Quantum no-scale regimes in string theory

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Based on

- T. Coudarchet, C. Fleming and H.P., Nucl. Phys. B930 (2018) 235 [arXiv:1711.09122].
- T. Coudarchet and H.P., Nucl. Phys. B933 (2018) 134 [arXiv:1804.00466].

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Outline

Introduction

■ **In QFT,** we consider classically a flat and static universe, with arbitrary particle content, and we compute quantum corrections of the associated fields. We do not wonder if the particle content imposes constraints for the flat universe to be stable.

■ To remain ∼flat and static **in presence of gravity,** the minima of the quantum potential should ∼vanish, $\langle \mathcal{V}_{\text{quantum}} \rangle \simeq 0$:

- **If susy spontaneously broken,** we can at least look for models at weak coupling where $\langle \mathcal{V}_{\text{1-loop}} \rangle \simeq 0$. [Abel, Dienes, Mavroudi,'15] [Kounnas, H.P.,'15] [Kachru, Kumar, Silverstein,'98] [Harvey,'98] [Shiu, Tye,'98] [Blumenhagen, Gorlich,'98] [Angelantonj, Antoniadis, Forger,'99] [Satoh, Sugawara, Wada,'15]

- **For generic $\mathcal{V}_{\text{1-loop}}$,**

Flat static classical background + $\mathcal{V}_{\text{1-loop}} \Longrightarrow$ **Flat FRW cosmology**

■ We study these **cosmological evolutions in a class of explicit string models.**

(I) Under which conditions a flat expanding universe keeps on growing ?

- A sufficient condition is that $\mathcal{V}_{\text{quantum}} \geq 0$.

- It can also be the case when $\mathcal{V}_{\text{quantum}} < 0$.

However, the presence of **moduli fields induces some kind of "instability" :** For most initial conditions the expansion stops and the universe collapses into a **Big Crunch.** This is so **unless the initial conditions are tuned in a tiny region of the phase space.**

$\Longrightarrow$ This suggests that **spectra leading to $\boldsymbol{\mathcal{V}_{\text{quantum}} \geq 0}$ are way more natural to describe expanding flat universe ?**

(II) What are the properties of these flat, expanding universes ?

- The susy breaking scale is a field $\boldsymbol{M \equiv e^{\alpha\Phi}}$.

$\diamond$ Classically : It is a flat direction of a potential $\mathcal{V}_{\text{class}} \geq 0$

$\Rightarrow$ $\boldsymbol{\Phi}$ is the no-scale modulus. [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

$\diamond$ **In the quantum flat cosmology :** The classical kinetic energies of M, the dilaton and some moduli are $\gg |\mathcal{V}_{\text{1-loop}}|$ and the kinetic energies of the remaining moduli.

$\Longrightarrow$ Asymptotically, the solution is identical to that found classically. No difference with the evolution found in the special models where $\mathcal{V}_{\text{1-loop}} = 0$.

$\Longrightarrow$ The No-Scale Structure present classically is restored cosmologically $\equiv$ The universe enters in **"Quantum No-Scale Regime" (QNSR)**

- **No Higgs-like instability occurs** in these regimes : The moduli can sit at minima, maxima, saddle points or anywhere.

Outline

Toy model with minimal set of fields

■ Heterotic string on : $\mathbb{R}^{1,d-1} \times \prod_{i=d}^{d+n-1} S^1(R_i) \times T^{10-d-n}$

with stringy Scherk-Schwarz mechanism along the n large S^1's, while the size of T^{10-d-n} is close to the string scale.

[Rhom,'84] [Kounnas, Porrati,'88] [Kounnas, Rostand,'90]

The susy breaking scale is $M_{(\sigma)} = \dfrac{M_\mathrm{s}}{(R_d \cdots R_{d+n-1})^{\frac{1}{n}}} \ll M_\mathrm{s}$

$\Longrightarrow$ No Hagedorn-like instability

■ The 1-loop effective potential is

$$\mathcal{V}^{(\sigma)}_{\text{1-loop}} \equiv -\frac{M_\mathrm{s}^d}{(2\pi)^d} \int_\mathcal{F} \frac{d^2\tau}{2\tau_2^2}\, Z$$

$$= (\boldsymbol{n_\mathbf{F}} - \boldsymbol{n_\mathbf{B}})\, v_d\, M_{(\sigma)}^d + \mathcal{O}\Big((M_\mathrm{s} M_{(\sigma)})^{\frac{d}{2}}\, e^{-M_\mathrm{s}/M_{(\sigma)}} \Big) \qquad (1)$$

