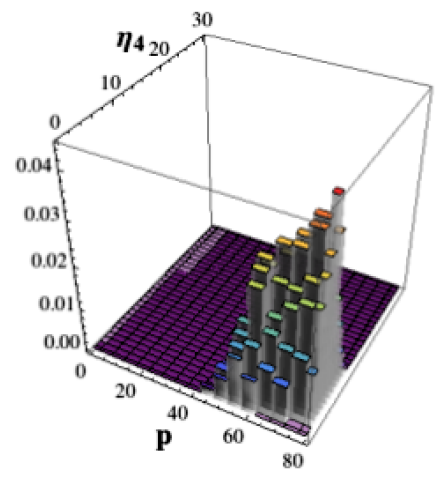
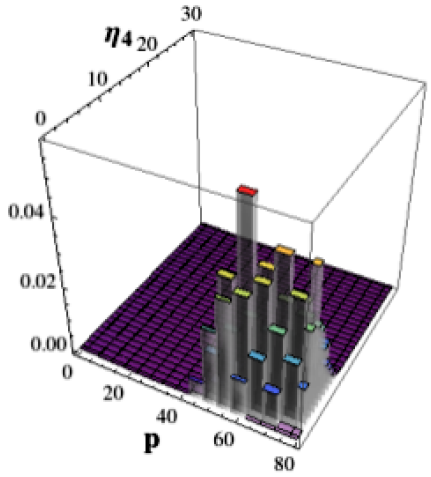
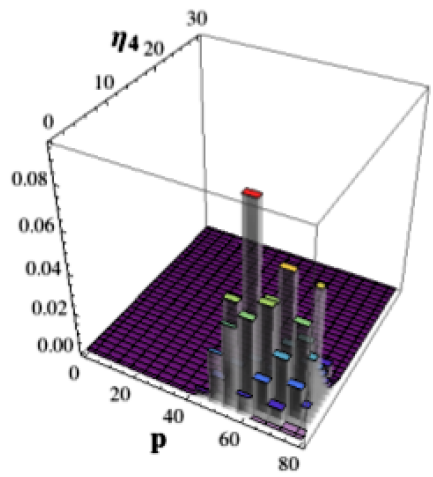
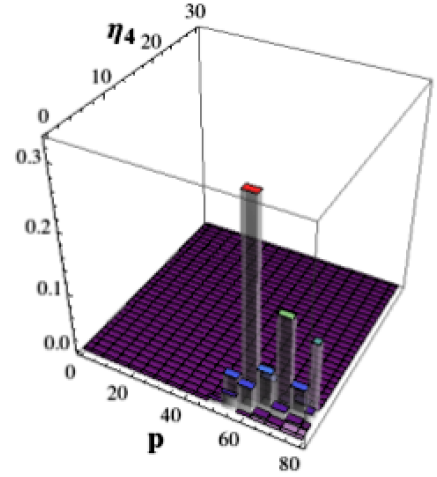
BA results with fictitious measuremets

13 km, 15 km			
15 km, 11 km			

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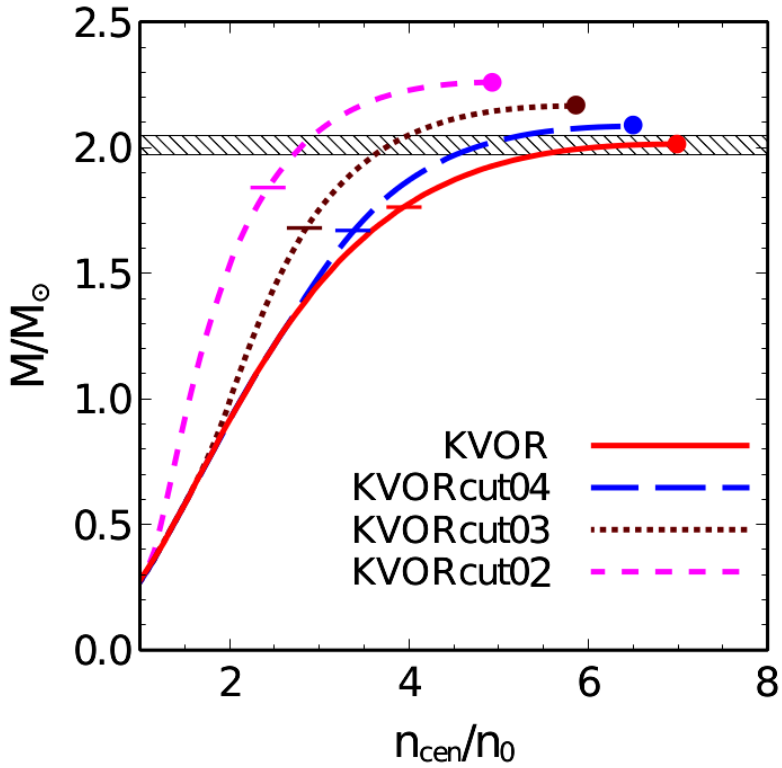
New preliminary results

New Hybrid EoS: Hadronic matter

For the hadronic pase the well known KVOR equation of state [Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)] with modification of stiffness is taken.

[K.A. Maslov, E.E. Kolomeitsev, D.N. Voskresensky, Nucl.Phys. A950 (2016)]

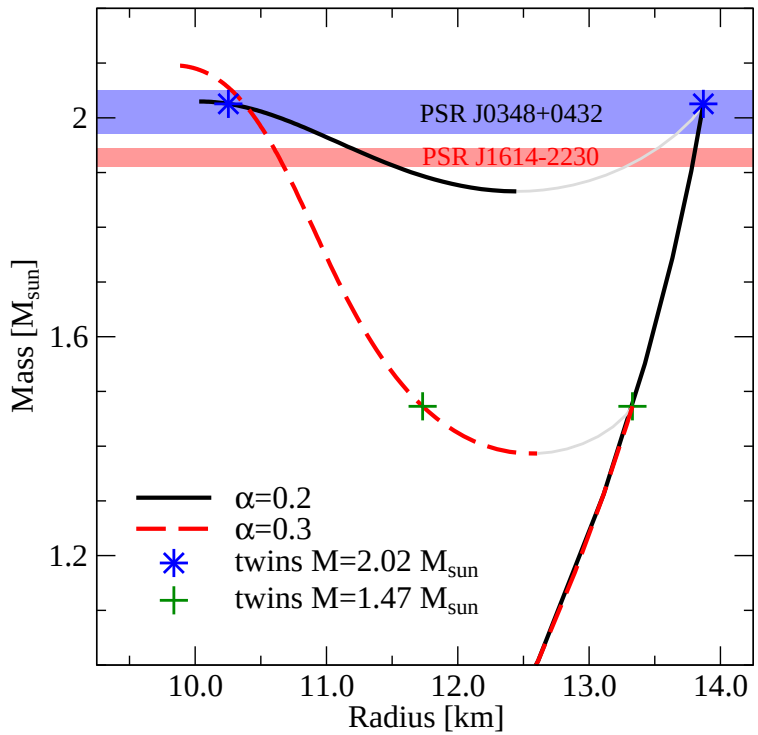


New hybrid EoS: Quark matter

For the quark phase the density functional for quark matter model is used with available volume fraction parameter α (varied in the range [0.1..0.3]).

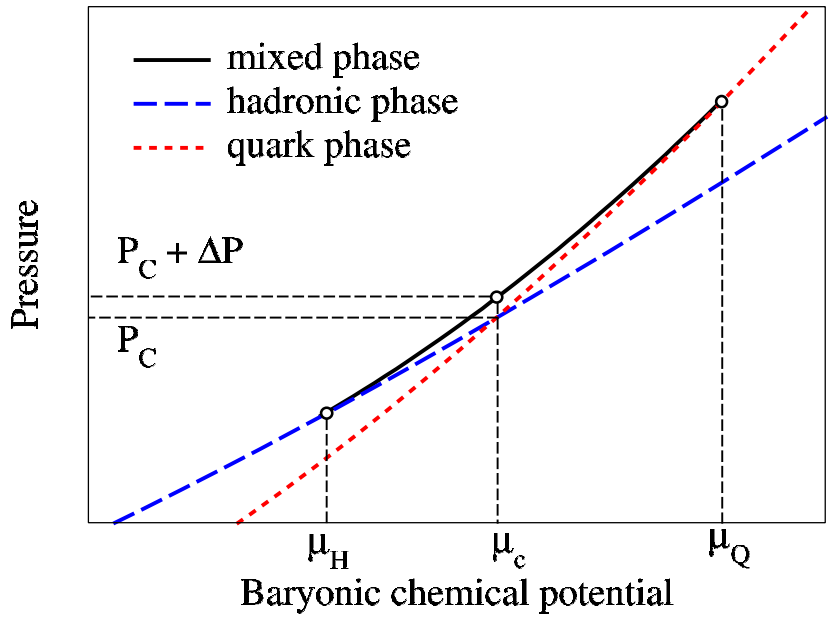
$$\Phi(n_{\mathrm{B}}) = \begin{cases} 1, & \text{if } n_{\mathrm{B}} < n_0 \\ e^{-\alpha(n_{\mathrm{B}}-n_0)^2}, & \text{if } n_{\mathrm{B}} > n_0 \end{cases},$$

where $\alpha = v|v|/2$ and v is the excluded volume parameter



[M. Kaltenborn, N.-U. Bastian, D. Blaschke, Phys. Rev. D 96, 056024 (2017)]

