

Multihadron Production: Universality, Correlations & Search for New Physics

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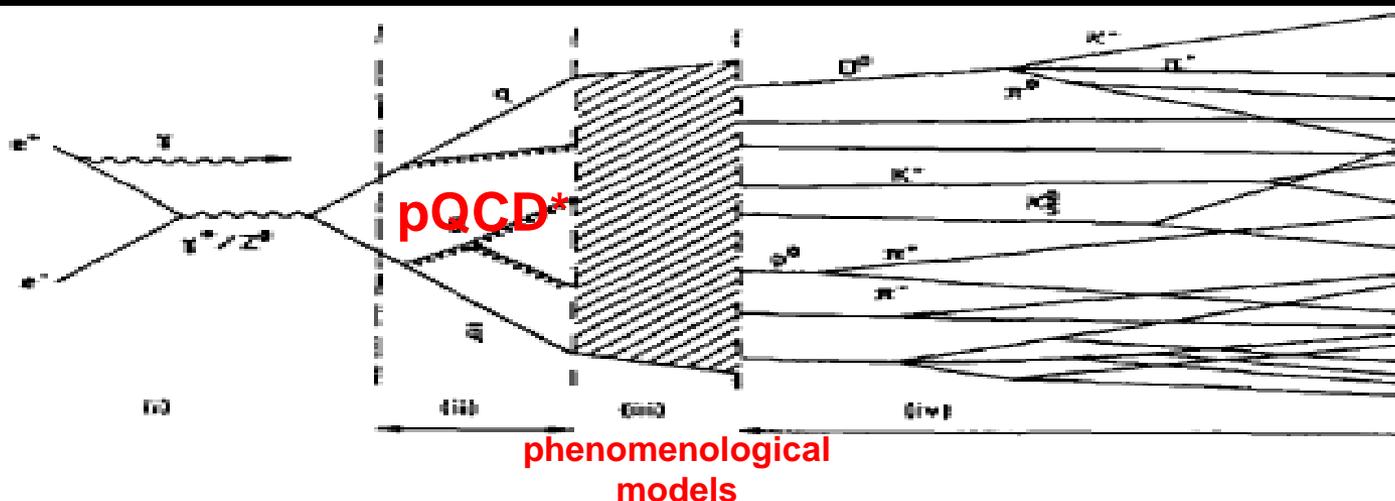
VII International Conference on New Frontiers in Physics

(ICNFP2018, Kolymbari, Crete, Greece, July 3-13, 2018)



Multihadron production

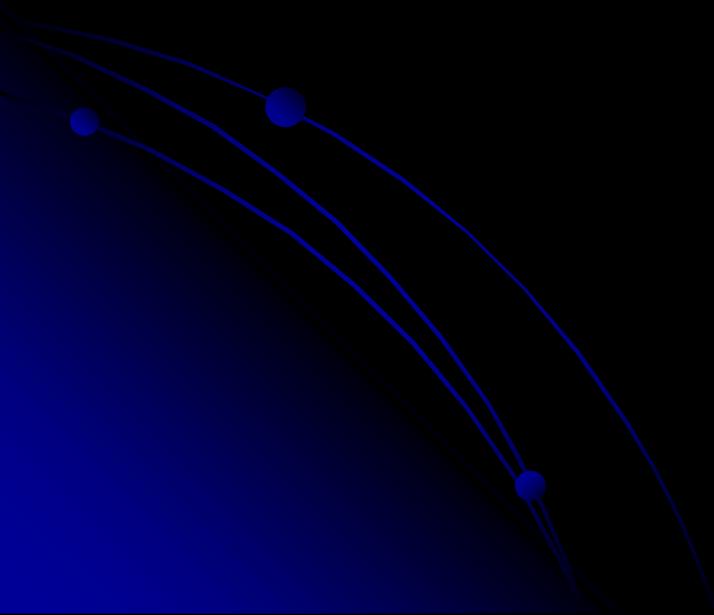
- Multihadron production process - an *important ingredient* of high energy physics, provides **underlying** features of strong interactions
- Multihadron production **still eludes** its complete **understanding**
- The theory of strong interactions, **QCD** well **reproduces** the multiplicity distribution, energy dependence, while **faces difficulties** in describing **multi-particle correlations**
- Different **phenomenological models** used to describe *parton to hadron* transition



Schematic illustration of a hadronic e^+e^- annihilation event

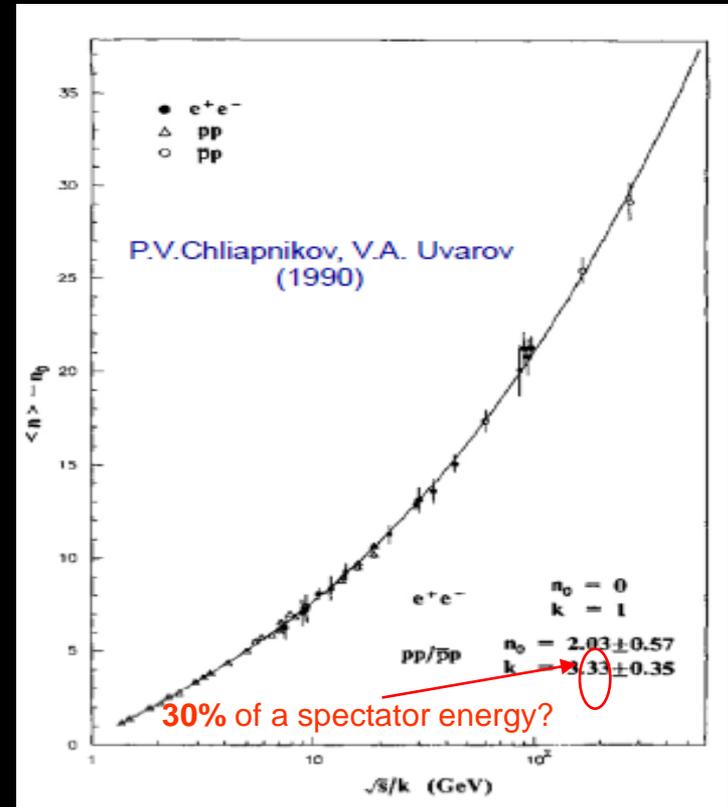
Bulk observables

- Bulk observables - mean **multiplicity** and **rapidity densities** - **control parameters** of the formation and evolution of the collision initial state
- ***extensively*** studied in **particle and nuclear** collisions



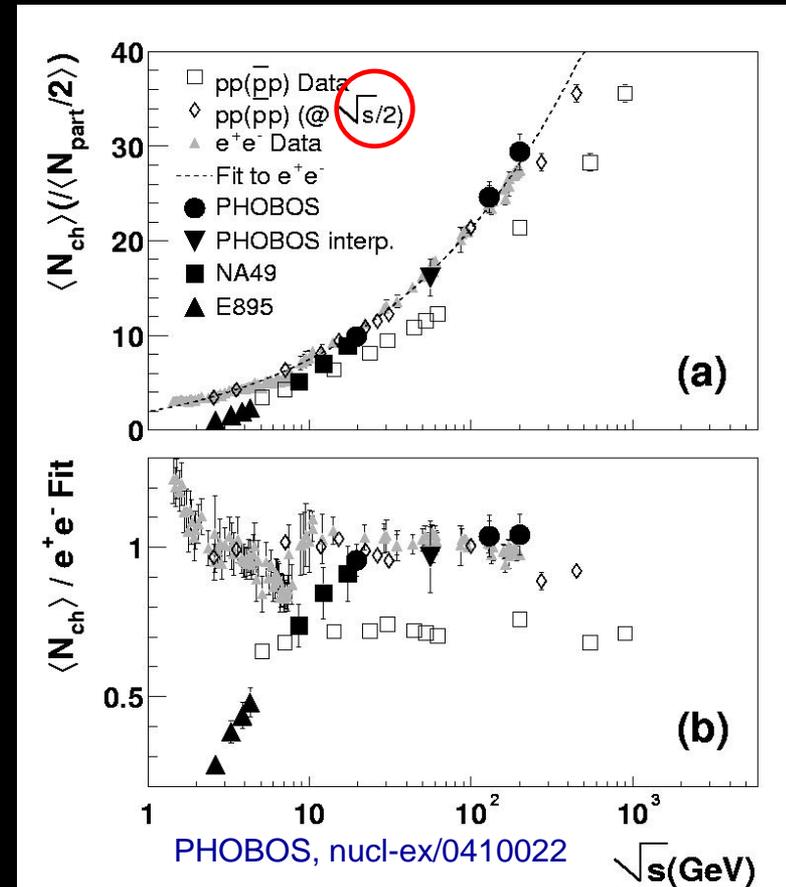
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- **similarities** in e^+e^- and pp data: **universality** in multihadron production



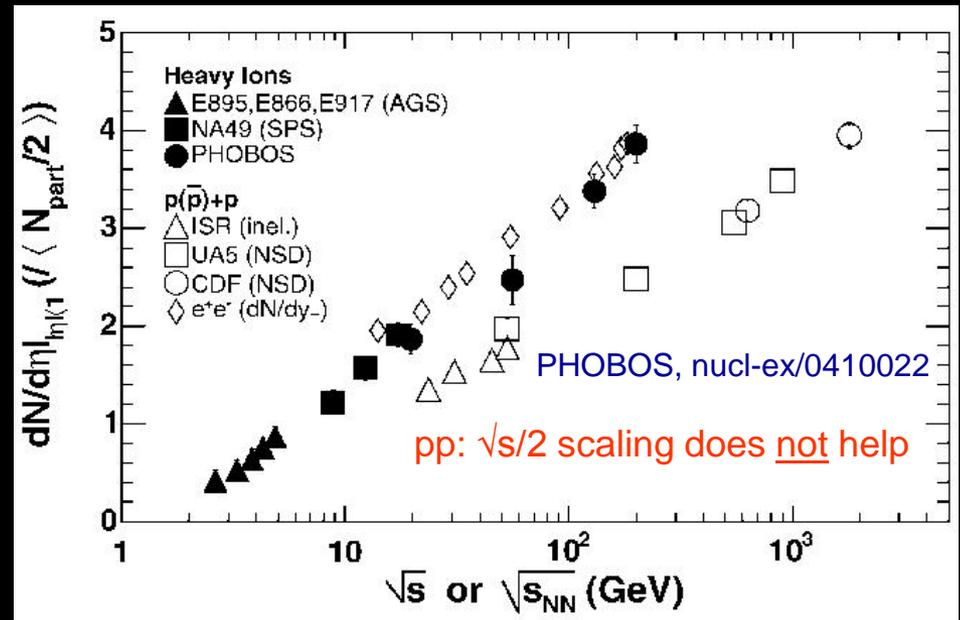
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- pp multiplicity data **needs to be scaled**



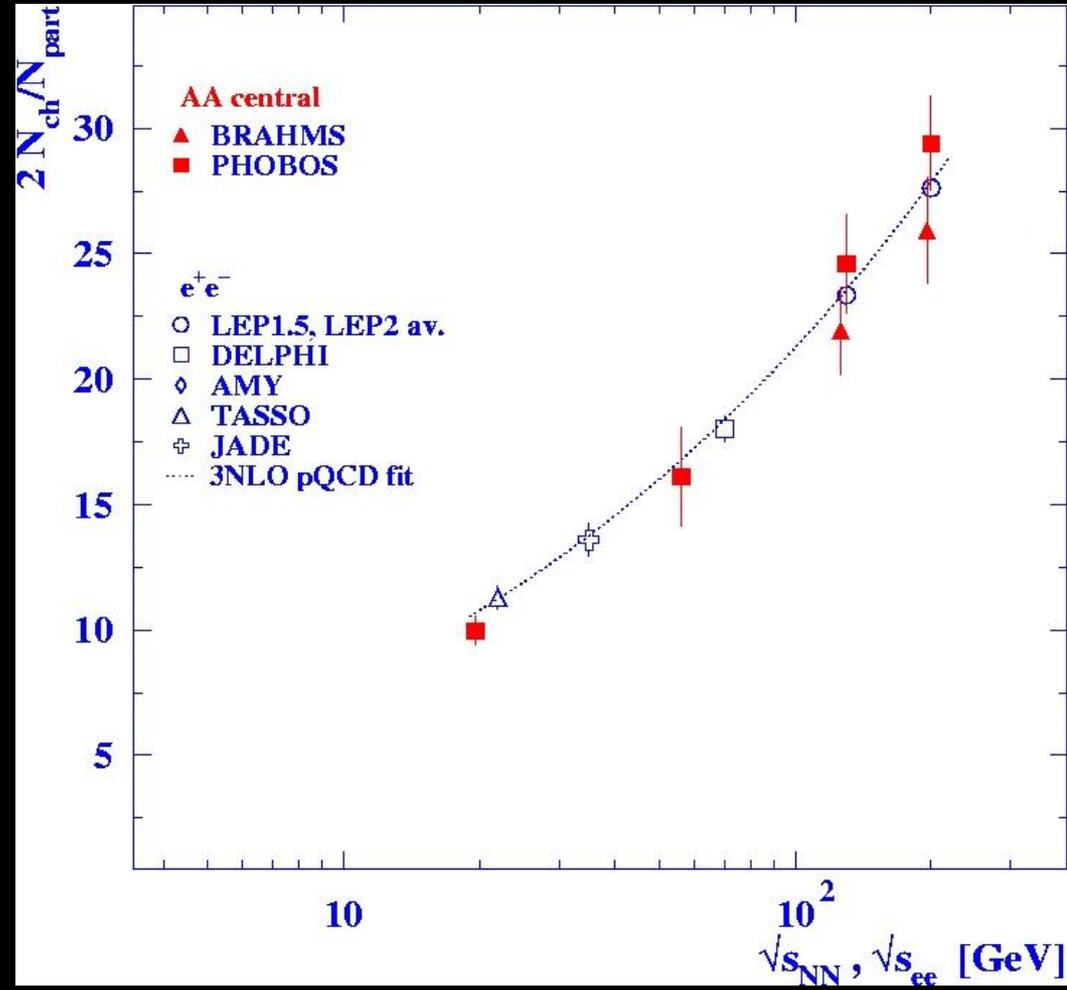
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- pp multiplicity data **to be scaled**
- **not the same** scaling for **both** variables



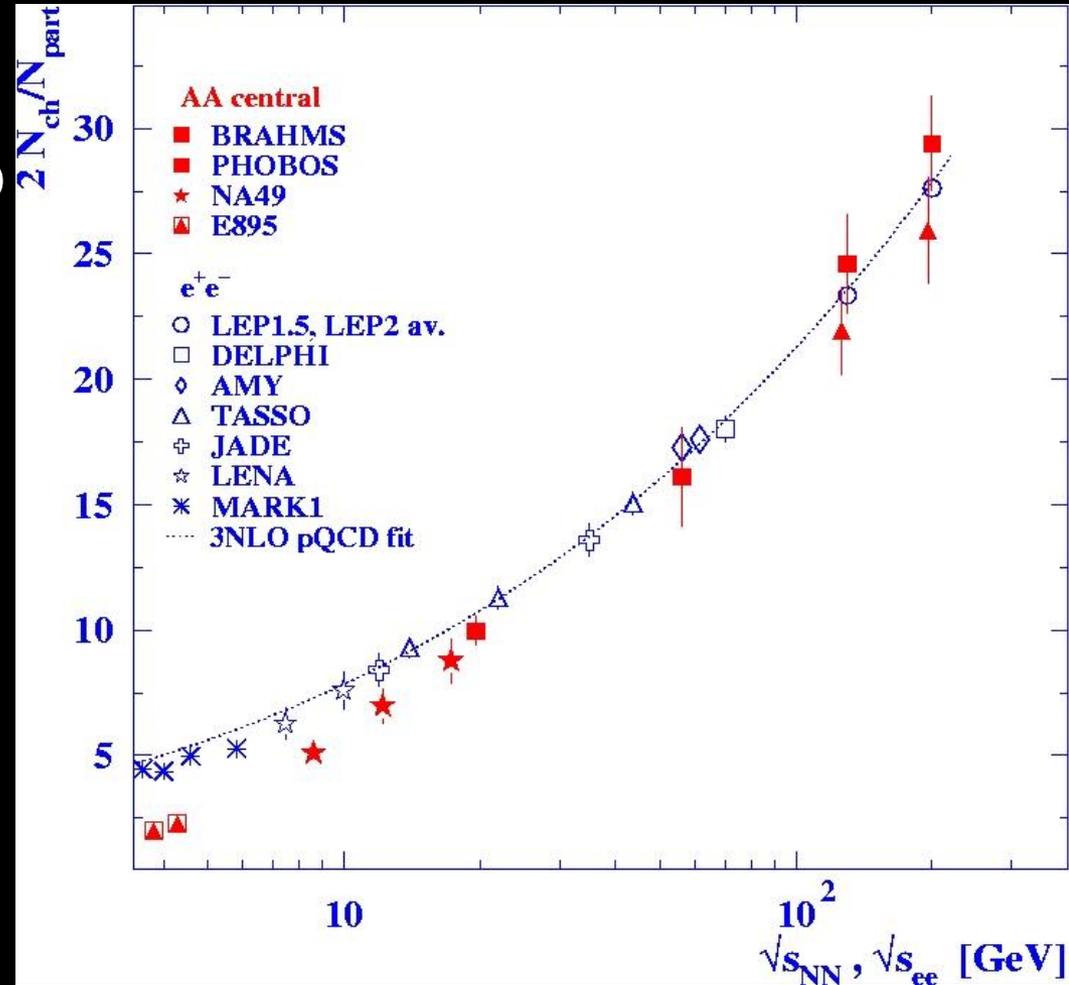
Multiplicity in e^+e^- and AA

- N_{ch} e^+e^- data *similar* to head-on AA data at RHIC energies
- well **reproduced** by 3NLO pQCD calculations



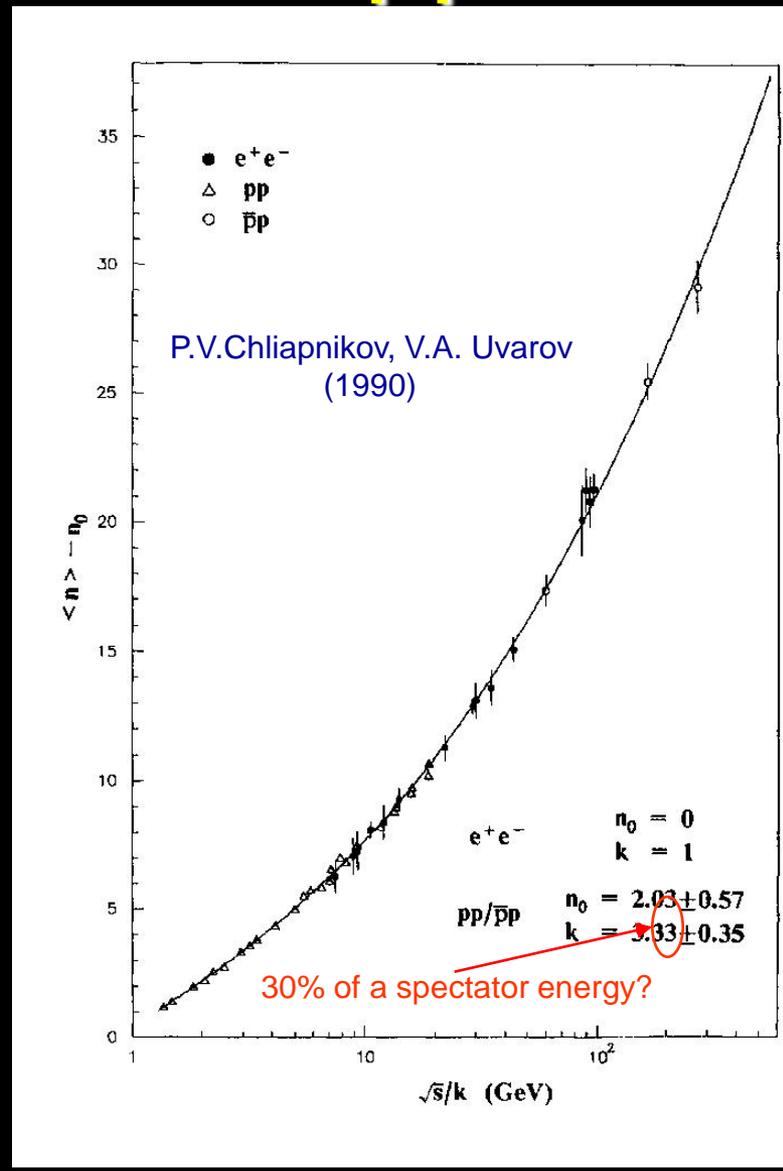
Multiplicity in e^+e^- and AA

- N_{ch} in e^+e^- *similar* to AA data
- well **reproduced** by 3NLO pQCD
- **difference** at the c.m. energy < 20 GeV: AA data lower than e^+e^- data (low energy fragments)



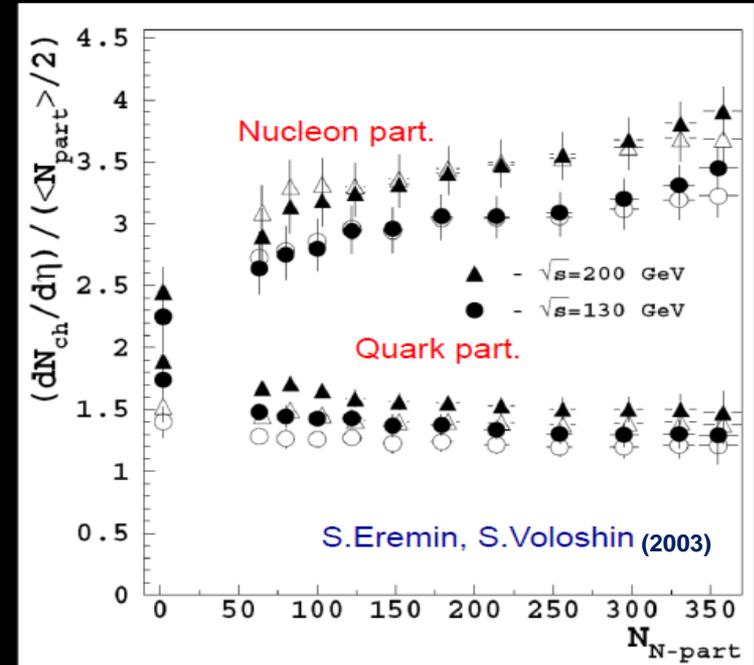
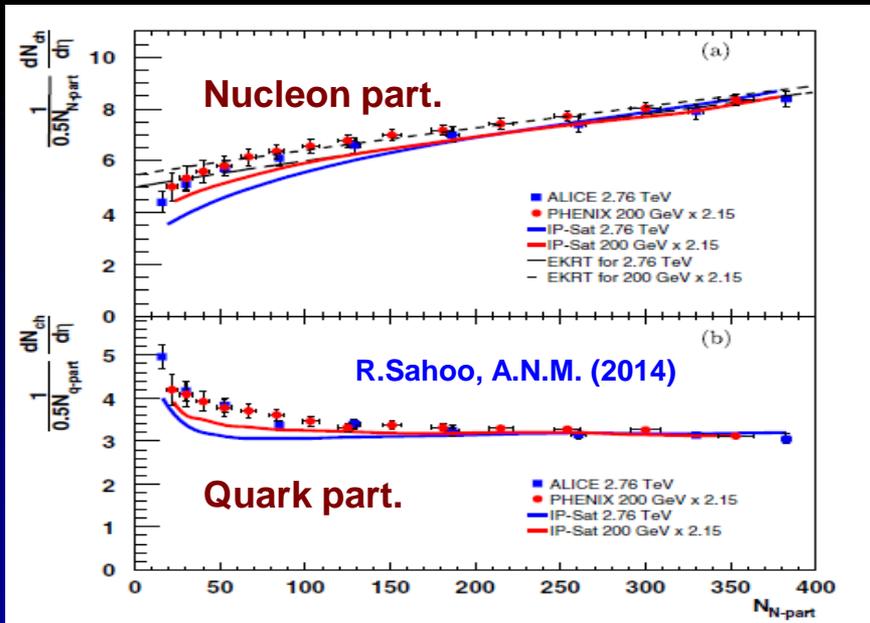
Multiplicity in e^+e^- and pp

- N_{ch} in e^+e^- *similar* to AA data
- well reproduced by 3NLO pQCD theory
- N_{ch} in pp *similar* to e^+e^- (and then to AA) as $\sqrt{s_{ee}} = \sqrt{s_{pp}}/3$ indicating quark-quark interaction plays a role
- observed at LEP starting days



Constituent quark framework

No nucleon participant dependence as soon as densities calculated in the **constituent quark framework**



Nucleon Participants:

