

Higgs Boson Alignment

Karim BENAKLI

LPTHE (CNRS / Sorbonne Université), Jussieu, Paris

I. Antoniadis, K. B., M. Quiròs, 2006

K. B., M. D. Goodsell, S. Williamson, 2018

Workshop on New physics paradigms after Higgs and gravitational wave
discoveries

ICNFP 2018, Kolymbari, Greece

Introduction

After Higgs boson discovery in 2012

Many open questions:

- Is the Higgs fundamental or composite ?
- What is the origin of the stability of the hierarchy between the electroweak scale (Higgs mass) and the Planck mass?
- Does the Higgs sector just consist of one doublet?

The "Higgs sector" properties = a way to explore possible Beyond Standard Model physics.

Some questions after Higgs boson discovery in 2012

Here:

- Is the Higgs **fundamental** or composite ?
- What is the origin of the stability of the hierarchy between the electroweak scale (Higgs mass) and the Planck mass?
- **Does the Higgs sector just consist of one doublet?**

Extending the Higgs Sector

Here:

- Does the Higgs sector just consist of one doublet? Minimal content?
Consider it is not \rightarrow **Extended Higgs sector**

Many constraints need to be satisfied by the Higgs sector:

- The ρ parameter should be very close to 1
- There should be a scalar of mass 125 GeV with properties very close to those of the SM Higgs
- The other states should be such that they escaped present experimental searches

Here, we will discuss the case of a Two Higgs Doublet Model (THDM)

THDM

Extending the Higgs Sector to THDM

Consider two Higgs doublets Φ_1, Φ_2 which mix. Rotate their neutral components to a basis where one Higgs only has a vev:

$$\begin{pmatrix} \text{Re}(\Phi_1^0) \\ \text{Re}(\Phi_2^0) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} v + \tilde{h} \\ \tilde{H} \end{pmatrix}$$

In the new basis, we can write the mass matrix as

$$\mathcal{M}_h^2 \equiv \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix},$$

The mass eigenstates are \tilde{h} and \tilde{H} if $Z_6 = 0$.

This is the condition for **alignment**: mass eigenstates align with the electroweak vacuum expectation value.

Extending the Higgs Sector

If $Z_6 \neq 0$, the two mass eigenstates h, H are obtained after a further rotation:

$$\begin{pmatrix} \tilde{h} \\ \tilde{H} \end{pmatrix} = \begin{pmatrix} s_{\beta-\alpha} & c_{\beta-\alpha} \\ c_{\beta-\alpha} & -s_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

In terms of the masses of the physical bosons $m_{h,H}$ this gives

$$Z_6 v^2 = s_{\beta-\alpha} c_{\beta-\alpha} (m_h^2 - m_H^2).$$

Coupling of the light Higgs to SM

The coupling of the light Higgs h is modified with respect to the SM Higgs' value.

- The h coupling to all up-type quarks is multiplied by:

$$\kappa_u = \frac{\cos \alpha}{\sin \beta},$$

- The h coupling to all down-type quarks is multiplied by:

$$\kappa_d = -\frac{\sin \alpha}{\cos \beta}$$

- The h coupling to vector bosons is multiplied by:

$$\kappa_V = \sin(\beta - \alpha).$$

Experimental constraint on Z_6

A combined ATLAS+CMS bound on the ratio:

$$\lambda_{Vu} \equiv \frac{\kappa_V}{\kappa_u} = 1^{+0.13}_{-0.12} = \frac{1}{1 + \frac{1}{t_\beta t_{\beta-\alpha}}}.$$

This is enough to constrain

$$t_\beta t_{\beta-\alpha} \gtrsim 7.3 \Rightarrow |Z_6| \lesssim \left| -\frac{7.3 t_\beta}{53 + t_\beta^2} \frac{m_H^2 - m_h^2}{v^2} \right|.$$

Example:

$$t_\beta \simeq 7.3 \Rightarrow |Z_6| \lesssim \left| -0.5 \frac{m_H^2 - m_h^2}{v^2} \right|.$$

The bound is much more stringent for large or small t_β .

