Supersymmetry breaking in Heterotic Strings and corrections to gauge couplings

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Based on work with
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Non-Supersymmetric String Theory

The most well known string theories are supersymmetric

However, the construction of non-supersymmetric string theories is not a new subject

- since the early days of string theory: e.g. temperature
- O(16)×O(16) heterotic string + many more examples

Nevertheless, non-supersymmetric string theories have not been exhaustively studied, due to technical difficulties in taming their radiative corrections

More recently: string phenomenology

progress in studying radiative corrections to gauge couplings

made possible by new mathematical techniques for studying string loop amplitudes

Angelantonj, IF, Tsulaia '14

Angelantonj, IF, Pioline '11,'12,'13,'15
if SUSY is broken in String Theory:

- quantum corrections to couplings in the effective action
- including the scalar potential

ALL perturbative states run in the loop

- oscillators
- Kaluza-Klein, winding states

To really make contact with low energies (Standard Model++)

one must take into account radiative corrections

\[
\frac{16\pi^2}{g^2(\mu)} = \frac{16\pi^2}{g^2(M_s)} + b \log \frac{M_s^2}{\mu^2} + \Delta
\]

threshold correction
Non-supersymmetric constructions

Two fundamental questions

- Tachyonic instabilities: either explicit, or spontaneous (Hagedorn, …)
- Destabilisation of the classical vacuum: one loop back-reaction

Tachyons simply mean that we are expanding the theory around an unstable point. If we were able to quantise string theory around the minimum, no tachyons would be found.

Some specialised constructions exist where would-be tachyons are projected out of the spectrum.

Angelantonj, Cardella, Irges ’06
Angelantonj, Kounnas, Partouche, Toumbas ’09
IF, Kounnas ’09
IF, Kounnas, Toumbas ’10
IF, Kounnas, Partouche, Toumbas ’11
Non-supersymmetric constructions

Further questions

• How do we break supersymmetry in String Theory?
• What is the SUSY breaking scale? what determines it?
• What happens to other moduli in the theory? (such as radii, etc)
A way to break supersymmetry

(stringy) Scherk-Schwarz mechanism

- Flat gauging of supergravity
- Spontaneous breaking of SUSY with exactly tractable worldsheet description
- Freely-acting orbifolds
Scherk Schwarz mechanism

Deformation of vertex operators / fields by symmetry $Q$

$$\Phi(X_5 + 2\pi R) = e^{iQ} \Phi(X_5)$$

Kaluza-Klein spectrum of charged states is shifted

$$\Phi(X_5) = e^{iQX_5/2\pi R} \sum_{m \in \mathbb{Z}} \Phi_m e^{imX_5/R}$$

Choose $Q=F$ (spacetime fermion number)

Assigns different boundary conditions (& masses) to states within the same supermultiplet: spontaneous breaking of supersymmetry

Breaking scale $\sim 1/R$, tied to the size of compact dimensions
What about the potential?

- Scherk-Schwarz breaking exhibits no-scale structure
- The scale of SUSY breaking is not determined at tree level

\[
V_{\text{tree}} = 4 \left( \frac{|T/2 - U|^2}{T_2 U_2} - 2 \right) \varphi^2 + 16 \left( \frac{|T/2 - U|^2}{T_2 U_2} + 1 \right) \varphi^4
\]

\[
m_{3/2} = \frac{|U|}{\sqrt{T_2 U_2}} \quad \text{T, U are moduli at tree level}
\]

- Loop corrections to the effective potential may (de)stabilise the no-scale moduli
- Dynamical determination of SUSY breaking scale

What is the morphology of the one loop effective potential in such models?
What about the potential?

- Fixed points under stringy symmetries (T-dualities) correspond to local extrema of the potential
  \[ V_{\text{loop}}(T_2) = V_{\text{loop}}(\ell_s^4/T_2) \]
- Natural scale in this problem: the string scale

**typical form of the 1-loop potential**

- SUSY is broken at the string scale
- No solution to hierarchy problem
- Huge negative cosmological constant
- Tachyons
What about the potential?