Open vs solid symbols: HIJING vs overlap model

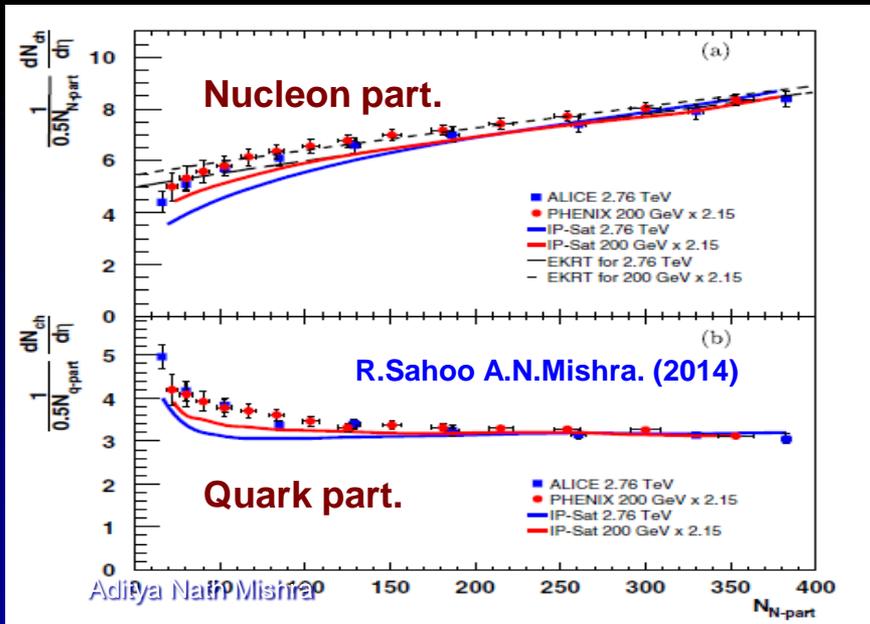
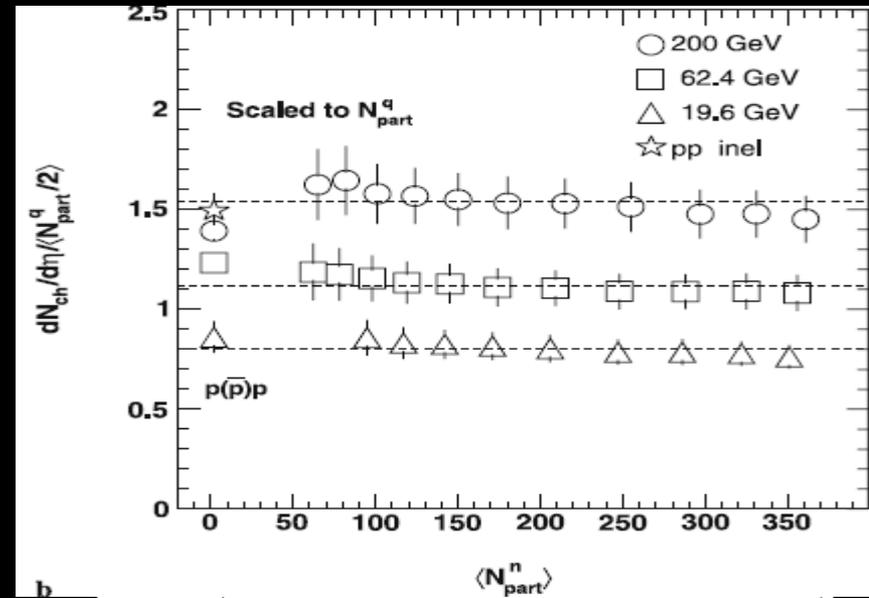
Quark Participants:

Open vs solid symbols: different σ_{pp}

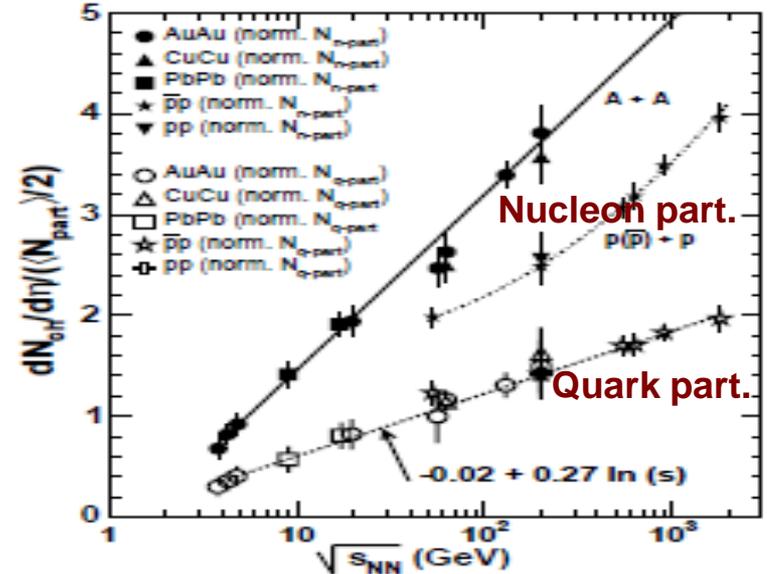
Constituent Quark Framework

No nucleon participant dependence as soon as densities calculated in the **constituent quark framework**

AA centrality data are **similar** to NSD **pp** measurements



R. Nouicer (2007), PHOBOS data

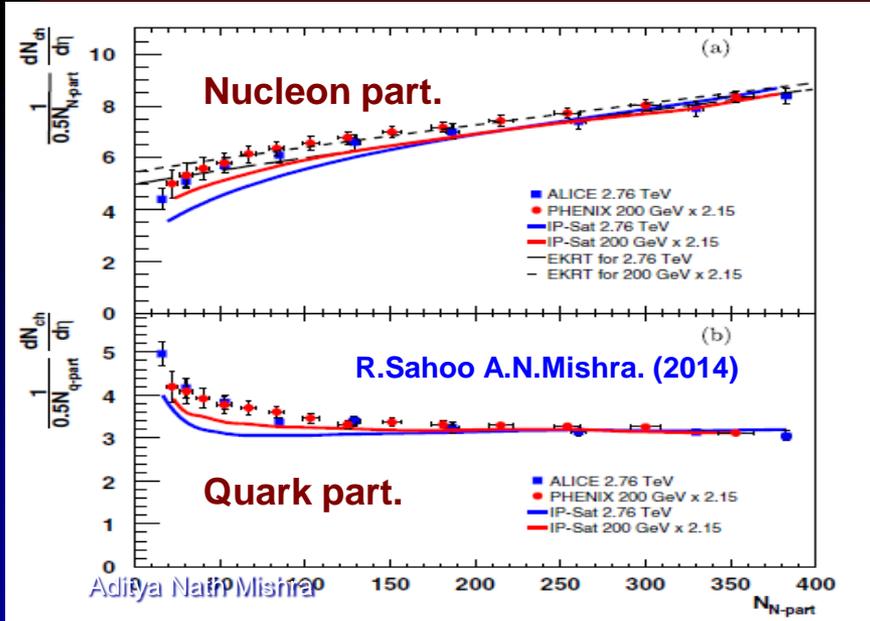
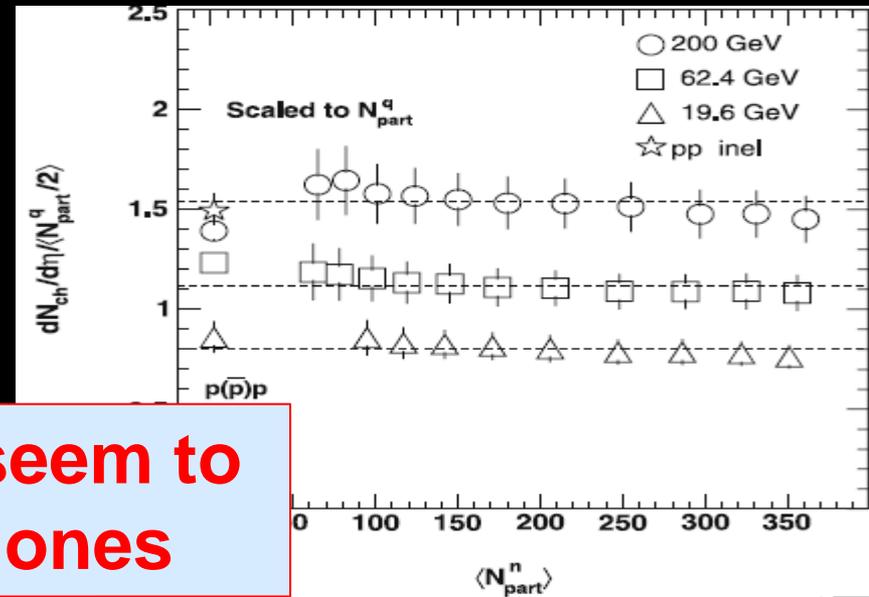


Constituent Quark Framework

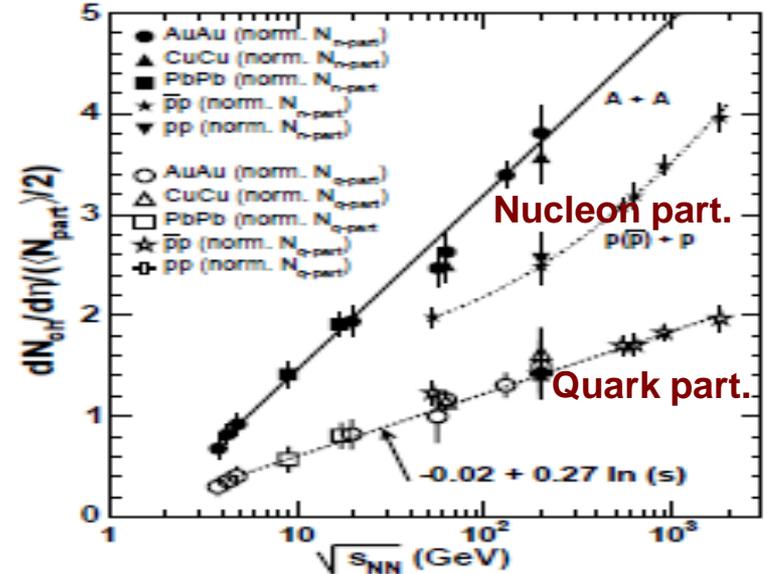
No nucleon participant dependence as soon as calculated in the **constituent quark framework**

AA centrality data are **similar** to NSD **pp** measurements

Quark degrees of freedom seem to play a role, not the nucleon ones



R. Nouicer (2007), PHOBOS data



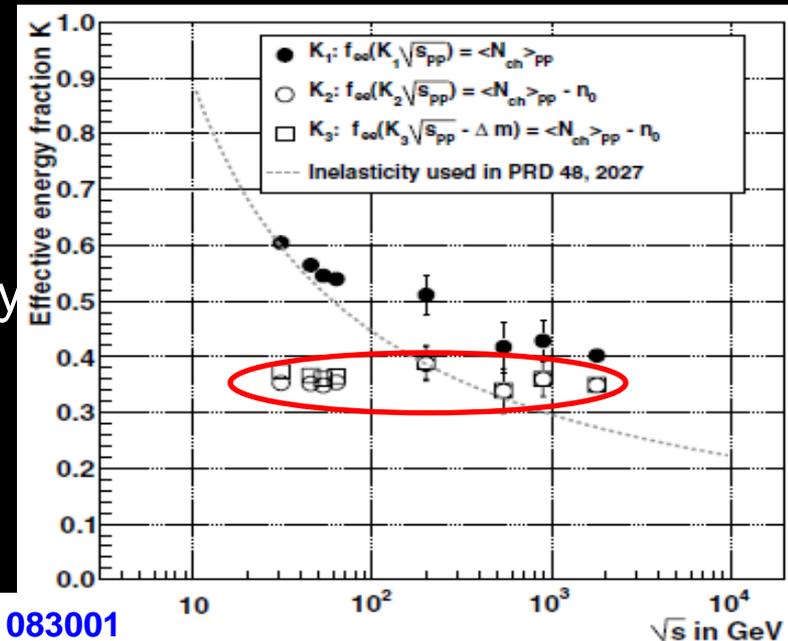
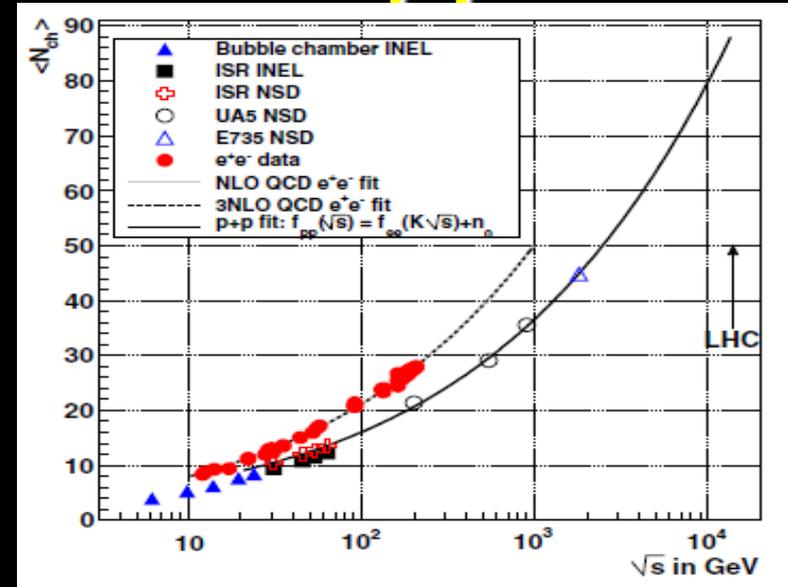
Energy scaling

- e^+e^- (**structureless particles**) annihilation - the **total** interaction **energy** is deposited in the initial state
- **pp** (**superposition of three pairs of constituents**) collision - **only** the **energy** of the interacting **single quark pair** is deposited in the initial state
- multiplicity and mid-rapidity density should be **similar** in pp at c.m. energy $\sqrt{s_{pp}}$ and e^+e^- at c.m. energy $\sqrt{s_{ee}} \approx \sqrt{s_{pp}}/3$
- **heavy ion (nuclear) collisions**: **more than one** quark per nucleon participates
- **head-on nuclear collisions**: **all three quarks** participate nearly simultaneously and deposit **their energy** coherently into initial state
- multiplicity and mid-rapidity density should be **similar** in pp at c.m. energy $\sqrt{s_{pp}}$ and head-on AA at c.m. energy $\sqrt{s_{NN}} \approx \sqrt{s_{pp}}/3$

EKGS & A. Sakharov (2004) : dissipating energy participants

Multiplicity in e^+e^- and pp

- N_{ch} in e^+e^- *similar* to AA data
- well reproduced by 3NLO pQCD theory
- N_{ch} in pp *similar* to e^+e^- (and AA)
as $\sqrt{s_{ee}} = \sqrt{s_{pp}}/3$
- pp data **similarly well** reproduced from e^+e^- theory (3NLO) fit *up to highest* LEP energies assuming $f_{pp} = f_{ee}(K\sqrt{s}) + n_0$:
 $n_0=2$ characterizes the *number of leading protons*, $K \approx 0.35$ is the inelasticity and characterizes the fraction of *effective* energy (of produced particles), i.e. $\sqrt{s_{pp}} = 3\sqrt{s_{ee}}$
- the inelasticity prefers the **0.35** value being *energy-independent*



Universality in e^+e^- , ep and pp up to LHC

- pp data *up to LHC* are **well reproduced** from QCD theory NNLO fit for multiplicities **and for the midrapidity densities, similar** to e^+e^- and ep multiplicity data

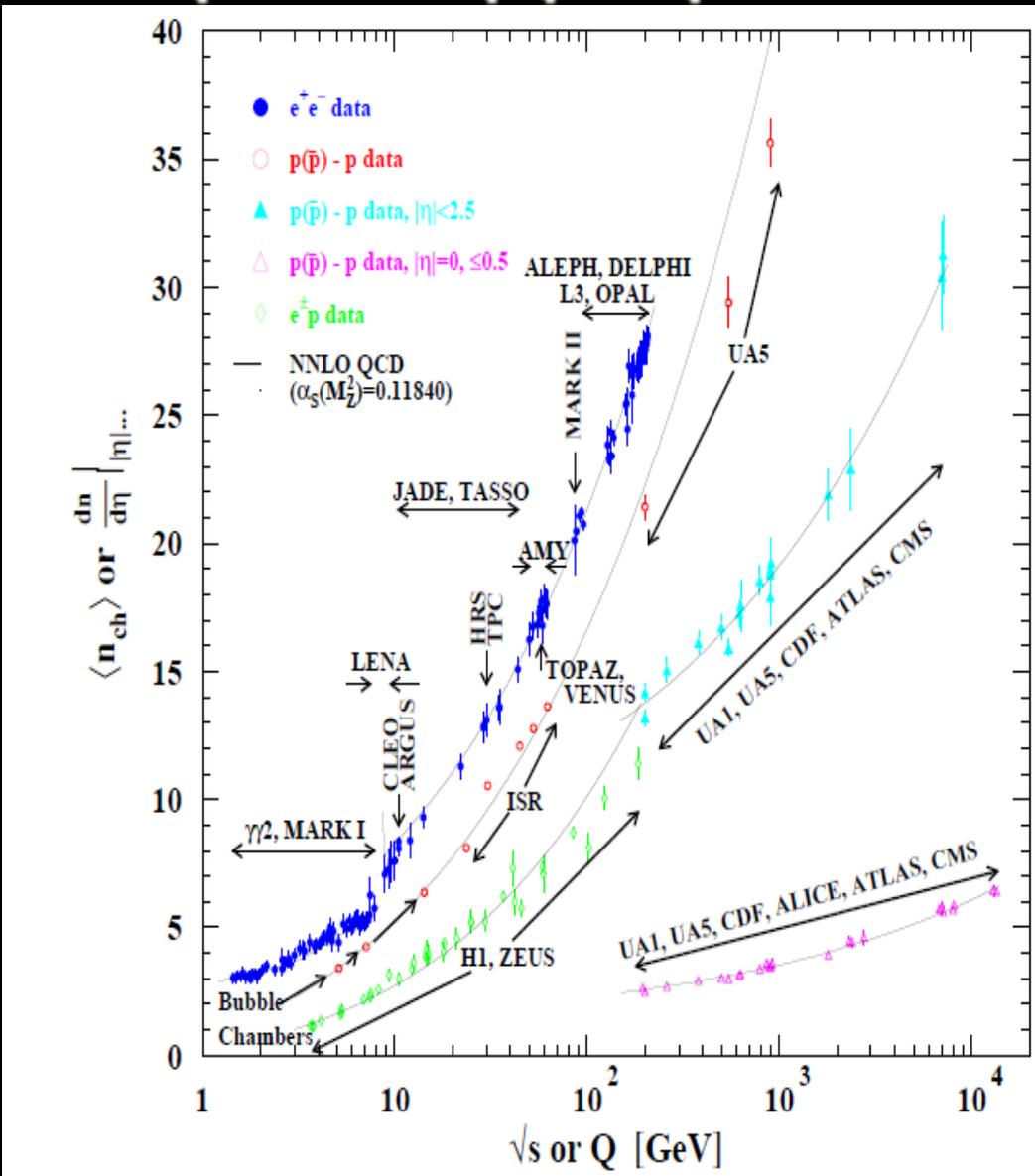
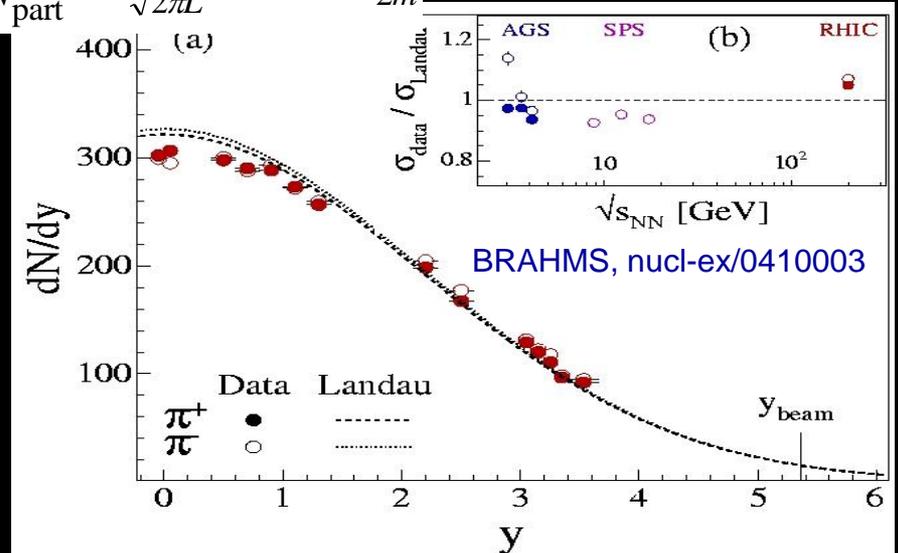
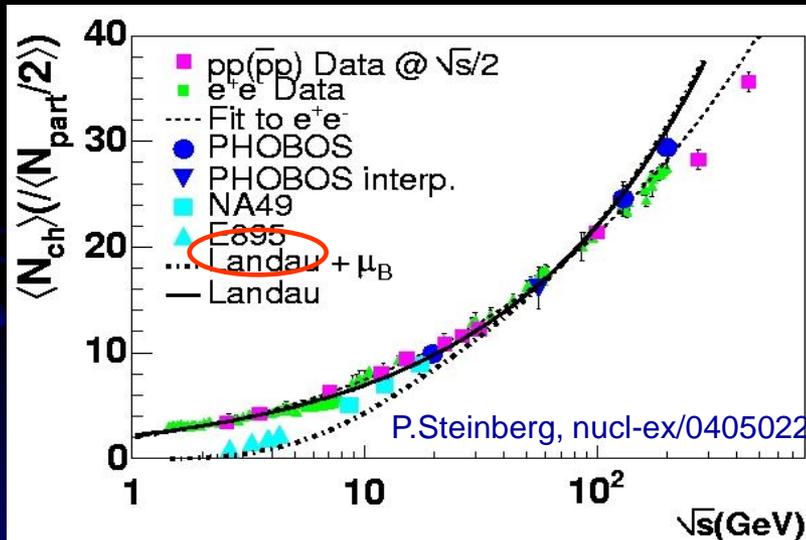


Figure 19.6: Average charged particle multiplicity $\langle n_{ch} \rangle$ as a function of \sqrt{s} or Q for e^+e^- and $p\bar{p}$ annihilations, and pp and ep collisions.

Hydrodynamics of collisions

- two head-on colliding Lorentz-contracted particles stop within overlapped zone
- formation of fully thermalised initial state at the collision moment
- the decay (expansion) of the initial state is governed by *relativistic hydrodynamics* - Landau model

$$\frac{2N_{\text{ch}}}{N_{\text{part}}} \frac{\exp(-y^2/2L)}{\sqrt{2\pi L}}, \quad L = \ln \frac{\sqrt{s}}{2m}$$



- the production of particles is *defined* by the energy deposited into the *initial state* (Heisenberg (1949), Fermi (1950), Landau (1953))

Hydrodynamics and energy scaling vs data

Landau Hydrodynamics + Constituent Quark approach

$$\rho(0) = \rho_{pp}(0) \frac{2N_{ch}}{N_{part} N_{ch}^{pp}} \sqrt{\frac{L_{pp}}{L_{NN}}}$$

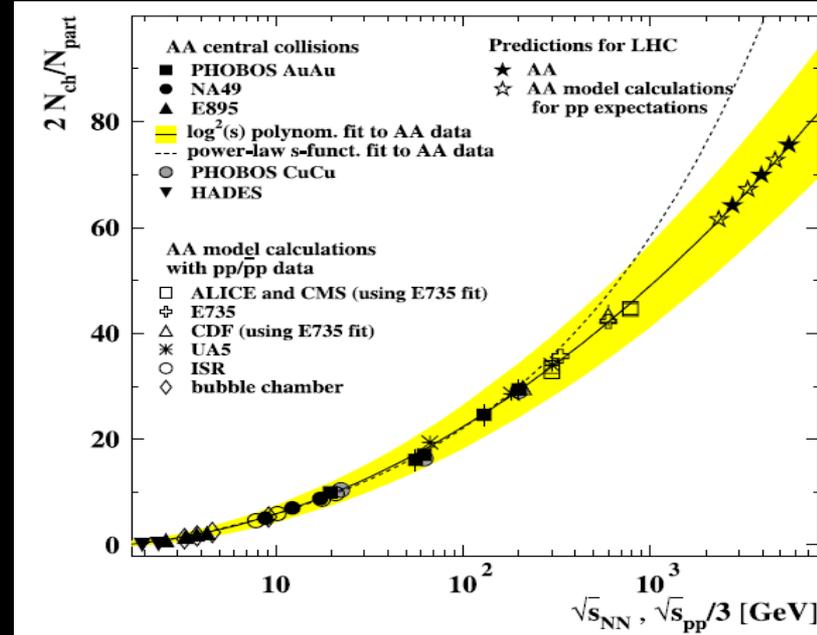
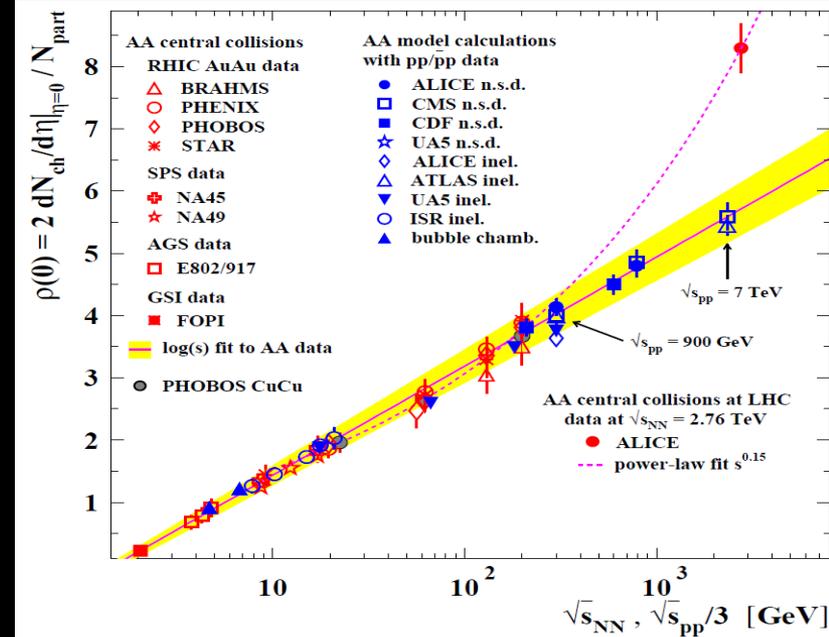
$$\sqrt{s_{NN}} = \sqrt{s_{pp}}/3$$

$$L = \ln \frac{\sqrt{s}}{2m}$$

$$m = m_p, \frac{1}{3} m_p$$

$$\rho(0) = \rho_{pp}(0) \frac{2N_{ch}}{N_{part} N_{ch}^{pp}} \sqrt{1 - \frac{4 \ln 3}{\ln(4m_p^2/s_{NN})}}$$