• $\boldsymbol{n_\mathbf{B}}$, $\boldsymbol{n_\mathbf{F}}$ massless bosons and fermions. They have light Kaluza - Klein modes associated to the large $S^1(R_i)$'s.

$$\mathcal{V}^{(\sigma)}_{\text{1-loop}} = (n_{\rm F} - n_{\rm B})\, v_d\, M^d_{(\sigma)} + \mathcal{O}\Big(\big(M_{\rm s} M_{(\sigma)}\big)^{\frac{d}{2}}\, e^{-M_{\rm s}/M_{(\sigma)}}\Big)$$

- $v_d > 0$ depends on the R_i/R_d, we treat as constants.
- All **other states are super massive.** They yield **exponentially suppressed** contributions we neglect.

■ **The effective action** involves the **dilaton** ϕ and **susy breaking scale.** **In Einstein frame,**

$$\alpha = \sqrt{\tfrac{1}{d-2} + \tfrac{1}{n}}$$

$$M \equiv e^{\frac{2}{d-2}\phi} M_{(\sigma)} \equiv e^{\alpha\Phi}\, M_{\rm s}\,, \qquad \phi_\perp = \frac{1}{\sqrt{d-2+n}}\left[2\phi + \ln\Big(\prod_{i=d}^{d+n-1} R_i\Big)\right]$$

$$S = \frac{1}{\kappa^2}\int d^dx\,\sqrt{-g}\left[\frac{\mathcal{R}}{2} - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}(\partial\phi_\perp)^2 - \kappa^2 \mathcal{V}_{\text{1-loop}}\right]$$

$$\text{where} \qquad \mathcal{V}_{\text{1-loop}} = (n_{\rm F} - n_{\rm B})\, v_d\, e^{d\alpha\Phi}\, M^d_{\rm s}$$

■ We look for **flat homogeneous and isotropic solutions**

$$ds^2 = -N(t)^2 dt^2 + a(t)^2\Big((dx^1)^2 + \cdots + (dx^{d-1})^2\Big)\,, \quad \Phi(t)\,, \quad \phi_\perp(t)$$

■ $\phi_\perp$ is a free field $\quad\Longrightarrow\quad \dot\phi_\perp = \sqrt{2}\,\frac{c_\perp}{a^{d-1}}$

■ $\frac{\partial \mathcal{V}_{\text{1-loop}}}{\partial \Phi} = d\alpha\, \mathcal{V}_{\text{1-loop}} \Longrightarrow$ a combination of fields is free :

$$\alpha\dot\Phi + \frac{\alpha^2}{2}\, d(d-2)\, H = \frac{c_\Phi}{a^{d-1}}$$

■ Eq. for $a(t)$: $\quad \frac{1}{2}\,(d-2)\,\dot H = -\frac{1}{2}\,\dot\Phi^2 - \frac{1}{2}\,\dot\phi_\perp^2$

Can be solved in terms of a new time τ

$$A\,\frac{da}{a} = -\frac{\tau\, d\tau}{\mathcal{P}(\tau)} \qquad \text{where} \qquad \tau \equiv \frac{2A}{d(d-1)c_\Phi}\,(a^{d-1})^{\cdot}$$

$$\mathcal{P}(\tau) = \tau^2 - 2\tau + \omega\left[1 + 2\alpha^2\left(\frac{c_\perp}{c_\Phi}\right)^2\right] \qquad \begin{array}{l} A = \frac{\omega}{4}\, d^2(d-2)\alpha^2 \\ \omega = 1 - \frac{4(d-1)}{d^2(d-2)\alpha^2} \end{array}$$

■ Friedmann Eq. for $N(t)$ fixes the 2d integration cst. of Φ *i.e.* M :