- Can we construct solutions with the opposite behaviour?
- Local maximum induces spontaneous decompactification

$V(T_2) \sim \ell_s^2$

modulus rolls away from the self-dual point

$T_2 \gg \ell_s^2$

non-perturbative effects

SUSY is recovered asymptotically

Opens the possibility for low scale SUSY breaking $m_{3/2} \sim 1/\sqrt{T_2}$

Favours large volume: no tachyons
What about the potential?

- Can we construct solutions with the opposite behaviour?
- Local maximum induces spontaneous decompactification

\[
V(T_2) \sim \ell_s^2
\]

modulus rolls away from the self-dual point

\[
T_2 \gg \ell_s^2
\]

SUSY is recovered asymptotically

\[
V_{\text{loop}}(T_2) \sim \frac{n_F - n_B}{T_2^2}
\]

asymptotically

For SUSY breaking at TeV range, the potential is still too large
What about the potential?

Possible way out: \( n_B = n_F \) at the massless level

\[
V_{\text{one-loop}} \sim \frac{n_F - n_B}{R^4} + \sum_N c(N) \sum_{m_i} \frac{U_2^{3/2}}{|m_1 + \frac{1}{2} + U m_2|^3} K_3 \left( 2\pi \sqrt{\frac{N T_2}{U_2}} \left| m_1 + \frac{1}{2} + U m_2 \right| \right)
\]

exponentially suppressed vacuum energy for large volume \( T_2 \gg 1 \)

\( V(T_2) \)

\( T_2 \gg \ell_s^2 \)

\( \sim \ell_s^2 \)

Itoyama, Taylor '87
Antoniadis '90
Abel, Dienes, Mavroudi '15, '16
Kounnas, Partouche '15, '16, '17

“super no-scale models”

- low SUSY breaking scale
- large volume, no tachyons
- small cosmological constant
- small back reaction
What about the potential?

**Question**: Is it possible to construct such chiral models?

- **Answer**: YES

**BUT**

although being necessary for suppressing the value of the cosmological constant, the condition for bose-fermi degeneracy is NOT sufficient

it turns out that **non level-matched states** around self-dual points crucially affect the shape of the potential, including its sign!
Example

Example: net chirality $12$ and $n_F = n_B$ at the generic point

$$T^2 \times T^2 \times T^2 / (\mathbb{Z}_2)^6$$

$$X^{1,2} \ X^{3,4} \ X^{5,6}$$

$$\mathbb{Z}_2^{(1)} : \ X^{1,2,5,6} \rightarrow -X^{1,2,5,6}$$

$$\mathbb{Z}_2^{(2)} : \ X^{3,4,5,6} \rightarrow -X^{3,4,5,6}$$

$$\mathbb{Z}_2^{(3)} : \ (-1)^{F_{\text{a.t.}} + F_2} \delta_1 \quad \delta_1 : \{X_1 \rightarrow X_1 + \pi R_1\}$$

$$\mathbb{Z}_2^{(4)} : \ (-1)^{F_2} \delta_3 \quad \delta_3 : \{X_3 \rightarrow X_3 + \pi R_3\}$$

$$\mathbb{Z}_2^{(5)} : \ (-1)^{F_1 + F_2} \delta_5 \quad \delta_5 : \{X^5 \rightarrow X^5 + \pi R_5\}$$

$$\mathbb{Z}_2^{(6)} : \ (-1)^{F_1} r \quad r : (0^8; 0^4, \frac{1}{2}^2)$$

+ a particular choice of discrete torsions

$$\varepsilon(2, 3), \ \varepsilon(2, 5), \ \varepsilon(4, 5), \ \varepsilon(5, 6)$$
Example: net chirality 12 and $n_F = n_B$ at the generic point

Figure 6: Numerical reconstruction of the one-loop potential for Model C as a smooth function of $T_2$ and $U_2$ within the allowed parameter space defined by eq. 6.10. To this end, we exploited the equivalence between fermionic and orbifold constructions at special points in moduli space, in order to scan a random sample of $10^6$ models subject to certain criteria, such as the presence of chiral matter and an observable SO(10) gauge group factor. Working in the interplay between the two formulations, it was possible to study the contributions of various states to the one-loop effective potential and derive a set of conditions (5.15) that guarantee its positivity. Our central observation is that massive and even non-level matched states play a significant role in determining the morphology of the effective potential around special self-dual points. This result, although counter-intuitive from a field theoretic perspective, was central to our analysis and resulted in the construction of the explicit example 'Model C' defined in (6.5) that illustrates the desired behaviour for the one-loop potential.

