Massive Graviton Geons and Dark Matter

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Ghost Free Bigravity Theory

ghost-free bigravity theory Two metrics *g*, *f*

$$S[g,f,\mathrm{matter}] = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}\,R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\,\mathcal{R}(f)$$

$$-\frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\,\mathscr{U}[g,f] + S_g^{[\mathrm{m}]}[g,\text{g-matter}] + S_f^{[\mathrm{m}]}[f,\text{f-matter}]$$

Twin Matter Fluids

Interaction term $\kappa^2 = \kappa_g^2 + \kappa_f^2$

$$\mathscr{U} = \sum_{k=0}^{4} b_k\,\mathscr{U}_k(\gamma),$$

$\{b_k\}$: 5 coupling constants

$\gamma^\mu{}_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$ λ_A : eigenvalues of $\gamma^\mu{}_\nu$

$$\mathscr{U}_0(\gamma) = -\frac{1}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 1 \qquad \mathscr{U}_1(\gamma) = -\frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\nu\rho\sigma}\gamma^\mu{}_\alpha = \sum_A \lambda_A$$

$$\mathscr{U}_2(\gamma) = -\frac{1}{2!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\rho\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta = \sum_{A<B}\lambda_A\lambda_B$$

$$\mathscr{U}_3(\gamma) = -\frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma = \sum_{A<B<C}\lambda_A\lambda_B\lambda_C$$

$$\mathscr{U}_4(\gamma) = -\frac{1}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\gamma^\mu{}_\alpha\gamma^\nu{}_\beta\gamma^\rho{}_\gamma\gamma^\sigma{}_\delta = \lambda_0\lambda_1\lambda_2\lambda_3$$

a flat space is a solution

m : graviton mass

$$b_0 = 4c_3 + c_4 - 6, \quad b_1 = 3 - 3c_3 - c_4,$$
$$b_2 = 2c_3 + c_4 - 1, \quad b_3 = -(c_3 + c_4), \quad b_4 = c_4.$$

$\{c_3\,, c_4\}$: 2 coupling constants

Basic equations Two Einstein equations

$$G_{\mu\nu} = \kappa_g^2 \left[T^{[\gamma]}_{\mu\nu} + T^{[\mathrm{m}]}_{\mu\nu} \right] \qquad \mathcal{G}_{\mu\nu} = \kappa_f^2 \left[\mathcal{T}^{[\gamma]}_{\mu\nu} + \mathcal{T}^{[\mathrm{m}]}_{\mu\nu} \right]$$

$$m_g^2 = \frac{\kappa_g^2}{\kappa^2} m^2$$
$$m_f^2 = \frac{\kappa_f^2}{\kappa^2} m^2$$

Energy-momentum tensor from interactions

$$\kappa_g^2 T^{[\gamma]\mu}{}_{\nu} = m_g^2 (\tau^\mu{}_\nu - \mathscr{U}\, \delta^\mu_\nu) \qquad \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_{\nu} = -m_f^2 \frac{\sqrt{-g}}{\sqrt{-f}} \tau^\mu{}_\nu \,,$$

$$\tau^\mu{}_\nu = \{b_1 \mathscr{U}_0 + b_2 \mathscr{U}_1 + b_3 \mathscr{U}_2 + b_4 \mathscr{U}_3\}\gamma^\mu{}_\nu - \{b_2 \mathscr{U}_0 + b_3 \mathscr{U}_1 + b_4 \mathscr{U}_2\}(\gamma^2)^\mu{}_\nu$$
$$+\{b_3 \mathscr{U}_0 + b_4 \mathscr{U}_1\}(\gamma^3)^\mu{}_\nu - b_4 \mathscr{U}_0 (\gamma^4)^\mu{}_\nu$$

$$\overset{(g)}{\nabla}_\mu T^{[\mathrm{m}]\mu}{}_\nu = 0, \qquad \overset{(f)}{\nabla}_\mu \mathcal{T}^{[\mathrm{m}]\mu}{}_\nu = 0$$

Conservation equations

Bianchi id.

$$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0. \qquad \overset{(f)}{\nabla}_\mu \mathcal{T}^{[\gamma]\mu}{}_\nu = 0$$