New hybrid EoS: Phase transition



An interpolation function has to go though three points

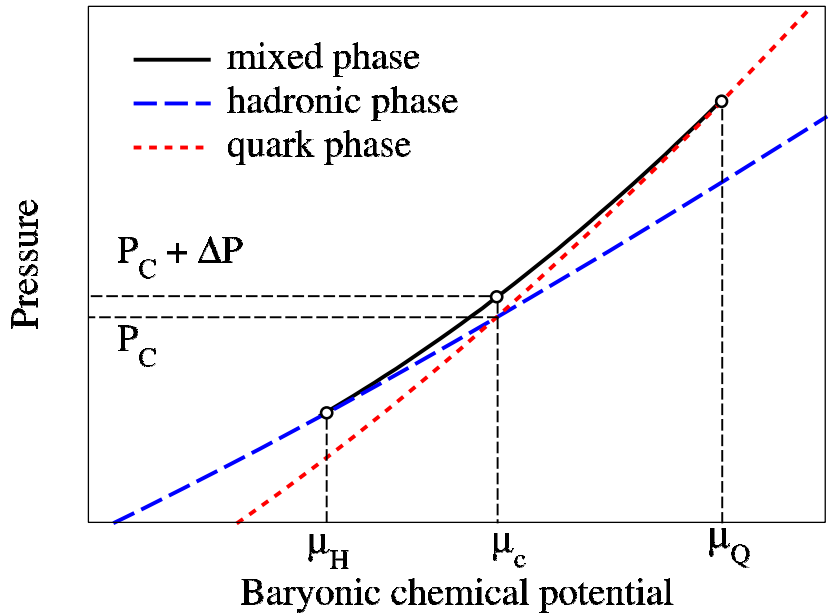
$$(\mu_H, P(\mu_H)),\ (\mu_c, P_M(\mu_c)),\ (\mu_Q, P(\mu_Q))$$

$$P_M(\mu) = \alpha(\mu - \mu_c)^2 + \beta(\mu - \mu_c) + (1 + \Delta_p)P_c,$$

[A. Ayriyan, H. Grigorian, EPJ WoC 173, 03003 (2018) arXiv:1701.04400]

[A. Ayriyan et al., PRC 97, 045802 (2018), arXiv:1711.03926]

New hybrid EoS: Phase transition



The values of α, β, μ_H and μ_Q could be found from the following system:

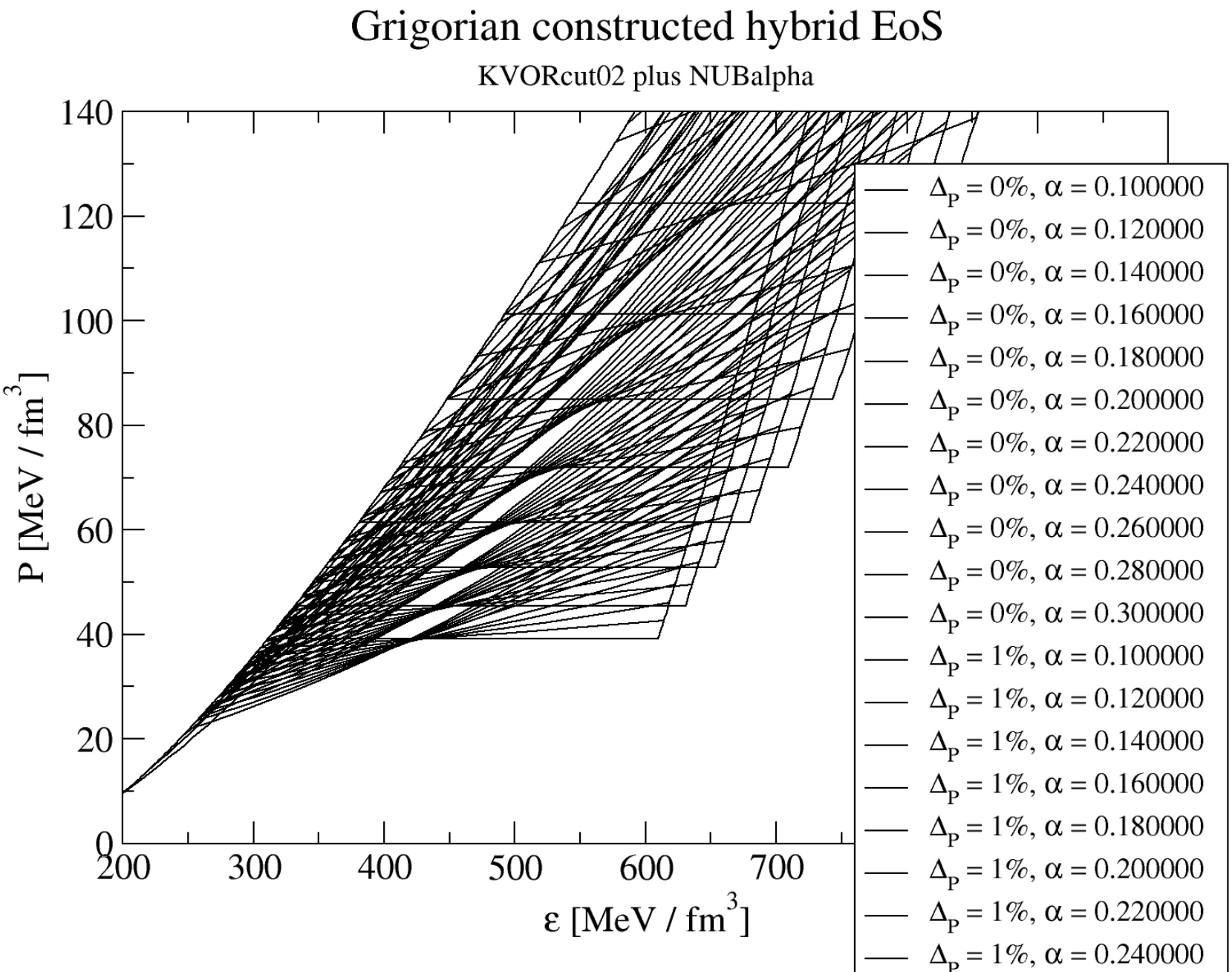
$$P_H(\mu_H) = P_M(\mu_H) \qquad n_H(\mu_H) = n_M(\mu_H)$$