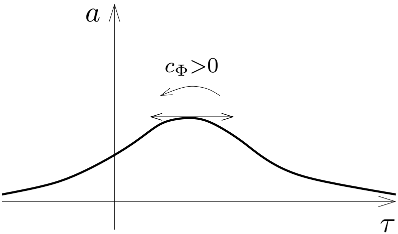
$$(n_{\rm F} - n_{\rm B})\, v_d\, \kappa^2 M^d = -\frac{c_\Phi^2}{2\alpha^2\omega}\, \frac{\mathcal{P}(\tau)}{a^{2(d-1)}}$$

$$\text{Let} \quad \gamma_c = \sqrt{\frac{1-\omega}{2\alpha^2\omega}}$$

■ **Supercritical case :** $\left|\frac{c_\perp}{\gamma_c c_\Phi}\right| > 1 \quad \Rightarrow \quad \mathcal{P}(\tau)$ **has no real roots.**

- Friedmann Eq

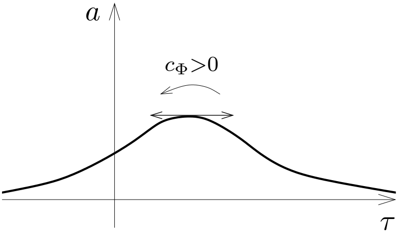
$\Longrightarrow \quad n_\mathrm{F} - n_\mathrm{B} < 0 \quad$ and $\quad$ the classical limit $\kappa \to 0$ does not exist.



- When $\tau \to \pm\infty$, we have $a(t) \sim \#|t - t_\pm|^{\frac{1}{A+d-1}}$

i.e. Big Bang $\to$ Big Crunch

$H^2 \sim \#\dot{\Phi}^2 \sim \#\kappa^2 |\mathcal{V}_{\text{1-loop}}| \gg \dot{\phi}_\perp^2 \quad$ *i.e.* $\quad$ **the universe is dominated by the kinetic energy and quantum potential of** M**.**



• The potential stops the expansion. At the maximum :

$$\ddot{a} \propto \mathcal{V}_{\text{1-loop}} < 0$$

• These solutions are valid far enough from the Big Bang and Big Crunch, for

- Kinetic Energies $\ll M_{\mathrm{s}}$ *i.e.* $a(t)$ not too small. (If not higher derivative terms must be included).
- String coupling remains weak : $e^{2d\alpha^2\phi} \sim \#|\tau|^{\frac{2}{\omega}}$.

Conclusion : The quantum potential allows new universes (no classical counterparts), however "unstable" in the sense that the quantum potential induces a collapse.

■ **Subcritical case : $\left|\frac{c_\perp}{\gamma_c c_\Phi}\right| < 1 \quad \Longrightarrow \quad \mathcal{P}(\tau)$ has 2 real roots**

$$\tau_\pm = 1 \pm r \qquad \text{where} \qquad r = \sqrt{1-\omega}\,\sqrt{1-\left(\frac{c_\perp}{\gamma_c c_\Phi}\right)^2}$$

- Friedmann Eq $\Longrightarrow \begin{cases} n_\text{F} - n_\text{B} > 0 & \Rightarrow \quad \tau_- < \tau < \tau_+ \\ n_\text{F} - n_\text{B} = 0 & \Rightarrow \quad \tau \equiv \tau_+ \text{ or } \tau_- \\ n_\text{F} - n_\text{B} < 0 & \Rightarrow \quad \tau < \tau_- \text{ or } \tau_+ < \tau \end{cases}$

- $\boldsymbol{n_\text{F} - n_\text{B} = 0} \quad \Longrightarrow \quad \mathcal{V}_\text{1-loop} = 0$

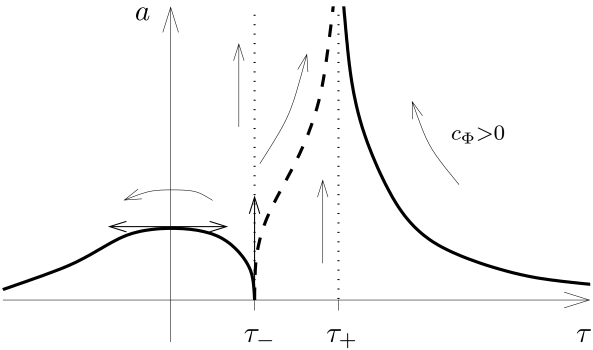
  $\Longrightarrow$ same solution found classically

