Towards basic gauging of orientation

- the sole seed of local field interactions, including gravity

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Abstract

Basic notions and references shall be assembled

in the initial phase of the present outline.

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1. Introduction

Before entering the actual topic of

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some tools, prepared in view of this lecture, shall be assembled and clarified. To this end we subdivide section 1 into several subsections

1a. A book project

After achieving to translate my thesis into english, the originally conceived Book-Project with the World Scientific Publishin Company was revived:

It is described in Fig. 1 below:

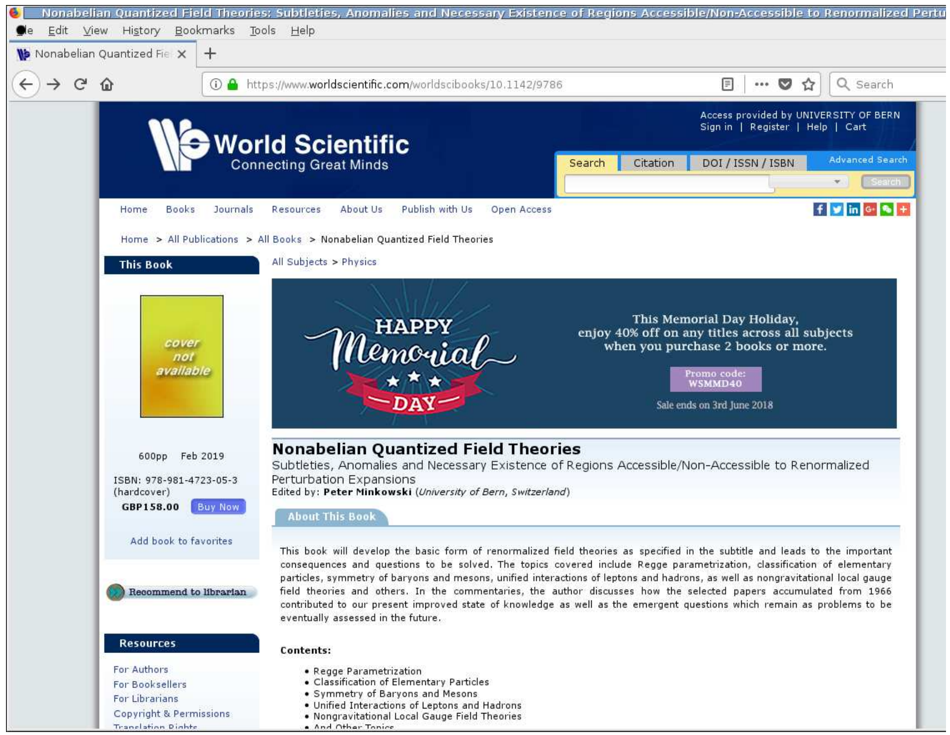

Fig. 1 : Book announcement by World Scientific Publishing Company, 2019

In connection with the book announcement, as shown in Fig. 1 above, I wish to state here, that a very long path is still lying ahead, before the announcement can be realized.

1b. The first discussions of 1-d spin chains by Werner Heisenberg in 1928 and Hans Bethe in 1931

The topic in this subsection, in refs. [1] and [2], was pioneered in early phases after the establishment of Quantum Mechanics.

We shall describe the milestones passed, on the way to the proof of two distinct regimes, given the presence of shortrange and longrange forces, before spontaneous symmetry breaking.

To this end I introduce as, mnemonic symbol for a 1 dimensional spin chain, an infinite chain of poplars, displayed in Fig. 2 below

Fig. 2 : 1 dimensional chain of poplars, symbolic for a 1 dimensional spin $\frac{1}{2}$ chain

The analog state, of spins $\frac{1}{2}$ perfectly aligned parallel, and to fix axes with spin up, as the common poplars orientation is, shall be denoted by

(1) $$\cdots \uparrow\uparrow \cdots \uparrow \cdots$$

On the operator level we introduce the double spin $\frac{1}{2}$ triplet of operators of the three Pauli matrices, one triplet each for every site, denoted by

(2) $$\begin{array}{l} -\infty < -j, \cdots < -1, -1, 0, 1 < 1 \cdots < k \cdots \infty \\ \cdots, \vec{\sigma}_J, \vec{\sigma}_{J+1}), \cdots \end{array}$$

A simple way, to generate spontaneous breaking

of ferromagnetic type with long range forces, without long range modes

is to consider the main Hamiltonian, while the choice of direction of the dynamically aligned spins is driven by a dipole term, involving the constant external polarization field $\vec{\mu}$

(3)
$$H = -J \left(\sum_{m=-\infty}^{+\infty} \vec{\sigma}_m\right) \cdot \left(\sum_{n=-\infty}^{+\infty} \vec{\sigma}_n\right) \; ; \; J : \textbf{constant} > 0$$
$$\delta(\vec{\mu})\, H = -\vec{\mu} \sum_{r=-\infty}^{+\infty} \vec{\sigma}_r$$

Preshow single spin precession

Preparing to follow a video, from You Tube, lets look at a preshow figure displayed in Fig. 3 below