✓ Nuclear data *both* on the **midrapidity density** and the **mean multiplicity** energy dependences **well reproduced** up to top RHIC energy



Hydrodynamics and energy scaling vs data

Landau Hydrodynamics

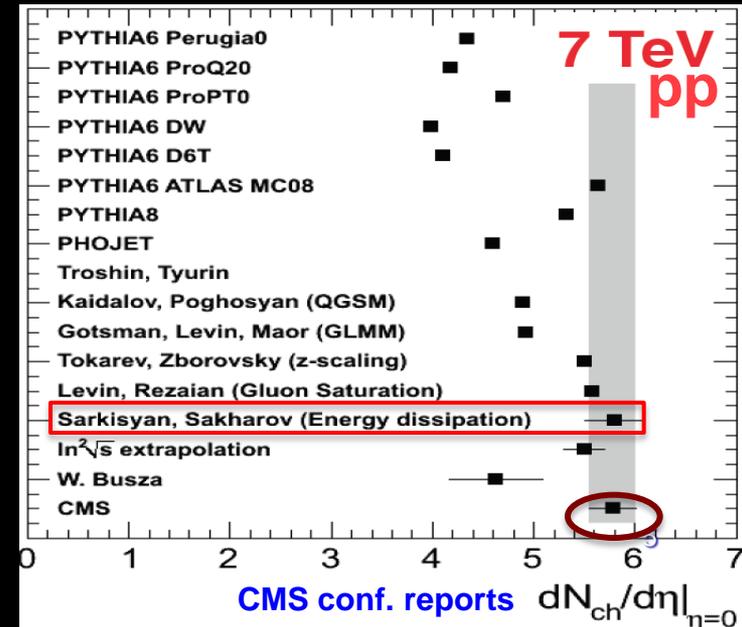
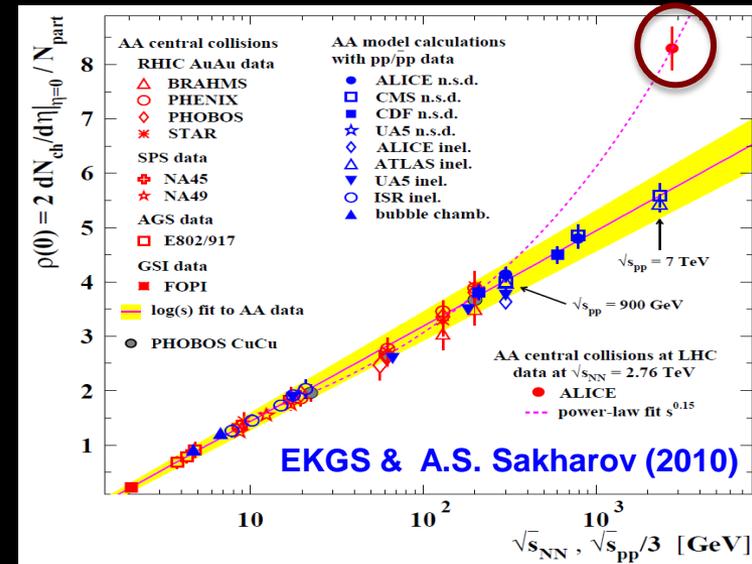
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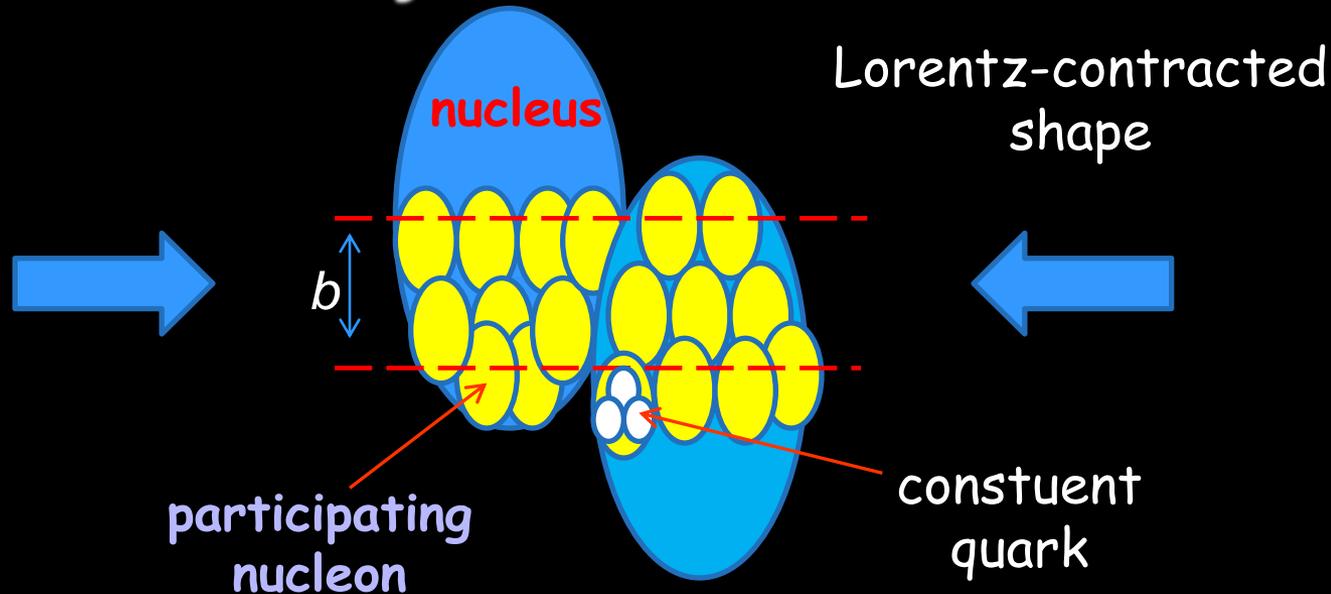
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- ✓ Nuclear data **both** on **midrapidity density** and **mean multiplicity** energy dependence well reproduced up to top RHIC energy
- ✓ pp data at the **LHC** energy of 2-7 TeV **well predicted**
- ✓ Heavy-ion collisions at the **LHC** indicate a transition to a possibly **new regime** with more degrees of freedom



Centrality in nuclear collisions



“**Centrality**” α characterizes the area of the overlap of the nuclei, described by the *impact factor*, b .

The *more central* the collision is *the smaller* the impact factor b , and then the centrality are. The centrality is measured in % characterizing the rate of cross-section.

The *number of participants* N_{part} *increases* as the centrality *decreases*.

Hydrodynamics and effective energy

Effective energy:

Effective energy can be calculated as following:

$$\epsilon_{\text{NN}} = \sqrt{s_{\text{NN}}} (1 - \alpha)$$

Here α is centrality percentile.

e.g. For 0-5% centrality collision, $\alpha = 0.025$

Hydrodynamics and effective energy

Effective energy:

Effective energy can be calculated as following:

$$\epsilon_{NN} = \sqrt{s_{NN}} (1 - \alpha)$$

this is "colliding" energy of participants

Here α is centrality percentile.

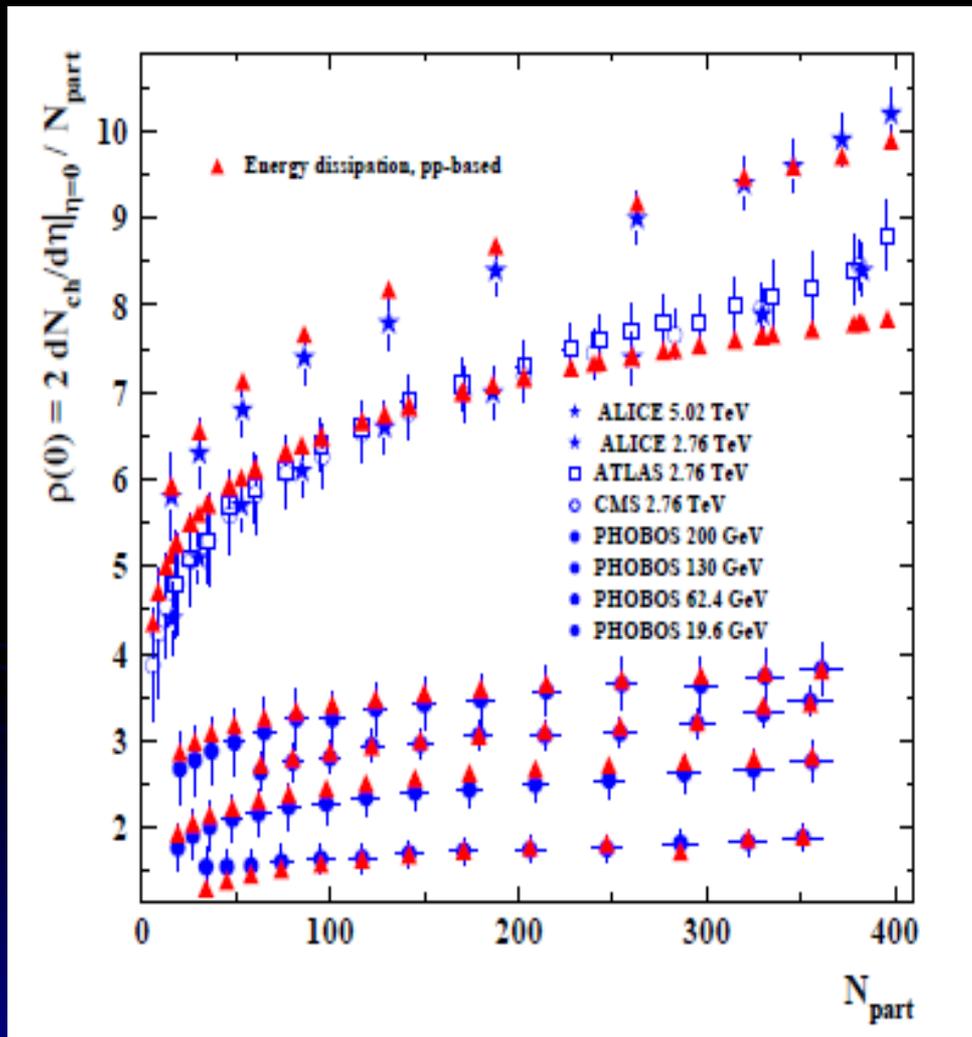
e.g. For 0-5% centrality collision, $\alpha = 0.025$

$$\rho(\mathbf{0}) = \rho_{pp}(\mathbf{0}) \frac{2N_{ch}}{N_{part} N_{ch}^{pp}} \sqrt{1 - \frac{2 \ln 3}{\ln(2m_p/\epsilon_{NN})}}$$

$$\epsilon_{NN} = \sqrt{s_{pp}} / 3$$

$N_{ch}/(N_{part}/2)$ comes from the *most central* collisions

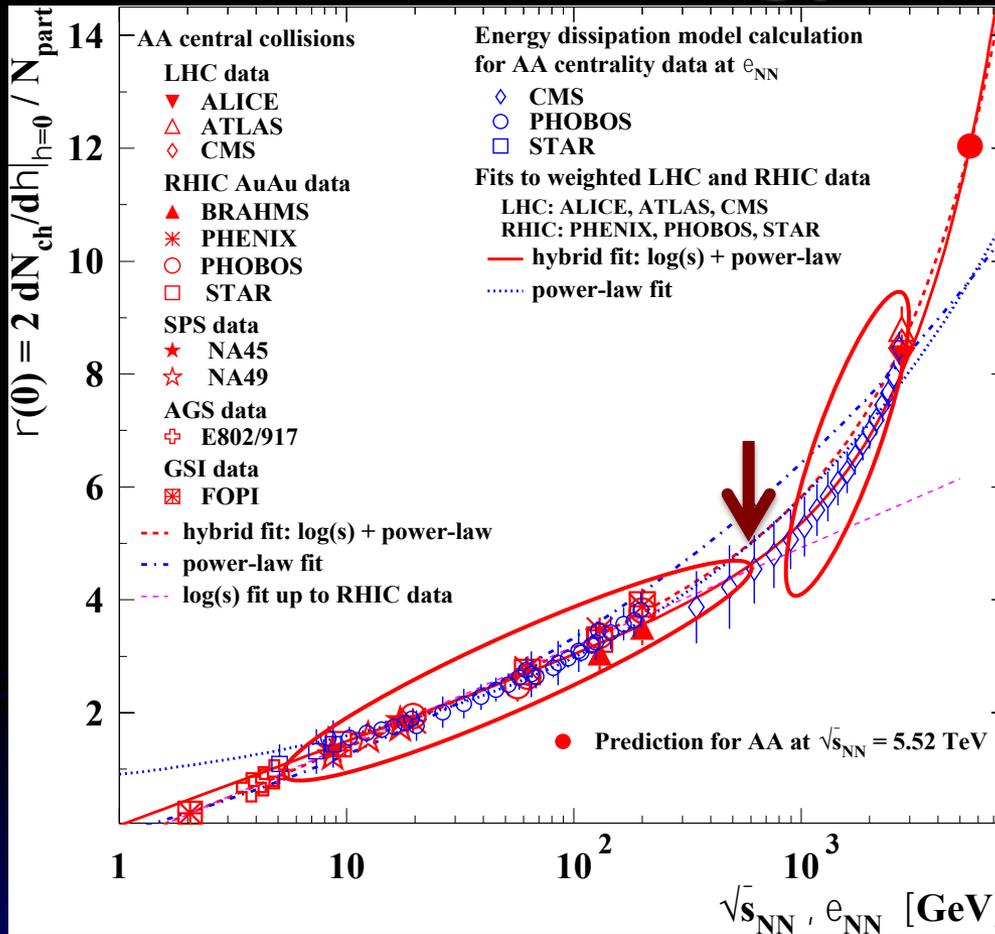
Effective energy vs data up to LHC



❖ *Effective-energy* (based on dissipating energy) *calculations* have a very good agreement with data up to LHC

❖ *Similarity* in all the data from peripheral to the most central ones

Effective energy vs data up to LHC



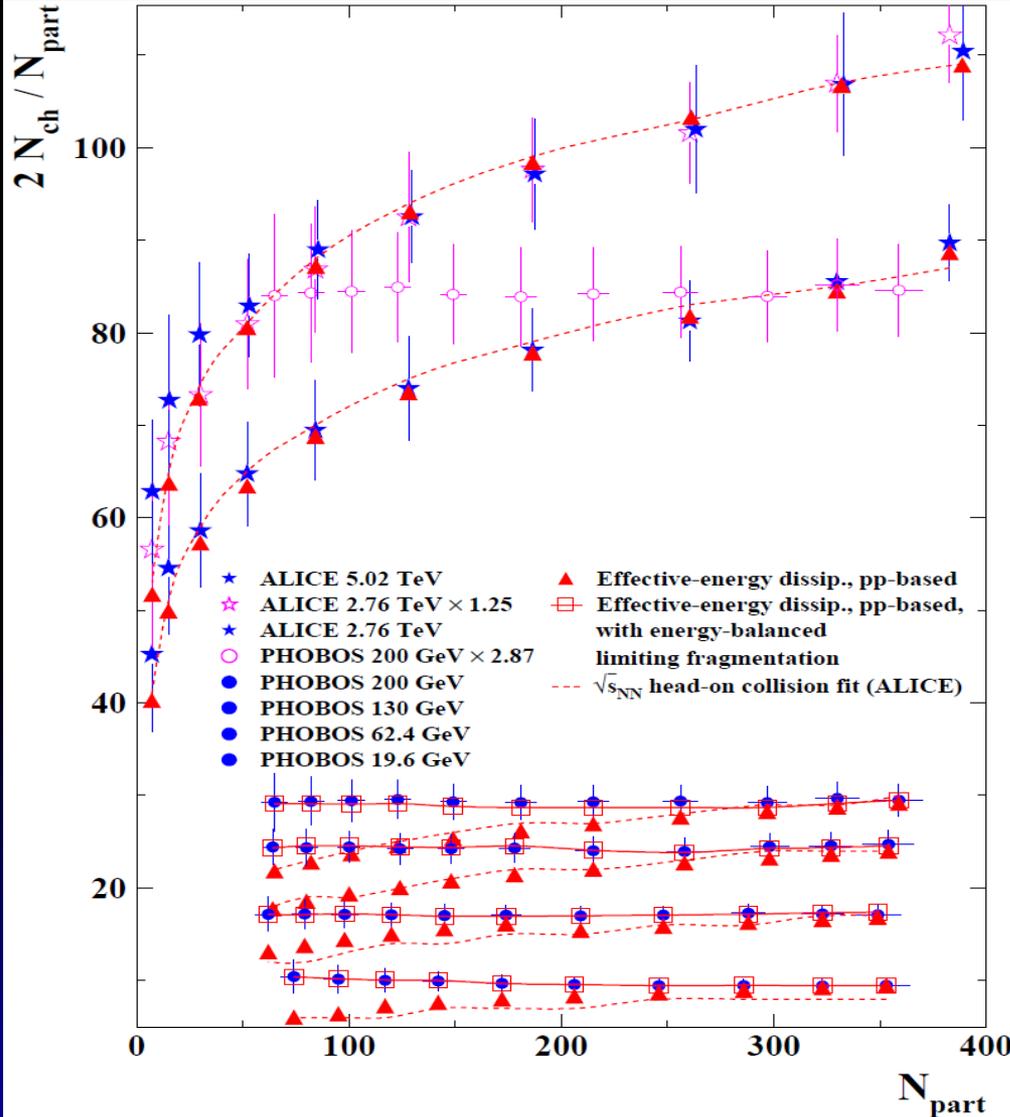
❖ *Effective-energy* (based on dissipating energy model) *calculations* have a very good agreement with data

❖ *Similarity* in all the data **from peripheral to the most central ones** follow the same energy behavior

❖ The combined data indicate *possible transition to a new regime* at $\sqrt{s_{NN}}=0.5-1.0$ TeV

Effective-energy approach stresses *underlying similarity* between **head-on and non-central** heavy-ion collisions

Total multiplicity centrality dependence



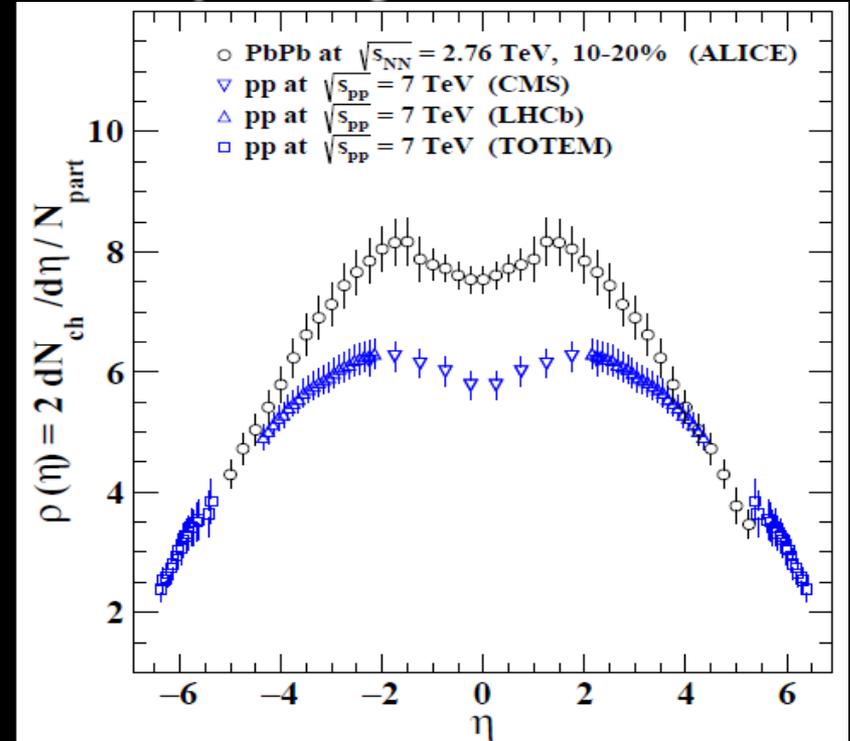
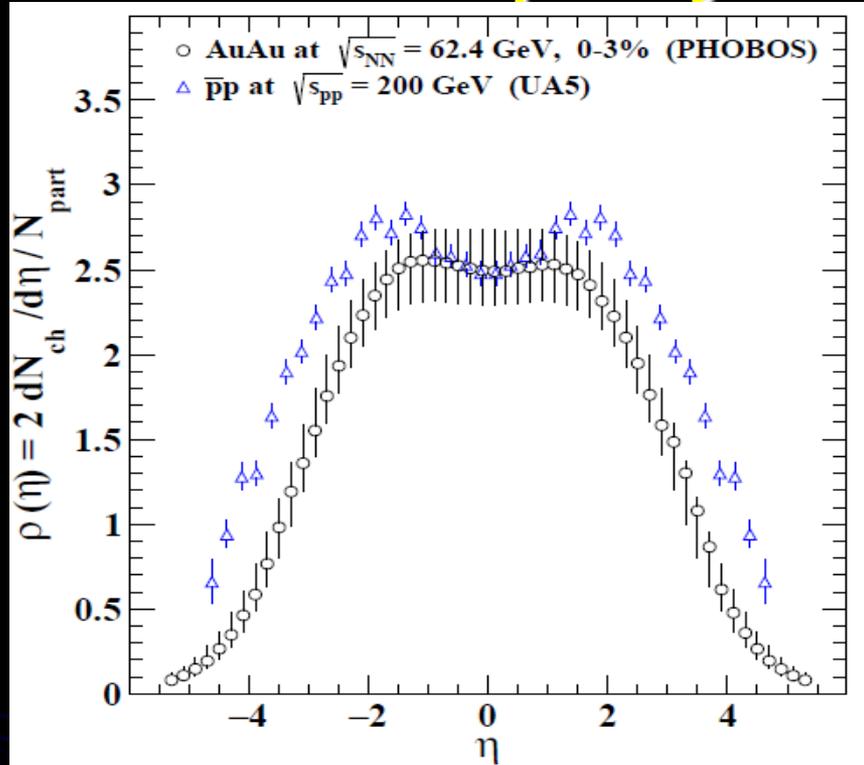
❖ *Effective-energy* (based on dissipating energy) *calculations* have a **very good agreement** with **LHC data**

❖ *Calculations* are **below** the **RHIC less central data**

❖ *Difference* between LHC (TeV) and RHIC (GeV) measurements

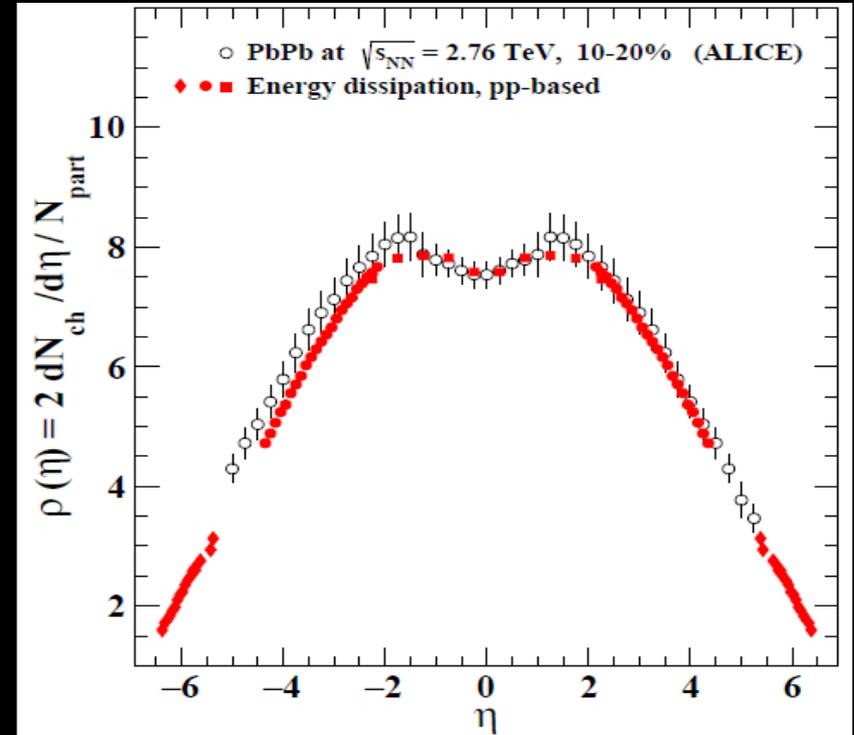
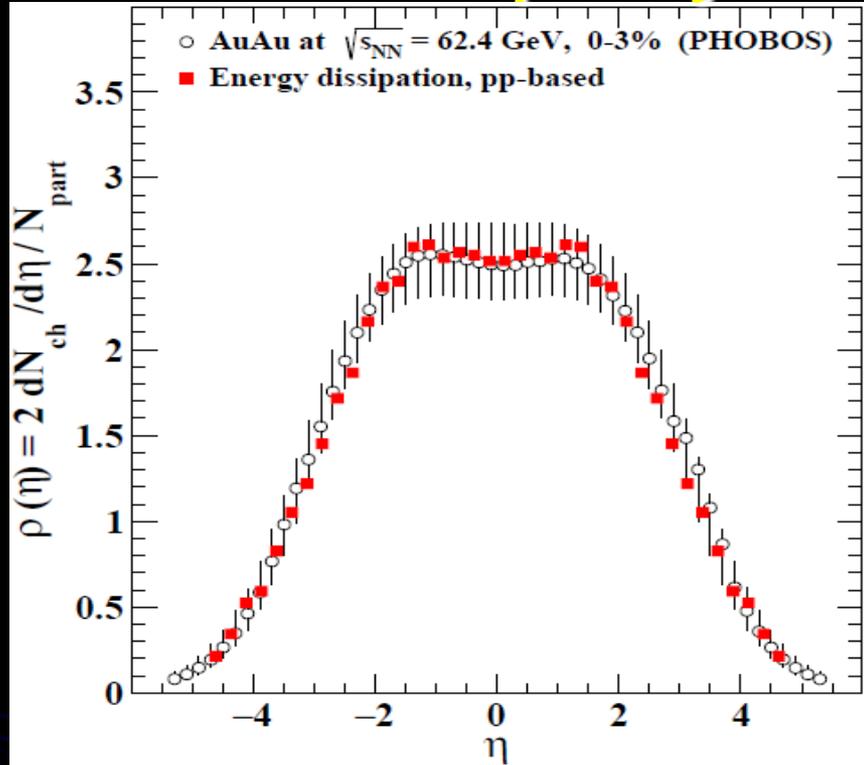
❖ ALICE $\sqrt{s_{\text{NN}}}$ (few GeV to few TeV) fit to **most central** data **follows** the effective energy calculations