Homothetic metrics:

$$f_{\mu\nu} = K^2 g_{\mu\nu} \Rightarrow \gamma^\mu{}_\nu = (\sqrt{g^{-1}f})^\mu{}_\nu = K\,\delta^\mu{}_\nu$$

$$\kappa_g^2 T^{[\gamma]\mu}{}_\nu = -\Lambda_g(K)\delta^\mu{}_\nu\,, \quad \kappa_f^2 \mathcal{T}^{[\gamma]\mu}{}_\nu = -\Lambda_f(K)\delta^\mu{}_\nu$$

$$\Lambda_g(K) = m_g^2\left(b_0 + 3b_1\,K + 3b_2\,K^2 + b_3\,K^3\right)$$

$$\Lambda_f(K) = m_f^2\left(b_1/K^3 + 3b_2/K^2 + 3b_3/K + b_4\right)$$

$$\overset{(g)}{\nabla}_\mu T^{[\gamma]\mu}{}_\nu = 0. \qquad \overset{(f)}{\nabla}_\mu \mathcal{T}^{[\gamma]\mu}{}_\nu = 0 \Rightarrow$$

K: constant

$$\Rightarrow \begin{array}{c}\Lambda_g(b_i)\\ \Lambda_f(b_i)\end{array}$$

cosmological constants

"GR" with a cosmological constant

$$G_{\mu\nu} + \Lambda_g g_{\mu\nu} = \kappa_g^2 T^{[\mathrm{m}]}_{\mu\nu}$$

$$\Lambda_g(K) = K^2\Lambda_f(K)$$: quartic equation for *K*

$$\mathcal{T}^{[\mathrm{m}]}_{\mu\nu} = K^2 T^{[\mathrm{m}]}_{\mu\nu}$$

perturbations around a homothetic solution (vacuum)

$$g_{\mu\nu} = \overset{(0)}{g}_{\mu\nu} + \epsilon h_{\mu\nu}\,, \quad f_{\mu\nu} := K^2 \tilde{f}_{\mu\nu} = K^2 \left(\overset{(0)}{g}_{\mu\nu} + \epsilon k_{\mu\nu} \right)$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}_{\rho\nu}(h) - \overset{(0)}{R}{}^{\rho(\mu} h_{\nu)\rho} = -\frac{m_g^2}{4}(b_1 K + 2b_2 K^2 + b_3 K^3)\left[2(h^\mu{}_\nu - k^\mu{}_\nu) + (h-k)\delta^\mu{}_\nu\right]$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}_{\rho\nu}(k) - \overset{(0)}{R}{}^{\rho(\mu} k_{\nu)\rho} = +\frac{m_f^2}{4K^2}(b_1 K + 2b_2 K^2 + b_3 K^3)\left[2(h^\mu{}_\nu - k^\mu{}_\nu) + (h-k)\delta^\mu{}_\nu\right]$$

linear combinations

$$\psi_{\mu\nu} := m_f^2 h_{\mu\nu} + K^2 m_g^2 k_{\mu\nu}$$

$$\varphi_{\mu\nu} := h_{\mu\nu} - k_{\mu\nu}$$

$$m_g^2 = \frac{\kappa_g^2}{\kappa^2} m^2$$

$$m_f^2 = \frac{\kappa_f^2}{\kappa^2} m^2$$

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}_{\rho\nu}(\psi) - \overset{(0)}{R}{}^{\rho(\mu} \psi_{\nu)\rho} = 0$$

massless mode

$$\overset{(0)}{g}{}^{\mu\rho} \overset{(1)}{R}_{\rho\nu}(\varphi) - \overset{(0)}{R}{}^{\rho(\mu} \varphi_{\nu)\rho} = -\frac{1}{4} m_{\text{eff}}^2 \left[2\varphi^\mu{}_\nu + \varphi \delta^\mu{}_\nu\right]$$