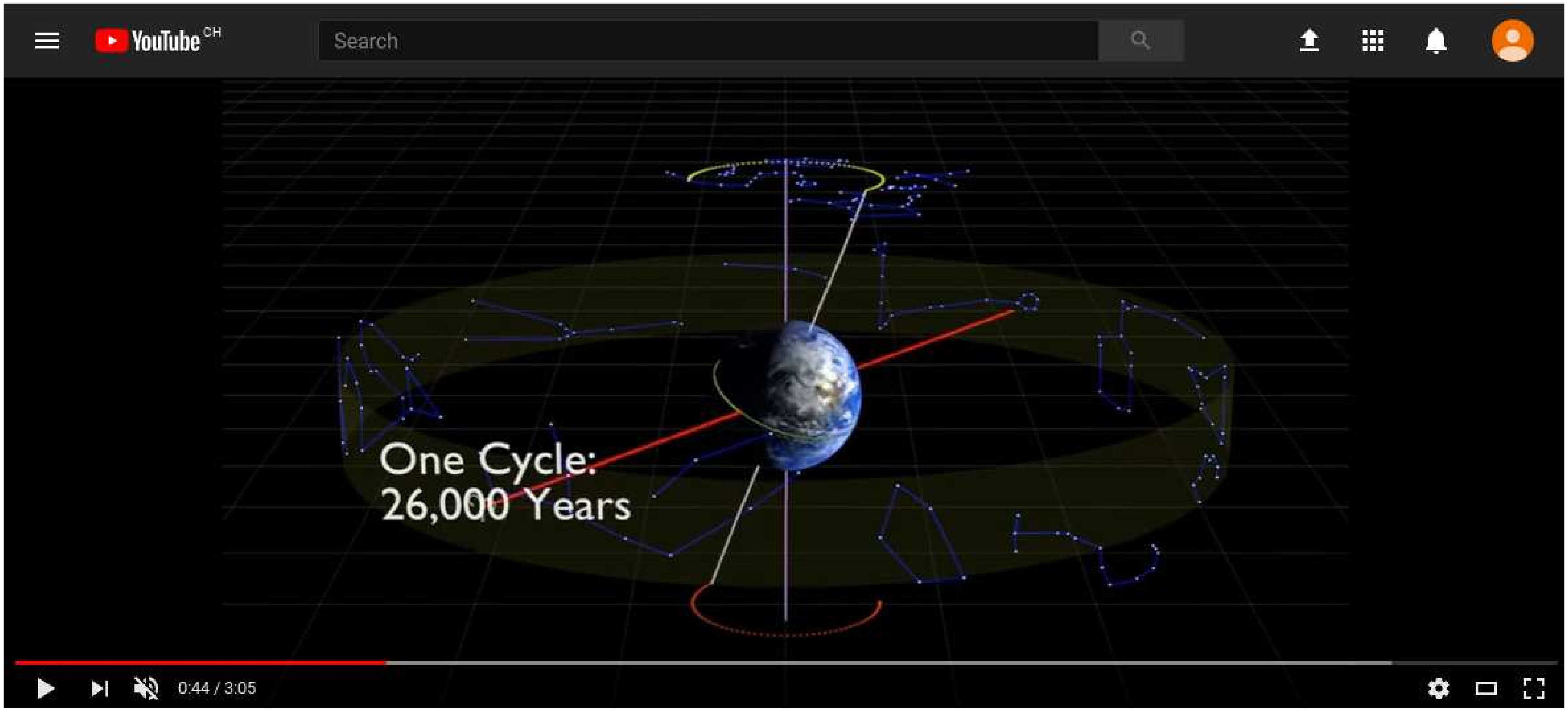

Fig. 3 : Precession of the Earth - Preshow https://www.youtube.com/watch?v=qlVgEoZDjok

’https://www.youtube.com/watch?v=qlVgEoZDjok’

The fate of an angle - $\Theta = \vartheta - \vartheta'$ - between the *preset* direction of the 1-d spin chain and the polarization vector $\vec{\mu}$

Using the variables defined in eq. 3, the situation is shown in Fig. 3, drawn in the plane formed by the two directons mentioned

$$\langle \vec{\sigma}_m \rangle = (0 , 0 , 1) \; ; \; m = -\infty \cdots -1 , 0 , 1 \cdots + \infty$$

$$\vec{\mu} = (\sin (\Theta) , 0 , \cos (\Theta))$$

(4)

... of ferromagnetic type with long range forces, without long range modes

that we are dealing with long range forces, without *any* long range modes.

Independently of (all of) this it follows that we are dealing with the ’equilibrium’ properties of the ground state. As a consequence the external polarisation field $\vec{\mu}$ is (to be) a configuration space-time independent one , at least as long as gravity effects are neglected.

The precession of the earth axis relative to the signs of the zodiac are taught today on the level of elementary school, the video, which will (hopefully) appear as illustration next, is not essential to the discussion of the angle Θ , shown in Fig. 3 here I reduce the discussion to its basic level.

The action functional for the ground state, in the limit of $G_F \rightarrow 0$, must be extremized relative to the spontaneous parameter ϑ' .

The action functional for the ground state, in the limit of $G_F \to 0$, must be extremized relative to the spontaneous parameter ϑ', independently of whether the discrete reflections CP , C or P are conserved or not, while ϑ must be considered given *and* fixed.

This is well known, with respect to the earth axis precession, as "Mach's principle" causing it through the precession being due to the motion relative to the 'distant stars'.

Ernst Mach, Austrian Physicist, born: February 18, 1838, Brno, Czech Republic, died: February 19, 1916, Munich, Germany. An illustration from google.ch is shown in Fig. 4, below.

Whether the variable ϑ is judicially chosen or not is equally irrelevant.

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The material presented in this subsection is a shorthand of the slide-file in ref. [4] .

Fig. 4 : Ernst Mach

1c. Translation to english of my thesis from 1966

In order to prove my involvement in preparing a framework for quantized gravity, since my thesis directed by my thesis adviser Markus Fierz, which was printed in german, in Helvetica Physica Acta in 1966 [5]. This *translation* **is presented as a pdf-slide file in ref. [6]. The reason, why this renewed effort, of translating my thesis into english, becomes evident, can be understood in my obituary, at the occasion of Markus Fierz's death, in the file: Fierzenglish.pdf in ref. [7] from 2006.**
Anyone interested to obtain a copy of the files, cited here as within my AEC-ITP file domain, is kindly asked to write to me an email, expressing this request.