From mid-rapidity to full-rapidity distribution



- ❖ pp data vs high-central AA data at $\sqrt{s_{pp}} \approx 3\sqrt{s_{NN}}$ (or $3\epsilon_{NN}$)
- ❖ At **LHC** energy, pp measurements from *three different* experiments

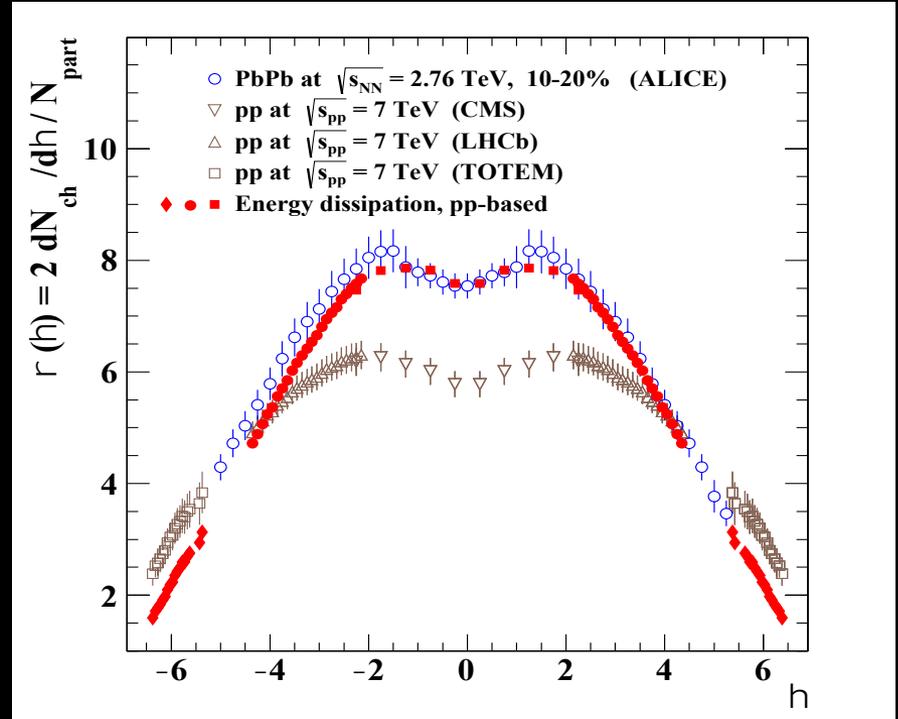
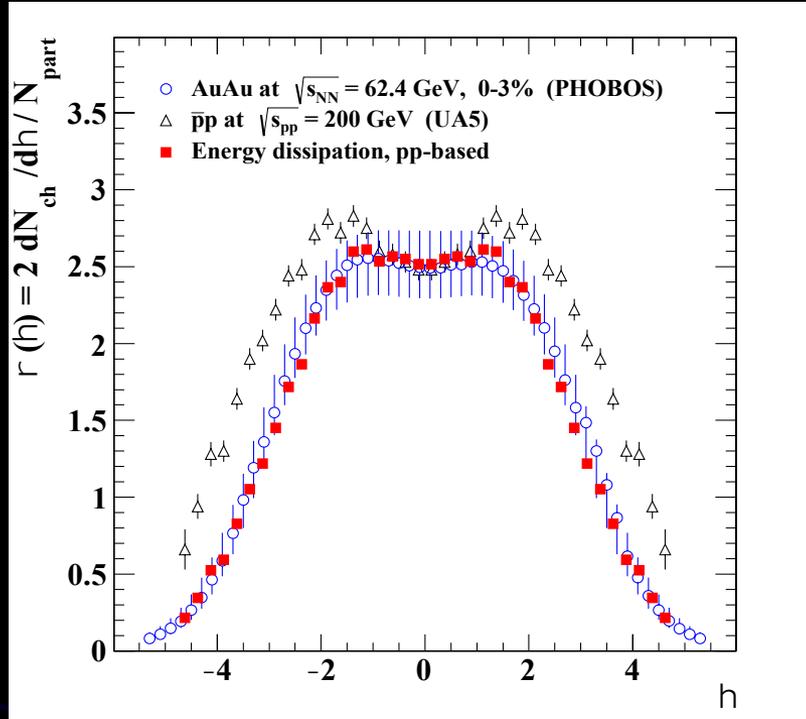
From mid-rapidity to full-rapidity distribution



$$\frac{\rho(\eta)}{\rho_{pp}(\eta)} = \frac{2N_{ch}}{N_{part}N_{ch}^{pp}} \sqrt{1 + \frac{2 \ln 3}{L_{NN}} \exp \left[\frac{-\eta^2}{L_{NN}(2 + L_{NN}/\ln 3)} \right]}$$

$$L = \ln(\sqrt{s}/2m) \quad \sqrt{s_{NN}} = \sqrt{s_{pp}}/3. \quad m = m_p, \frac{1}{3}m_p$$

From mid-rapidity to full-rapidity distribution

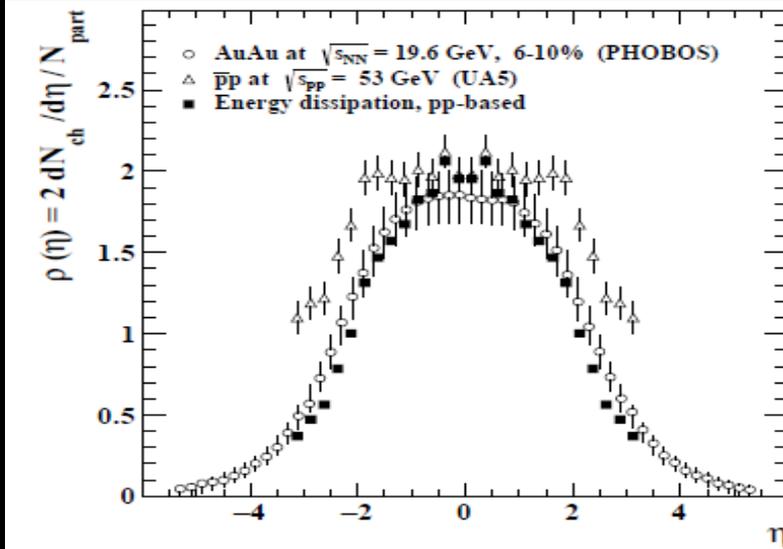


- ❖ Calculations for high-central collisions are in **very good agreement** with the measurements.

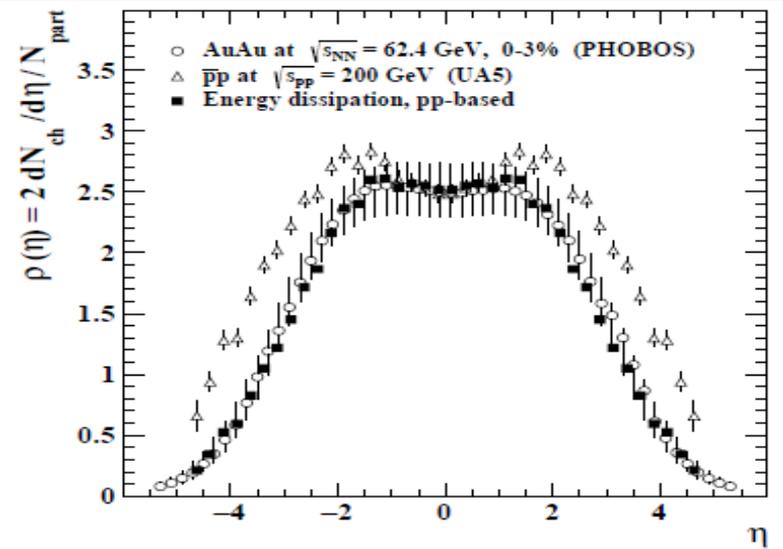
Agreement found for all available data $\sqrt{s_{NN}} = 20$ GeV to a few TeV

- ❖ At **LHC** energy, pp measurements from *three different* experiments are used and **reproduce** AA data

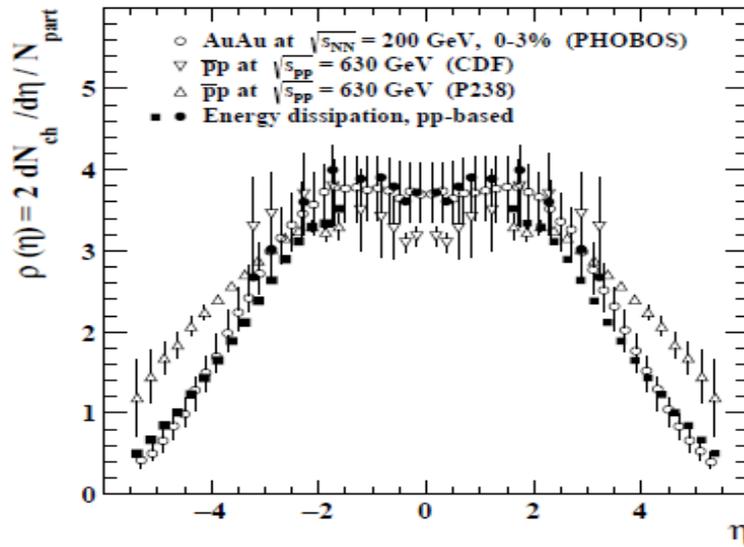
From mid-rapidity to full-rapidity distribution



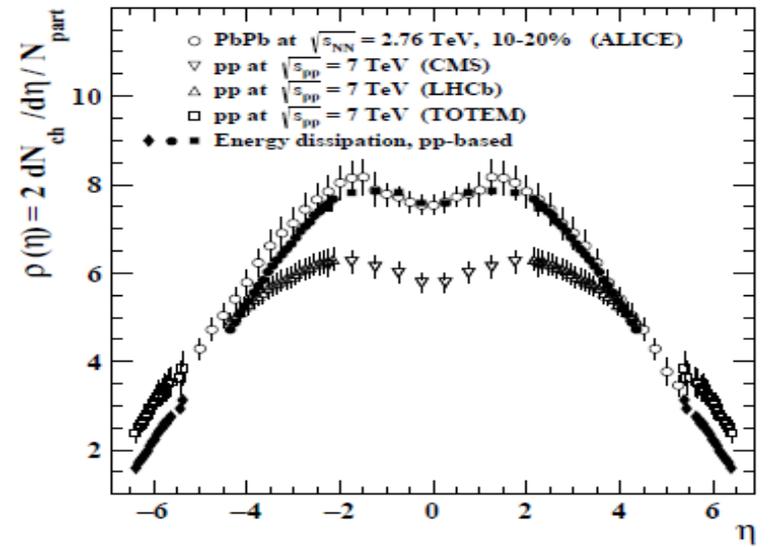
a)



b)

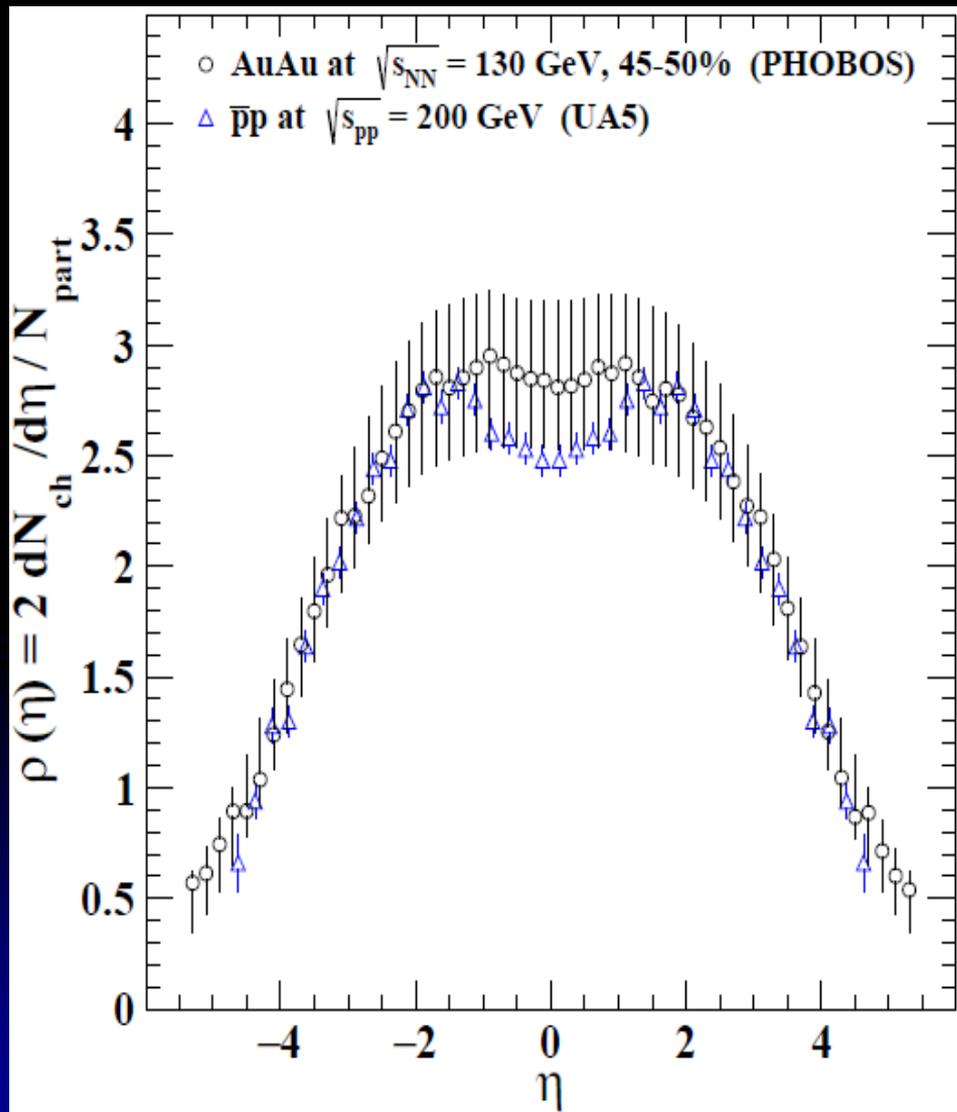


c)



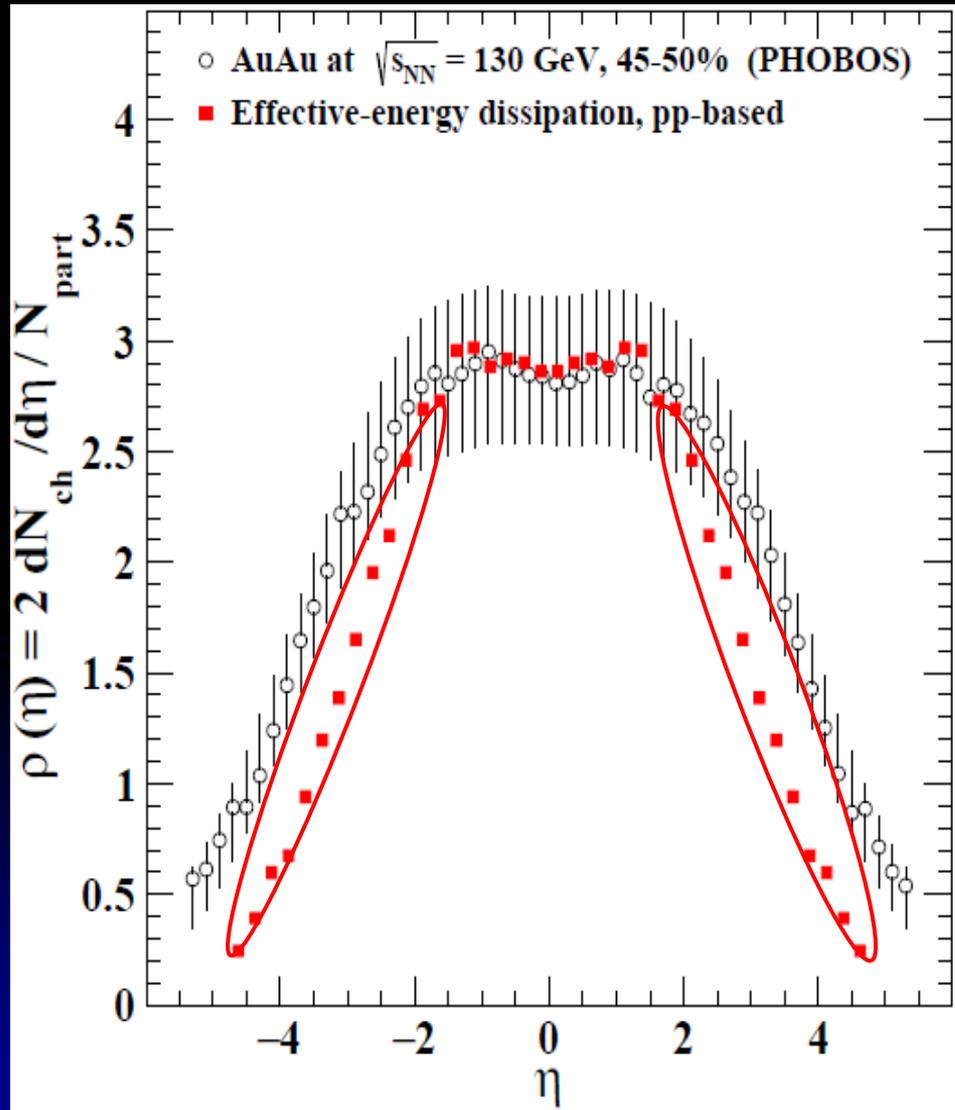
d)

Non-central collisions



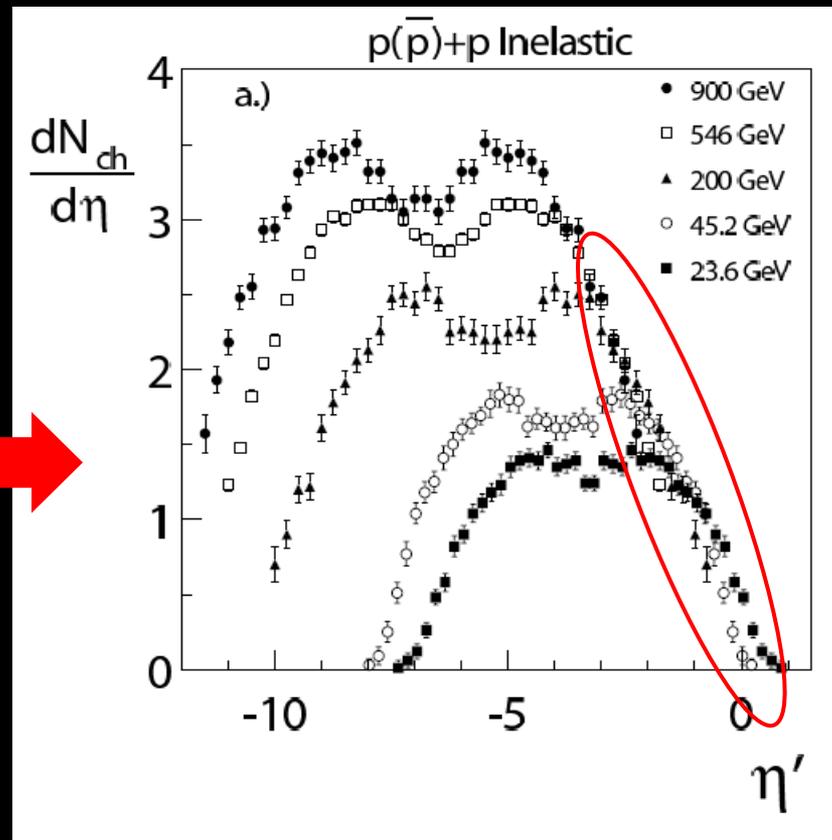
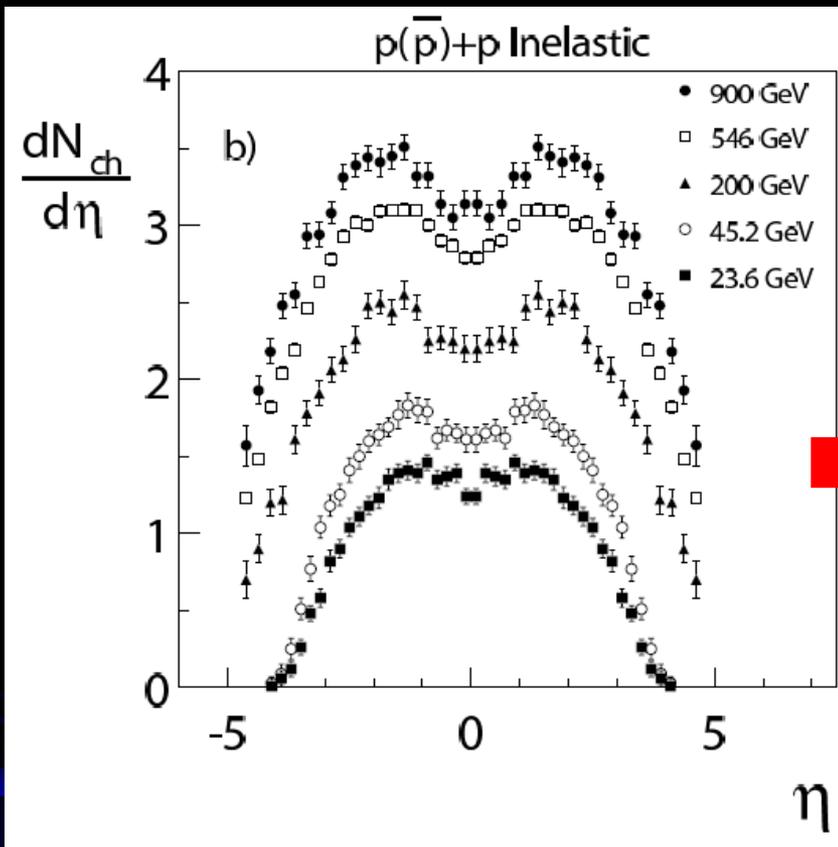
❖ pp data vs. non-central AA data at $\sqrt{s_{pp}} \approx 3\epsilon_{NN}$

Non-central collisions



- ❖ Calculations for non-central collisions agree well with the measurements in the central η region while **fall below the data outside this region**

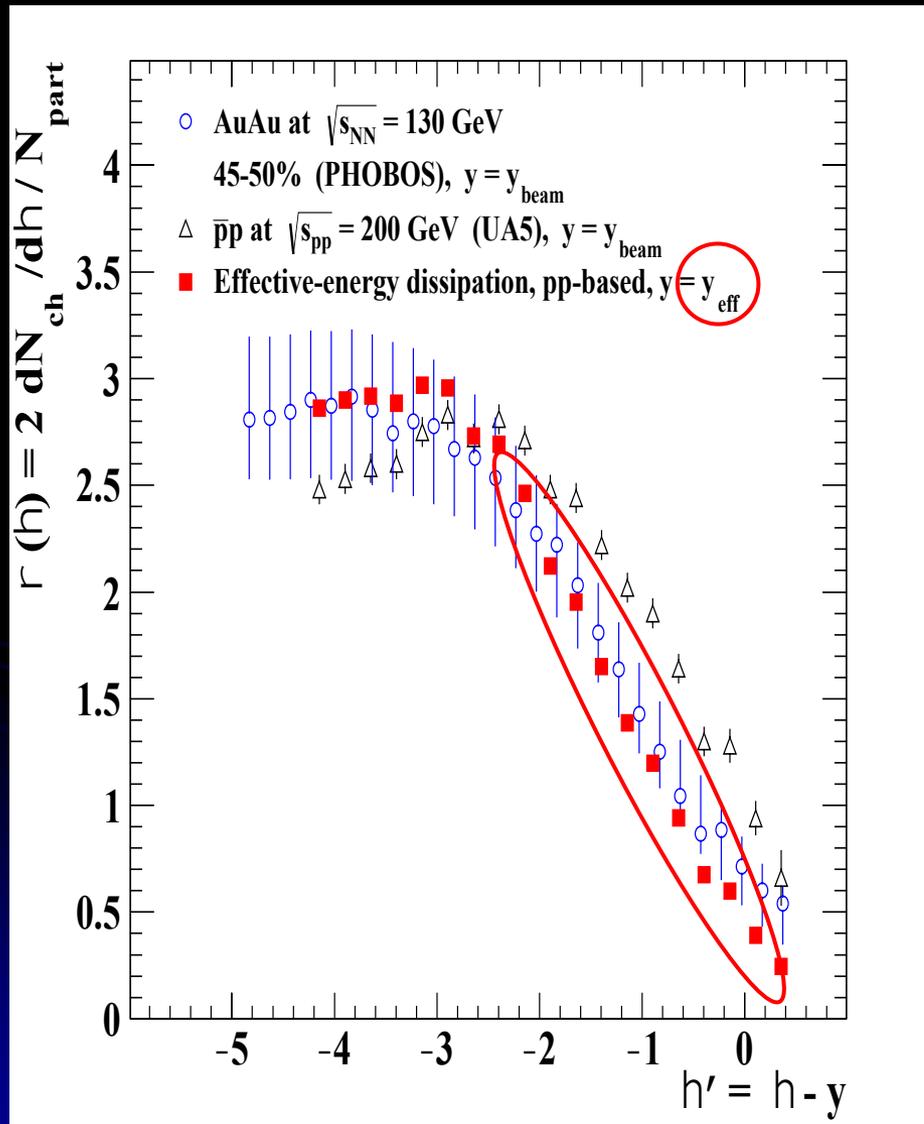
Limiting fragmentation scaling



- The *fragmentation area* of $\rho(\eta)$ is **collision-energy-independent** in the **beam (target) rest frame**, i.e. under $\rho(\eta)$ transformation to (shift by) $\eta' = \eta - y_{eff}$, where $y_{eff} = \ln(\sqrt{s_{pp}}/m_p)$
- Holds for *all* types of collisions