Of course, our present analysis is only a first step in this very interesting direction and there are several open questions that deserve future investigation. On the one hand, the specific construction of Model C is by no means unique but only a particular solution to our computer-aided scan in a random sample of models. For $T_2 > 2.20$, stabilisation of $U_2$ at its fermionic value and the potential is dynamically stable.
Example

These are the first examples of heterotic string models with a dynamical attraction to low SUSY breaking scales

- stabilisation of other moduli not participating in the breaking, e.g. U2
- dynamical protection against tachyons
- exponentially small cosmological constant
- chirality and SO(10) GUT gauge group, as a first example

What about radiative corrections to other couplings?

- Gravitational couplings: $R$, $R^2$
  
  Kiritsis, Kounnas '95-'99
  I.F. 2016

- Gauge couplings
  

Until very recently, nothing was known about how such couplings renormalise in non-supersymmetric string theory
Gauge coupling corrections
One loop corrections

Running coupling associated to gauge group $G$

$$\frac{16\pi^2}{g^2_G(\mu)} = \frac{16\pi^2}{g^2_s} + \beta_G \log \frac{M_s^2}{\mu^2} + \Delta_G$$

threshold correction

$$\frac{16\pi^2}{g^2_G} = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^2} \int_{\text{torus}} d^2 z \langle \mathcal{V}^a(z, \bar{z}) \mathcal{V}^b(0) \rangle_{\text{CFT}}$$

one loop correction to gauge couplings

$(z, \bar{z}) \in \Sigma_1$
Moduli Dependent Contributions

Focus on their dependence on the compactification moduli in the case of unbroken SUSY

\[
\Delta_1 - \Delta_2 = b_{12} \int \frac{d^2 \tau}{\tau_2} \sum_{\text{states}} e^{-\pi t M^2} = -b_{12} \log T_2 U_2 |\eta(T)\eta(U)|^4
\]

moduli dependence through KK and winding excitations

\[
M^2 = \left( \frac{m}{R} \right)^2 + \left( \frac{n R}{\ell_s^2} \right)^2
\]

supersymmetric universality

Dixon, Kaplunovsky, Louis ’91
Moduli Dependent Contributions

Focus on their dependence on the compactification moduli

in the case of spontaneously broken SUSY

\[ \Delta_1 - \Delta_2 = \alpha \log T_2 U_2 |\eta(T)\eta(U)|^4 \]
\[ + \beta \log T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4 \]
\[ + \gamma \log |\hat{j}_2(T/2) - \hat{j}_2(U)|^4 |j_2(U) - 24|^4 \]

non-supersymmetric universality

- model independent form

- model dependence only in constant coefficients \(\alpha, \beta, \gamma\)

fully universal form later found for \(\Delta\) itself (not only differences) and also for gravitational thresholds by exploiting modular symmetries of string theory
The decompactification problem

Typically, the large volume limit of gauge thresholds is dominated by 6d

\[ \Delta_a = -\frac{k_a}{48} Y + \hat{\beta}_a \Delta + \ldots \]

“universal part”

“running part”

N=2 beta function coeff.
governing 6d physics

typically, one absorbs \( Y \) into a redefinition of the tree level string coupling

in the large volume scenario \( T_2 \gg 1 \)

\[ \Delta = -\log T_2 U_2 |\eta(T)\eta(U)|^4 \rightarrow \frac{\pi}{3} T_2 - \log T_2 + \ldots \]

• if \( \beta > 0 \), effectively decouples

• if \( \beta < 0 \), strong coupling regime

decompactification problem: when \( T_2 \gg 1 \), couplings effectively behave as 6d ones
A solution of the decompactification problem