2. From the translation to english of my thesis from 1966

Attempt of a consistent theory of a spin 2 meson

english translation of the original version of the thesis of the author, in german

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The propriety structure of the underlying german original

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Attempt of a consistent theory of a spin 2 meson

by P. Minkowski

Seminar for theoretical physics, ETH Zurich

(7. VI. 66)

Summary. The electromagnetic interaction of a charged spin 2 meson is considered. Several canonical transformations of both the meson- and photon-field lead to an interaction representation. The Hamiltonian expressed by the interaction picture fields is a noncovariant power series in the charge. On the other hand the time-ordered products of the fields and their derivatives contain noncovariant terms of a $\delta^{(4)}(x)$ type, which give rise to higher order vertices compensating the noncovariant contributions from the Hamiltonian. The latter is calculated in second order approximation and reduced to a simple form making use of covariant contractions. The multipole moments in nonrelativistic limit are obtained. The 'minimal coupling' admits a free parameter α which corresponds to an anomalous magnetic moment

(e: charge, M: meson mass, $\vec{S}$: meson spin).

$$\vec{\mu} = - \frac{\alpha}{2} \frac{e}{2\,M} \vec{S}$$

2.1 Formulation of the problem

In the following pages reference numbers refer to the bibliography on page 'PMthesis-Refs.' below.

The fundamental paper(s) by M. Fierz and W. Pauli [1] have demonstrated the difficulties, which arise with higher spin fields. P. A. M. Dirac [2] has discussed for c-numbers the Hamilton-Formalism with subsidiary conditions. There is an extended analogy between c-number- and q-number-theory, which will allow us to apply the results of the cited papers. Only the notion of subsidiary condition has to be defined more precisely. We propose to understand a subsidiary condition not as an operator identity, but as a function of the fields, which vanishes on a suitable subspace of the linear space considered, with indefinite metric.

This definition of subsidiary condition is analogous to the method introduced by S. Gupta [3] and K. Bleuler [4] for the treatment of the Lorentz-condition in a space with indefinite metric.

The interaction of particles with spin ¿ 1 was recently investigated within the general framework of covariant scattering amplitudes , whereby only properties are used, which do not depend on a specific model. M. Jacob and G. C. Wick [5] have described the reduction of matrixelements in the helicity formalism. K. Hepp [6] and D. Williams [7] gave the decomposition of an analytic function into a sum of covariant standard polynomials. D. Zwanziger [8] describes the electrodynamic interaction of particles with restmass and general spin. Independently of a model he defines building on V. Bargmann, L. Michel and V. L. Telegdi [9] a 'minimal coupling' that corresponds to the g-factor g = 2. The universal g-factor g = 2 implies that the

T. Regge [10] has calculated for a special magnetic moment the cross section for Coulomb scattering of a (charged) spin 2 meson. In going over to the interaction picture, Regge obtains an invariant Hamiltonoperator. This operated generates a scattering matrix that at first is not lorentzinvariant, because the T-products involved contain besides covariant also noncovariant expressions, which depend on a spacelike surface. H. Umezawa and Y. Takahashi [11,12,13] have shown, that $\mathcal{H}_{INT}$, in the interaction representation, likewise contains surface terms, which cancel against the noncovariant T-products. The transformation from the Heisenberg to the interaction-representation is generated by an operator $U(\sigma)$ that can be constructed under 'ad hoc' premises.

T. D. Lee and C. N. Yang [14] have described the electromagnetic interaction of a vector meson with arbitrary magnetic moment. Doing this they counterpose to the canonical formalism a limiting procedure. Both methods lead to the same results. They succeed to explicitely delimit the mentioned compensations, and to show that the deviation of $\mathcal{H}_{INT}$ from $-/\mathcal{L}_{INT}$ can be reduced to the fact, that the ordering of products of field operators is not (uniquely) determined, whic leads to so called 'tadpoles'. Also the interaction of a vector meson with a leptonic current satisfies the hypothesis of Umezawa and Takahashi, which can be verified directly. With the help of these hypotheses S. Weinberg [15] is able to lay a bridge to the invariant perturbation expansion and to derive Feynman rules for arbitrary spins.

In this work it shall be attempted to treat the electromagnetic

With the help of these hypotheses S. Weinberg [15] is able to lay a bridge to the invariant perturbation expansion and to derive Feynman rules for arbitrary spins.
In this work it shall be attempted to treat the electromagnetic interaction of a charged spin 2 meson , with arbitrary magnetic moment, as a consistent canonical theory.

It succeeds to derive the Hamiltonoperator as a power series in the coupling constant e and simultaneously to transform to the interaction representation. Subsequently it is shown, that from the construction of the interaction representation the hypotheses follow naturally from which Umezawa and Takahashi deduced the relation $\mathcal{H}_{INT} = -/\mathcal{L}_{INT}$.

It is revealed, that the canonical formalism to determine the Hamiltonoperator is equivalent on one hand to the S-matrix in the interaction representation according to S. Tomonaga [16] and and J. Schwinger [17], and on the other hand to the construction of a unitary and causal S-matrix according to E. C. G. Stückelberg [18,19] .The methods used here for the electromagnetic interaction to treat the subsidiary conditions of fields with spin 2, can be used,

for arbitrary couplings, for instance to a current of strongly interacting particles. However the fact that we are here dealing with 'nonderivative coupling' brings no simplification.