$$P_Q(\mu_Q) = P_M(\mu_Q) \qquad n_Q(\mu_Q) = n_M(\mu_Q)$$

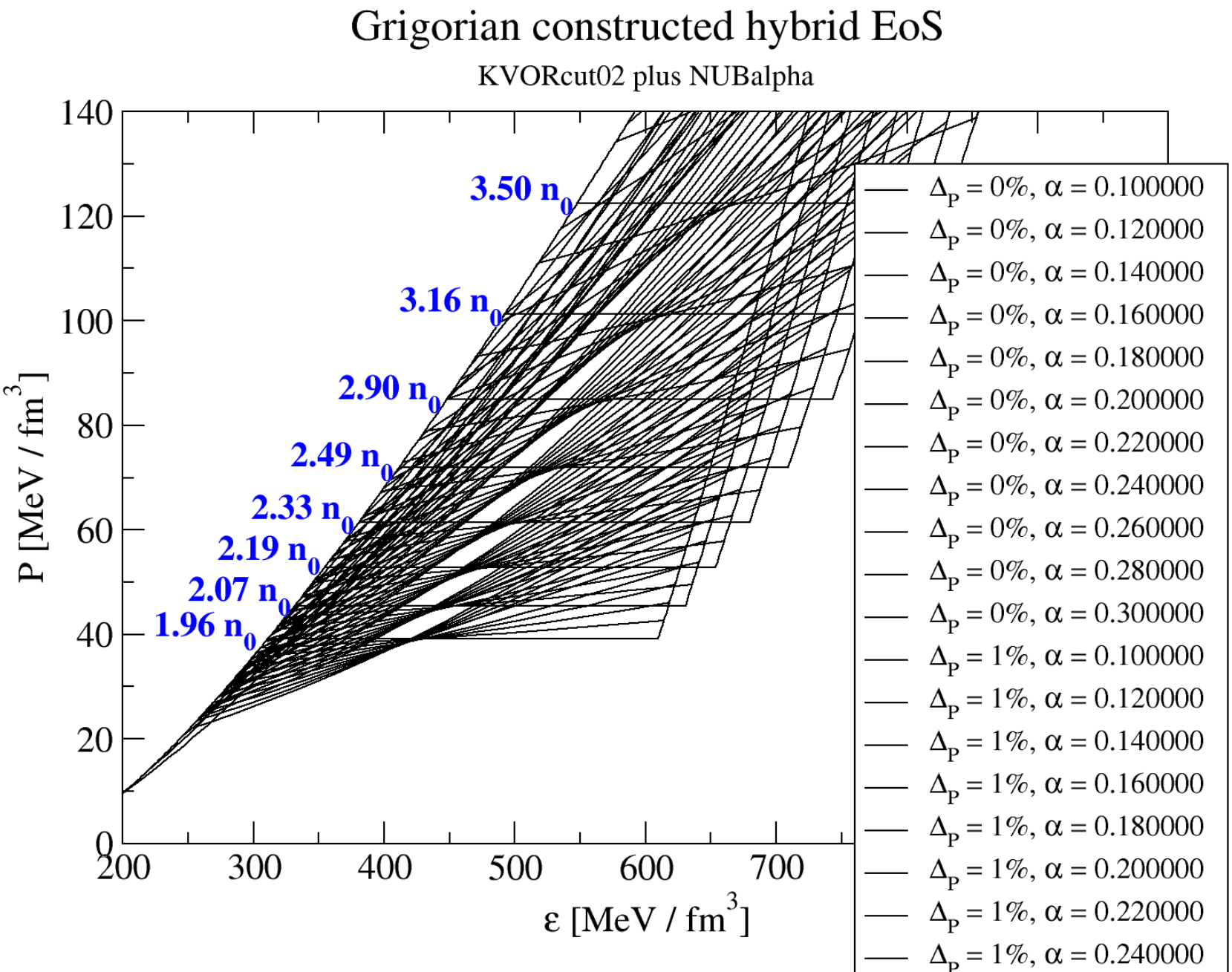
[A. Ayriyan, H. Grigorian, EPJ WoC 173, 03003 (2018) arXiv:1701.04400]

[A. Ayriyan et al., PRC 97, 045802 (2018), arXiv:1711.03926]

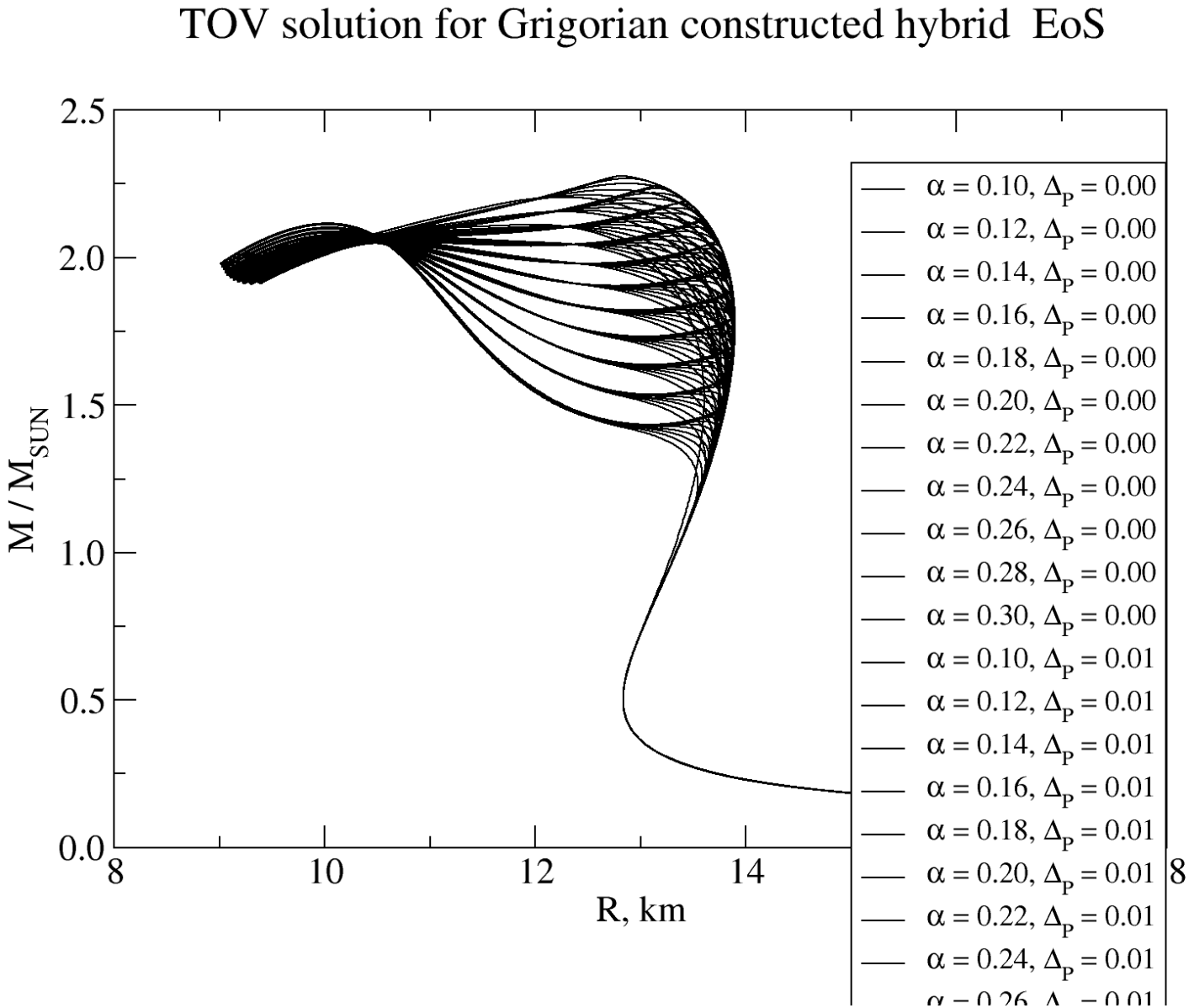
Hybrid EoS


Grigorian constructed hybrid EoS
KVORcut02 plus NUBalpha
P [MeV / fm^3]
ε [MeV / fm^3]
0
20
40
60
80
100
120
140
200
300
400
500
600
700
Δ_P = 0%, α = 0.100000
Δ_P = 0%, α = 0.120000
Δ_P = 0%, α = 0.140000
Δ_P = 0%, α = 0.160000
Δ_P = 0%, α = 0.180000
Δ_P = 0%, α = 0.200000
Δ_P = 0%, α = 0.220000
Δ_P = 0%, α = 0.240000
Δ_P = 0%, α = 0.260000
Δ_P = 0%, α = 0.280000
Δ_P = 0%, α = 0.300000
Δ_P = 1%, α = 0.100000
Δ_P = 1%, α = 0.120000
Δ_P = 1%, α = 0.140000
Δ_P = 1%, α = 0.160000
Δ_P = 1%, α = 0.180000
Δ_P = 1%, α = 0.200000
Δ_P = 1%, α = 0.220000
Δ_P = 1%, α = 0.240000