It is relevant when the two states masses approach degeneracy.

Stronger experimental constraint on Z_6

$$\lambda_{du} \equiv \frac{\kappa_d}{\kappa_u} = 0.92 \pm 0.12 = \frac{1 - \frac{t_\beta}{t_{\beta-\alpha}}}{1 + \frac{1}{t_\beta t_{\beta-\alpha}}}$$

and, since from the previous constraint we know that the denominator is nearly equal to one, we have

$$-0.04 \lesssim \frac{t_\beta}{t_{\beta-\alpha}} \lesssim 0.2$$

which in turn implies $t_{\beta-\alpha} \gg t_\beta$ and so $s_{\beta-\alpha} c_{\beta-\alpha} \simeq \frac{1}{t_{\beta-\alpha}}$ and

$$-0.04 \frac{m_H^2 - m_h^2}{t_\beta v^2} \lesssim Z_6 \lesssim 0.2 \frac{m_H^2 - m_h^2}{t_\beta v^2}.$$

This leads to a sensible constraint; for example, for $m_H = 600$ GeV and $t_\beta = 5$ it leads to $Z_6 \lesssim 0.2$.

Towards alignment

$$-0.04 \frac{m_H^2 - m_h^2}{t_\beta v^2} \lesssim Z_6 \lesssim 0.2 \frac{m_H^2 - m_h^2}{t_\beta v^2}.$$

This leads to a sensible constraint; for example, for $m_H = 600$ GeV and $t_\beta = 5$ it leads to $Z_6 \lesssim 0.2$.

- So we see that either we should take the mass m_H to be large, in which case we have **decoupling**
- we keep the new sates light in order to possibly detect them at the LHC, in which case we need **alignment without decoupling**

Towards alignment

$$\mathcal{M}_h^2 \equiv \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix},$$

and

$$m_A^2 + Z_5 v^2 \gg Z_6 v^2$$

- **Decoupling:** we take the mass m_H large: $m_A^2 \gg v^2$
- **Alignment without decoupling:** m_H not large, Z_6 small. We keep the new states lighter in order to possibly detect them at the LHC,

Alignment without decoupling

$$\mathcal{M}_h^2 \equiv \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix},$$

Making Z_6 small:

- Accidental / tuned as such (example MSSM).
- Some symmetry

Easy to find a symmetry for the Higgs scalar potential. Less easy is to find one that is not spoiled by the couplings to matter.

THDM parameters

The THDM parameters

The THDM scalar potential can be parametrised as:

$$\begin{aligned}
 V = & \quad m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] \\
 & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
 & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left[\frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right],
 \end{aligned}$$

where Φ_1, Φ_2 are the two Higgs doublets

THDM mass matrix elements

To begin with, the mass-matrices for the CP-even neutral scalars in the two-Higgs doublet model can be parametrised in the alignment basis where

$$\begin{pmatrix} \text{Re}(\Phi_1) \\ \text{Re}(\Phi_2) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} v+h \\ H \end{pmatrix}$$

is

$$\mathcal{M}_h^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix},$$

where, using $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ we have

$$Z_1 \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2} \lambda_{345} s_{2\beta}^2, \quad Z_5 \equiv \frac{1}{4} s_{2\beta}^2 [\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_5$$

$$Z_6 \equiv -\frac{1}{2} s_{2\beta} [\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}].$$

THDM tree-level masses

The parameter m_A is the pseudo-scalar mass, given by

$$m_A^2 = -\frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2,$$

while the charged Higgs mass is

$$m_{H^\pm}^2 = \frac{1}{2}(\lambda_5 - \lambda_4)v^2 + m_A^2.$$

The neutral Higgs masses are

$$m_{H,h}^2 = \frac{1}{2} \left[m_A^2 + (Z_1 + Z_5)v^2 \pm \sqrt{(m_A^2 + (Z_5 - Z_1)v^2)^2 + 4Z_6^2 v^4} \right].$$

Alignment w/o decoupling

Supersymmetric THDM

SM \rightarrow Minimal Higgs sector: 1 Higgs doublet

Supersymmetry \rightarrow at least: 2 Higgs doublets

In the minimal version, MSSM, only alignment with decoupling is natural.