Content structure

In a first subsection starting from the Lagrange density of a free meson with mass M using a 'minimal coupling' the Lagrange density is obtained, which should describe the electromagnetic interaction. The associated Euler equations imply 5 subsidiary conditions, which render the contact transformation, which leads from the Lagrange- to the Hamilton-density, to become degenerate.
In the second subsection the Lagrangefunction is replaced by a new one, in such a way, that the new equations of motion are compatible

with the 5 subsidiary conditions and the degeneracy of the contact transformation is lifted.
If we now set the subsidiary conditions equal 0, we obtain back the old equations of motion. Expressing the subsidiary conditions and their first (partial) derivatives with respect to time through the canonical variables and transiting to the Hamiltonfunction, we can use the theorems of Dirac [2] for the Hamilton formalism with subsidiary conditions.
The Hamilton operator is only defined modulo subsidiary conditions and we content ourselves for the moment with an arbitrary representative. Besides this, we give general functionals of canonical transformations.

In the third subsection the subsidiary conditions are brought by a series of canonical transformations to thid fotm, which corresponds to the free equations. This is the most important and furthermore most complex part of the reduction of $\mathcal{H}_{INT}$ to $-/\mathcal{L}_{INT}$. From the construction of these transformations it follows, that they do not depend (explicitely) on the interactions.

Once the subsidiary conditions attain the sought form, in the fourth subsection the transition to the interaction representation is performed, led by the subsidiary conditions corresponding to the free meson. It succeeds to find a singled out Hamiltonoperator, which commutes with all (10) subsidiary conditions, from which it follows the vanishing of the latter follows from a suitable choice of initial conditions.

In so proceeding, the space of states, which carries an indefinite matric, appears split in the form of a direct product $R_1 \otimes R_2$.
The subsidiary conditions have the representation $N = \P \odot N_2$.
The physical partial space $\mathcal{H}_P$ is formed by the product states $x \odot \Omega_2$, whereby x lies in R_1 and Ω_2 corresponds to the ground state in R_2. $\mathcal{H}_P$ is a Hilbert space in which the subsidiary conditions vanish.
In subsection five with $\mathcal{H}_{INT}$ in first and second order approximation is discussed. In the nonrelativistic limit the multipole moments are calculated. The anomalos magnetic moment

$$\vec{\mu} = -\frac{\alpha}{2}\frac{e}{2M}\vec{S}$$

does not lead to any specific difficulties, why a special value of the g-factor is not suggested, contrary to the case to the case of the electron. All multipolemoments up to the order 2^4 depend on the parameters e, α, such that only two (out of 4) can be chosen independently. The hogher multipole moments vanish. The second order gives rise to the compensations mentioned. These are classified by means of associated Feynman-graphs and calculated for a special case.

In the sixth subsection, finally the connection to the papers by Umezawa and Takahashi [11,12,13] is established. The canonical formalism generates in a natural way the surface terms in the Hamilton operator, which compensate the noncovariant terms of the timeordered products, whereby it is guaranteed, that the unitary

and causal transformation

$$U(t_2, t_1) = T^* \exp\left[i \int_{t_1}^{t_2} \mathfrak{L}_{INT}(x)\; d^4x \right]$$

describes the time evolution of the states in the interaction representation, which follows from the equations of motion. Hereby T^* describes the timeordering operation, which only respects covariant contractions.

Notations and Definitions

Electromagnetic Potentials:	A_ϱ
Canonically conjugate Momenta:	ω_ϱ
Mesonfields:	$\psi_{\mu\nu} = \psi_{\nu\mu}$
Thereto canonicallyconjugate Momenta:	$\phi^{*\mu\nu} = \phi^{*\nu\mu}$
Electromagnetic Field Strength:	$F_{\varrho\sigma} = -F_{\sigma\varrho} = \partial_\sigma A_\varrho - \partial_\varrho A_\sigma$

$$F_{\varrho\sigma} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}.$$

Greek Indices run from 0 to 3, Latin one from 1 to 3

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & 0 & \\ 0 & & -1 & \\ & & & -1 \end{pmatrix}$$

'Minimal coupling' :
Momentum

$$p_\mu \longrightarrow p_\mu - \frac{e}{c} A_\mu$$

$$\downarrow$$

$$\hbar\, i\, \partial_\mu \longrightarrow \hbar\, i\, \partial_\mu - \frac{e}{c} A_\mu$$

$$\partial_\mu \to D_\mu = \partial_\mu + i\, e\, \frac{1}{\hbar\, c} A_\mu$$

D_μ : covariant derivative

equations of motion of a point charge in an electromagnetic field

$$M\, \dot{u}^\mu = -\frac{e}{c} F^{\mu\nu} u_\nu$$

$\cdot$: derivative relative to proper time

$$M\, u^\mu = p^\mu$$

$$\dot{p}^\mu = -\frac{e}{M\, c} F^{\mu\nu} p_\nu$$

M : Mass of the Particle

$$\mathfrak{L} = \mathfrak{L}_{elm} + \mathfrak{L}_{Mat} + \mathfrak{L}_{INT}$$

$\mathfrak{L}$: Lagrangedensity for the electromagnetic- and the matter field

$$\mathfrak{L}_{elm} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

or with the help of the Lorentz condition

$$\mathfrak{L}_{elm} = -\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu$$

we choose rational units

$$c = \hbar = 1$$

$$\mathfrak{L}_{Mat} = \partial^\varrho \psi^{*\mu\nu} \partial_\varrho \psi_{\mu\nu} - 2\, \partial^\mu \psi^*_{\mu\nu} \partial_\varrho \psi^{\varrho\nu}$$
$$+ \partial^\nu \psi^* \partial^\mu \psi_{\mu\nu} + \partial^\mu \psi^*_{\mu\nu} \partial^\nu \psi - \partial^\nu \psi^* \partial_\nu \psi$$
$$- M^2 [\psi^{*\mu\nu} \psi_{\mu\nu} - \psi^* \psi] \qquad \text{(A.1)}$$

From eq. (A.1) the equations and subsidiary conditions follow:

$$(\Box + M^2)\, \psi_{\mu\nu} = 0\,, \quad \partial_\mu \psi^{\mu\nu} = 0\,, \quad \psi = 0$$

The subsidiary conditions are necessary, in order to prohibit the appearance of particles with spin 1 or 0. If they are fulfilled, then in addition the energy is positive definite. The conditions for

The subsidiary conditions are necessary, in order to prohibit the appearance of particles with spin 1 or 0. If they are fulfilled, then in addition the energy is positive definite. The conditions for

$\psi_{0i}:\ \partial_\nu \psi^{\nu i} = 0$ *are analogous to the vanishing of the divergence of vectorfields in a theory with spin 1.*

$$\partial^k \partial^l (\psi_{lk} - g_{lk} \psi^n_n) - M^2 \psi^n_n = 0$$

imply the derived relation in the previous line.