A new proposal was put forward very recently

\[ \Delta_a = -\frac{k_a}{48} Y + \hat{\beta}_a \Delta + \ldots \]

Observation: It's not only the “running part” \( \Delta \) that grows linearly in the large volume limit, but also the “universal part” \( Y \)

Split the “universal part” \( Y \) into its linear growth and absorb the rest into the tree level coupling

\[ \Delta_a = \left( \frac{\beta_a}{3} - k_a \right) \pi T_2 + \text{logarithmic} \]

Use the universal part to cancel the linear growth of the running part

In other words, choose the \( N=2 \) subsector (sensitive to Kaluza-Klein and winding) such that it exactly cancels the linear volume term of the “universal part”

The actual chiral matter of the theory comes from \( N=1 \) sectors, is moduli independent and it has the desired logarithmic running!
A solution of the decompactification problem

These conditions can be imposed at the 6d limit of the theory (which produces the N=2 subsector after compactification)

For an SO(2n) gauge group factor at level one

\[ N_V + 2^{n-4} N_S = 2n + 4 \]

After cancellation of the linear volume term, the gauge couplings run logarithmically

\[ \frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_s^2} + \beta_a \log \frac{M_s^2}{\mu^2} + \beta'_a \log \left( \frac{2e^{1-\gamma} M_{KK}^2}{3\pi \sqrt{3} M_s^2} \right) + \ldots \]

- determined in terms of the twisted chiral N=1 matter
- contribution of KK-winding towers from N=2 exact + N=2→0 remnants and N=4→0, N=4→2 sectors

Many explicit heterotic super no-scale models with chirality have already been constructed, where the decompactification problem is absent, according to this large volume scenario

Preliminary scan of $10^8$ models \( \sim 40\% \) do not suffer from the decompactification problem
It is now a question of model building to construct theories with all desired properties and realistic values.

Suppose we could find an idealised string model with $\beta$'s and $k$'s equal to their Standard Model values, and $(\beta_1', \beta_2', \beta_3') = (-15/2, -43/6, -23/3)$.

there exist regions in parameter space consistent with experimental bounds for the Weinberg angle and $\alpha$-strong.
Comprehensive scan of $10^8$ models, out of which 72896 satisfy initial constraints.

$W_0^G = 0 \quad V_0 > 0$

Number of models versus net number of generations for models with $W_0^G = 0$ and $V_0 > 0$.

Super no scale models with positive potential.

40% do not have the decompactification problem.
The fate of the Planck mass

Does the Einstein-Hilbert term renormalise at 1-loop?

For unbroken SUSY, this question was answered by Kiritsis and Kounnas in the ’90s

For spontaneously broken SUSY

R term is still protected against moduli dependent corrections

in type II theories, it has a constant, topological 1-loop correction (regardless of SUSY)

\[ \frac{\chi(X)}{4} \]

actually, this is a property of string theory on Minkowski space

in curved spaces, non-trivial corrections do arise!
The decompactification problem

An interesting way of curing the 6d linear growth is to remove the N=2 subsector, i.e. replace the theory with 8-supercharges that is obtained in the 6d limit, by a theory with 16-supercharges (or a spontaneously broken version of it)

i.e. for high enough energy scales, an effective N=4 theory is recovered and gauge couplings do not run

This indeed removes the linear growth and “solves” the decompactification problem

However, the examples presented in the literature so far, based on Z2 orbifolds, spoil the requirement for the presence of chiral matter

see follow-up talk by Carlo Angelantonj

Kiritsis, Kounnas, Petropoulos, Rizos 1996
Faraggi, Kounnas, Partouche 2014
Outlook

- One-loop radiative corrections to gauge couplings in heterotic strings
- Supersymmetry spontaneously broken by Scherk-Schwarz flux

- Exact Universality for gauge and gravitational thresholds
- Planck Mass does not renormalise
- Chiral super no-scale models with spontaneous decompactification
- The decompactification problem is not a problem, but a selection criterion
- String model building?
- Other couplings?
Thank you!