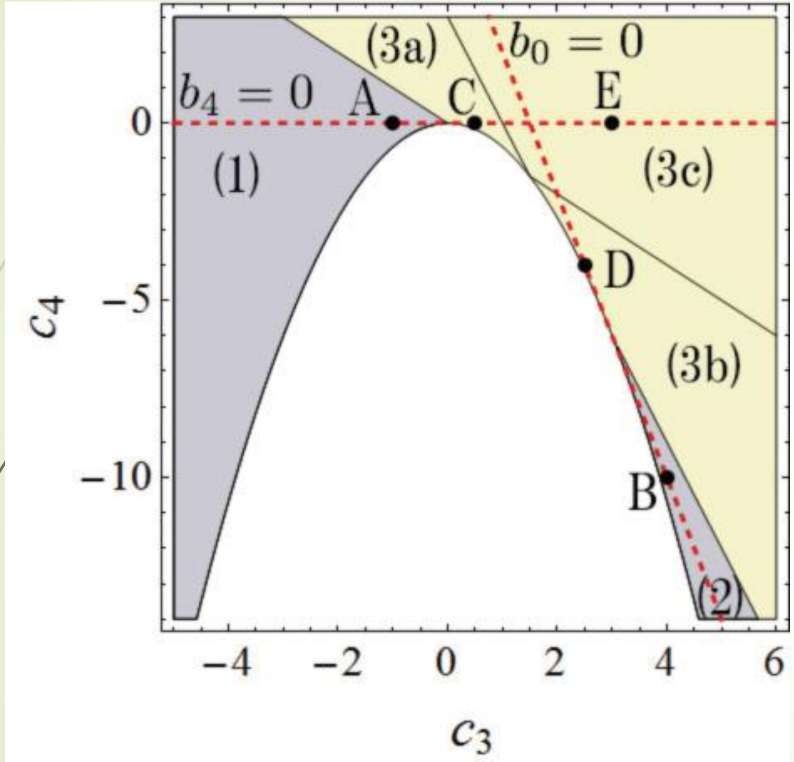
massive mode

graviton mass

$$m_{\text{eff}}^2 = \left(m_g^2 + \frac{m_f^2}{K^2} \right)(b_1 K + 2b_2 K^2 + b_3 K^3)$$

Homothetic solution without matter

Parameter region for dS



1 dS
1 M
2 AdS

dS spacetime is an attractor solution

It could be the present acceleration of the universe if $m \sim 10^{-33}$ eV

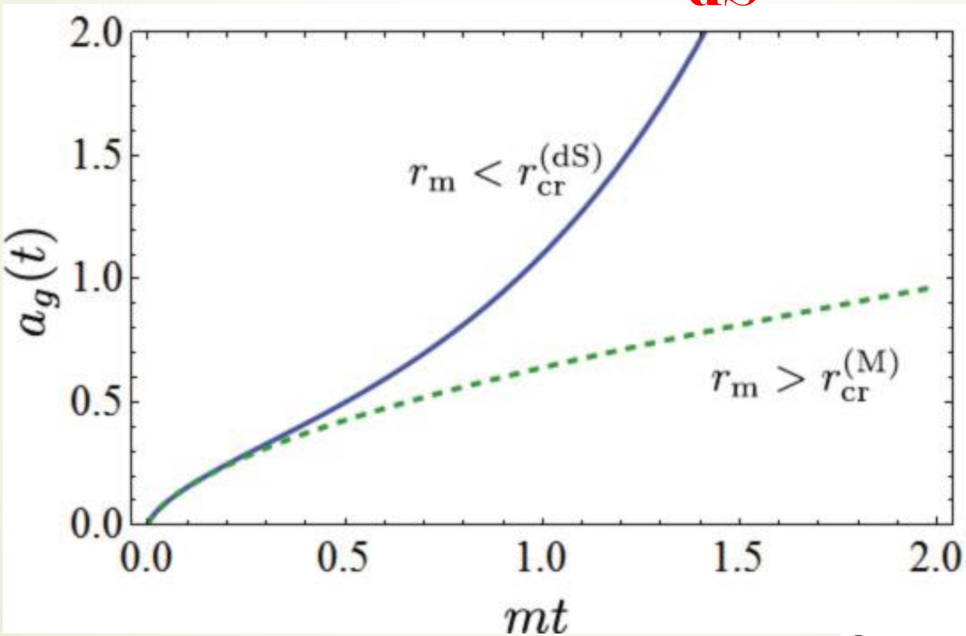
There are two attractor solutions

dS: de Sitter

M: matter dominant universe

The fate depends on the ratio of two matter fluids $r_{\rm m} \propto \left.\frac{\rho_f}{\rho_g}\right|_0$