Extend the MSSM with:

- A symmetry of the Higgs potential
- The required new states to enforce realistic model

Check its experimental viability.

Introduce more supersymmetries: $N = 2$

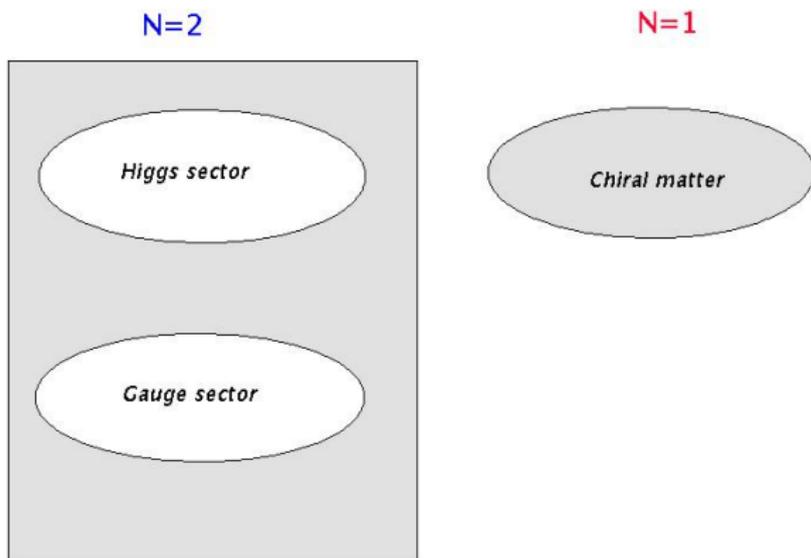


Figure: $N=2/N=1$ sectors.

The added new states S and T

- Make the two MSSM Higgs doublets (H_u, H_d) an $N = 2$ supersymmetry hypermultiplet.
- Introduce two chiral supermultiplets S and T adjoint of $U(1)$ and $SU(2)$.

S and T coupling in the Higgs superpotential

New couplings to the Higgs:

$$W_{\text{Higgs}} = \mu \mathbf{H}_u \cdot \mathbf{H}_d + \lambda_S \mathbf{S} \mathbf{H}_u \cdot \mathbf{H}_d + 2\lambda_T \mathbf{H}_d \cdot \mathbf{T} \mathbf{H}_u$$

The role of $N = 2$ is to fix:

$$\lambda_S = \frac{g'}{\sqrt{2}}$$
$$\lambda_T = \frac{g^2}{\sqrt{2}}$$

Decoupling part of the added new states S and T

- S and T fermionic components combine with $U(1)$ and $SU(2)$ to generate Dirac masses for gauginos
- S and T bosonic components get vevs: small contribution to ρ parameters \rightarrow small vevs \rightarrow TeV mass for S and T bosonic components

1 - The (pseudo)scalars in S and T made massive:

They are **decoupled** \rightarrow **effective THDM**

2 - **Fermions in S and T remain light: extended neutralinos and charginos sectors.**

Higgs potential

$$V_{EW} = V_0 + V_1 + V_2$$

The first part:

$$V_0 = \frac{(m_{H_u}^2 + \mu^2)}{2} h_u^2 + \frac{(m_{H_d}^2 + \mu^2)}{2} h_d^2 - B_\mu h_u h_d + \frac{g^2 + g'^2}{32} (h_u^2 - h_d^2)^2$$

is the MSSM contribution. The second,

$$V_1 = \frac{\lambda_S^2 + \lambda_T^2}{4} h_u^2 h_d^2$$

is a quartic term.