Expanding (contracting) **universe driven by the kinetic energy of free fields**

$$H^2 \propto \dot{\phi}_\perp^2 \propto \dot{\Phi}^2 \propto \frac{1}{a^{2(d-1)}}$$

$$a \propto |t - t_\pm|^{\frac{1}{d-1}} \;, \qquad M^d \propto \frac{1}{a^{2(d-1)+K_\pm}} \quad \text{where} \quad K_\pm = \pm\frac{2Ar}{1 \pm r}$$

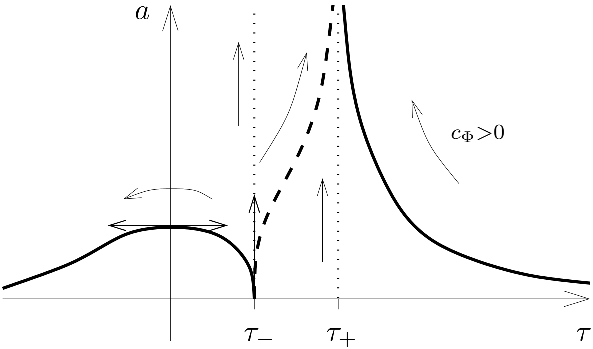
- $n_\mathrm{F} - n_\mathrm{B} \neq 0$



◇ **2 branches for $n_\mathrm{F} - n_\mathrm{B} < 0$ (solide lines)**
1 branch for $n_\mathrm{F} - n_\mathrm{B} > 0$ (dashed line)

◇ All solutions start/end with a Big Bang or Big Crunch.

◇ At $\tau \to \pm\infty$, no difference with the supercritical case.

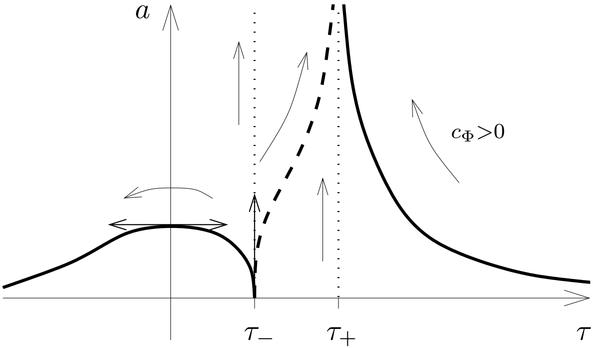


$\diamond$ All solutions approach τ_+ or τ_- *i.e.* those found in the case $n_\mathrm{F} - n_\mathrm{B} = 0$, or in the classical limit $\kappa \to 0$.

$$H^2 \sim \#\,\dot\phi_\perp^2 \sim \#\,\dot\Phi^2 \sim \frac{\#}{a^{2(d-1)}} \gg |\mathcal{V}_{\text{1-loop}}| \sim \frac{\#}{a^{2(d-1)+K_\pm}}$$

$\Longrightarrow$ **classical No-Scale Structure is restored cosmologically.**

$\equiv$ The universe enters in a **"Quantum No-Scale Regime" : *M* becomes free.**



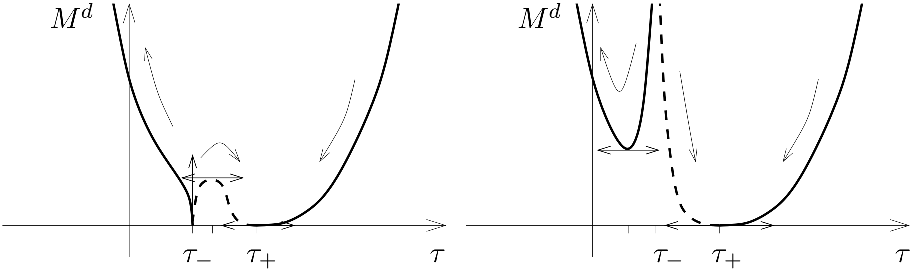
⋄ **In models with $\mathcal{V}_{\text{1-loop}} > 0$, the evolution is always attracted to the forever expanding Quantum No-Scale Regime.**

When $\mathcal{V}_{\text{1-loop}} < 0$, there is no preference between the good and bad branches. This will be different when all moduli fields are included.

⋄ The Quantum No-Scale Regimes are compatible with string weak coupling (see below).