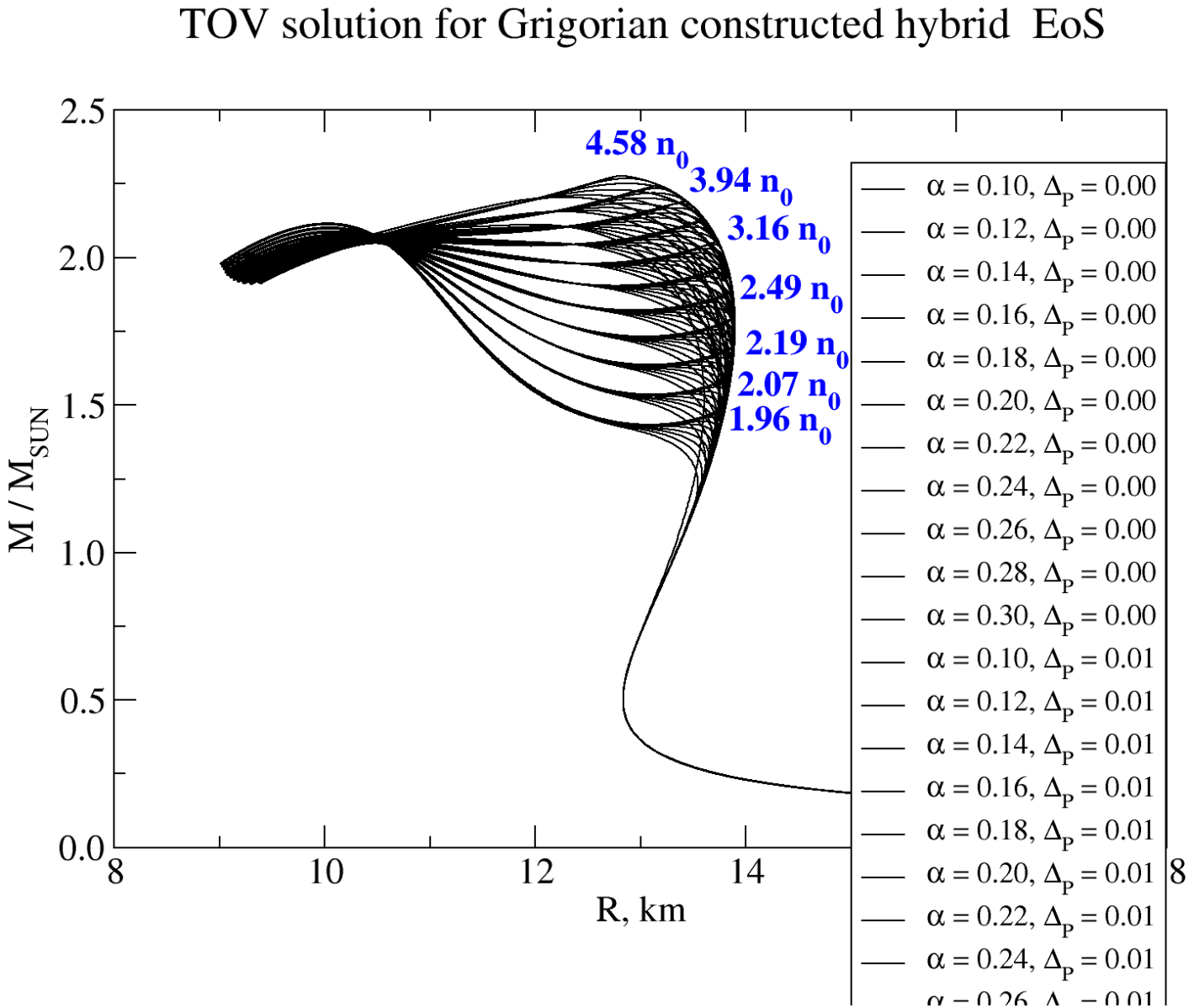
Hybrid EoS


Grigorian constructed hybrid EoS
KVORcut02 plus NUBalpha
P [MeV / fm^3]
ε [MeV / fm^3]
0
20
40
60
80
100
120
140
200
300
400
500
600
700
3.50 n0
3.16 n0
2.90 n0
2.49 n0
2.33 n0
2.19 n0
2.07 n0
1.96 n0
Δp = 0%, α = 0.100000
Δp = 0%, α = 0.120000
Δp = 0%, α = 0.140000
Δp = 0%, α = 0.160000
Δp = 0%, α = 0.180000
Δp = 0%, α = 0.200000
Δp = 0%, α = 0.220000
Δp = 0%, α = 0.240000
Δp = 0%, α = 0.260000
Δp = 0%, α = 0.280000
Δp = 0%, α = 0.300000
Δp = 1%, α = 0.100000
Δp = 1%, α = 0.120000
Δp = 1%, α = 0.140000
Δp = 1%, α = 0.160000
Δp = 1%, α = 0.180000
Δp = 1%, α = 0.200000
Δp = 1%, α = 0.220000
Δp = 1%, α = 0.240000

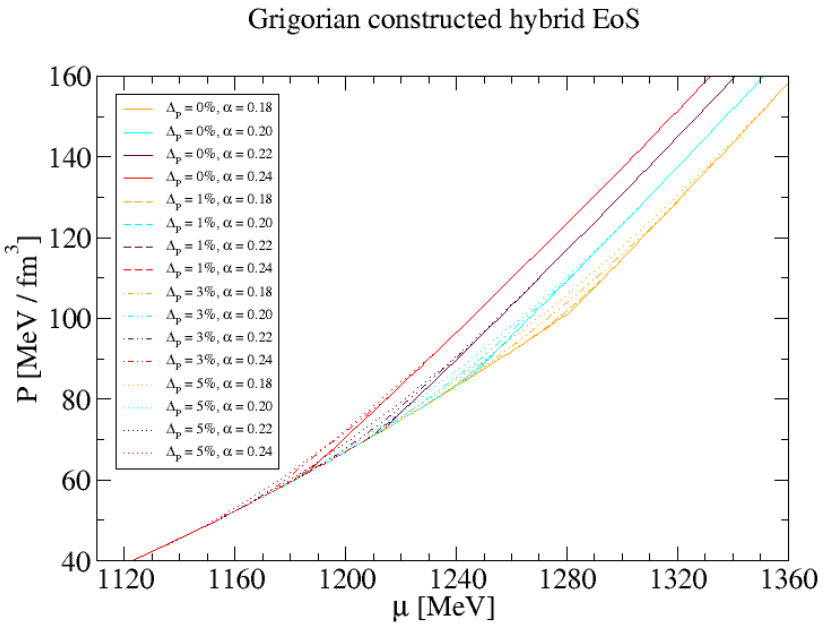
Hybrid EoS

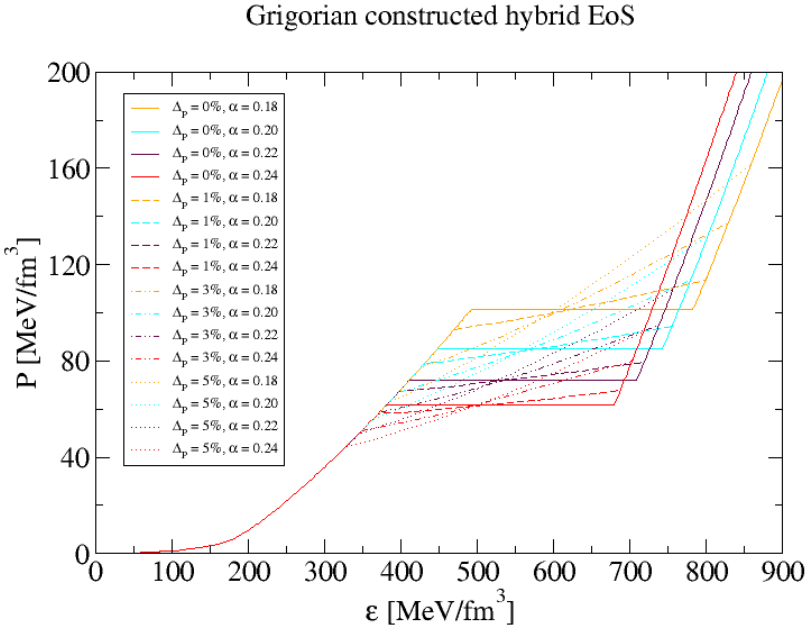

TOV solution for Grigorian constructed hybrid EoS
M / M_SUN
R, km
2.5
2.0
1.5
1.0
0.5
0.0
8
10
12
14
8
α = 0.10, Δ_P = 0.00
α = 0.12, Δ_P = 0.00
α = 0.14, Δ_P = 0.00
α = 0.16, Δ_P = 0.00
α = 0.18, Δ_P = 0.00
α = 0.20, Δ_P = 0.00
α = 0.22, Δ_P = 0.00
α = 0.24, Δ_P = 0.00
α = 0.26, Δ_P = 0.00
α = 0.28, Δ_P = 0.00
α = 0.30, Δ_P = 0.00
α = 0.10, Δ_P = 0.01
α = 0.12, Δ_P = 0.01
α = 0.14, Δ_P = 0.01
α = 0.16, Δ_P = 0.01
α = 0.18, Δ_P = 0.01
α = 0.20, Δ_P = 0.01
α = 0.22, Δ_P = 0.01
α = 0.24, Δ_P = 0.01