dS



M

$c_3 = 4\,, \quad c_4 = -10$

Effective Friedmann Equation

$$B = \frac{a_f}{a_g}$$

Near an attractor point (B_{dS}),

$$\kappa_g^2 \left[\rho_g^{[\gamma]}(\tilde{B}) + \rho_g(a_g)\right] - {\kappa_f}^2 \tilde{B}^2 \left[\rho_f^{[\gamma]}(\tilde{B}) + \rho_f(a_f)\right] = 0$$

$$\Rightarrow \quad \tilde{B} - \tilde{B}_{dS} \propto \frac{\alpha}{a_g^3} + \frac{\beta}{a_f^3} + O\left(\frac{1}{a_g^6}\right)$$

$$\Downarrow$$

$$H_g^2 + \frac{k}{a_g^2} \approx \frac{\Lambda_g}{3} + \frac{\kappa_{\rm eff}^2}{3}\left[\rho_g + \rho_{\rm D}\right]$$

$$\kappa_{\rm eff}^2 = \kappa_g^2 \left[1 - \frac{1}{\left(1 - \frac{2\Lambda_g}{3m_{\rm eff}^2}\right)\left(1 + \frac{\kappa_f^2}{\tilde{B}_{dS}^2 \kappa_g^2}\right)}\right]$$

$$\rho_{\rm D} = \frac{\kappa_f^2 \tilde{B}_\ell^2}{\kappa_g^2 \left[\left(1 - \frac{2\Lambda_g}{3m_{\rm eff}^2}\right)\left(1 + \frac{\kappa_f^2}{\tilde{B}_{dS}^2 \kappa_g^2}\right) - 1\right]} \boxed{\rho_f}$$

twin matter

= DM

Dark Matter

twin matter is dark matter ? $\rho_D/\rho_g \sim 5$ (if ρ_g is baryon)

Model	ρ_D/ρ_g
B	$0.16261\, r_m$
D	$0.32278\, r_m$
E	$44.9613\, r_m$

X

○

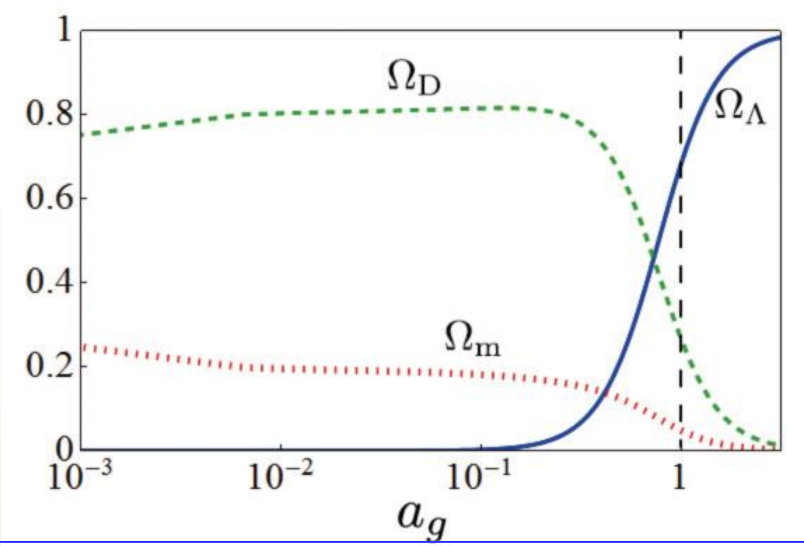
attractor condition

$r_m < r_m^{(cr)}$

Model H

$c_3 = -4\,, \quad c_4 = -10$

$$\frac{\kappa_f^2}{\kappa_g^2} = 1000\,, \quad r_m = 3000$$



Twin matter can be "dark matter" in Friedmann eq.

Twin matter can really be dark matter ?