- **Non-trivial dymamics of M**

$$\left|\frac{c_\perp}{\gamma_c c_\Phi}\right| < \sqrt{\omega} \qquad\qquad \sqrt{\omega} < \left|\frac{c_\perp}{\gamma_c c_\Phi}\right| < 1$$



$\diamond$ **In the branch $\tau > \tau_+$, $\mathcal{V}_{\text{1-loop}} < 0$ and M climbs the potential** as the universe expands. Similar to [Dudas, Kitazawa, Sagnotti,'10]

$\diamond$ **In the branch $\tau_- < \tau < \tau_+$, $\mathcal{V}_{\text{1-loop}} > 0$,** M climbs and then descends its potential. Around the maximum, **a can accelerate, e-fold $\lesssim 1$.** See also [Townsend, Wohlfarth,'03] [Ohta,'03] [Roy,'03] [Emparan, Garriga,'03]

Outline

Including moduli fields

■ On T^{10-d}, $$\frac{SO(10-d,26-d)}{SO(10-d)\times SO(26-d)}$$

$$(G+B)_{Ij} = (G^{(0)}+B^{(0)})_{Ij} + \sqrt{2}\, y_{Ij}\ , \qquad I,j = d,\dots,9$$

$$Y_{Ij} = Y^{(0)}_{Ij} + y_{Ij}\ , \qquad j = 10,\dots,25$$

- $\boldsymbol{y_{Ij}}$ **is the Wilson line of the** $\boldsymbol{j}$**-th** $\boldsymbol{U(1)}$ **along** $\boldsymbol{X^I}$**.**
- **Stringy Scherk Schwarz along** $\boldsymbol{X^d}$ $\Longrightarrow$ $M_{(\sigma)} = \sqrt{G^{dd}}\, M_{\rm s}$

■ In fact, $\boldsymbol{n_{\rm F}}$**,** $\boldsymbol{n_{\rm B}}$ **are functions of** $\boldsymbol{y}$**'s.** Taylor expanding :

$$\mathcal{V}_{\text{1-loop}} = (n_{\rm F}-n_{\rm B})\, v_d\, M^d +$$

$$M^d\,\frac{v_{d-2}}{2\pi}\sum_{j=d+1}^{25}\left(C_{\rm B}^{(j)}-C_{\rm F}^{(j)}\right)\left[(d-1)y_{dj}^2+\frac{1}{G^{dd}}\left(y_{d+1,j}^2+\cdots+y_{9j}^2\right)\right]+\cdots$$

$C_{\rm B}^{(j)}$ is the sum of (charges)^2 of the bosons charged under the j-th $U(1)$. Same for $C_{\rm F}^{(j)}$ with fermions.

■ Let us construct models with **"massive" or "tachyonic" moduli.**

- Usual Scherk Schwarz breaking along X^d :

Bosons are periodic, Fermions antiperiodic $\Longrightarrow$ momenta $m_d + \frac{F}{2}$

Lightest Bosons are massless, lightest Fermions have mass M

$$P_d = m_d + \frac{1}{2}\,F + \cdots + (G+B)_{dj} n_j$$
$$= m_d + \frac{1}{2}\Big[F + 2(G^{(0)} + B^{(0)})_{dj} n_j\Big] + \cdots + \sqrt{2}\, y_{dj} n_j$$

- If $2(G^{(0)} + B^{(0)})_{dj} n_j$ is an odd integer :

Lightest Fermions are massless, lightest Bosons have mass M

- Example : Background $b \in \mathbb{Z}$,

$$(G+B)_{Ij} = \begin{pmatrix} R_d^2 & \frac{b}{2} + \sqrt{2}\, y_{d,d+1} \\ -\frac{b}{2} + \sqrt{2}\, y_{d+1,d} & 1 + \sqrt{2}\, y_{d+1,d+1} \end{pmatrix}$$

$\diamond$ When $R_d \to +\infty$, susy is restored :

For b even, $SU(2)$ enhancement when the y's vanish.