Despite the fact that the above subsidiary conditions bring nonlocal quantities into the theory, as we will see, this is not the case.Even if interaction sets in, all components $\psi_{\mu\nu}$ remain relatively local.
We will return to a more precise characterization of the subsidiary con- in appendix B.

The expression $-2\,\partial^{\mu}\,\psi^*_{\mu\nu}\,\partial_{\varrho}\,\psi^{\,\varrho\nu}$ **is for the free field equivalent to** $-(1+\alpha)\,\partial^{\mu}\,\psi^*_{\mu\nu}\,\partial_{\varrho}\,\psi^{\,\varrho\nu}\;-\;(1-\alpha)\,\partial^{\varrho}\,\psi^*_{\mu\nu}\,\partial^{\mu}\,\psi_{\,\varrho\nu}$ **. f we substitute** $\partial_{\mu}\;\rightarrow\;D_{\mu}\;=\;\partial_{\mu}+i\,e\,A_{\mu}$ **following the principle of 'minimal coupling', we obtain the following Lagrangean density**

$$
\begin{aligned}
\mathfrak{L}(e,\alpha) = &-\frac{1}{2}\,\partial^{\mu}A^{\nu}\,\partial_{\mu}A_{\nu} + D^{*\varrho}\,\psi^{*\mu\nu}\,D_{\rho}\,\psi_{\mu\nu} \\
&-(1+\alpha)\,D^{*\mu}\,\psi^{*}_{\mu\nu}\,D_{\rho}\,\psi^{\varrho\nu} - (1-\alpha)\,D^{*}_{\varrho}\,\psi^{*}_{\mu\nu}\,D^{\mu}\,\psi^{\varrho\nu} \\
&+D^{*\nu}\,\psi^{*}\,D^{\mu}\,\psi_{\mu\nu} + D^{*\mu}\,\psi^{*}_{\mu\nu}\,D^{\nu}\,\psi - D^{*\nu}\,\psi^{*}\,D_{\nu}\,\psi \\
&-M^{2}\,(\psi^{*\mu\nu}\,\psi_{\mu\nu} - \psi^{*}\,\psi)\,. \qquad\qquad (A.2)
\end{aligned}
$$

For the electromagnetic potentials a Lorentz condition $(\,\partial_{\mu}\,A^{\mu}\,)_{+}\,|\,z\,\rangle_{p}\;=\;0$ **shall hold for all physical states** $|\,z\,\rangle_{p}$**; with** $(\,\partial_{\mu}\,A^{\mu}\,)_{+}$ **: positive frequency part of** $(\,\partial_{\mu}\,A^{\mu}\,)$ **.**

$\mathcal{L}\,(\,e\,,\,\alpha\,)$ **gives rise to the equations of motion**

$$
\begin{aligned}
&D^2\psi_{\mu\nu} - \frac{1+\alpha}{2}\,(D_\mu\,D^\varrho\psi_{\varrho\nu} + D_\nu\,D^\varrho\psi_{\mu\varrho}) - \frac{1-\alpha}{2}\,(D^\varrho\,D_\mu\psi_{\varrho\nu} + D^\varrho\,D_\nu\psi_{\mu\varrho}) \\
&\quad + g_{\mu\nu}\,D^\alpha\,D^\beta\,\psi_{\alpha\beta} + \frac{1}{2}\,(D_\mu\,D_\nu + D_\nu\,D_\mu)\,\psi - g_{\mu\nu}\,D^2\psi + M^2\,(\psi_{\mu\nu} - g_{\mu\nu}\,\psi) = 0 \\
&\quad \Box\,A^\nu = j^\nu = -\,\mathfrak{L}_{,A\nu}
\end{aligned}
\tag{A.3}
$$

From (A.3) the following subsidiary conditions are derived:

$$
N_\nu(e,\alpha) = D^\mu\psi_{\mu\nu} - D_\nu\psi - n_\nu(e,\alpha) = 0 \qquad \nu = 0,1,2,3 \qquad N_4(e,\alpha) = \psi - n_4(e,\alpha) = 0
$$

$$
n_\nu = \frac{i\,e}{M^2}\left[\begin{array}{l}
\frac{3+\alpha}{2}\,F^{\mu\varrho}\,D_\varrho\psi_{\mu\nu} - \frac{3-\alpha}{2}\,F^\varrho_\nu\,D^\mu\psi_{\mu\varrho} \\
-\,\frac{1-\alpha}{2}\,(\partial^\varrho F^\mu_\nu)\,\psi_{\varrho\mu} + \frac{1+\alpha}{2}\,j^\mu\psi_{\mu\nu} \\
+\,\frac{3}{2}\,F^\varrho_\nu\,D_\varrho\psi - \frac{1}{2}\,j_\nu\,\psi
\end{array}\right]
$$

$$
n_4 = \frac{2}{3\,M^4}\left[\begin{array}{l}
i\,e\,(1+\alpha)\,(\partial^\nu F^{\mu\varrho})\,D_\varrho\psi_{\mu\nu} - \frac{3+\alpha}{2}\,e^2\,F^{\mu\varrho}\,F^\nu_\varrho\,\psi_{\mu\nu} \\
+\,i\,e\,\alpha\,F^{\mu\varrho}\,D_\varrho\,D^\nu\psi_{\mu\nu} - i\,e(1-\alpha)\,j^\varrho\,D^\mu\psi_{\mu\varrho} \\
+\,i\,e\,\alpha(\partial^\nu\,j^\mu)\,\psi_{\mu\nu} + \frac{3\,e^2}{4}\,F^{\varrho\nu}\,F_{\nu\varrho}\,\psi + i\,e\,j^\varrho\,D_\varrho\psi
\end{array}\right]
\tag{A.4}
$$