Dark matter is required in three situations:

- “Dark matter” in Friedmann equation ○
- Dark matter at a galaxy scale

  rotation curve
- Dark matter in structure formation

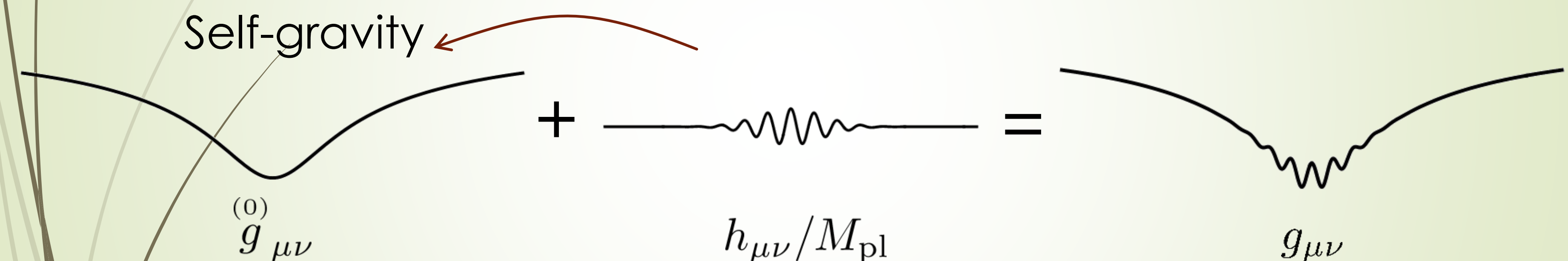
Massive Gravity Geon in Bigravity Theory

: Another possibility of dark matter

GWs have their gravitational energy!

Due to the nonlinearities of the Einstein equation,
GWs (=perturbations) themselves change the background geometry.

Is it possible to realize self-gravitating gravitational waves?



Gravitational "Geons"

The original idea of "geon" is a gravitational electromagnetic entity.
= a realization of classical "object" by gravitational attraction.

Wheeler, 1955.

Gravitational Geons

Gravitational geons are singularity-free time periodic vacuum solutions to GR.

Brill and Hartle, 1964, Anderson and Brill, 1997.

not stable and decay in time.

Gibbons and Stewart, 1984.

Gravitational geons

This may not be the case in modified gravity.

Geons may exist beyond GR?

We consider gravitational geons in bigravity theory.

Graviton $T^{\mu\nu}$ in Bigravity

Assuming $|\partial^2 g_{\mu\nu}| \ll m^2$ (no Vainshtein effect) and taking Isaacson average,

we find the Einstein and Klein-Gordon equations

$$G^{\mu\nu}[\overset{(0)}{g}] \simeq \frac{1}{M_{\rm pl}^2}\left(\langle T_{\rm gw}^{\mu\nu}\rangle_{\rm low} + \langle T_G^{\mu\nu}\rangle_{\rm low}\right)$$

$$\Box h_{\mu\nu} \simeq 0\,, \quad (\Box - m^2)\varphi_{\mu\nu} \simeq 0 \qquad \text{+ TT conditions}$$

where $T_{\rm gw}^{\mu\nu} \sim (\partial h_{\mu\nu})^2\,, \quad T_G^{\mu\nu} \sim (\partial\varphi_{\mu\nu})^2 + m^2\varphi_{\mu\nu}^2$

$$M_{\rm pl} = \frac{\kappa}{\kappa_g \kappa_f}\,, M_G = \frac{\kappa}{\kappa_g^2}$$

The metrics are given by

$$g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} + \frac{\varphi_{\mu\nu}}{M_G}\,, \quad f_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} - \frac{\varphi_{\mu\nu}}{\alpha M_G}\,,$$

$$(\alpha = M_{\rm pl}^2/M_G^2)$$

We can ignore the massless gravitational waves $h_{\mu\nu}$.

Newtonian limit of bigravity

We then assume that the massive gravitons are non-relativistic.

$$\overset{(0)}{g}_{\mu\nu}dx^\mu dx^\nu = -(1+2\Phi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j$$

$$\varphi_{\mu\nu} = \begin{pmatrix} \psi_{00} & \psi_{0i} \\ * & \frac{\psi_{\rm tr}}{3}\delta_{ij} + \psi_{ij} \end{pmatrix} e^{-imt} + \text{c.c.}\,,$$

↑ traceless, $\psi^i{}_i = 0$

where $\Phi, \psi_{..}$ are slowly varying functions.