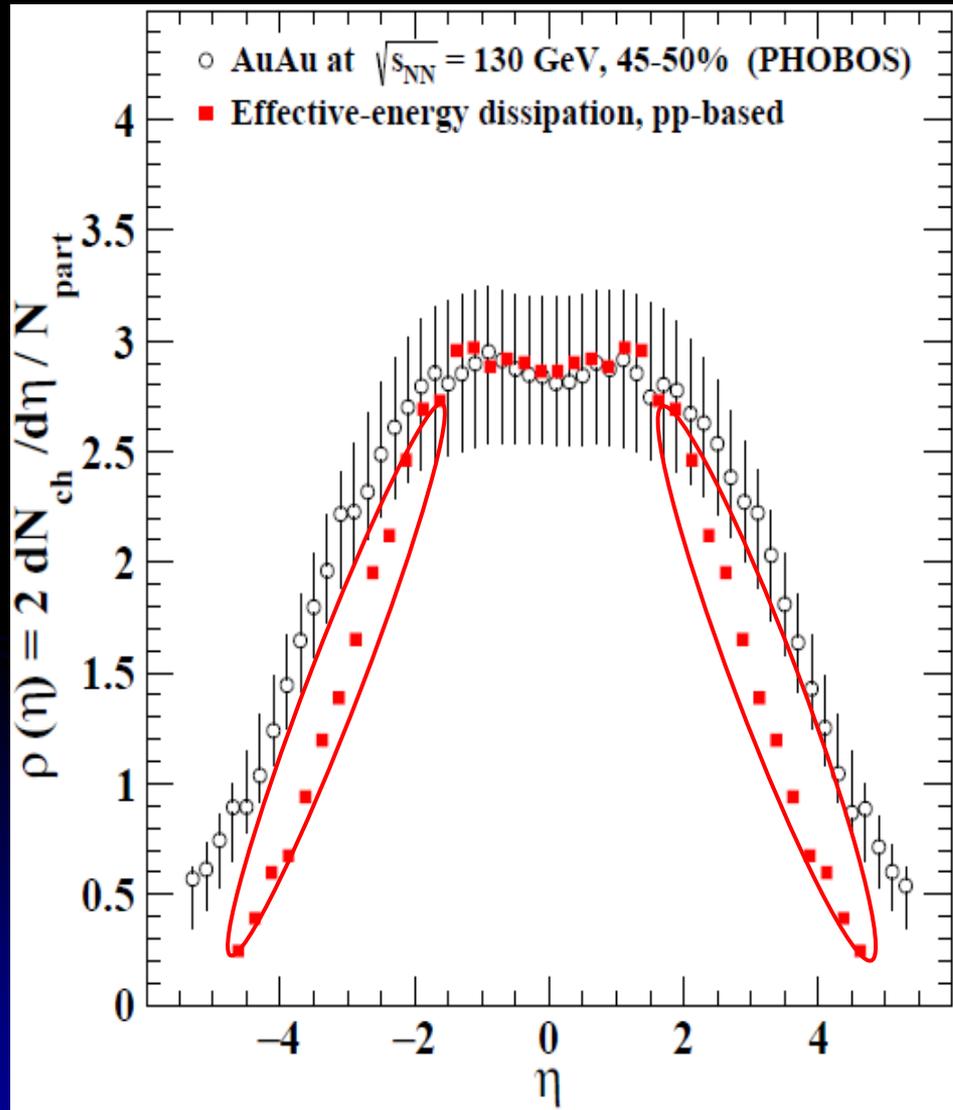
J. Benecke, T.T. Chou, C.N. Yang, E. Yen (1969)

Energy-balanced Limiting Fragmentation



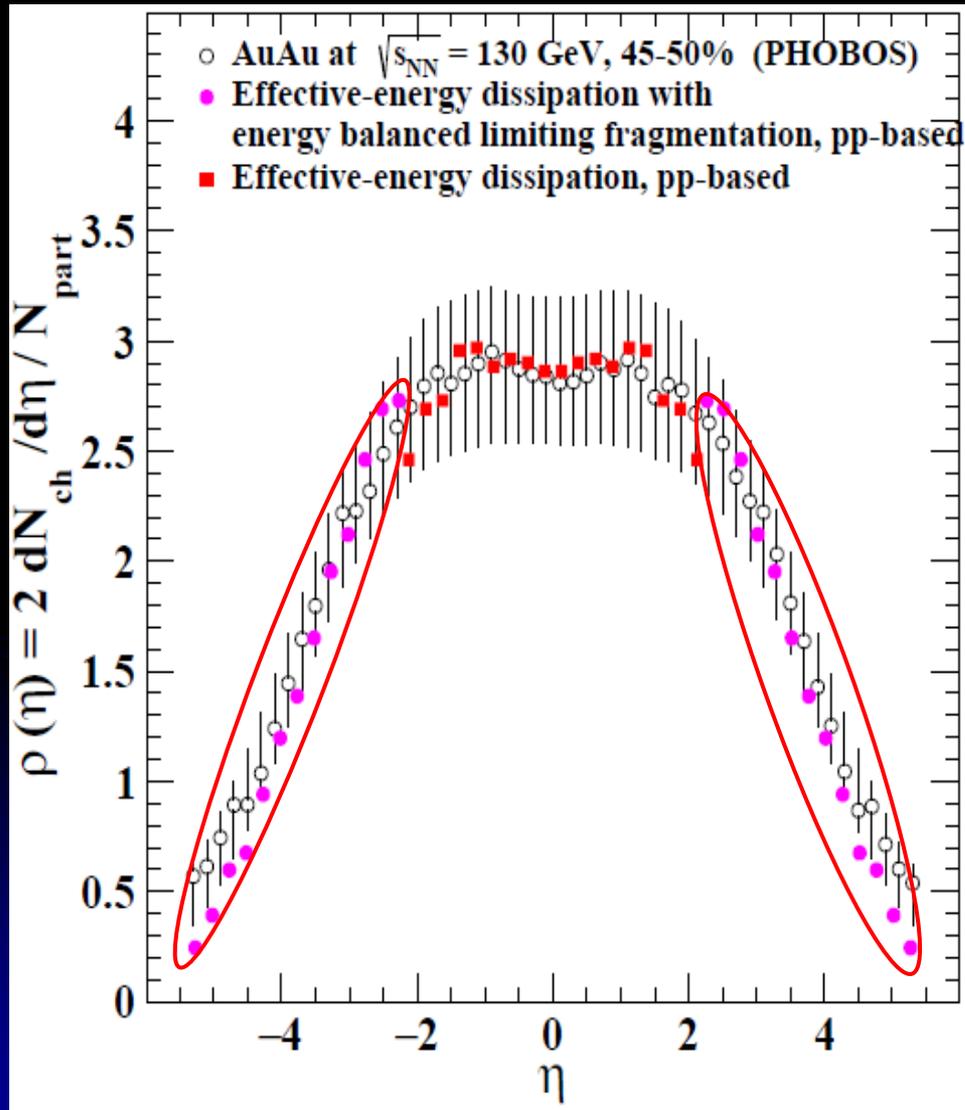
- ❖ Within the effective-energy approach, one expects the **limiting fragmentation scaling** of $\rho(\eta)$ (*fragmentation area of $\rho(\eta)$ independence of collision energy in the beam/target rest frame*) measured at $\sqrt{s_{NN}}$ to be **similar** to that of the calculated distribution but **taken at the effective energy ϵ_{NN}** , i.e. $\eta \rightarrow \eta - y_{\text{eff}}$, where $y_{\text{eff}} = \ln(\epsilon_{NN}/m_p)$

Non-central collisions



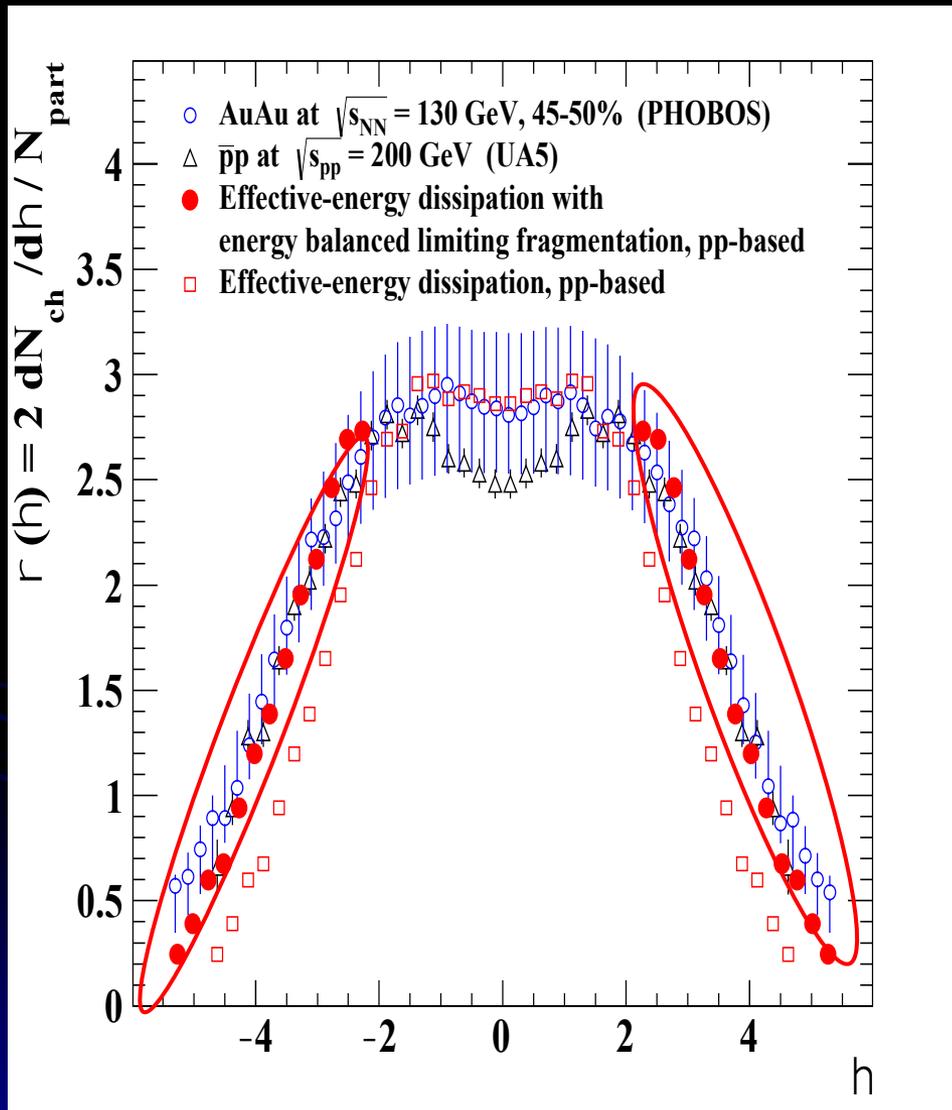
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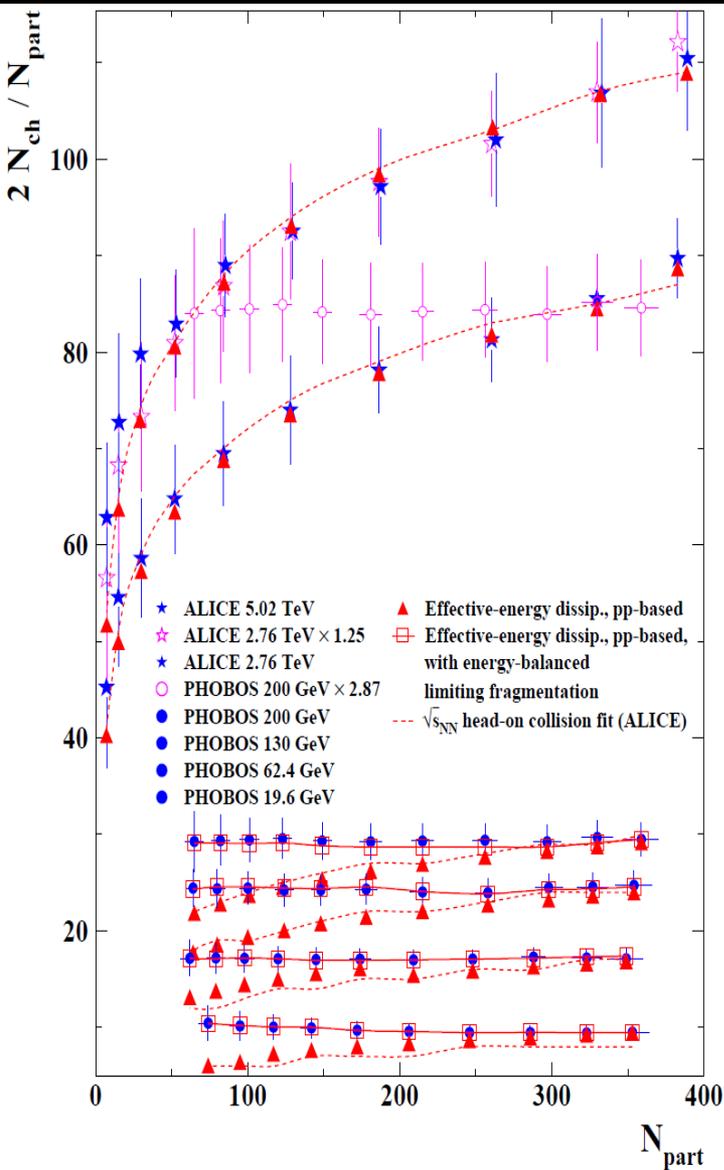
- ❖ The measured distribution $\rho(\eta)$ is shifted by the beam rapidity, y_{beam} , while the calculated distribution is shifted by $y_{\text{eff}} = \ln(\epsilon_{NN}/m_p)$ and becomes a function of $\eta' = \eta - y_{\text{eff}}$
- ❖ The newly calculated distribution $\rho(\eta)$ needs to be shifted by the difference ($y_{\text{eff}} - y_{\text{beam}}$) in the fragmentation region: $\eta \rightarrow \eta - (y_{\text{eff}} - y_{\text{beam}}) = \eta - \ln(1 - \alpha)$. This represents the “energy balanced limiting fragmentation scaling”

Energy-balanced Limiting Fragmentation



- ❖ The measured distribution $\rho(\eta)$ is shifted by the beam rapidity, y_{beam} , while the calculated distribution is shifted by $y_{\text{eff}} = \ln(\epsilon_{NN}/m_p)$ and becomes a function of $\eta' = \eta - y_{\text{eff}}$
- ❖ The newly calculated distribution $\rho(\eta)$ needs to be shifted by the difference ($y_{\text{eff}} - y_{\text{beam}}$) in the fragmentation region: $\eta \rightarrow \eta - (y_{\text{eff}} - y_{\text{beam}}) = \eta - \ln(1 - \alpha)$. This represents the “energy balanced limiting fragmentation scaling”

Energy-balanced Limiting Fragmentation



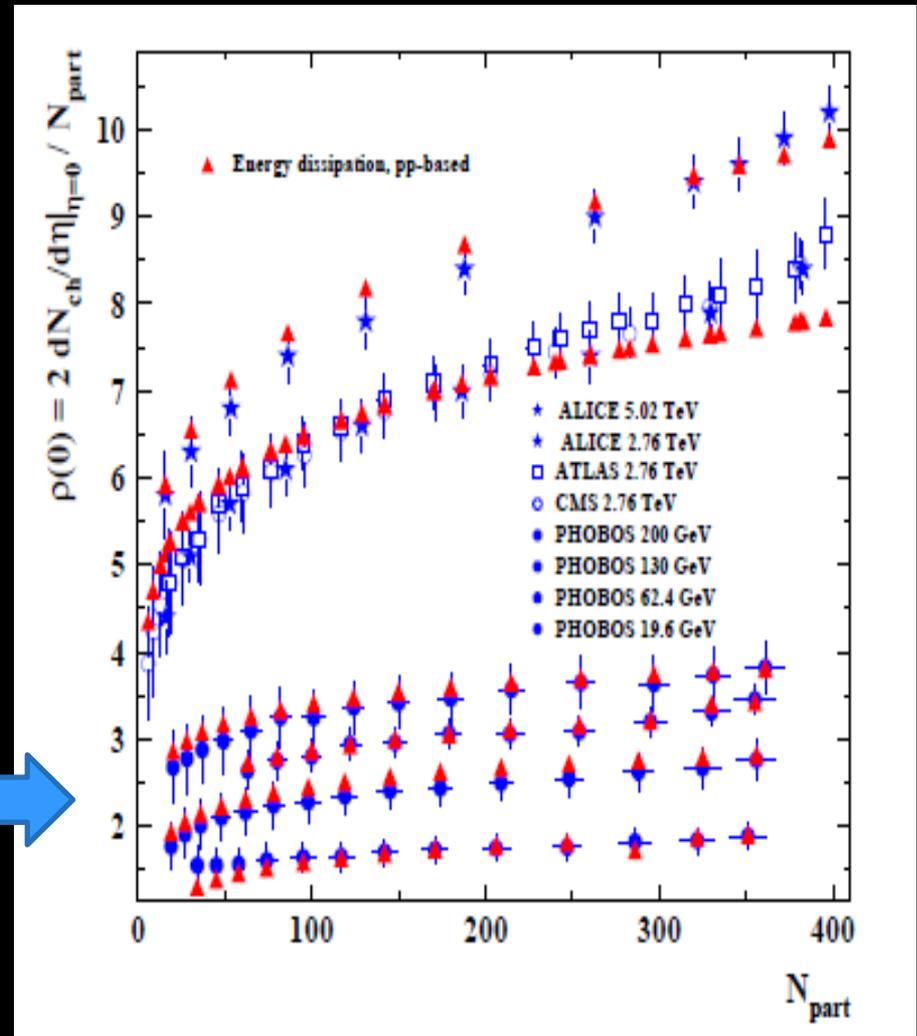
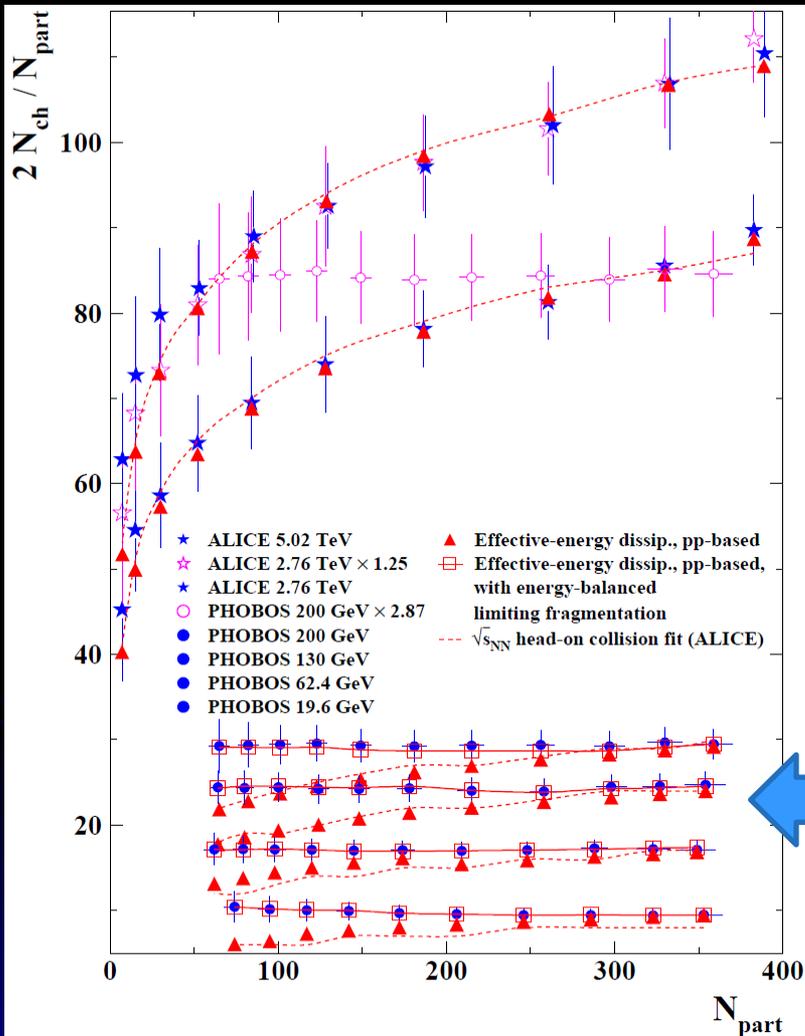
❖ The shift adds the needed energy balanced ingredient to the calculations providing the description of the measured pseudorapidity density distribution in *the full η range in non-central* heavy-ion collision

❖ With the new scaling, which adds a needed ingredient to balance *the energy of a collision and of nucleon participants*, the measured $\rho(\eta)$ distribution is reproduced for *all centralities*

❖ In order to describe the LHC mean multiplicity data, almost **no additional contribution is needed** for the participant dissipating energy calculations

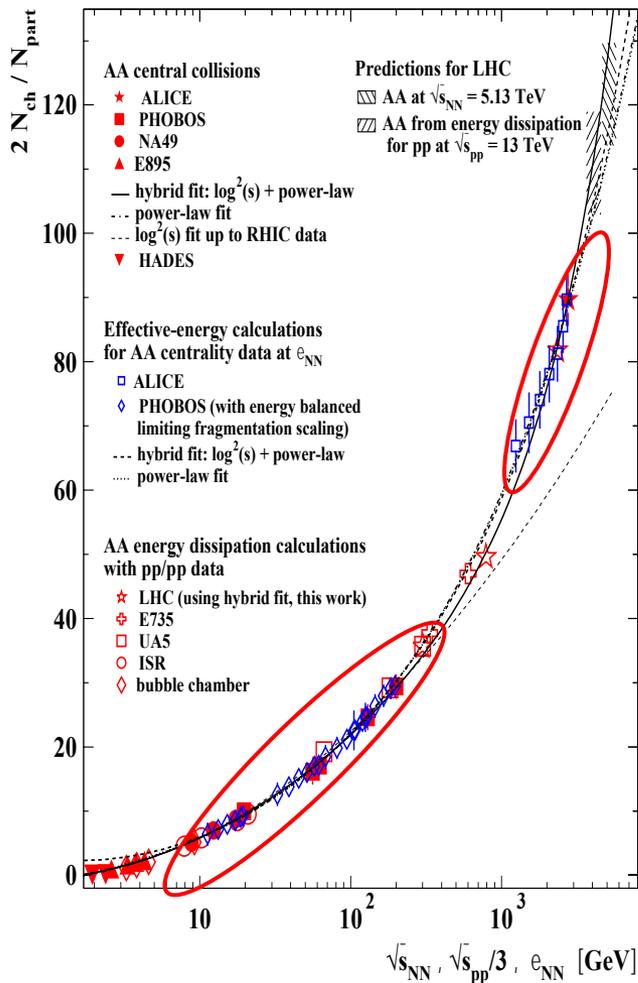
❖ The calculations, driven by the centrality-defined effective c.m. energy **well reproduce measurements from RHIC after removing energy-balanced contribution**

Energy-balanced Limiting Fragmentation



Explains the **difference** in the centrality dependence of the **multiplicity** and the **midrapidity density at RHIC** (“RHIC puzzle”), not seen at LHC

Energy-balanced Limiting Fragmentation



❖ The shift adds the needed energy-balanced ingredient to the calculations providing the description of the measured pseudorapidity density distribution in *the full η range in non-central heavy-ion collision*

❖ With the new scaling, which adds a needed ingredient to balance *the energy of a collision and of nucleon participants*, the measured $\rho(\eta)$ distribution is reproduced for **all centralities**

❖ **Effective-energy dependent centrality data are compliment to c.m. energy dependent head-on collision data**

(Intermediate) Conclusions

- ❖ The **universality** of hadroproduction process is pointed out based on the picture of the ***effective dissipating energy of participants***
- ❖ The AA measurements are **well reproduced** under the assumption of the ***effective energy driving the multiparticle production*** process and pointing to the **same energy behaviour for all types of heavy-ion collisions**, from peripheral to the most central collisions
- ❖ A **new scaling**, called **the energy-balanced limiting fragmentation scaling**, which takes into account the *balance between the collision energy and the energy shared by the participants*, is introduced
- ❖ Energy-balanced limiting fragmentation scaling provides a **solution of the RHIC “puzzle”** of the difference between the centrality independence of the mean multiplicity vs. the monotonic decrease of the midrapidity pseudorapidity density with the increase of centrality
- ❖ Under the concept of the effective energy and using the energy-balanced limiting fragmentation scaling, ***the centrality data are found to follow the head-on collisions $\sqrt{s_{NN}}$ dependence***
- ❖ A possible **transition to a new regime** at $\sqrt{s_{NN}} \sim 1$ TeV is indicated