V_2 contains the explicit dependence on the mass parameters of the S and T adjoints. In the case of interest: $m_S \rightarrow \infty$ and $m_T \rightarrow \infty$

$$\begin{aligned} V_1 &\rightarrow \frac{\lambda_S^2 + \lambda_T^2}{4} h_u^2 h_d^2 \\ V_2 &\rightarrow 0 \end{aligned}$$

the MSSM scalar potential and a quartic term.

Mapping this model to generic THDM: **masses**

To map our model onto the THDM make the identification:

$$\Phi_2 = H_u, \quad \Phi_1^i = -\epsilon_{ij}(H_d^j)^* \leftrightarrow \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} \Phi_1^0 \\ -(\Phi_1^+)^* \end{pmatrix}$$

from which we can write down

$$m_{11}^2 = m_{H_d}^2 + \mu^2, \quad m_{22}^2 = m_{H_u}^2 + \mu^2, \quad m_{12}^2 = B_\mu.$$

Mapping this model to generic THDM: couplings

The (pseudo)scalars in S and T massive are decoupled \rightarrow effective THDM
The parameters λ_i are given at tree-level ($m_S \rightarrow \infty$ and $m_T \rightarrow \infty$):

$$\lambda_1 = \frac{1}{4}(g_2^2 + g_Y^2)$$

$$\lambda_2 = \frac{1}{4}(g_2^2 + g_Y^2)$$

$$\lambda_3 = \frac{1}{4}(g_2^2 - g_Y^2) + 2\lambda_T^2$$

$$\lambda_4 = -\frac{1}{2}g_2^2 + \lambda_S^2 - \lambda_T^2$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0.$$

Higgs alignment without decoupling from $N=2$

$$W_{\text{Higgs}} = \mu \mathbf{H}_u \cdot \mathbf{H}_d + \lambda_S \mathbf{S} \mathbf{H}_u \cdot \mathbf{H}_d + 2\lambda_T \mathbf{H}_d \cdot \mathbf{T} \mathbf{H}_u$$

The scalars mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix},$$

with at tree-level:

$$Z_6 = -\frac{1}{2} s_{2\beta} c_{2\beta} \left[\frac{(g^2 + g'^2)}{2} - (\lambda_S^2 + \lambda_T^2) \right]$$

$\rightarrow Z_6 = 0$ for the $N = 2$ THDM at tree-level

$N=2 \implies$ Alignment without decoupling

I. Antoniadis - K.B. - A. Delgado - M. Quiros, '06

This has experimental consequences: constraints and signals

K.B.- M. D. Goodsell - S. Williamson '18 / J. Ellis - J. Quevillon - V. Sanz, '16

Challenges

Higgs misalignment from $N=2$ to $N=1$

The alignment without decoupling was a consequence of $N = 2$ supersymmetry in the gauge/Higgs sector. BUT:

- this can be realised above some scale $M_{N=2}$.
- below $M_{N=2}/M_{SUSY}$ till M_{SUSY} the theory can be at most $N = 1$ because it has chiral matter.
- below M_{SUSY} the theory must be $N = 0$.

We need the alignment w/o decoupling at the LHC scales

Not enough to achieve alignment at a very high scale.

Higgs misalignment from $N=2$ to $N=1$

The presence of chiral fields generates some misalignment:

$$Z_6(M_{SUSY}) = \frac{1}{4} s_{2\beta} c_{2\beta} \left[(2\lambda_S^2 - g_Y^2) + (2\lambda_T^2 - g_2^2) \right] + \text{threshold corrections.}$$

The misalignment due to runnings of λ_S and g' , and λ_T and g :

$$\begin{aligned} [2\lambda_S^2 - g'^2]_{\Lambda_{N=1}} &= -\frac{2g'^2}{16\pi^2} [3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 10g'^2] \log\left(\frac{\Lambda_{N=2}}{\Lambda_{N=1}}\right), \\ [2\lambda_T^2 - g^2]_{\Lambda_{N=1}} &= -\frac{2g^2}{16\pi^2} [3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 6g^2] \log\left(\frac{\Lambda_{N=2}}{\Lambda_{N=1}}\right). \end{aligned}$$