The transverse-traceless condition leads to $|\psi_{00}|, |\psi_{\rm tr}| \ll |\psi_{0i}| \ll |\psi_{ij}|$

Finally, we obtain the Poisson-Schrodinger equations

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \quad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

Self-gravitating bound state

The bound state of the Poisson-Schrodinger eqs. with intrinsic spin.

$$\psi_{ij}(t,\mathbf{x}) = \psi_{ij}(\mathbf{x})e^{-iEt}\,, \quad i\frac{\partial}{\partial t} \to E$$

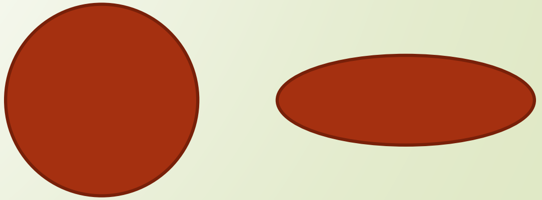
$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \quad E\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,, \qquad \text{Spin-2}$$

Cf. $$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*\psi\,, \quad E\psi = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi\,, \qquad \text{Spin-0}$$

Only difference is the intrinsic spin

ψ_{ij} : symmetric traceless tensor $\quad \psi$: scalar

What is the most stable configuration?



Stable? Unstable?

Angular momentum of bound state

Maybe… spherically symmetric configuration (monopole)?

However, it is NOT because of the intrinsic spin!

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \quad E\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

The most stable = The lowest energy eigenvalue

= The lowest angular momentum

$$\Delta = \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2}$$

There are total angular momentum j and orbital angular momentum ℓ

The monopole configuration: $j = 0$ but $\ell = 2$

A quadrupole configuration: $j = 2$ but $\ell = 0$ **Lowest energy**

(cf. The monopole configuration in spin-0 case: $j = 0$ and $\ell = 0$)

Monopole geon and Quadrupole geon

The monopole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_0(r)e^{-iEt}(T^{-2}_{0,0})_{ij}\,,$$

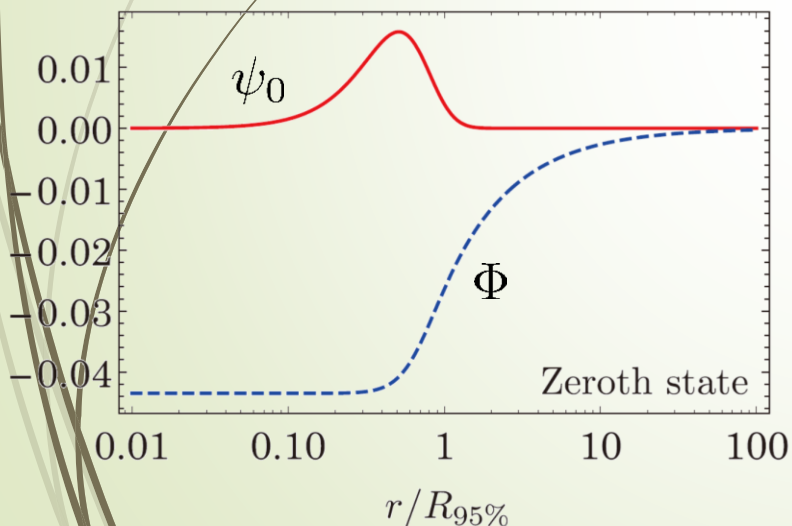
$$\tilde{E} = \frac{E}{(GM)^2m^3} = -0.027$$

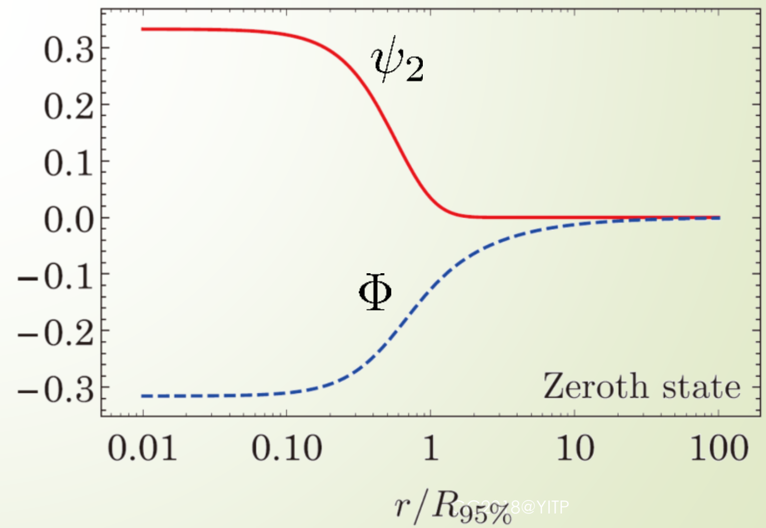
$>$

The quadrupole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_2(r)e^{-iEt}\sum_{j_z} a_{j_z}(T^{+2}_{2,j_z})_{ij}\,,$$