Two-particle rapidity correlations

$$C_2(y_1, y_2) = \rho(y_1, y_2) - \rho(y_1)\rho(y_2)$$

← 2-particle rapidity correlation function

$$\rho(y) = \frac{1}{\sigma_{in}} \frac{d\sigma_{in}}{dy}, \quad \rho_2(y_1, y_2) = \frac{1}{\sigma_{in}} \frac{d^2\sigma_{in}}{dy_1 dy_2}$$

one- and two-particle densities

$$\int dy_1 dy_2 C(y_1, y_2) = D^2 - \langle n \rangle \quad (= 0 \text{ for independent emission})$$

$$K_2(y_1, y_2) = \frac{C_2(y_1, y_2)}{\rho(y_1)\rho(y_2)} = \frac{1}{\sigma_{in}} \frac{d^2\sigma_{in}}{dy_1 dy_2} / \frac{1}{\sigma_{in}^2} \frac{d\sigma_{in}}{dy_1} \frac{d\sigma_{in}}{dy_2} - 1$$

$$F_2 = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} = \frac{D^2}{\langle n \rangle^2} - \frac{1}{\langle n \rangle} + 1, \quad D^2 = \langle n^2 \rangle - \langle n \rangle^2$$

Scaled factorial moment

Generalization to *higher-orders* is straightforward:

I.M.Dremin and W.J.Gary, Phys. Rept.349 (2001) 301

E.A. De Wolf, I.M. Dremin, W. Kittel, Phys. Rep. 270 (1996) 1

2-particle azimuthal and (pseudo)rapidity correlations

$$R(\Delta\eta, \Delta\phi) = \frac{S(\Delta\eta, \Delta\phi)}{B(\Delta\eta, \Delta\phi)}$$

$$\Delta\eta = \eta_1 - \eta_2 \quad ; \quad \Delta\phi = \phi_1 - \phi_2$$

$S(\Delta\eta, \Delta\phi)$: particle pair distribution from **the same** event

$B(\Delta\eta, \Delta\phi)$: particle pair distribution from **different** events

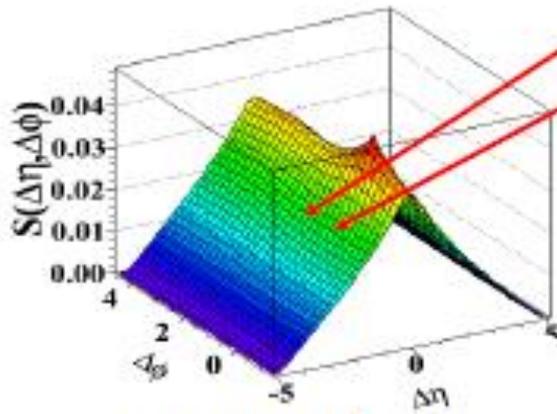
Complex structure of 2-dimensional plot in pp, pA and AA collisions

seen by ALICE, ATLAS, and CMS at the LHC

2-particle azimuthal and (pseudo)rapidity correlations

Signal distribution:

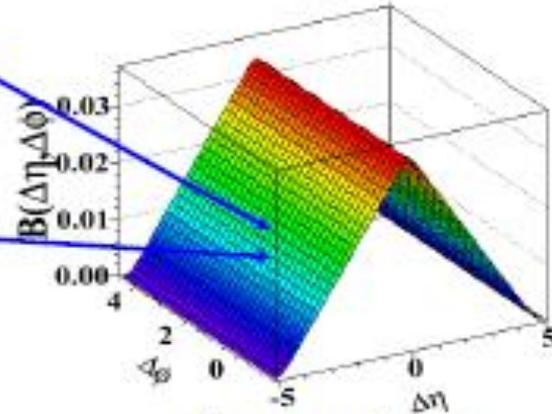
$$S(\Delta\eta, \Delta\phi) = \frac{1}{N_{\text{trig}}} \frac{d^2 N^{\text{assoc}}}{d\Delta\eta d\Delta\phi}$$



same event pairs

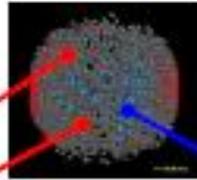
Background distribution:

$$B(\Delta\eta, \Delta\phi) = \frac{1}{N_{\text{trig}}} \frac{d^2 N^{\text{mix}}}{d\Delta\eta d\Delta\phi}$$

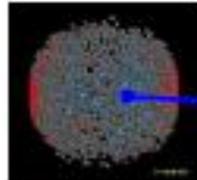


mixed event pairs

Event 1

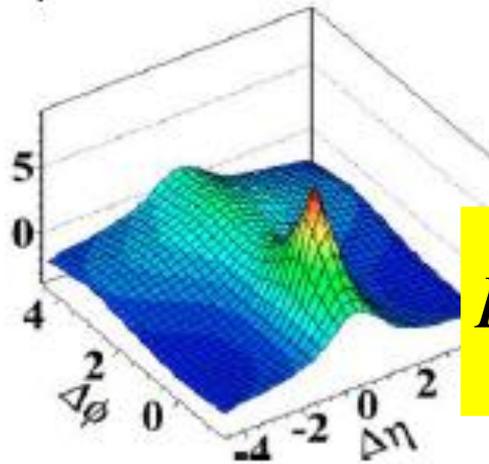


Event 2



$$\Delta\eta = \eta^{\text{assoc}} - \eta^{\text{trig}}$$

$$\Delta\phi = \phi^{\text{assoc}} - \phi^{\text{trig}}$$



$$R(\Delta\eta, \Delta\phi) = \frac{S(\Delta\eta, \Delta\phi)}{B(\Delta\eta, \Delta\phi)}$$

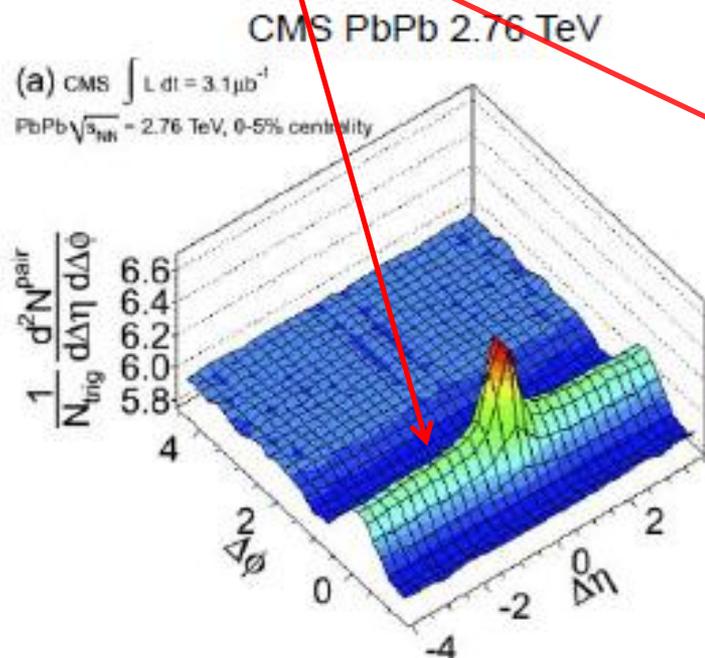
Divide signal by background

$S(\Delta\eta, \Delta\phi)$: particle pair **signal** distribution from **the same** event

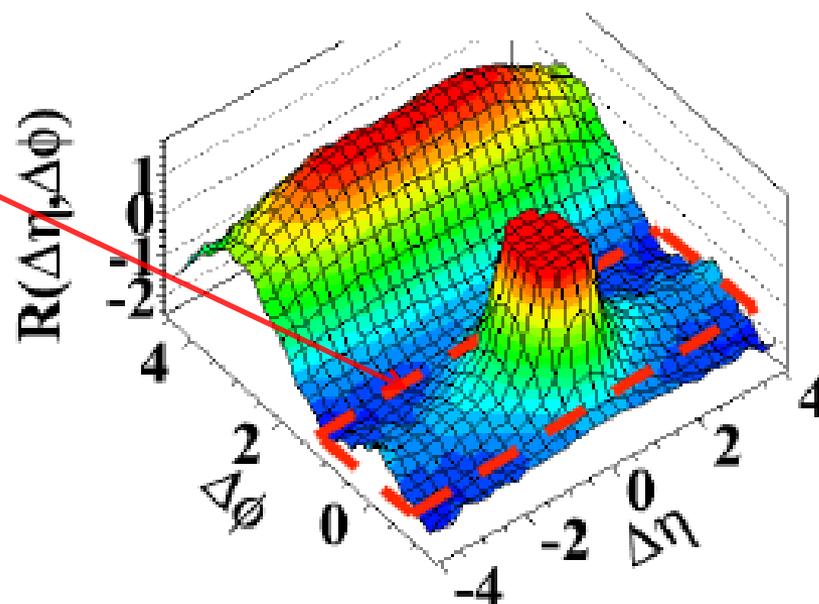
$B(\Delta\eta, \Delta\phi)$: particle pair **background** distribution from **different** events

Ridge structure

“Ridge” structure extending over $\Delta\eta$ at $\Delta\phi = 0$



High multiplicity pp ($N > 110$)

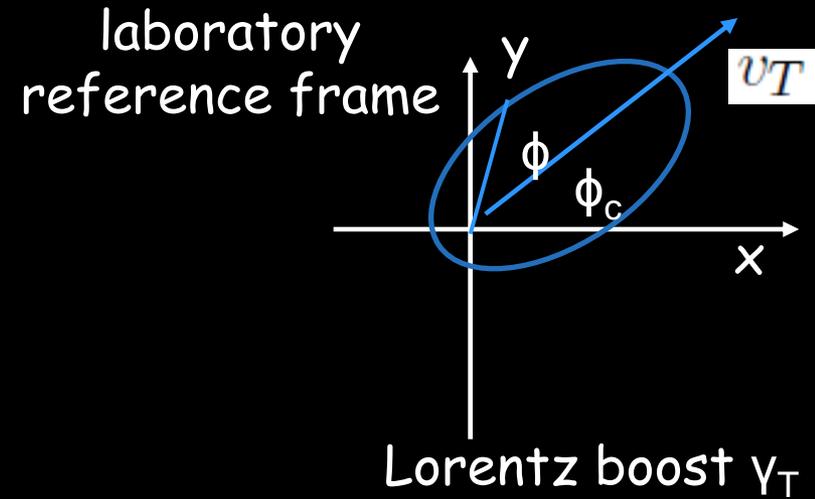
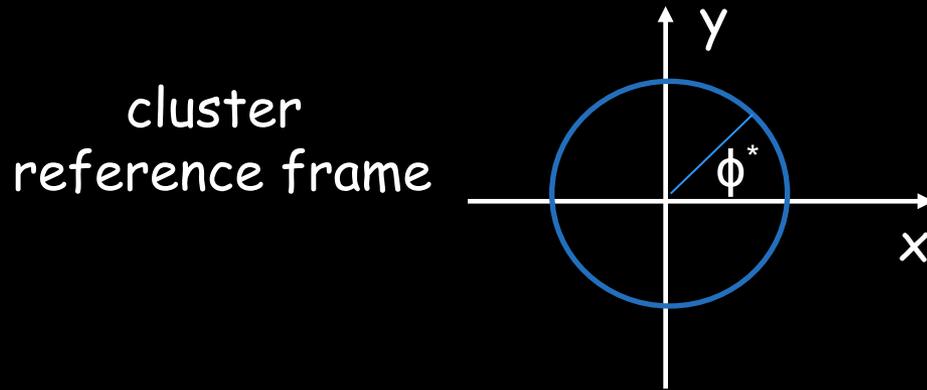


Intermediate p_T : 1-3 GeV/c

CMS Collab. J. High Energy Phys. 1009 (2010) 091

- **Expected** in heavy-ion collisions (*hydro, high density*)
- **Unexpected** in pp (and pA) interactions
- **Similarity in pp and heavy-ion collisions!**
- **No explanation** so far, while many models proposed

Correlated-cluster model



- Azimuthal dependence

1) *Isotropic* cluster emission

2) *Isotropic* particle emission in clusters: $w^*(\phi^*) = \text{constant}$

- **Gaussians** for *cluster and particle distributions* inside clusters

$$\rho^{(c)}(y_c, \phi_c) \sim \exp\left[-\frac{y_c^2}{2\delta_{cy}^2}\right], \rho^{(1)}(y, \phi; y_c, \phi_c) \sim \exp\left[-\frac{(y - y_c)^2}{2\delta_y^2}\right] \exp\left[-\frac{(\phi - \phi_c)^2}{2\delta_\phi^2}\right]$$

The cluster correlation length $\delta_{cy}^2 \gg \delta_y^2 \lesssim 1$ the cluster decay "width",
and the cluster azimuthal decay "width"

$$\delta_\phi \sim \frac{1}{v_T \gamma_T}$$

Correlated-cluster model

$$\rho_2(y_1, \phi_1, y_2, \phi_2) = \langle N_c \rangle \bar{\rho}^{(1)2} E_s^{\text{SR}}(y_1, \phi_1, y_2, \phi_2) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)2} E_s^{\text{LR}}(y_1, \phi_1, y_2, \phi_2)$$

$\bar{\rho}^{(1)}$ average particle density for single-cluster decay

$\langle N_c \rangle$ average number of clusters

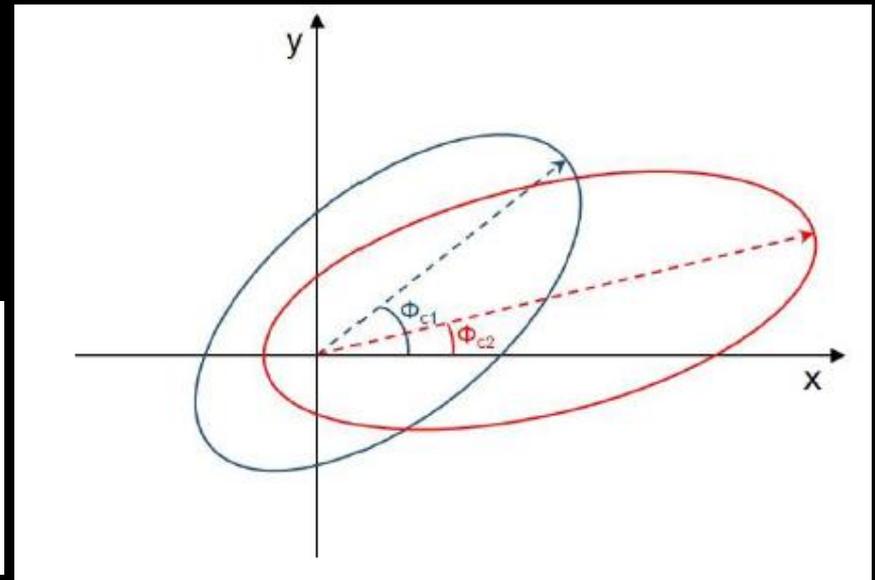
$$E_b(y_1, \phi_1, y_2, \phi_2) = E_b^L(y_1, y_2) \cdot E_b^T(\phi_1, \phi_2)$$

$$E_s(y_1, \phi_1, y_2, \phi_2) = E_s^L(y_1, y_2) \cdot E_s^T(\phi_1, \phi_2)$$

Factorization

Cluster correlation function considers (partial) longitudinal momentum conservation and implements cluster azimuthal correlations (transverse plane)

$$\rho_2^{(c)}(y_{c1}, \phi_{c1}, y_{c2}, \phi_{c2}) \sim \exp\left[-\frac{(y_{c1} + y_{c2})^2}{2\delta_{cy}^2}\right] \exp\left[-\frac{(\phi_{c1} - \phi_{c2})^2}{2\delta_{c\phi}^2}\right]$$



Correlated-cluster model

$$\rho_2(y_1, \phi_1, y_2, \phi_2) = \langle N_c \rangle \bar{\rho}^{(1)2} E_s^{\text{SR}}(y_1, \phi_1, y_2, \phi_2) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)2} E_s^{\text{LR}}(y_1, \phi_1, y_2, \phi_2)$$

Short-range
Long-range

Factorization

$$E_b(y_1, \phi_1, y_2, \phi_2) = E_b^L(y_1, y_2) \cdot E_b^T(\phi_1, \phi_2)$$

$$E_s(y_1, \phi_1, y_2, \phi_2) = E_s^L(y_1, y_2) \cdot E_s^T(\phi_1, \phi_2)$$

Upon the integration over cluster rapidity and azimuth:

- for two particles from *the same* cluster (SR),

$$E_s^L(y_1, y_2) \sim \exp \left[-\frac{\delta_{cy}^2 (y_1 - y_2)^2}{2\delta_y^2 (\delta_y^2 + 2\delta_{cy}^2)} \right] \exp \left[-\frac{(y_1^2 + y_2^2)}{2(\delta_y^2 + 2\delta_{cy}^2)} \right]$$

$$E_s^T(\phi_1, \phi_2) \sim \exp \left[-\frac{(\phi_1 - \phi_2)^2}{4\delta_\phi^2} \right]$$

- for two particles from *two different* clusters (LR)

$$E_s^L(y_1, y_2) \sim \exp \left[-\frac{(y_1 + y_2)^2}{2(2\delta_y^2 + \delta_{cy}^2)} \right]$$

$$E_s^T(\phi_1, \phi_2) \sim \exp \left[-\frac{(\phi_1 - \phi_2)^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)} \right]$$

- for background function

$$E_b^L(y_1, y_2) \sim \exp \left[-\frac{(y_1^2 + y_2^2)}{2(\delta_y^2 + \delta_{cy}^2)} \right]$$

$$E_b^T(\phi_1, \phi_2) \sim \text{const.} \quad (\text{isotropic decay})$$

Correlated-cluster model

Signal (s):

$$\rho_2(y_1, \phi_1, y_2, \phi_2) = \langle N_c \rangle \bar{\rho}^{(1)2} E_s^{\text{SR}}(y_1, \phi_1, y_2, \phi_2) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)2} E_s^{\text{LR}}(y_1, \phi_1, y_2, \phi_2)$$

Background (b):

$$\rho_{\text{mixed}}(y_1, \phi_1, y_2, \phi_2) = \langle N_c \rangle^2 \bar{\rho}^{(1)2} E_b(y_1, \phi_1, y_2, \phi_2)$$

Upon the integration over the particle pairs rapidity and azimuthal angles, given $\Delta y = y_1 - y_2$ and $\Delta\phi = \phi_1 - \phi_2$:

$$R(\Delta y, \Delta\phi) = \frac{s(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)}$$

$$= \frac{s^{\text{SR}}(\Delta y, \Delta\phi) + s^{\text{LR}}(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)} = 1 + \frac{h^{\text{SR}}(\Delta y, \Delta\phi)}{\langle N_c \rangle} + \frac{\text{near-side ridge}}{\langle N_c \rangle^2} h^{\text{LR}}(\Delta\phi)$$

Correlated-cluster model

$$R(\Delta y, \Delta\phi) = \frac{s(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)}$$

$$= \frac{s^{\text{SR}}(\Delta y, \Delta\phi) + s^{\text{LR}}(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)} = 1 + \frac{h^{\text{SR}}(\Delta y, \Delta\phi)}{\langle N_c \rangle} + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^2} h^{\text{LR}}(\Delta\phi)$$

Short-range contribution:

$$h^{\text{SR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{SR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} = \exp \left[-\frac{\delta_{cy}^2}{4\delta_y^2(\delta_y^2 + \delta_{cy}^2)} (\Delta y)^2 \right] \exp \left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2} \right]$$

Long-range contribution:

$$h^{\text{LR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{LR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} \simeq \exp \left[\frac{(\Delta y)^2}{4(\delta_y^2 + \delta_{cy}^2)} \right] \exp \left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)} \right]$$

near-side ridge

Correlated-cluster model

$$R(\Delta y, \Delta\phi) = \frac{s(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)}$$

$$= \frac{s^{\text{SR}}(\Delta y, \Delta\phi) + s^{\text{LR}}(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)} = 1 + \frac{h^{\text{SR}}(\Delta y, \Delta\phi)}{\langle N_c \rangle} + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^2} h^{\text{LR}}(\Delta\phi)$$

Reduces at large $\langle N_c \rangle$

Short-range contribution:

$$h^{\text{SR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{SR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} = \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right] \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right]$$

Long-range contribution:

$$h^{\text{LR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{LR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} \simeq \exp\left[\frac{(\Delta y)^2}{4(\delta_y^2 + \delta_{cy}^2)}\right] \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)}\right]$$

near-side ridge

For $\delta_{cy}^2 \gg \delta_y^2$

≈ 0.1 radians ($p_T \approx 1$ GeV)

Correlated-cluster model

$$R(\Delta y, \Delta\phi) = \frac{s(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)}$$

$$= \frac{s^{\text{SR}}(\Delta y, \Delta\phi) + s^{\text{LR}}(\Delta y, \Delta\phi)}{b(\Delta y, \Delta\phi)} = 1 + \frac{h^{\text{SR}}(\Delta y, \Delta\phi)}{\langle N_c \rangle} + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^2} h^{\text{LR}}(\Delta\phi)$$

Reduces at large $\times N_c$

Short-range contribution:

$$h^{\text{SR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{SR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} = \exp\left[-\frac{(\Delta y)^2}{4\delta_y^2}\right] \exp\left[-\frac{(\Delta\phi)^2}{4\delta_\phi^2}\right]$$

Long-range contribution:

$$h^{\text{LR}}(\Delta y, \Delta\phi) = \frac{e_s^{\text{LR}}(\Delta y, \Delta\phi)}{e_b(\Delta y, \Delta\phi)} \simeq \exp\left[\frac{(\Delta y)^2}{4(\delta_y^2 + \delta_{cy}^2)}\right] \exp\left[-\frac{(\Delta\phi)^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)}\right]$$

near-side ridge

For $\delta_{cy}^2 \gg \delta_y^2$

≈ 0.1 radians ($p_T \approx 1$ GeV)

MAIN RESULT: The ridge effect of 2-particle correlations at small $\Delta\phi$ over a wide (pseudo)rapidity range is naturally explained within a model of clusters correlated in the transverse plane

3-particle correlations

$$C_3(1, 2, 3) = \rho_3(1, 2, 3) + 2\rho(1)\rho(2)\rho(3) - \rho_2(1, 2)\rho(3) - \rho_2(2, 3)\rho(1) - \rho_2(1, 3)\rho(2)$$

$$\rho_3(y_1, y_2, y_3, \phi_1, \phi_2, \phi_3) = \frac{1}{\sigma_{\text{in}}} \frac{d^6\sigma}{dy_1 dy_2 dy_3 d\phi_1 d\phi_2 d\phi_3}$$

**3-particle
density**

Correlation function ratio:

$$c_3(\vec{\Delta y}, \vec{\Delta\phi}) = \frac{s_3 + 2b_3 - s_{123} - s_{231} - s_{132}}{b_3},$$

$\vec{\Delta y}, \vec{\Delta\phi}$ for $\Delta y_{ij}, \Delta\phi_{ij}$

$\vec{y} = (y_1, y_2, y_3), \vec{\phi} = (\phi_1, \phi_2, \phi_3)$

Signal (s):

$$s_3(\vec{\Delta y}, \vec{\Delta\phi}) = \int d\vec{y} d\vec{\phi} \vec{\delta}(\Delta y) \vec{\delta}(\Delta\phi) \rho_3(\vec{y}, \vec{\phi})$$

$$d\vec{y} d\vec{\phi} = dy_1 dy_2 dy_3 d\phi_1 d\phi_2 d\phi_3$$

$$\vec{\delta}(\Delta y) = \delta(\Delta y_{12} - y_1 + y_2) \delta(\Delta y_{13} - y_1 + y_3)$$

Background (b):

$$b_3(\vec{\Delta y}, \vec{\Delta\phi}) = \int d\vec{y} d\vec{\phi} \vec{\delta}(\Delta y) \vec{\delta}(\Delta\phi) \rho(y_1, \phi_1) \rho(y_2, \phi_2) \rho(y_3, \phi_3)$$

$$s_{123}(\vec{\Delta y}, \vec{\Delta\phi}) = \int d\vec{y} d\vec{\phi} \vec{\delta}(\Delta y) \vec{\delta}(\Delta\phi) \rho(y_1, \phi_1) \rho_2(y_2, \phi_2, y_3, \phi_3)$$

+ permutations

Correlated-cluster model: 3 clusters

$$\rho_3(\vec{y}, \vec{\phi}) = \langle N_c \rangle \bar{\rho}^{(1)3} E_s^{(1)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)3} E_s^{(2)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1)(N_c - 2) \rangle \bar{\rho}^{(1)3} E_s^{(3)}(\vec{y}, \vec{\phi})$$