K.B. - M. Goodsell - S. Williamson, '18

Higgs misalignment from $N=2$ to $N=1$

These equations are only useful for small $M_{N=2}/M_{SUSY}$, because for large ratios the top Yukawa coupling can change by a factor of two or more, but it gives an indication of the amount of misalignment:

$$Z_6(M_{SUSY}) \sim -\mathcal{O}(0.1) \frac{t_\beta}{1+t_\beta^2} \left(\frac{t_\beta^2-1}{t_\beta^2+1} \right) \left(\frac{\log M_{N=2}/M_{SUSY}}{10} \right). \quad (1)$$

Small !!!!

Higgs misalignment from $N=1$ to $N=0$

Misalignment induced from:

- the threshold corrections at M_{SUSY}
- the running between M_{SUSY} and the scale of the THDM

These can both be approximated at one loop by corrections to the $\delta\lambda_i$:

$$\begin{aligned}
 \delta\lambda_1 &= \frac{1}{16\pi^2} \log \frac{M_\Sigma^2}{\mu^2} \left[\lambda_S^4 + 3\lambda_T^4 + 2\lambda_S^2\lambda_T^2 \right] \\
 \delta\lambda_2 &= \delta\lambda_1 + \frac{3y_t^4}{8\pi^2} \log \frac{m_t^2}{\mu^2} \\
 \delta\lambda_3 &= \frac{1}{16\pi^2} \log \frac{M_\Sigma^2}{\mu^2} \left[\lambda_S^4 + 3\lambda_T^4 - 2\lambda_S^2\lambda_T^2 \right] \\
 \delta\lambda_4 &= \frac{1}{16\pi^2} 4\lambda_S^2\lambda_T^2 \log \frac{M_\Sigma^2}{\mu^2}, \tag{2}
 \end{aligned}$$

Higgs misalignment from $N=1$ to $N=0$

We then find the remarkable result:

- The singlet/triplet scalar contributions to Z_6 exactly cancel out!
- The dominant contribution to Z_6 is that coming from the stops:

$$Z_6(v) \simeq Z_6(M_{SUSY}) + s_\beta^3 c_\beta \times \frac{3y_t^4}{8\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2},$$

Although the magnitude of this is the same as the loop contribution to Z_6 in the MSSM, the misalignment thus induced is **much smaller**, because

- there is no tree-level contribution
- it is also proportional to the stop correction to the Higgs mass, which is smaller than in the MSSM due to the **tree-level boost to the Higgs mass from $\lambda_{S,T}$** .

Higgs misalignment without decoupling precisely

A precision study using numerical tools shows that the misalignment in the model is even smaller than the above naive estimate

$$\begin{aligned}
 Z_6 &= \frac{s_\beta c_\beta}{v^2(m_A^2 s_\beta^2 + M_Z^2 c_\beta^2 - m_h^2)} \left[(m_A^2 - m_h^2)(m_h^2 - M_Z^2) - v^2 \hat{\delta} \lambda_{345} (m_A^2 s_\beta^2 - M_Z^2) \right] \\
 &\approx \frac{0.12}{t_\beta} - \frac{1}{2} \frac{t_\beta}{1 + t_\beta^2} \left[(2\lambda_S^2 - g_Y^2) + (2\lambda_T^2 - g_2^2) \right]. \quad (3)
 \end{aligned}$$

The departure of $\lambda_{S,T}$ from their $N = 2$ value accidentally cancels the other contributions making Z_6 smaller !!!