For
$$\Box\,\psi_{\mu\nu} - (\partial_\mu\,\partial^\varrho\psi_{\varrho\nu} + \partial_\nu\,\partial^\varrho\psi_{\mu\varrho}) + g_{\mu\nu}\,\partial^\alpha\,\partial^\beta\,\psi_{\alpha\beta}$$
$$+ \partial_\mu\,\partial_\nu\,\psi - g_{\mu\nu}\,\Box\,\psi + M^2\,(\psi_{\mu\nu} - g_{\mu\nu}\,\psi) = 0$$
$$\Box\,A^\nu = 0 \qquad \partial_\mu\psi^{\mu\nu} = 0\,, \qquad \psi = 0 \tag{A.5}$$

From (A.5) finally it follows: $(\Box + M^2)\,\psi_{\mu\nu} = 0$.

B. Canonical formalism, canonical transformations, Hamilton operator

The Legendre transformation belonging to $\mathcal{L}(e, \alpha)$ is as a consequence of the subsidiary conditions dege

$$\mathcal{L}'(e, \alpha) = \mathcal{L}(e, \alpha) + \sum_{s,t=0}^{9} c_{st}\,N_s^*\,N_t \tag{B.1}$$

$(N_5, \ldots, N_9) = (\partial_0 N_0, \ldots, \partial_0 N_4)$, $c_{st} = c_{ts}^*$, c_{st}: **constants**

It is consistent to require that the Euler equations pertinent to $\mathcal{L}'$ satisfy the subsidiary conditions $N_0, \cdots, N_9) = 0$, because for $N_0, \cdots, N_9) = 0$ the Euler equations pertaining to $\mathcal{L}'$ go over in those stemming from $\mathcal{L}$, from which equations again $N_0, \cdots, N_9) = 0$ follows.

We determin the Hamilton operator $\mathcal{H}'$ associated with the Lagrangean density $\mathcal{L}'$

The field pairs $\psi_{\mu\nu}$, $\phi^{\alpha\beta}$ and A_ϱ , ω^σ shall be canonically quantized.

$$[\psi_{\mu\nu}(x), \phi^{*\alpha\beta}(y)]_{x^0=y^0} = i\, \delta^{\alpha\beta}_{\mu\nu}\, \delta^{(3)}\, (\vec{x} - \vec{y}) \qquad [A_\varrho(x), \omega^\sigma(y)]_{x^0=y^0} = i\, \delta^\sigma_\varrho\, \delta^{(3)}\, (\vec{x} - \vec{y}) \quad \text{(B.2)}$$

the other commutators = 0

$$\delta^{\alpha\beta}_{\mu\nu} = \frac{1}{2}\, (\delta^\alpha_\mu\, \delta^\beta_\nu + \delta^\beta_\mu\, \delta^\alpha_\nu)$$

We solve the variational equations:

$$\delta \int dt\, d^3x\, (\phi^{*\mu\nu}\, \dot{\psi}_{\mu\nu} + \dot{\psi}^*_{\mu\nu}\, \phi^{\mu\nu} - \mathcal{H}' - v^m\, N_m) = 0\,.$$

The ν^m are Lagrangean multipliers, which allow to vary without respecting the subsidiary conditions. We collect the quantities $(\psi_{\mu\nu}, \phi_{\mu\nu}\, A_\varrho, \omega_\varrho)$ into a vector X_α . Then we obtain the following canonical equations:

$$H' = \int_t d^3x\, \mathcal{H}' \qquad \left[H' + \sum_{m=0}^{9} \int_{y0} d^3x\, v^m\, N_m\,,\quad X_\alpha(y)\right] = -\, i\, \partial_0\, X_\alpha(y)$$

$$N_k = 0 \quad k = 0, 1, \dots, 9 \qquad \text{(B.3)}$$

In B.3 the Hamilton function is not uniquely determined. If we change $\mathcal{H}'$ by multiples of the subsidiary conditions:

$$\mathcal{H}'' = \mathcal{H}' + \sum \lambda^m N_m$$

λ^m : arbitrary functions of the fields;

then we obtain the the equations equivalent, yet different from B.3

$$\left[H'' + \sum_{m=0}^{9} \int_{y0} d^3x \, (\nu^m - \lambda^m) \, N_m \, , \; X_\alpha(y)\right] = -\, i \, \partial_0 \, X_\alpha(y) \qquad N_k = 0 \quad H'' = \int_t \mathcal{H}'' \, d^3x \, .$$

The linear space R, in which the commutation relations (B.2) are represented carries an indefinite metric; $\psi^*_{\mu\nu}$ is, relative to this metric, the conjugate operator to $\psi_{\mu\nu}$. Because of the commutation relations of the subsidiary conditions among themselves, they cannot vanish identically. It will be shown, that R decomposes into a direct product $R_1 \otimes R_2$ whereby the subsidiary conditions $N_k = \P \otimes N_k^{(2)}$ do not act on the subspace R_1 . The physical subspace $\mathcal{H}_p$, which we want to envisage, consists of the state vectors $|\, Z \,\rangle_p = Z_1 \odot \Omega_2$, where Ω_2 is the vacuum state in the factor space R_2 .