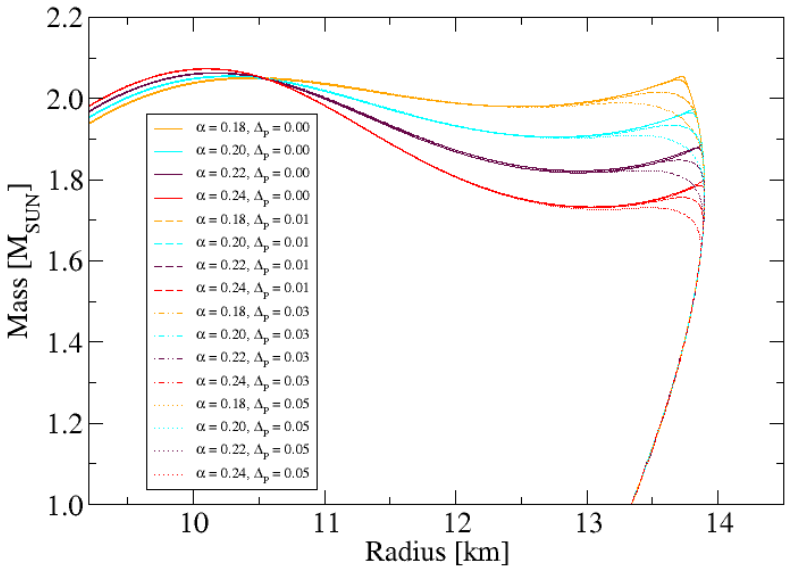
Hybrid EoS

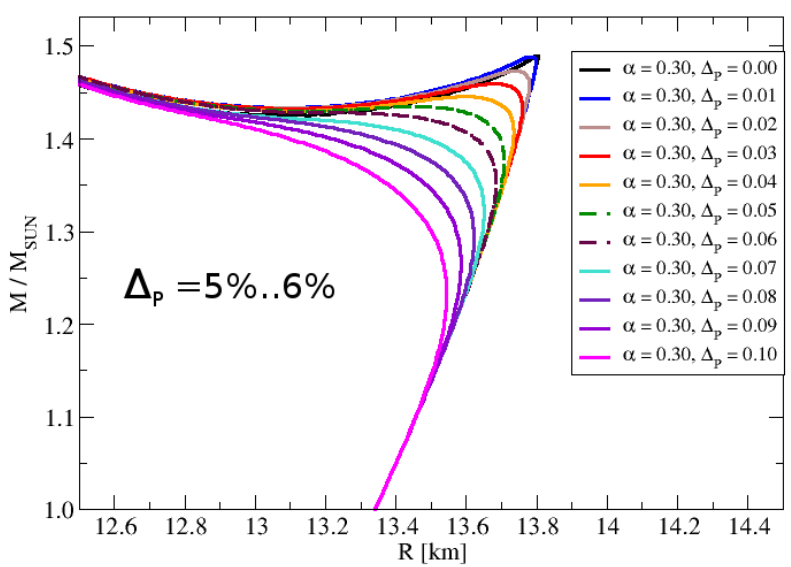

TOV solution for Grigorian constructed hybrid EoS
M / M_SUN
R, km
4.58 n0
3.94 n0
3.16 n0
2.49 n0
2.19 n0
2.07 n0
1.96 n0
α = 0.10, Δ_P = 0.00
α = 0.12, Δ_P = 0.00
α = 0.14, Δ_P = 0.00
α = 0.16, Δ_P = 0.00
α = 0.18, Δ_P = 0.00
α = 0.20, Δ_P = 0.00
α = 0.22, Δ_P = 0.00
α = 0.24, Δ_P = 0.00
α = 0.26, Δ_P = 0.00
α = 0.28, Δ_P = 0.00
α = 0.30, Δ_P = 0.00
α = 0.10, Δ_P = 0.01
α = 0.12, Δ_P = 0.01
α = 0.14, Δ_P = 0.01
α = 0.16, Δ_P = 0.01
α = 0.18, Δ_P = 0.01
α = 0.20, Δ_P = 0.01
α = 0.22, Δ_P = 0.01
α = 0.24, Δ_P = 0.01

Hybrid EoS

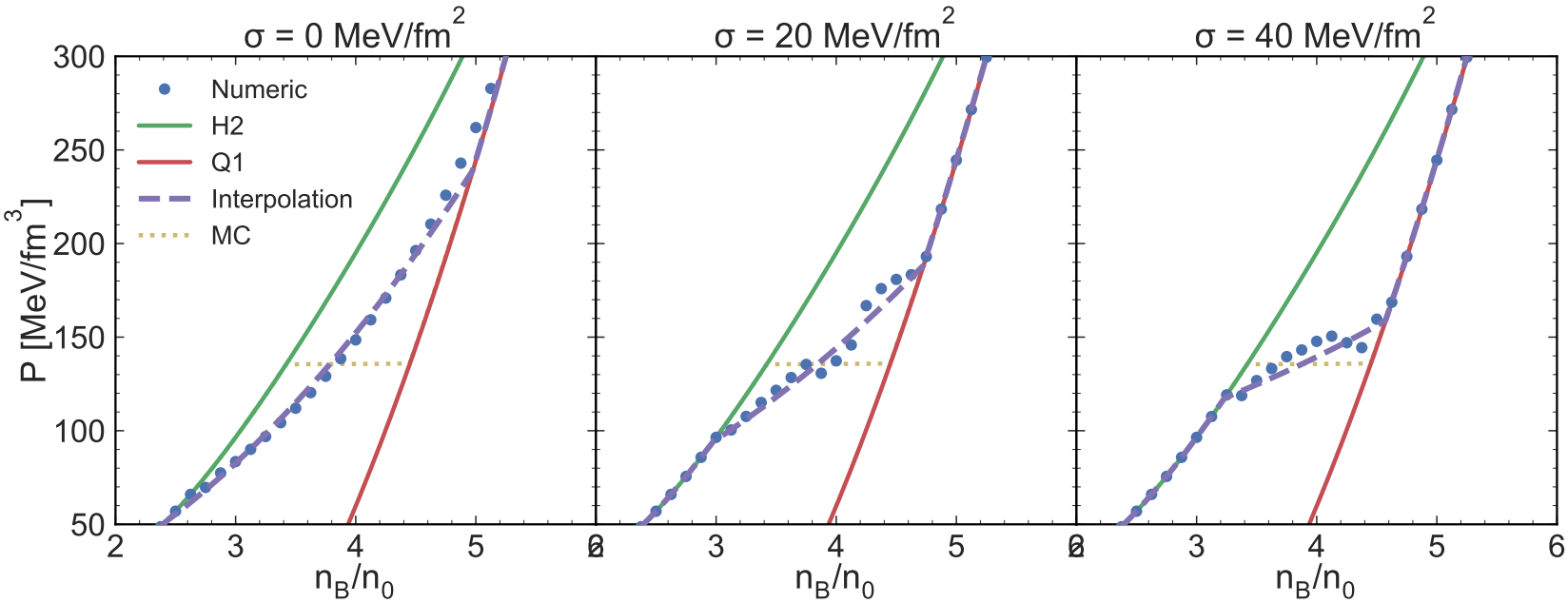

Grigorian constructed hybrid EoS
P [MeV / fm^3]
μ [MeV]
Δ_P = 0%, α = 0.18
Δ_P = 0%, α = 0.20
Δ_P = 0%, α = 0.22
Δ_P = 0%, α = 0.24
Δ_P = 1%, α = 0.18
Δ_P = 1%, α = 0.20
Δ_P = 1%, α = 0.22
Δ_P = 1%, α = 0.24
Δ_P = 3%, α = 0.18
Δ_P = 3%, α = 0.20
Δ_P = 3%, α = 0.22
Δ_P = 3%, α = 0.24
Δ_P = 5%, α = 0.18
Δ_P = 5%, α = 0.20
Δ_P = 5%, α = 0.22
Δ_P = 5%, α = 0.24


Grigorian constructed hybrid EoS
P [MeV/fm^3]
ε [MeV/fm^3]
Δ_P = 0%, α = 0.18
Δ_P = 0%, α = 0.20
Δ_P = 0%, α = 0.22
Δ_P = 0%, α = 0.24
Δ_P = 1%, α = 0.18
Δ_P = 1%, α = 0.20
Δ_P = 1%, α = 0.22
Δ_P = 1%, α = 0.24
Δ_P = 3%, α = 0.18
Δ_P = 3%, α = 0.20
Δ_P = 3%, α = 0.22
Δ_P = 3%, α = 0.24
Δ_P = 5%, α = 0.18
Δ_P = 5%, α = 0.20
Δ_P = 5%, α = 0.22
Δ_P = 5%, α = 0.24