$\bar{\rho}^{(1)}$ average particle density for single-cluster decay

$\langle N_c \rangle$ average number of clusters

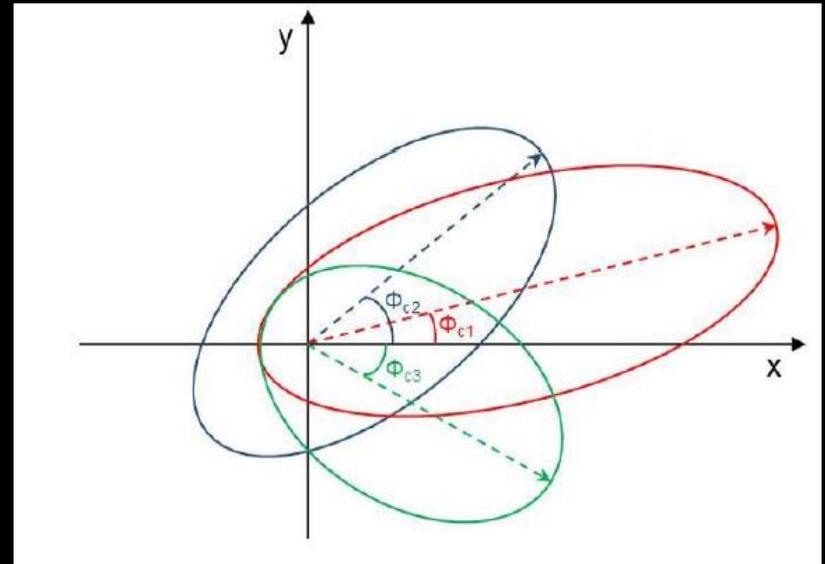
Factorization

$$E_b(\vec{y}, \vec{\phi}) = E_b^L(\vec{y}) \cdot E_b^T(\vec{\phi})$$

$$E_s(\vec{y}, \vec{\phi}) = E_s^L(\vec{y}) \cdot E_s^T(\vec{\phi})$$

Cluster correlation function considers (partial) longitudinal momentum conservation and implements cluster azimuthal correlations (transverse plane)

$$\rho_3^{(c)}(\vec{y}_c, \vec{\phi}_c) \sim \exp \left[-\frac{(y_{c1} + y_{c2} + y_{c3})^2}{2\delta_{cy}^2} \right] \times \exp \left[-\frac{(\phi_{c1} - \phi_{c2})^2 + (\phi_{c1} - \phi_{c3})^2 + (\phi_{c2} - \phi_{c3})^2}{2\delta_{c\phi}^2} \right]$$



Correlated-cluster model: 3 clusters

$$\rho_3(\vec{y}, \vec{\phi}) = \langle N_c \rangle \bar{\rho}^{(1)3} E_s^{(1)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)3} E_s^{(2)}(\vec{y}, \vec{\phi}) \\ + \langle N_c(N_c - 1)(N_c - 2) \rangle \bar{\rho}^{(1)3} E_s^{(3)}(\vec{y}, \vec{\phi})$$

Upon the integration over cluster rapidities and azimuths, particle pair rapidities and azimuths:

- for three particles from *the same* cluster (1),

$$E_s^{(1)}(\vec{\Delta y}) \sim \exp \left[-\frac{(\Delta y_{12})^2 + (\Delta y_{13})^2 + (\Delta y_{23})^2}{6\delta_y^2} \right]$$

$$E_s^{(1)}(\vec{\Delta \phi}) \sim \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 + (\Delta \phi_{23})^2}{6\delta_\phi^2} \right]$$

- for three particles from *two different* clusters (2)

$$E_s^{(2)}(\vec{y}) \sim \exp \left[-\frac{(\Delta y_{13})^2}{4\delta_y^2} \right] + \exp \left[-\frac{(\Delta y_{12})^2}{4\delta_y^2} \right] + \exp \left[-\frac{(\Delta y_{23})^2}{4\delta_y^2} \right]$$

$$E_s^{(2)}(\vec{\Delta \phi}) \sim \exp \left[-\frac{\delta_{c\phi}^2 (\Delta \phi_{23})^2}{2\delta_\phi^2 (3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right] \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 + (\Delta \phi_{23})^2}{2(3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right]$$

$$\exp \left[-\frac{\delta_{c\phi}^2 (\Delta \phi_{13})^2}{2\delta_\phi^2 (3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right] \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 + (\Delta \phi_{23})^2}{2(3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right],$$

$$\exp \left[-\frac{\delta_{c\phi}^2 (\Delta \phi_{12})^2}{2\delta_\phi^2 (3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right] \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 + (\Delta \phi_{23})^2}{2(3\delta_\phi^2 + 2\delta_{c\phi}^2)} \right]$$

Correlated-cluster model: 3 clusters

$$\rho_3(\vec{y}, \vec{\phi}) = \langle N_c \rangle \bar{\rho}^{(1)3} E_s^{(1)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)3} E_s^{(2)}(\vec{y}, \vec{\phi}) \\ + \langle N_c(N_c - 1)(N_c - 2) \rangle \bar{\rho}^{(1)3} E_s^{(3)}(\vec{y}, \vec{\phi})$$

Upon the integration over cluster rapidities and azimuths, particle pair rapidities and azimuths:

- for three particles from *three correlated clusters* (3)

$$E_s^{(3)}(\vec{\Delta}y) \sim \text{const.}$$

$$E_s^{(3)}(\vec{\Delta}\phi) \sim \exp \left[-\frac{(\Delta\phi_{12})^2 + (\Delta\phi_{13})^2 + (\Delta\phi_{23})^2}{2(3\delta_\phi^2 + \delta_{c\phi}^2)} \right]$$

- for three particles from *two (out of three) correlated clusters* (3)

$$E_s^{(3)}(\vec{\Delta}y) \sim \text{const.}$$

$$E_s^{(3)}(\vec{\Delta}\phi) \sim \exp \left[-\frac{(\Delta\phi_{12})^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)} \right] + \exp \left[-\frac{(\Delta\phi_{13})^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)} \right] + \exp \left[-\frac{(\Delta\phi_{23})^2}{2(2\delta_\phi^2 + \delta_{c\phi}^2)} \right]$$

- for three particles from *three independently emitted clusters* (3)

$$E_s^{(3)}(\vec{\Delta}\phi) \sim \text{const.}$$

- for background function

$$E_b^{(3)}(\vec{\Delta}y) \sim \exp \left[-\frac{(\Delta y_{12})^2 + (\Delta y_{13})^2 + (\Delta y_{23})^2}{6(\delta_y^2 + \delta_{cy}^2)} \right]$$

$$E_b^T(\phi_1, \phi_2, \phi_3) \sim \text{const.}$$

(isotropic decay)

Correlated-cluster model: 3 clusters

Signal (s):

$$\rho_3(\vec{y}, \vec{\phi}) = \langle N_c \rangle \bar{\rho}^{(1)3} E_s^{(1)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1) \rangle \bar{\rho}^{(1)3} E_s^{(2)}(\vec{y}, \vec{\phi}) + \langle N_c(N_c - 1)(N_c - 2) \rangle \bar{\rho}^{(1)3} E_s^{(3)}(\vec{y}, \vec{\phi})$$

Background (b):

$$\rho_{\text{mixed}}(\vec{y}, \vec{\phi}) = \rho(y_1, \phi_1)\rho(y_2, \phi_2)\rho(y_3, \phi_3) = \langle N_c \rangle^3 \bar{\rho}^{(1)} E_b(\vec{y}, \vec{\phi})$$

Three-particle correlation function:

$$c_3(\Delta\vec{y}, \Delta\vec{\phi}) = \frac{s_3^{(1)}(\Delta\vec{y}, \Delta\vec{\phi}) + s_3^{(2)}(\Delta\vec{y}, \Delta\vec{\phi}) + s_3^{(3)}(\Delta\vec{y}, \Delta\vec{\phi})}{b_3(\Delta\vec{y}, \Delta\vec{\phi})}$$

ridge effect

$$= \frac{1}{\langle N_c \rangle^2} h^{(1)}(\Delta\vec{y}, \Delta\vec{\phi}) + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^3} h^{(2)}(\Delta\vec{y}, \Delta\vec{\phi}) + \frac{\langle N_c(N_c - 1)(N_c - 2) \rangle}{\langle N_c \rangle^3} h^{(3)}(\Delta\vec{y}, \Delta\vec{\phi})$$

Correlated-cluster model: 3 clusters

$$c_3(\vec{\Delta}y, \vec{\Delta}\phi) = \frac{s_3^{(1)}(\vec{\Delta}y, \vec{\Delta}\phi) + s_3^{(2)}(\vec{\Delta}y, \vec{\Delta}\phi) + s_3^{(3)}(\vec{\Delta}y, \vec{\Delta}\phi)}{b_3(\vec{\Delta}y, \vec{\Delta}\phi)}$$

$$= \frac{1}{\langle N_c \rangle^2} h^{(1)}(\vec{\Delta}y, \vec{\Delta}\phi) + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^3} h^{(2)}(\vec{\Delta}y, \vec{\Delta}\phi) + \frac{\langle N_c(N_c - 1)(N_c - 2) \rangle}{\langle N_c \rangle^3} h^{(3)}(\vec{\Delta}y, \vec{\Delta}\phi)$$

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Reducing at large $\langle N_c \rangle$ ridge effect

Three-particle three-cluster contribution

for $\delta_{cy}^2 \gg \delta_y^2$ and $\delta_{c\bar{\phi}}^2 \gg \delta_{\bar{\phi}}^2$

$$h^{(3)}(\Delta y_{12}, \Delta y_{13}, \Delta \phi_{12}, \Delta \phi_{13}) \sim \exp \left[\frac{(\Delta y_{12})^2 + (\Delta y_{13})^2 - (\Delta y_{12})(\Delta y_{13})}{3\delta_{cy}^2} \right]$$

$$\times \left(\exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 - \Delta \phi_{12}\Delta \phi_{13}}{\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{12})^2}{2\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{13})^2}{2\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 - 2\Delta \phi_{12}\Delta \phi_{13}}{2\delta_{c\bar{\phi}}^2} \right] \right)$$

Correlated-cluster model: 3 clusters

$$c_3(\vec{\Delta y}, \vec{\Delta \phi}) = \frac{s_3^{(1)}(\vec{\Delta y}, \vec{\Delta \phi}) + s_3^{(2)}(\vec{\Delta y}, \vec{\Delta \phi}) + s_3^{(3)}(\vec{\Delta y}, \vec{\Delta \phi})}{b_3(\vec{\Delta y}, \vec{\Delta \phi})}$$

$$= \frac{1}{\langle N_c \rangle^2} h^{(1)}(\vec{\Delta y}, \vec{\Delta \phi}) + \frac{\langle N_c(N_c - 1) \rangle}{\langle N_c \rangle^3} h^{(2)}(\vec{\Delta y}, \vec{\Delta \phi}) + \frac{\langle N_c(N_c - 1)(N_c - 2) \rangle}{\langle N_c \rangle^3} h^{(3)}(\vec{\Delta y}, \vec{\Delta \phi})$$

Reducing at large $\langle N_c \rangle$

ridge effect

Three-particle three-cluster contribution

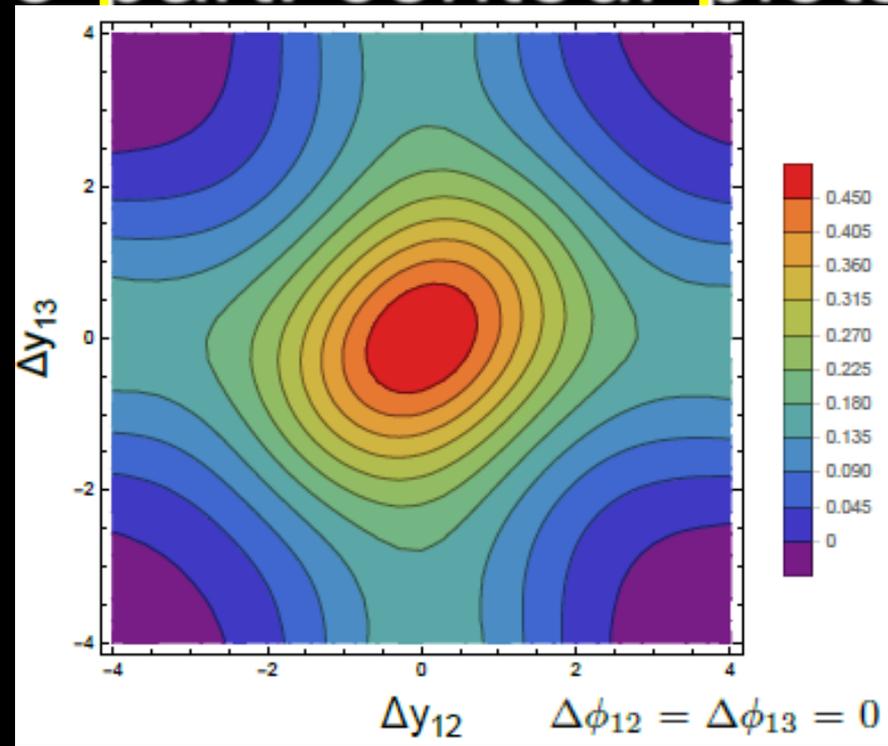
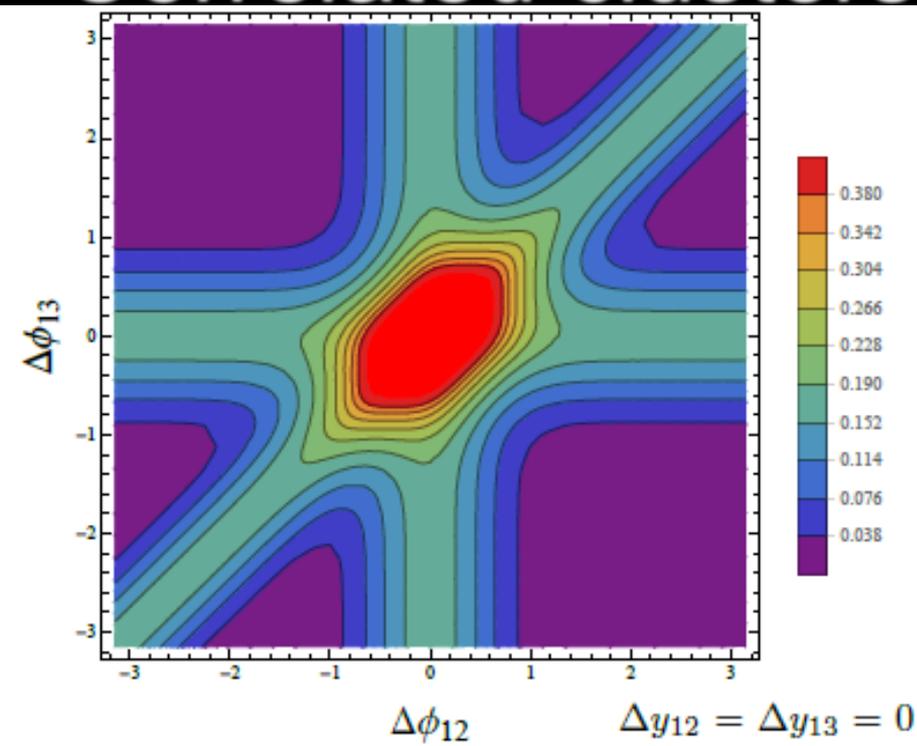
for $\delta_{cy}^2 \gg \delta_y^2$ and $\delta_{c\bar{\phi}}^2 \gg \delta_{\bar{\phi}}^2$

$$h^{(3)}(\Delta y_{12}, \Delta y_{13}, \Delta \phi_{12}, \Delta \phi_{13}) \sim \exp \left[\frac{(\Delta y_{12})^2 + (\Delta y_{13})^2 - (\Delta y_{12})(\Delta y_{13})}{3\delta_{cy}^2} \right]$$

$$\times \left(\exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 - \Delta \phi_{12}\Delta \phi_{13}}{\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{12})^2}{2\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{13})^2}{2\delta_{c\bar{\phi}}^2} \right] + \exp \left[-\frac{(\Delta \phi_{12})^2 + (\Delta \phi_{13})^2 - 2\Delta \phi_{12}\Delta \phi_{13}}{2\delta_{c\bar{\phi}}^2} \right] \right)$$

MAIN RESULT: The *ridge effect* of 3-particle correlations at small $\Delta \phi$ over a wide (pseudo)rapidity range is natural and to be observed as predicted in **model of clusters correlated in the transverse plane**

Correlated clusters: 3-part. contour plots



$$\delta_y = 0.9, \delta_{cy} = 4, \delta_\phi = 0.14, \delta_{c\phi} = 0.5$$

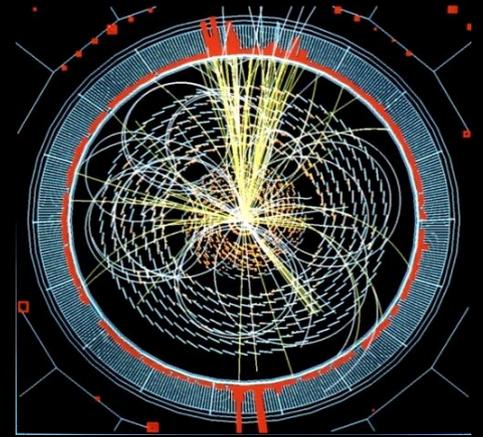
Left panel: structured asymmetric two-dimensional plot, results from the two correlation scales - a short-range azimuthal correlation scale set by single cluster decay vs. long-range correlation length from $h^{(3)}$ term of three cluster formation, the **ridge effect due to transversely correlated-cluster emission**

Right panel: rather structureless plot dominating by single cluster decay short-range correlation scale

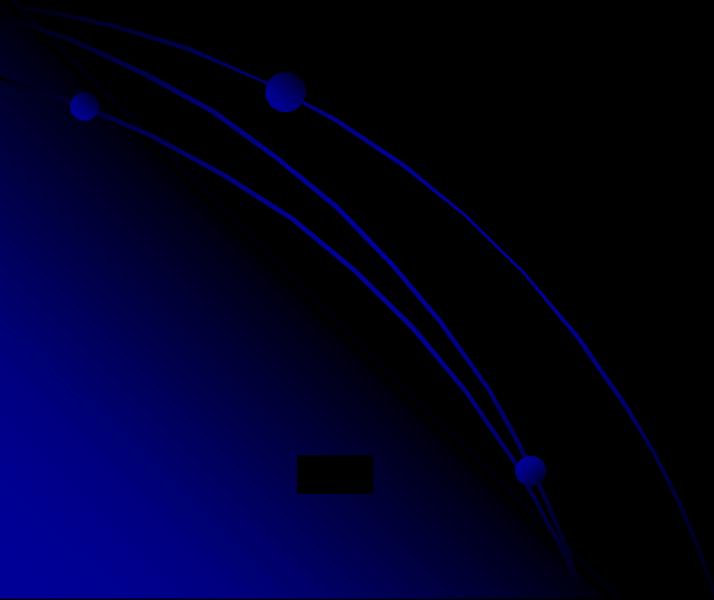
Usually expected signatures of New Physics @ LHC

Mainly on the transverse plane:

- > Lower background
- > Expected signatures such as
 - high- p_T jets, leptons or photons
 - missing transverse energy/momentum
 - displaced vertices ...
 - mass peaks



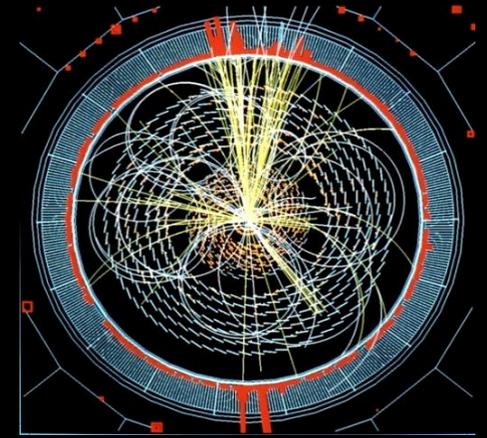
**LHC potential
must be fully used**



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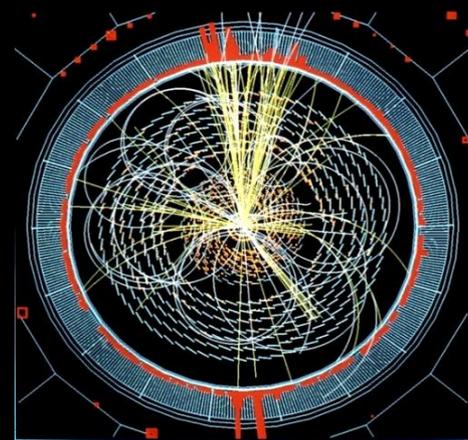
Novel signals should not be overlooked however, e.g.

- related to multiparticle production (*soft* physics)
 - but *tagged by hard signals*
- } *diffuse signal*

Usually expected signatures of New Physics @ LHC

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LHC potential
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Novel signals should not be overlooked however, e.g.

- related to multiparticle production (*soft* physics)
 - but *tagged by hard signals*
- } *diffuse signal*

May be helpful for *discovery* of a **new stage of matter** (Hidden/Dark Sector) manifesting in the **parton cascade** of high-energy pp collisions.

Not an easy task!

Techniques related to the quest for QGP in heavy-ion collisions

Hidden Valley + SM shower

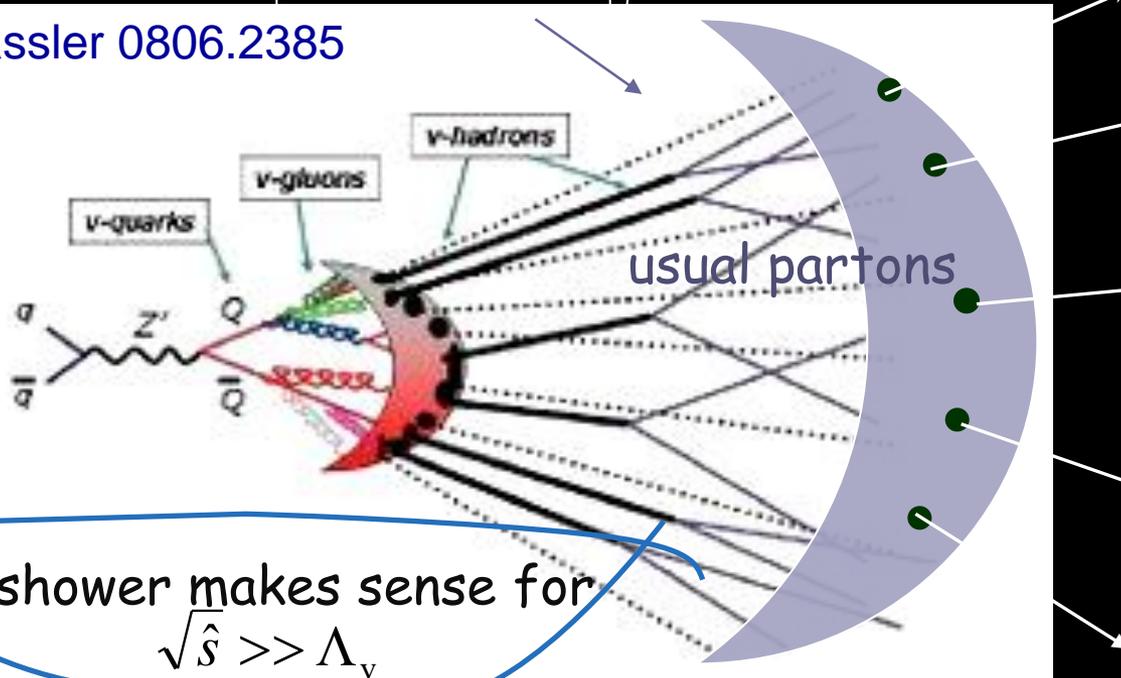
Depending on the model parameters

Some v -particles can be stable, decay outside the detectors, or promptly decay back to SM quarks and gluons

QCD parton cascade

Unseen v -particles

Strassler 0806.2385



Kind of diffuse signal

Final state SM particles

v -shower makes sense for $\sqrt{\hat{s}} \gg \Lambda_v$

Final non-perturbative hadronization

One more (and different) step than in conventional QCD-parton showers

Hidden Valley + SM shower

Depending on the model parameters

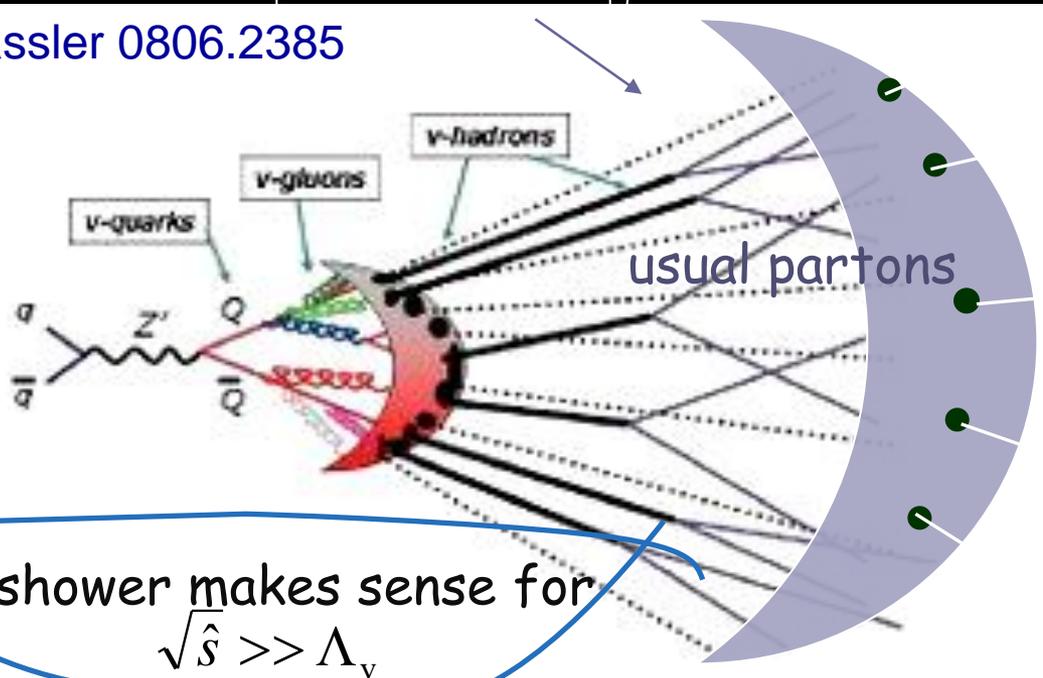
Some v -particles can be stable, decay outside the detectors, or promptly decay back to SM quarks and gluons