$\mathcal{H}_p$ is a Hilbert space.

For $|\, Z \,\rangle_p$ the subsidiary conditions vanish as soon as $\langle \Omega_2 | \; N_k^{(2)} \Big| \, \Omega_2 \rangle = 0$.

$$\langle Z' \,|\, N_k \,|\, Z'' \rangle_p = \langle Z_1' \otimes \Omega_2 \,|\, 1 \otimes N_k^{(2)} \,|\, Z'' \otimes \Omega_2 \rangle = \langle Z_1' \,|\, Z_1'' \rangle_1 \, \langle \Omega_2 \,|\, N_k^{(2)} \,|\, \Omega_2 \rangle_2 = 0 \, .$$

The scalar product used here corresponds to the indefinite metric:

$$\langle Z_1' \otimes Z_2' \,|\, Z_1 \otimes Z_2 \rangle = \langle\!\langle Z_1' \otimes Z_2'; \, \eta; \, Z_1 \otimes Z_2 \rangle\!\rangle \, .$$

η : hermitian operator, $<<>>$ positive definite scalar product in R. If we understand $N_k = 0$ in the way described above, the results of Dirac [2] can be applied also to (B.3).

Among the operators $\mathcal{H}' + \sum \lambda^m N_m$ (λ^m : arbitrary functions of the fields) one is singled out, through the prope

$$[\mathcal{H}, N_k] = 0 \qquad k = 0, 1, \ldots, 9$$

Then equations B.3 can be reduced to

$$H = \int_t d^3x\, \mathcal{H} \qquad [H, X_\alpha(y)] = -i\, \partial_0 X_\alpha(y) \qquad \text{(B.4)}$$

The subsidiary conditions follow from the corresponding initial conditions $N_k(t) = 0$; $\mathcal{H}$ will be determined in subsection D.

Initially we select a representative among the operators $\mathcal{H}' + \sum \lambda^m N_m$. For this it thus suffices to, give the fields X_α modulo subsidiary conditions, for which the shorthand notation m (N) is used.

In order to determine $\mathcal{H}$ it will be necessary to perform canonical transformations, i.e. such transformations, upon which the commutation rules (B.2) remain unchanged. Such a transformation is generated by a functional S. For S, among others, we have the following possibilities:

$$S \to S_1 = S_1(\psi, \phi', A, \omega')$$
$$S \to S_2 = S_2(\psi', \phi, A', \omega)$$

We have for S_1 : $\psi' - \psi = \frac{\delta S_1}{\delta \phi^{*\prime}}$; $\phi - \phi' = \frac{\delta S_1}{\delta \psi^*}$

and S_2 : $\psi' - \psi = \frac{\delta S_2}{\delta \phi^*}$; $\phi - \phi' = \frac{\delta S_2}{\delta \psi^{*\prime}}$

written in extenso : $\psi'_{\mu\nu}(\vec{x}) - \psi_{\mu\nu}(\vec{x}) = \frac{\delta S_1}{\delta \phi^{\mu\nu *\prime}(\vec{x})}$

similar formulae hold for: $A' - A, \omega - \omega'$

$$K(X'_\alpha) = \int_t \mathcal{H}\, d^3x + \partial_0\, S_2^1 \qquad \text{(B.5)}$$

Hereby $\delta / \delta \phi^{\mu\nu *\prime}(\vec{x})$

denotes the functional derivative with respect to $\phi^{\,\mu\nu*'}\,(\,\vec{x}\,)$.

We do not use the constants c_{st} in B.1 by setting

$$
\begin{aligned}
{}^{(0)}\phi_{\mu\nu} &= \mathfrak{L}',\, \partial_0 \psi^{*\mu\nu} \\
{}^{(0)}\phi_{00} &= D_0\psi - n_9 - \alpha\, D^k \psi_{k\,0} \\
{}^{(0)}\phi_{0\,i} &= D^{\varrho}\,\psi_{\varrho\,i} - D_i\psi - n_i - \frac{1+\alpha}{2}\, D^k\psi_{k\,i} + \frac{1}{2}\, D_i\psi^n_n + \frac{\alpha}{2}\, D_i\psi_{00} \\
{}^{(0)}\phi_{i\,k} &= D_0\,(\psi_{i\,k} - g_{i\,k}\,\psi^n_n) - \frac{1-\alpha}{2}\,(D_i\psi_{0\,k} + D_k\psi_{0\,i}) + g_{i\,k}\, D^m\psi_{m\,0} \qquad m(N) \qquad \text{(B.7)} \\
\omega_{\varrho} &= -\ \mathbf{M}\rho
\end{aligned}
$$

Denoting by ${}^{(0)}\phi_{\mu\nu}$ the the fields canonically conjugate to $\phi^{*\mu\nu}$ relative to $\mathcal{L}'$, we derive

$$
\begin{aligned}
{}^{(0)}\phi_{00} - \phi_{00} &= -\,\alpha\, D^k\psi_{k\,0} \\
{}^{(0)}\phi_{0\,i} - \phi_{0\,i} &= -\,\frac{1+\alpha}{2}\, D^k\psi_{k\,i} + \frac{1}{2}\, D_i\psi^n_n + \frac{\alpha}{2}\, D_i\psi_{0\,0} \\
{}^{(0)}\phi_{i\,k} - \phi_{i\,k} &= \frac{1+\alpha}{2}\,(D_i\psi_{0\,k} + D_k\psi_{0\,i}) - g_{i\,k}\, D^n\psi_{n\,0} \\
{}^{(0)}\omega_i - \omega_i &= [-\,(1+\alpha)\; i\; e\, \psi_{i\,k} + i\; e\; g_{i\,k}\,\psi^n_n + i\; e\; \alpha\; g_{i\,k}\,\psi_{00}]\; \psi^{*0\,k} + \text{h.c.} \\
{}^{(0)}\omega_0 &= \omega_0 \qquad \text{(B.9)}
\end{aligned}
$$