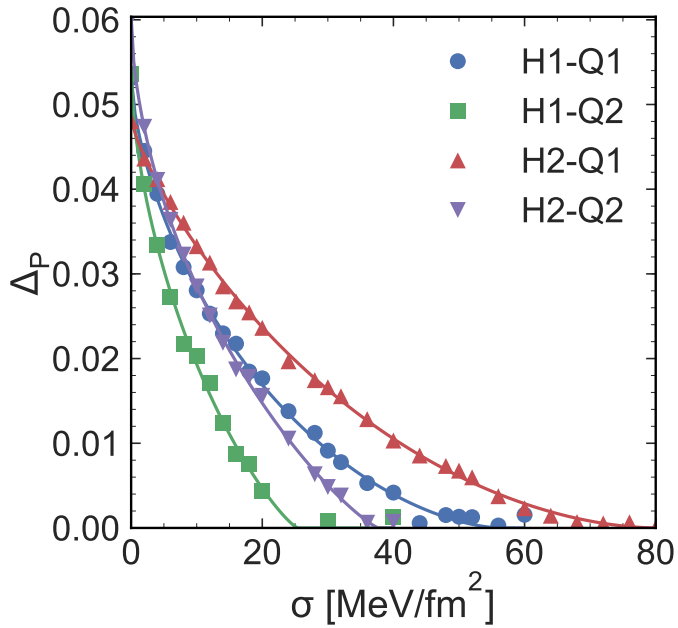

TOV solution for Grigorian constructed hybrid EoS
Mass [M_SUN]
Radius [km]
α = 0.18, Δ_P = 0.00
α = 0.20, Δ_P = 0.00
α = 0.22, Δ_P = 0.00
α = 0.24, Δ_P = 0.00
α = 0.18, Δ_P = 0.01
α = 0.20, Δ_P = 0.01
α = 0.22, Δ_P = 0.01
α = 0.24, Δ_P = 0.01
α = 0.18, Δ_P = 0.03
α = 0.20, Δ_P = 0.03
α = 0.22, Δ_P = 0.03
α = 0.24, Δ_P = 0.03
α = 0.18, Δ_P = 0.05
α = 0.20, Δ_P = 0.05
α = 0.22, Δ_P = 0.05
α = 0.24, Δ_P = 0.05


TOV solution for Grigorian constructed hybrid EoS
M / M_SUN
R [km]
Δ_P =5%..6%
α = 0.30, Δ_P = 0.00
α = 0.30, Δ_P = 0.01
α = 0.30, Δ_P = 0.02
α = 0.30, Δ_P = 0.03
α = 0.30, Δ_P = 0.04
α = 0.30, Δ_P = 0.05
α = 0.30, Δ_P = 0.06
α = 0.30, Δ_P = 0.07
α = 0.30, Δ_P = 0.08
α = 0.30, Δ_P = 0.09
α = 0.30, Δ_P = 0.10

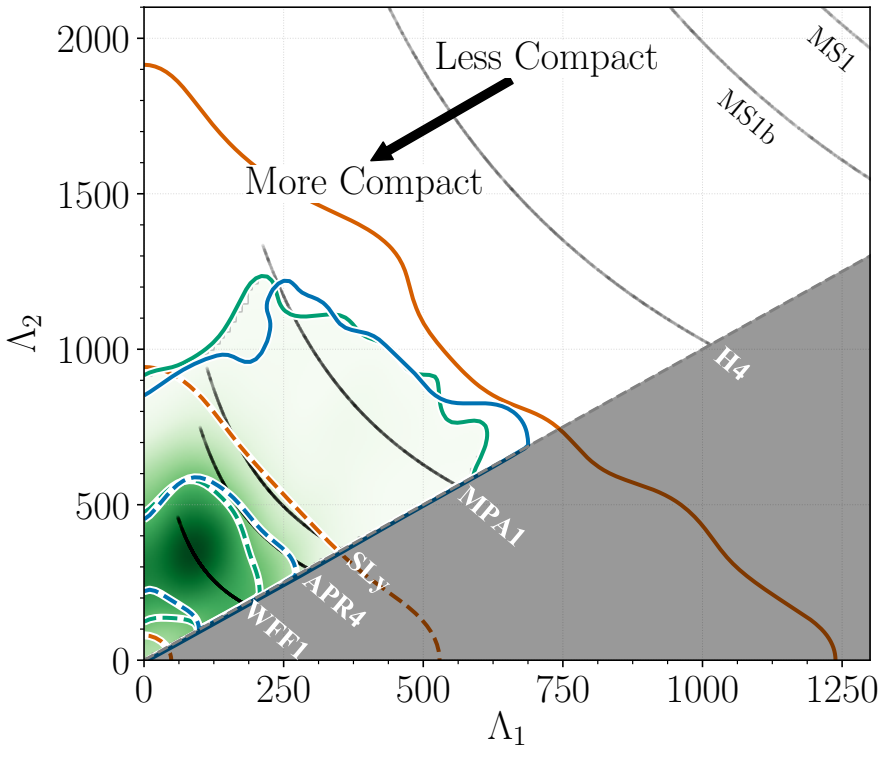
Hybrid EoS



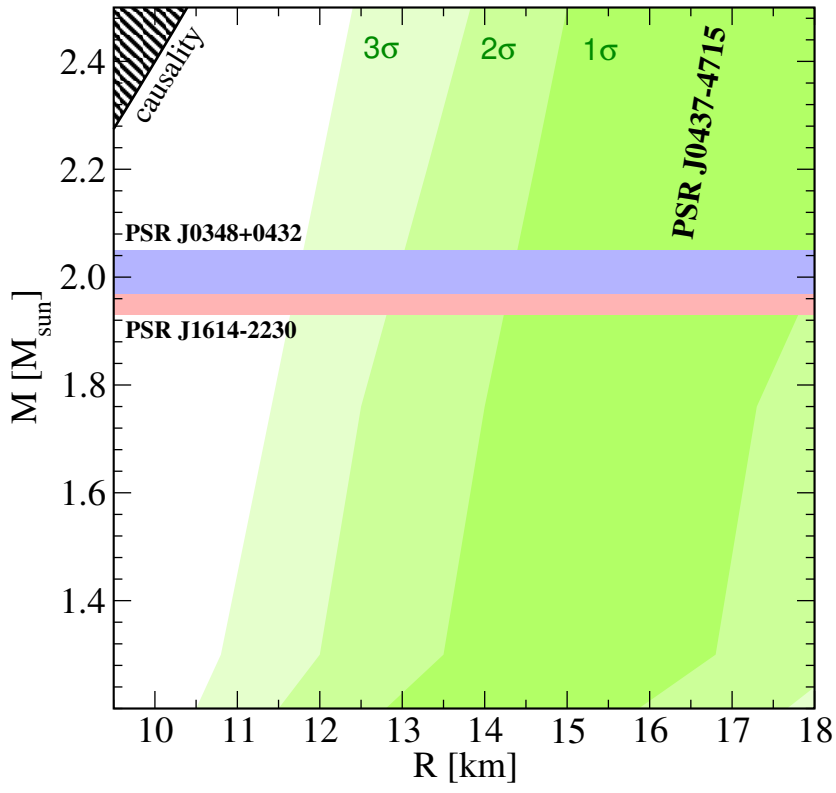
Grigorian construction of the pasta phase supports third family!



Constraints



LIGO collab. arXiv:1805.11581 (2018)



Antoniadis et al. Sci.340, 6131

Bayesian inference for the EoS models

Vector of Parameters

$\overrightarrow{\pi}_i = \{\alpha_{(p)}, \Delta_P(q)\}$,
$i = N_2 \times p + q$ and $p = 0..N_1 - 1$, $q = 0..N_2 - 1$

Likelihood for the GW170817 $\Lambda_1 - \Lambda_2$ Constraint for given π_i

$P(E_{GW}|\pi_i) = \int_I \beta(\Lambda_1(\tau), \Lambda_1(\tau))d\tau$, here $\beta(\Lambda_1, \Lambda_1)$ is the PDF given from GW170817, and τ is a free parameter, like central density of a star [Antoniadis et al. Science 340].