$\diamond$ At finite R_d,

- for *b* even, the bosons of the $SU(2)$ non-Cartan multiplets are massless

$$SU(2) \text{ enhancement : } \left(C_{\rm B}^{(j)} - 0\right) = \mathbf{+16}$$

- for *b* odd, the fermions of charges $\pm\sqrt{2}$ under the $U(1)$ are massless :

$$\left(0 - C_{\rm F}^{(j)}\right) = \mathbf{-16}$$

- The potential of the example is, for all other moduli set to 0

$$\mathcal{V}^{(\sigma)}_{\text{1-loop}} = \big(n_{\rm F} - n_{\rm B} + (-1)^b\, 8\times 2\big)\, v_d\, M^d_{(\sigma)} - (-1)^b\, 16\, \frac{2M^d_{(\sigma)}}{(2\pi)^{\frac{3d+1}{2}}} \sum_k \frac{\cos\!\big(2\pi(2k+1)z\big)}{|2k+1|^{d+1}}\, \mathcal{H}\Big(2\pi|2k+1|\frac{\mathcal{M}}{M_{(\sigma)}}\Big) + \mathcal{O}\Big(\big(cM_{\rm s}M_{(\sigma)}\big)^{\frac{d}{2}} e^{-2\pi c M_{\rm s}/M_{(\sigma)}}\Big)$$

where $\mathcal{M} = \dfrac{\sqrt{2}\,|y_{d+1,d+1}|}{\sqrt{1+\sqrt{2}\,y_{d+1,d+1}}}$

tells us if the $SU(2)$ non-Cartan susy multiplets are light or heavy :

◇ For $\mathcal{M} > M_{(\sigma)}$, exponential suppression of $\mathcal{H}(x) \equiv x^{\frac{d+1}{2}} K_{\frac{d+1}{2}}(x)$.

◇ For $\mathcal{M} < M_{(\sigma)}$, $\mathcal{H}$ has a U-shape.

$$z = \sqrt{2}\left(y_{d,d+1} - \frac{y_{d,d+1} + y_{d+1,d}}{\sqrt{2}\,\big(1+\sqrt{2}\,y_{d+1,d+1}\big)}\, y_{d+1,d+1}\right)$$

◇ The potential oscillates as $\cos(2\pi z)$.

◇ There is a flat direction, $\sqrt{2}\, y_{d+1,d} + \cdots$

Small Wilson line deformations

■ We want to show Quantum No-Scale Regimes exist in presence of Wilson lines. It is enough to show it for small deformations

$$|y_{d+1,d+1}| \ll \sqrt{G^{dd}} \;\Rightarrow\; \text{Not flat directions}\,, \qquad |y_{d,d+1}|,\; |y_{d+1,d}| \ll 1$$

$$S_{\text{1-loop}} = \frac{1}{\kappa^2}\int d^dx\,\sqrt{-g}\,\Big[\frac{\mathcal{R}}{2} - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}(\partial\phi_\perp)^2 - \frac{1}{4}(\partial y_{d+1,d+1})^2 - \frac{G^{dd}}{4}(\partial y_{d,d+1})^2 - \frac{G^{dd}}{4}(\partial y_{d+1,d})^2 + \cdots - \kappa^2\mathcal{V}_{\text{1-loop}}\Big]$$

$$\mathcal{V}_{\text{1-loop}} = e^{d\alpha\Phi} M_{\text{s}}^d\Big[(n_{\text{F}}-n_{\text{B}})\,v_d + (-1)^b\,\frac{8}{\pi}v_{d-2}\,\Big((d-1)y_{d,d+1}^2 + \frac{y_{d+1,d+1}^2}{G^{dd}}\Big)\Big] + \cdots$$

$$\text{where} \qquad G^{dd} = e^{\frac{2}{\alpha}\Phi}\,e^{-\frac{2}{\sqrt{d-1}}\phi_\perp}$$

■ Look for Quantum No-Scale Regimes

$$a(t) \underset{t\to+\infty}{\longrightarrow} +\infty \quad \text{or} \quad a(t) \underset{t\to t_-}{\longrightarrow} 0\,, \qquad \kappa^2 M_{\rm s}^d\, e^{d\alpha\Phi} = \mathcal{O}\left(\frac{H^2}{a^{K_\pm}}\right)\,, \quad \pm K_\pm > 0$$

- $\boldsymbol{a \sim \#|t - t_\pm|^{\frac{1}{d-1}}}$ (where $t_+{=}0$)

- $y_{d,d+1}$ and $y_{d+1,d}$ have non-trivial friction term.