K.B. - M. Goodsell - S. Williamson, '18

Numerical value of Z_6

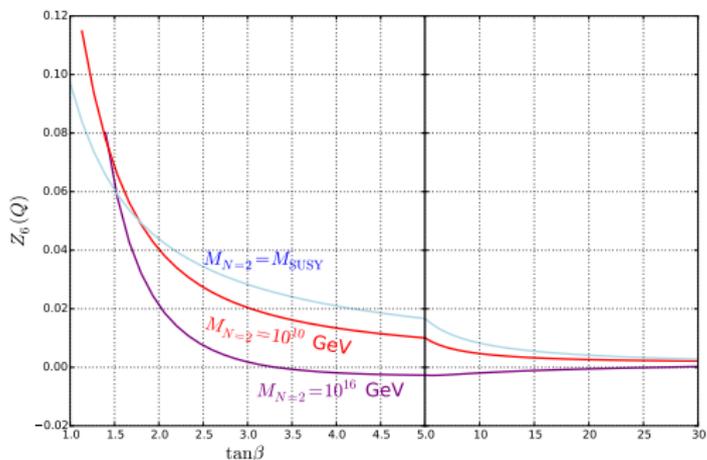


Figure: $Z_6(Q)$ against $\tan\beta$, where $Q = 400 \text{ GeV}$ is our low-energy matching scale. We find that the model shows good alignment for all values of $\tan\beta > 1.5$, with the surprising conclusion that raising the $N = 2$ scale improves the alignment.

Fitting the Higgs mass

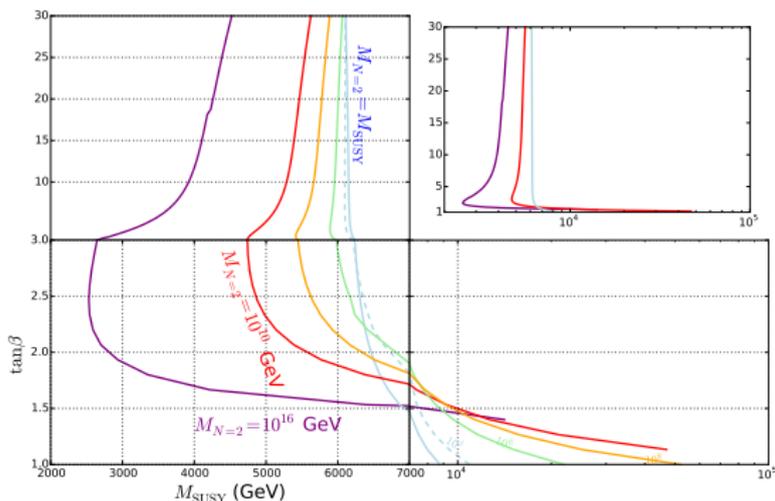


Figure: SUSY scale that fits $m_h = 125.2$ GeV against $\tan \beta$. The cases $M_{N=2} = \{M_{SUSY}, 10^{10} \text{ GeV}, 10^{16} \text{ GeV}\}$ are the solid lines in blue, red and purple respectively and are labelled in full; the cases $M_{N=2} = \{10^4, 10^6, 10^8\} \text{ GeV}$ are respectively shown in blue dashed, solid green and solid orange curves and only labelled with $\{10^4, 10^6, 10^8\}$.

Experimental constraints from ρ parameter

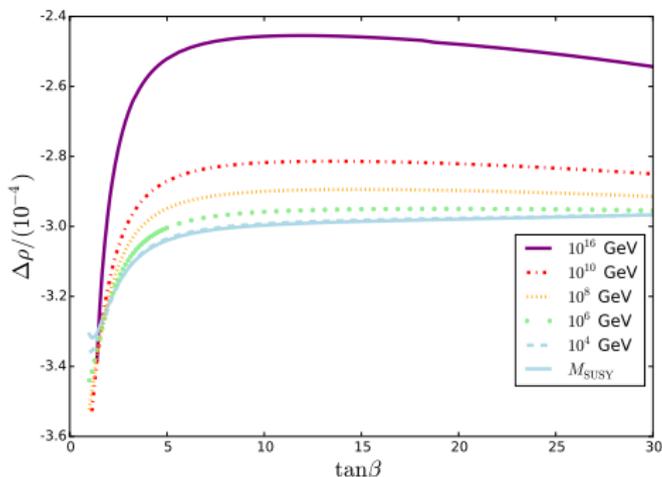


Figure: $\Delta\rho$ calculated at one-loop in the low-energy theory, for different values of $M_{N=2}$ given in the legend. We see that the magnitude is roughly equal to the experimental error, and we are always well within 3σ of the experimental central value (which is anyway above the Standard Model value by 1.6σ).