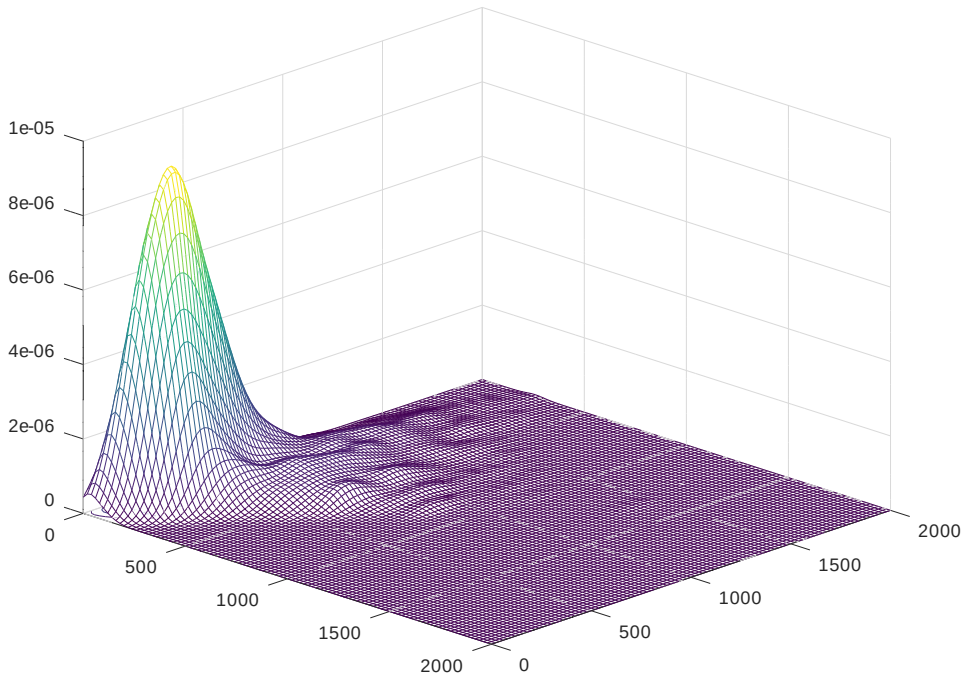
Likelihood for the Mass Constraint for given π_i

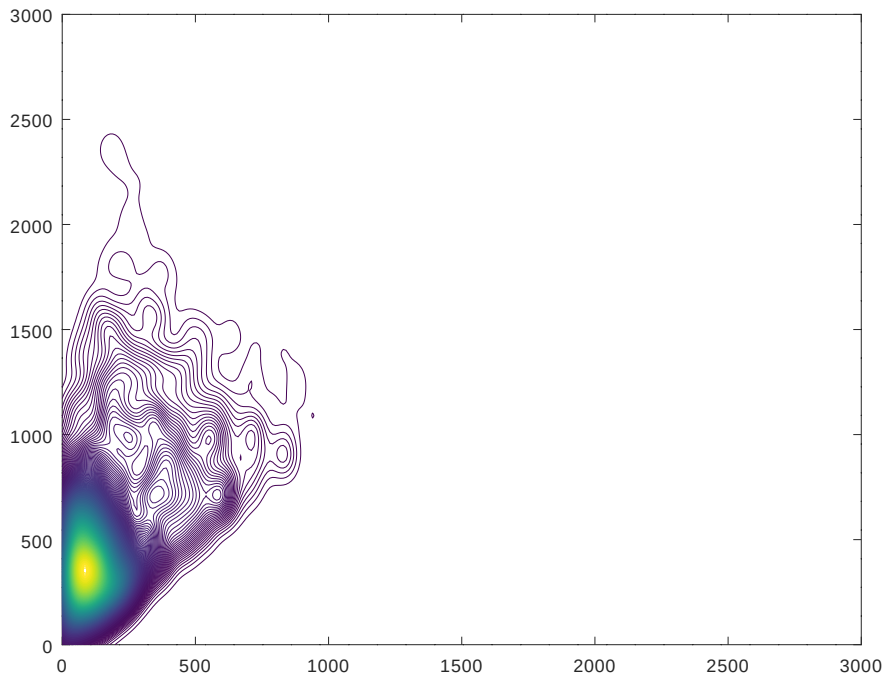
$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$.

Overall Likelihood and Posterior

$P(E|\overrightarrow{\pi}_i) = \prod_\alpha P(E_\alpha|\overrightarrow{\pi}_i)$., $P(\overrightarrow{\pi}_i|E) = \frac{P(E|\overrightarrow{\pi}_i)P(\overrightarrow{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\overrightarrow{\pi}_j)P(\overrightarrow{\pi}_j)}$.