$y_{d,d+1}$ has a positive or negative mass term, irrelevant asymptotically

$$\dot{y}_{d+1,d} = \frac{\#}{a^{d-1}G^{dd}}\,, \qquad \dot{y}_{d,d+1} \sim \frac{\#}{a^{d-1}G^{dd}}$$

⋄ In l.h.s. of Friedmann Eq, $H^2 \sim \dfrac{\#}{t - t_\pm}$

⋄ In r.h.s., kinetic energy $\sim \dfrac{\#}{(t-t_\pm)G^{dd}}$ and potential negligible

$\Longrightarrow$ $G^{dd} \sim \#(t-t_\pm)^{J_\pm}\,,$ $\boldsymbol{\pm J_\pm > 0}$ *i.e.* $M_{(\sigma)}$ increases

$\Longrightarrow$ $\boldsymbol{y_{d+1,d},\ y_{d,d+1}}$ **converge to arbitrary constants**

• $y_{d+1,d+1}$ has a positive or negative mass term, irrelevant asymptotically

$$\dot{y}_{d+1,d+1} \sim \frac{2\, \boldsymbol{c_y}}{a^{d-1}} \quad \Longrightarrow \quad |y_{d+1,d+1}| \sim \#|\ln(t-t_\pm)| \ll \sqrt{G^{dd}}$$

$\boldsymbol{y_{d+1,d+1}}$ effectively approaches the minimum or maximum of $\boldsymbol{\mathcal{V}_{\text{1-loop}}}$ (!)

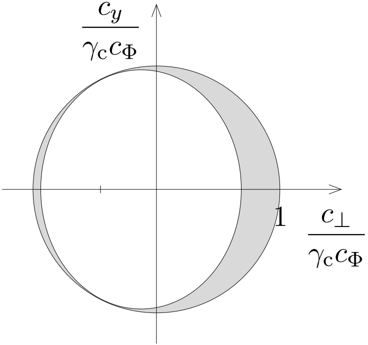
• $\phi_\perp$ and Φ couple to the kinetic and mass terms of y's, but are free asymptotically

$$\dot{\phi}_\perp \sim \sqrt{2}\, \frac{\boldsymbol{c_\perp}}{a^{d-1}}\,, \qquad \alpha\dot{\Phi} + \frac{\alpha^2}{2}\, d(d-2)\, H \sim \frac{\boldsymbol{c_\Phi}}{a^{d-1}}$$

• **$\boldsymbol{K_\pm}$ and $\boldsymbol{J_\pm}$ now depend on $\boldsymbol{c_\perp/c_\Phi}$ and $\boldsymbol{c_y/c_\Phi}$**

◇ The **subcritical region is now a disk**

◇ **Restrict to regions $\pm J_{\pm} > 0$.**



$\boldsymbol{H^2 \sim \# \, \dot{\phi}_\perp^2 \sim \# \, \dot{\Phi}^2 \sim \# \, \dot{y}^2_{d+1,d+1} \sim \frac{\#}{a^{2(d-1)}}}$**, which dominate $\mathcal{V}_{\text{1-loop}}$ and the kinetic energies of $y_{d,d+1}$, $y_{d+1,d}$**

◇ The $a \to \infty$ **QNSR is perturbative.** The $a \to 0$ almost always.

◇ The true left shell is extremely small (10^{-3}–10^{-2}). **Fine tuning ?**

Numerical simulations

■ **For the QNSR $a \to \infty$, $D = 4$**

$$c_\perp^{\text{sim}} = \frac{a^{d-1}}{\sqrt{2}}\,\dot{\phi}_\perp\,, \qquad c_\Phi^{\text{sim}} = a^{d-1}\left(\alpha\dot{\Phi} + \frac{\alpha^2}{2}\,d(d-2)H\right), \qquad c_y^{\text{sim}} = \frac{a^{d-1}}{2}\,\dot{y}_{55}$$

are expected to converges to $c_\perp$, c_Φ, c_y.