$$\Delta\rho = (3.7 \pm 2.3) \times 10^{-4}. \quad (4)$$

experimental constraints

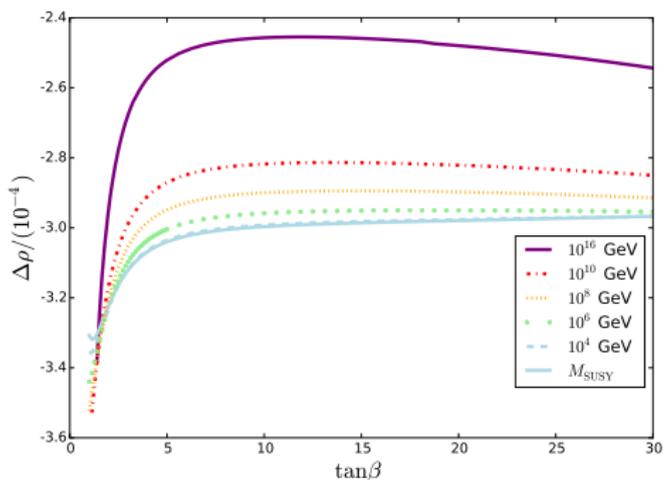


Figure: $\Delta\rho$ calculated at one-loop in the low-energy theory, for different values of $M_{N=2}$ given in the legend. We see that the magnitude is roughly equal to the experimental error, and we are always well within 3σ of the experimental central value (which is anyway above the Standard Model value by 1.6σ).

Experimental constraints on M_H

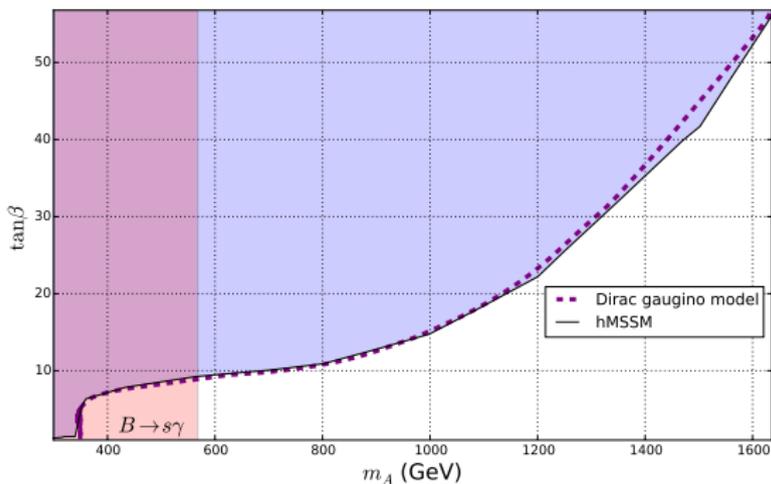


Figure: Bounds from $pp \rightarrow H/A \rightarrow \tau^+ \tau^-$ (blue region) and $B \rightarrow s\gamma$ (red region, $m_A \lesssim 568$ GeV).

The present limit on M_H puts it in the region where it corresponds roughly to decoupling (i.e. the value needed to fit the precision on couplings for alignment).

Summary

Take-away

- For THDM: LHC data require alignment of the light Higgs with the direction having a vev
- Alignment is realised either with or without decoupling
- $N = 2$ susy of the Higgs sector leads naturally to alignment without decoupling
- An alignment without decoupling to an impressive amount remains after quantum corrections !!!
- Alignment without decoupling important if the data improves precision of Higgs couplings measurements