QCD parton cascade

Unseen v -particles

Multiplicity distributions of final state particles and rapidity/azimuthal correlations can be affected by the extra step in the cascade

Strassler 0806.2385



Kind of diffuse signal

Final state SM particles

v -shower makes sense for $\sqrt{\hat{s}} \gg \Lambda_v$

Final non-perturbative hadronization

One more (and different) step than in conventional QCD-parton showers

Effect of NP contribution in 3-step cascade

Two-particle density $\frac{1}{\sigma_{\text{in}}} \frac{d^2\sigma}{d\phi_1 d\phi_2} = \int d\phi_s \rho^{(s)}(\phi_s)$

$\times \left[\int d\phi_c \rho^{(c)}(\phi_c; \phi_s) \rho_2^{(1)}(\phi_1, \phi_2; \phi_c) + \int d\phi_{c1} d\phi_{c2} \rho_2^{(c)}(\phi_{c1}, \phi_{c2}; \phi_s) \rho^{(1)}(\phi_1; \phi_{c1}) \rho^{(1)}(\phi_2; \phi_{c2}) \right]$

$+ \int d\phi_{s1} d\phi_{s2} \rho_2^{(s)}(\phi_{s1}, \phi_{s2}) \approx e^{-\frac{(\phi_{s1} - \phi_{s2})^2}{2\delta_{s\phi}^2}}$

$\times \int d\phi_{c1} d\phi_{c2} \rho^{(c)}(\phi_{c1}; \phi_{s1}) \rho^{(c)}(\phi_{c2}; \phi_{s2}) \rho^{(1)}(\phi_1, \phi_{c1}) \rho_1^{(1)}(\phi_2; \phi_{c2})$

We use again **Gaussians** to parametrize the effect of a hidden/dark sector

$$C(\Delta\phi) \approx \exp \left[-\frac{(\Delta\phi)^2}{2(\delta_{s\phi}^2 + 2\delta_{c\phi}^2 + 2\delta_{\phi}^2)} \right], \quad \delta_{s\phi}^2 \gg \delta_{c\phi}^2 \gg \delta_{\phi}^2$$

$$\delta_{s\phi}^2$$

3-particle correlations in 3-step cascade

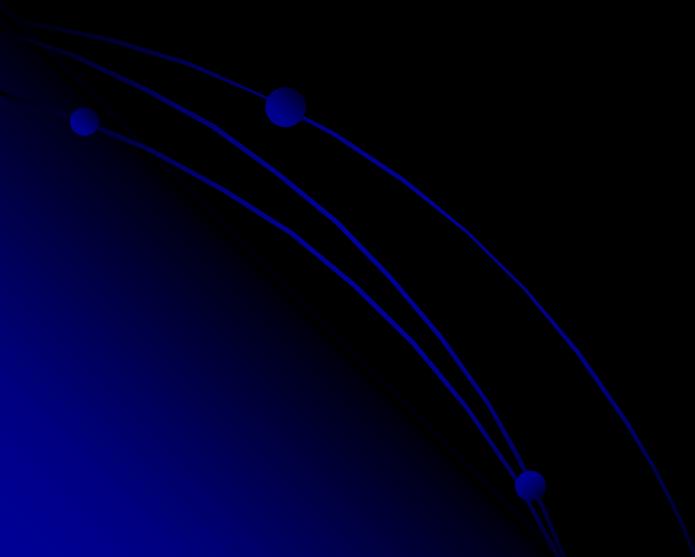
$$\Delta y_{12} = y_1 - y_2 \quad , \quad \Delta y_{12} = y_1 - y_2, \quad \Delta\phi_{12} = \phi_1 - \phi_2 \quad , \quad \Delta\phi_{13} = \phi_1 - \phi_3$$

Focusing on azimuthal variable

$$\begin{aligned} & \frac{1}{\sigma_{\text{in}}} \frac{d^3\sigma}{d\phi_1 d\phi_2 d\phi_3} = \int d\phi_s \rho^{(s)}(\phi_s) \\ & \times \left[\rho^{(c)}(\phi_c; \phi_s) \rho_3^{(1)}(\phi_1, \phi_2, \phi_3; \phi_c) + \rho_2^{(c)}(\phi_{c1}, \phi_{c2}; \phi_s) \rho^{(1)}(\phi_1; \phi_{c1}) \rho_2^{(1)}(\phi_2, \phi_3; \phi_{c2}) \right. \\ & \left. + \rho_3^{(c)}(\phi_{c1}, \phi_{c2}, \phi_{c3}; \phi_s) \rho^{(1)}(\phi_1; \phi_{c1}) \rho^{(1)}(\phi_2; \phi_{c2}) \rho^{(1)}(\phi_3; \phi_{c3}) \right] + \int d\phi_{s1} d\phi_{s2} \rho_2^{(s)}(\phi_{s1}, \phi_{s2}) \\ & \times \left\{ \left[\rho^{(c)}(\phi_{c1}; \phi_{s1}) \rho^{(c)}(\phi_{c2}; \phi_{s2}) \rho^{(1)}(\phi_1, \phi_{c1}) \rho_2^{(1)}(\phi_2, \phi_3; \phi_{c2}, \phi_{c3}) + \text{combinations} \right] \right. \\ & \left. \left[+ \rho^{(c)}(\phi_{c1}; \phi_{s1}) \rho_2^{(c)}(\phi_{c2}, \phi_{c3}; \phi_{s2}) \rho^{(1)}(\phi_1; \phi_{c1}) \rho^{(1)}(\phi_2; \phi_{c2}) \rho^{(1)}(\phi_3; \phi_{c3}) + \text{combinations} \right] \right\} \\ & + \int d\phi_{s1} d\phi_{s2} d\phi_{s3} \rho_3^{(s)}(\phi_{s1}, \phi_{s2}, \phi_{s3}) \\ & \times \left[\rho^{(c)}(\phi_{c1}; \phi_{s1}) \rho^{(c)}(\phi_{c2}; \phi_{s2}) \rho^{(c)}(\phi_{c3}; \phi_{s3}) \rho^{(1)}(\phi_1; \phi_{c1}) \rho^{(1)}(\phi_2; \phi_{c2}) \rho^{(1)}(\phi_3; \phi_{c3}) \right] \end{aligned}$$

3-particle correlations from three hidden sources

$$c_3(\Delta\phi_{12}, \Delta\phi_{13}) = \frac{1}{\langle N_s \rangle^2} h^{(1)}(\Delta\phi_{12}, \Delta\phi_{13}) + \frac{1}{\langle N_s \rangle} h^{(2)}(\Delta\phi_{12}, \Delta\phi_{13}) + h^{(3)}(\Delta\phi_{12}, \Delta\phi_{13})$$



3-particle correlations from three hidden sources

$$c_3(\Delta\phi_{12}, \Delta\phi_{13}) = \frac{1}{\langle N_s \rangle^2} h^{(1)}(\Delta\phi_{12}, \Delta\phi_{13}) + \frac{1}{\langle N_s \rangle} h^{(2)}(\Delta\phi_{12}, \Delta\phi_{13}) + h^{(3)}(\Delta\phi_{12}, \Delta\phi_{13})$$

Reducing at large $\langle N_c \rangle$

Three-particle contribution from three hidden sources

for $\delta^2_{s\bar{\Phi}} \gg \delta^2_{c\bar{\Phi}} \gg \delta^2_{\bar{\Phi}}$

$$h^{(3)}(\Delta\phi_{12}, \Delta\phi_{13}) \sim \exp \left[-\frac{(\Delta\phi_{12})^2 + (\Delta\phi_{13})^2 - \Delta\phi_{12}\Delta\phi_{13}}{3\delta_{c\bar{\Phi}}^2 + \delta_{s\bar{\Phi}}^2} \right]$$

$$+ \exp \left[-\frac{(\Delta\phi_{12})^2}{2(2\delta_{c\bar{\Phi}}^2 + \delta_{s\bar{\Phi}}^2)} \right] + \exp \left[-\frac{(\Delta\phi_{13})^2}{2(2\delta_{c\bar{\Phi}}^2 + \delta_{s\bar{\Phi}}^2)} \right] + \exp \left[-\frac{(\Delta\phi_{12})^2 + (\Delta\phi_{13})^2 - 2\Delta\phi_{12}\Delta\phi_{13}}{2(2\delta_{c\bar{\Phi}}^2 + \delta_{s\bar{\Phi}}^2)} \right]$$

3-particle correlations from three hidden sources

$$c_3(\Delta\phi_{12}, \Delta\phi_{13}) = \frac{1}{\langle N_s \rangle^2} h^{(1)}(\Delta\phi_{12}, \Delta\phi_{13}) + \frac{1}{\langle N_c \rangle \langle N_s \rangle} h^{(2)}(\Delta\phi_{12}, \Delta\phi_{13}) + h^{(3)}(\Delta\phi_{12}, \Delta\phi_{13})$$

Reducing at large $\langle N_c \rangle$

Three-particle contribution from three hidden sources

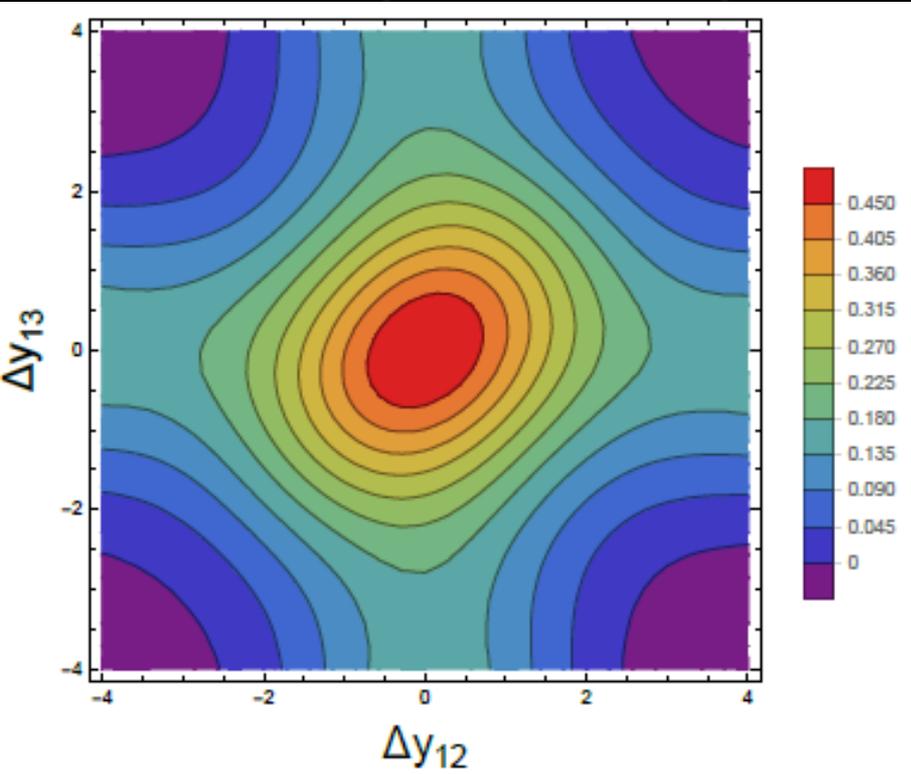
for $\delta^2_{s\bar{\Phi}} \gg \delta^2_{c\bar{\Phi}} \gg \delta^2_{\bar{\Phi}}$

$$h^{(3)}(\Delta\phi_{12}, \Delta\phi_{13}) \sim \exp \left[-\frac{(\Delta\phi_{12})^2 + (\Delta\phi_{13})^2 - \Delta\phi_{12}\Delta\phi_{13}}{3\delta_{c\bar{\Phi}}^2 + \delta_{s\bar{\Phi}}^2} \right]$$

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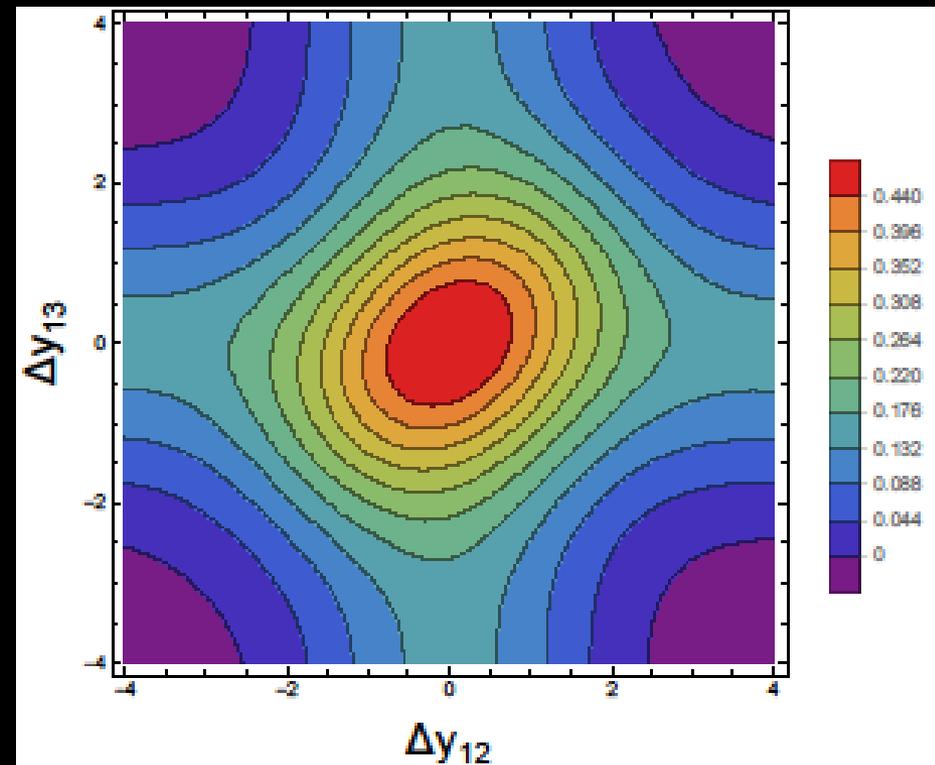
MAIN RESULT: The *effect of NP to be observed in the three-particle correlations on top of the ridge phenomenon is predicted in the model of clusters correlated in the transverse plane*

Three-particle pseudorapidity correlations



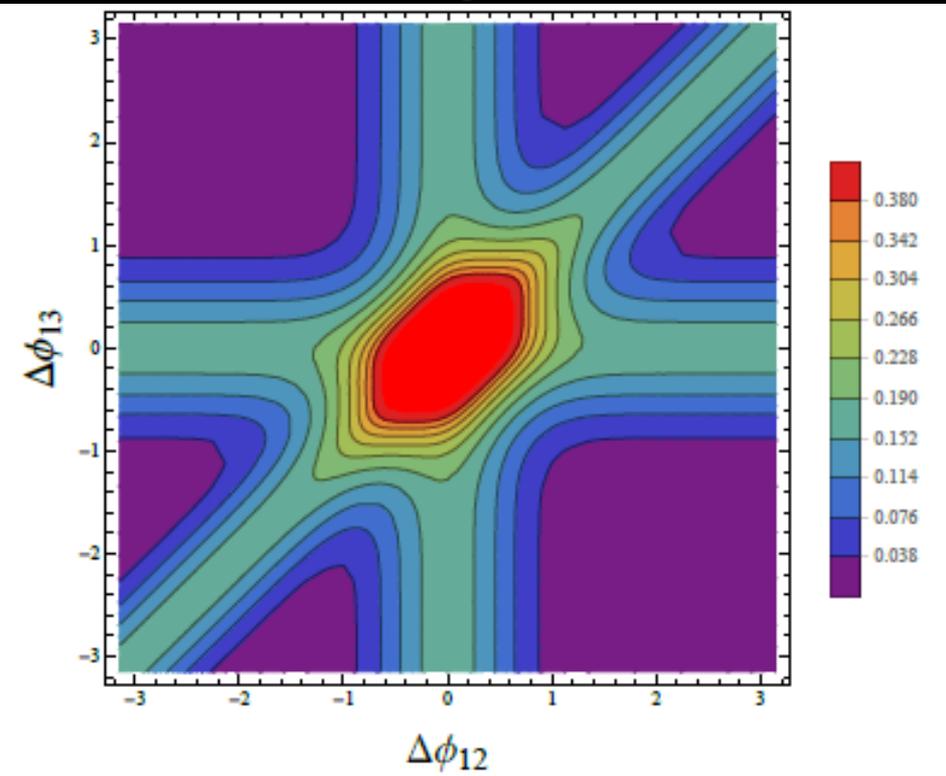
Almost no difference

NEW PHYSICS



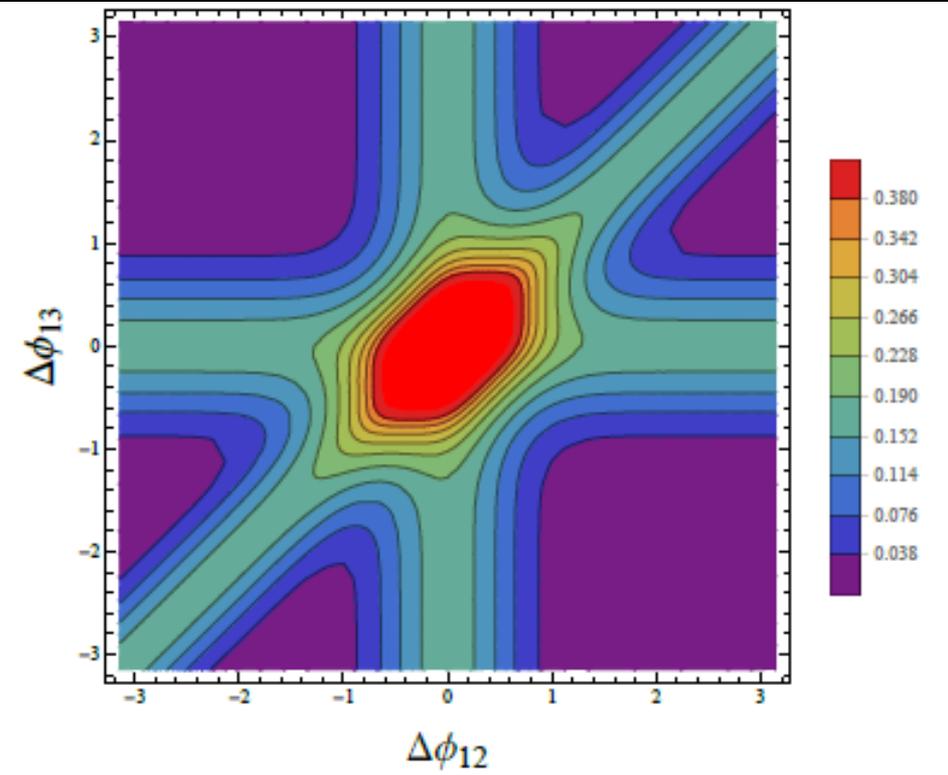
SM

Three-particle azimuthal correlations




SM

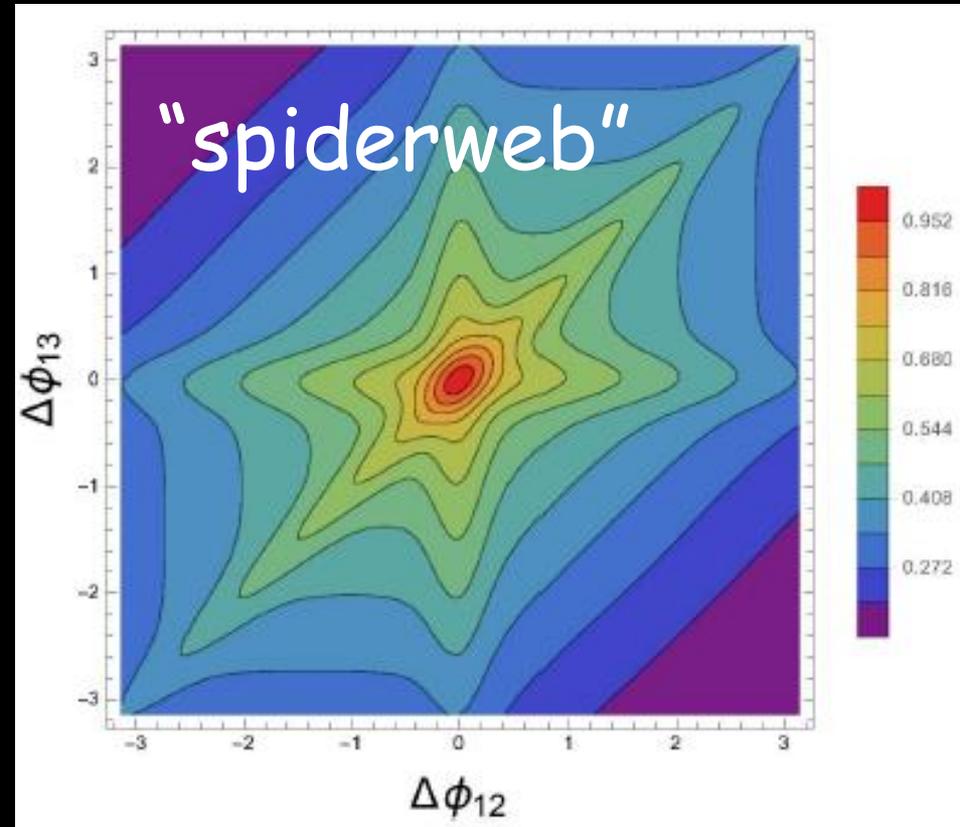
Three-particle azimuthal correlations



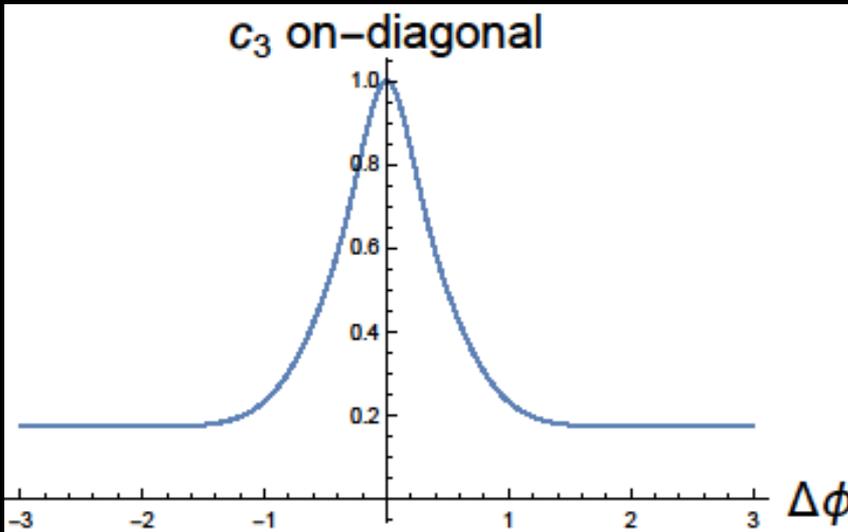
SM

NP effects should rather manifest in azimuth!

NEW PHYSICS

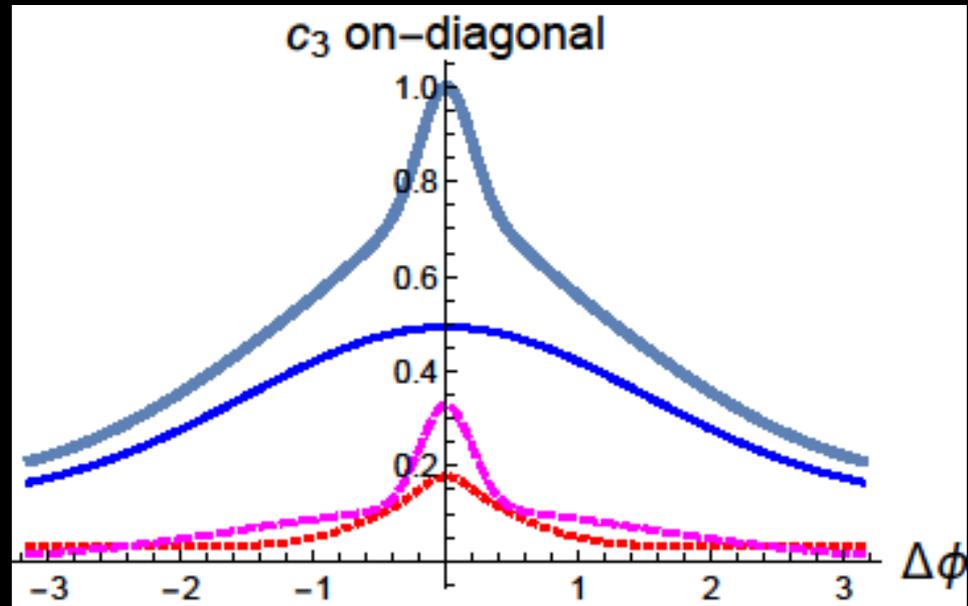


On-diagonal projection



SM

NEW PHYSICS



Can provide an estimate of $\delta_{s\phi}$

Summary

- ❖ The **universality of hadroproduction** in *different* types of collisions – from leptonic to nuclear collisions - as seen **already from first-to-come observables**, is confirming by the “*ridge effect*” and *J/psi suppression* recently **observed in pp interactions** at LHC while *believed to be* the **features of nuclear matter**
- ❖ **A model of the clusters correlated in the transverse plane** provides an **explanation** of the two-particle *ridge* effect and **predicts** the ridge phenomenon to hold in **three particle correlations**
- ❖ **New physics (hidden/dark sector) signatures** are **shown to be directly tested** by experiments using **(multi)particle correlations** (with the selection cuts to *enhance NP effect*)

Summary

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THANK YOU !