■ **If we start in the shell, with low velocities,** they freeze : **ok**

- Climbing effect of M when $\mathcal{V}_{\text{1-loop}} < 0$: **ok**

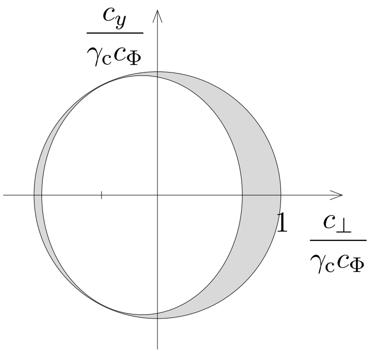
- No destabilization of the background, even when the y's are at maxima : **ok**

■ **What if we start far from the shell, or high velocitites ?** The y's are expected to explore large distances in moduli space.
We use the full potential to simulate $a(t)$, $\Phi(t)$, $\phi_\perp(t)$, $y_{45}(t)$, $y_{54}(t)$, keeping $y_{55} \equiv 0$.

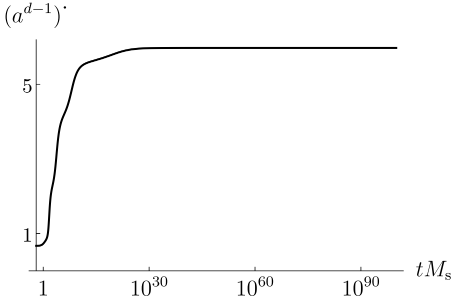
• **if $\mathcal{V}_{\text{1-loop}} < 0$ for some y_{45}** (y_{54} is a flat direction), **the initially expanding flat universe stops growing and then collapses** (unless we sit in the shell).

$\Longrightarrow$ **In presence of Wilson lines, the set of initial conditions yielding a forever expanding universe is drastically reduced !**

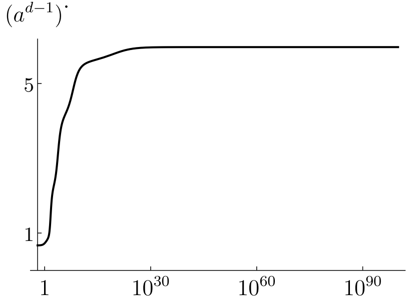
• **If $\mathcal{V}_{\text{1-loop}} \geq 0$ for all y_{45}, $\left|\frac{c_\perp^{\text{sim}}}{\gamma_c c_\Phi^{\text{sim}}}\right|_t$ is always attracted towards the shell where it stops.**



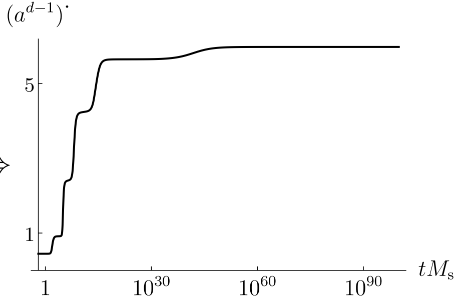
Remind $a^{d-1} \sim \# \, t$



◇ For small $\dot{y}_{45}(0)$, $\dot{y}_{54}(0)$:



$\Longrightarrow$



(1) As in the toy model ⇒ attracted to a QNSR ⇒ 1st plateau.

(2) Since $J_+ < 0$, the kinetic energies of y_{45}, y_{54} end up dominating ⇒ leave the 1st plateau.

(3) We showed that this domination cannot last for good (no such asymptotic solution) ⇒ y_{45}, y_{54} have to release their kinetic energy ⇒ Back to step (1), until $J_+ > 0$.

Conclusion

■ We have shown that the **flat cosmological evolutions found at the quantum level in generic No-Scale Models ($n_F - n_B \neq 0$) are identical (asymptotically) to those found for $n_F - n_B = 0$ or classically.**

$\Longrightarrow$ Restoration of the classical No-Scale Structure

■ **Automatic when $\mathcal{V}_{\text{1-loop}} \geq 0$.**
Requires the initial conditions to be tuned in a **tiny region of the phase space when $\mathcal{V}_{\text{1-loop}}$ can be negative.**

■ **The kinetic energies of M, the dilaton and Wilson lines y_{ij} (i, j not in the Scherk-Schwarz directions) dominate $\mathcal{V}_{\text{1-loop}}$ and the kinetic energies of the Wilson lines having one index in a Scherk-Schwarz direction.**

■ **The Wilson lines y_{ij} can be "trapped" at minima, maxima or saddle points (!) of $\mathcal{V}_{\text{1-loop}}$. The remaining moduli freeze at arbitrary values.**