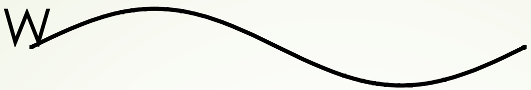
$$\tilde{E} = \frac{E}{(GM)^2m^3} = -0.16$$





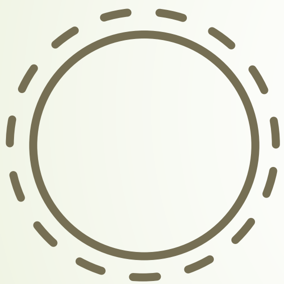
Stability of geons

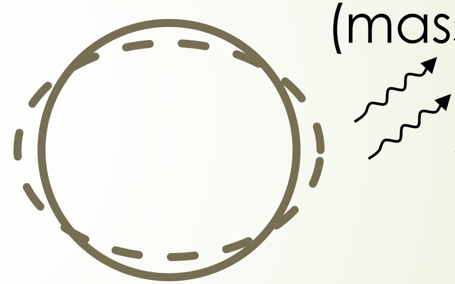
Coherent massive GW



Jeans instability
(KA and Maeda, '18)

Transit?





monopole

quadrupole

$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.027 \quad > \quad \tilde{E} = \frac{E}{(GM)^2 m^3} = -0.16$$

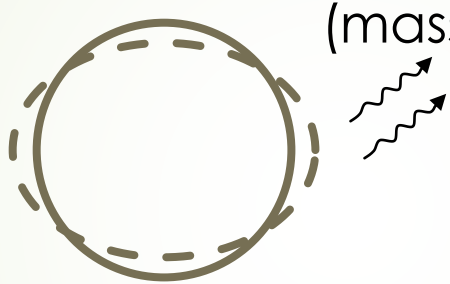
It may transit to the quadrupole geon by releasing binding energy.

The monopole geon is unstable against quadrupole mode perturbations.

The final state must be the quadrupole geon.

It could emit GWs due to non-spherically symmetric oscillations.

(massless and/or massive) GWs?



But, the emission is small because of the large hierarchy between the time and the length scales.

Anisotropic pressure $\sim T^{\mathrm{TT}}_{G,ij}(\mathbf{x})e^{-2imt}\,,\quad \partial_k T^{\mathrm{TT}}_{G,ij}(\mathbf{x}) \ll m T^{\mathrm{TT}}_{G,ij}(\mathbf{x})$

(GWs are emitted if $\omega^2 = k^2$ or $\omega^2 = k^2 + m^2$)

$\rightarrow$ The non-relativistic quadrupole geon is an (approximately) stable object.

Geons as field (fuzzy) dark matter

If a mass is $\sim 10^{-21}$eV, massive graviton can be a fuzzy dark matter.

Ultralight axion: spin-0 DM
Massive graviton: spin-2 DM

In FDM, the central part of DM halos is given by the "soliton" (=geon).

Although the field configuration is not spherically symmetric,
the energy distribution is spherically symmetric.

ψ_{ij} : not spherical $\qquad\qquad$ $\psi^*_{ij}\psi^{ij}$: spherical

and the energy distribution is exactly the same as that of spin-0 case.

Spin-2 FDM could share the successes of spin-0 FDM.

Summary

Twin matter could dark matter in bigravity theory

Massive graviton geons = self-gravitating massive GWs

New vacuum solutions in bigravity theory.

The ground state must be non-spherical.

Spin-0: ground state = monopole ⇒ $\ell = j = 0$

Spin-2: ground state = quadrupole ⇒ $\ell = 0,\ j = 2$

Ultralight massive graviton can be FDM as well.

Note that DM is not new "particle" but spacetime itself

$$g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{\varphi_{\mu\nu}}{M_G},$$

Possible prospects: Hairy BHs?, Geon as BE condensate? etc…