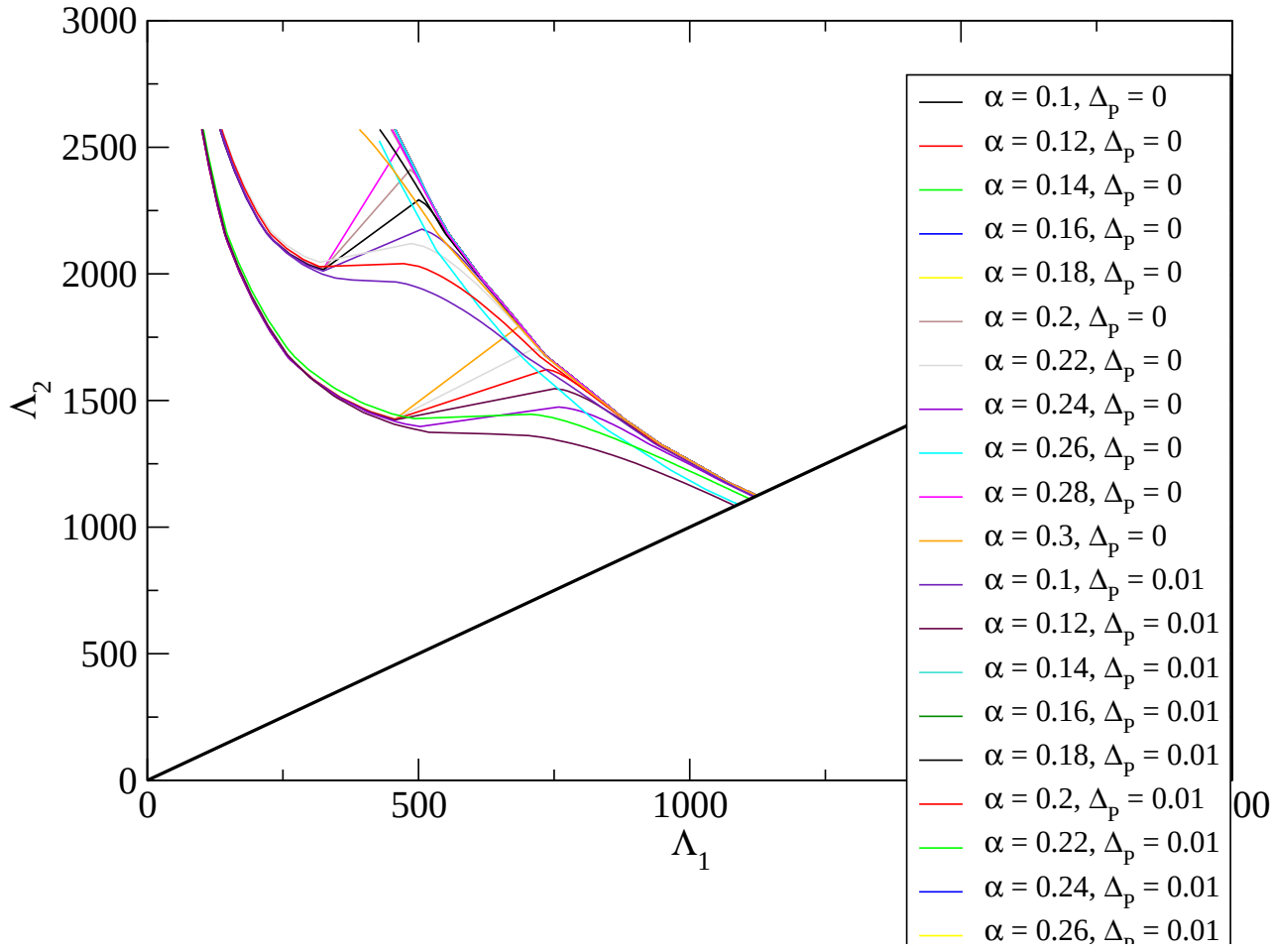
$\beta(\Lambda_1, \Lambda_2)$





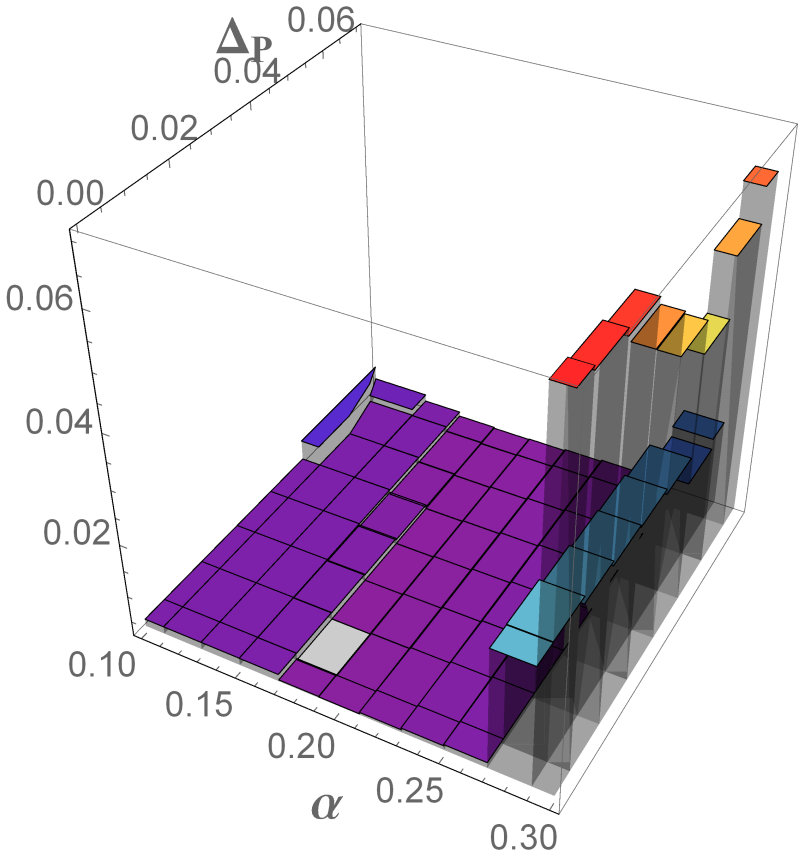
The PDF is build by Gaussian kernel density estimation with $\Lambda_1 - \Lambda_2$ data given at LIGO webpage
https://dcc.ligo.org/LIGO-P1800115/public

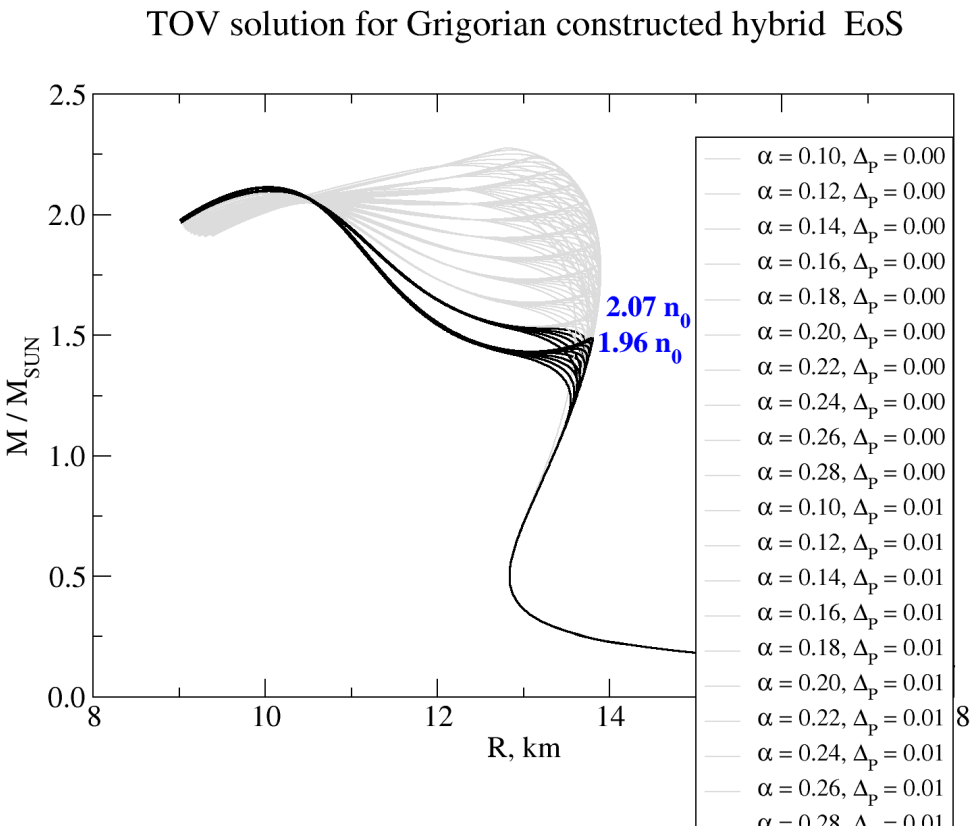
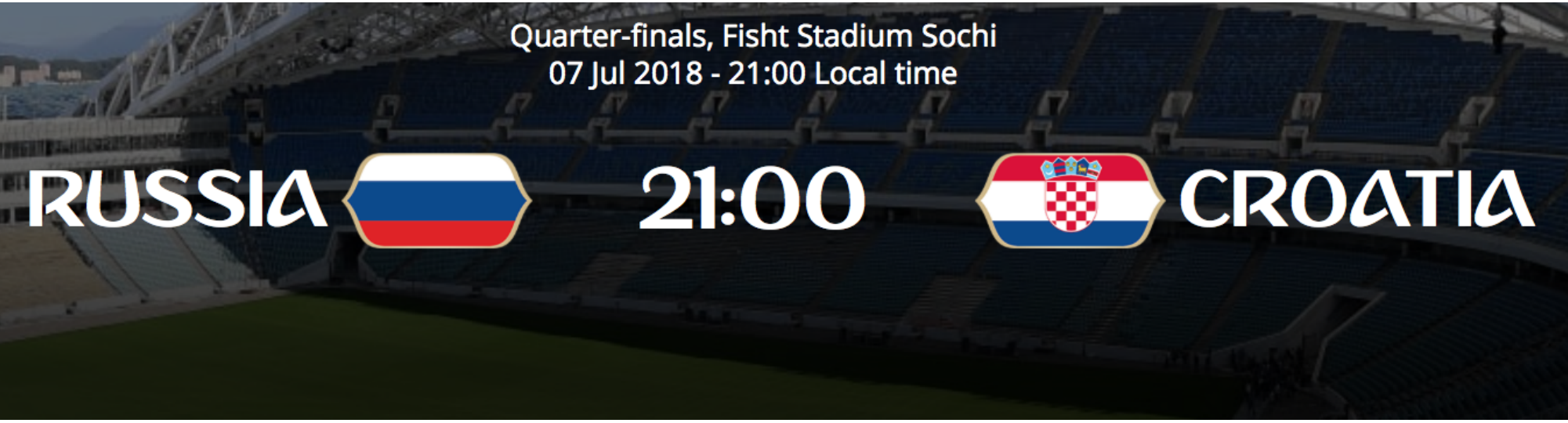
$\Lambda_1 - \Lambda_2$



da KVORcut02 + SFM($\alpha = 0.1(0.02)0.3$) + $\Delta_P = 0(0.01)0.06$.

Results


Δ_P
0.06
0.04
0.02
0.00
0.06
0.04
0.02
0.10
0.15
0.20
0.25
0.30
α


TOV solution for Grigorian constructed hybrid EoS
M / M$_{SUN}$
R, km
2.07 n$_0$
1.96 n$_0$
α = 0.10, Δ$_P$ = 0.00
α = 0.12, Δ$_P$ = 0.00
α = 0.14, Δ$_P$ = 0.00
α = 0.16, Δ$_P$ = 0.00
α = 0.18, Δ$_P$ = 0.00
α = 0.20, Δ$_P$ = 0.00
α = 0.22, Δ$_P$ = 0.00
α = 0.24, Δ$_P$ = 0.00
α = 0.26, Δ$_P$ = 0.00
α = 0.28, Δ$_P$ = 0.00
α = 0.10, Δ$_P$ = 0.01
α = 0.12, Δ$_P$ = 0.01
α = 0.14, Δ$_P$ = 0.01
α = 0.16, Δ$_P$ = 0.01
α = 0.18, Δ$_P$ = 0.01
α = 0.20, Δ$_P$ = 0.01
α = 0.22, Δ$_P$ = 0.01
α = 0.24, Δ$_P$ = 0.01
α = 0.26, Δ$_P$ = 0.01

Thank you for your attention!