We note that even for e = 0, ${}^{(0)}S$ generates a transformation of the variables ${}^{(0)}\omega_0 = \omega_0$; $\psi_{\mu\nu}$.
Thus we have expressed 5 subsidiary conditions, through the canonical variables :

$$N_i\colon \phi_{0i} = 0 \longleftrightarrow D^\mu \psi_{\mu i} - D_i \psi - n_i = 0$$
$$N_4\colon \psi - n_4 = 0$$
$$N_9\colon \phi_{00} = 0 \longleftrightarrow D_0 \psi - n_9 = 0$$

It holds:

$$\phi_{ik} = D_0 (\psi_{ik} - g_{ik}\psi^n_n) - (D_i \psi_{0k} + D_k \psi_{0i}) + 2 g_{ik} D^m \psi_{m0}$$
$$D_0 \psi^n_n = -\frac{1}{2}\phi^n_n + 2 D^n \psi_{n0}$$
$$D_0 \psi_{ik} = \phi_{ik} - \frac{1}{2} g_{ik} \phi^n_n + D_i \psi_{0k} + D_k \psi_{0i} \qquad \text{(B.10)}$$

$D^\mu \psi_{\mu 0} - D_0 \psi - n_0$ becomes $\frac{1}{2}\psi^n_n - D^n \psi_{n0} - n_0$. The equations A.3 for $\psi_{0\nu}$ yield 4 further subsidiary condition. In this way we obtain the 10 subsidiary conditions:

$$N_0: \quad \frac{1}{2}\,\phi^n_n - D^n\psi_{n0} - n_0 = 0 \longleftrightarrow D^\mu\psi_{\mu 0} - D_0\psi - n_0 = 0$$

$$N_{1,2,3}: \phi_{0i} = 0 \qquad \longleftrightarrow D^\mu\psi_{\mu i} - D_i\psi - n_i = 0$$

$$N_4: \quad \psi - n_4 = 0$$

$$N_5: \quad D^i\,D^k\,(\psi_{ik} - g_{ik}\,\psi^n_n) - M^2\,\psi^n_n + i\,e\,\alpha\,{}^{(0)}F^{0k}\,\psi_{0k} = 0$$

$$\hookrightarrow \partial_0 N_0 = 0$$

$$N_{6,7,8}: M^2\,\psi_{0i} - \partial^k\phi_{ki} - p_i = 0 \longleftrightarrow \partial_0 N_{1,2,3} = 0$$

$$N_9: \qquad \phi_{00} = 0 \qquad \longleftrightarrow D_0\psi - n_9 = 0 \qquad \text{(B.11)}$$

$$p_i(e,\alpha) = i\,e\,A^k\,\phi_{ki} - \frac{i\,e}{2}\,{}^{(0)}F^{0k}\,(\psi_{ki} - g_{ki}\,\psi^n_n) + i\,e\,\frac{3-\alpha}{2}\,{}^{(0)}F_i{}^k\psi_{k0}$$
$$+\frac{i\,e\,\alpha}{2}\,{}^{(0)}F_{0i}\,\psi_{00} - \frac{i\,e\,\alpha}{2}\,{}^{(0)}F^{0k}\,\psi_{ki} \qquad \text{(B.11a)}$$

We should take into account ${}^{(0)}F_{0i} = \partial_i A_0 + {}^{(0)}\omega_i$ differing from

$F_{0i} = \partial_i A_0 + \omega_i$

The Lorentz condition $(\omega_0 - \partial^k A_k)_+ |Z\rangle_p = 0$ remained unchanged by ${}^{(0)}S$.
In (B.11) the functions n_o, $n_4 p_i$ appear. The arguments of these functions contain also the quantities $\psi_{0\nu}$, $\partial_0 \psi_0$. These can be expressed with rthe help of

$$D^\mu \psi_{\mu\nu} - D_\nu \psi - n_\nu = 0 \quad \psi - n_4 = 0 \quad M^2 \psi_{0i} - \partial^k \phi_{ki} - p_i = 0$$

through ψ_{ik}, ψ^*_{ik}, $\phi_{ik}\, \phi^*_{ik}$, A_ϱ, ω_ϱ and their spatial derivatives. By successive replacement of the above arguments,we obtain n_ν, n_{4a}, p_i, as a power series in e.
$N_0, \cdots, N_9$ decompose in two groups: $(N_{1,2,3}, N_4, N_{6,7,8}, N_9)$ and (N_0, N_5).
The first 8 subsidiary conditions can be used to express ψ_0, $m(N)$ as dependent quantities through (ψ_{ik}, A_ϱ, ω_ϱ, analogous to U_0 $(U_1, U_2, U_3, A_\varrho, \omega_\varrho)$ in the case of the vector meson.
// U_μ : field operators of the vector mesons. // N_0, N_5 can be used to eliminate the traces of ψ_{ik}, ϕ_{ik}

$$\psi_{ik} = \chi_{ik} + \frac{1}{3} g_{ik} \psi^n_n; \quad \chi^n_n = 0$$

$$\phi_{ik} = \eta_{ik} + \frac{1}{3} g_{ik} \phi^n_n; \quad \eta^n_n = 0$$

N_0 , N_5 can be brought to the form ϕ_{ik}

$$\psi_{ik} = \chi_{ik} + \frac{1}{3}\, g_{ik}\, \psi^n_n; \quad \chi^n_n = 0$$

$$\phi_{ik} = \eta_{ik} + \frac{1}{3}\, g_{ik}\, \phi^n_n; \quad \eta^n_n = 0$$

$$N_0\colon \frac{M^2}{2}\, \phi^n_n - \partial^i\, \partial^k\, \phi_{ik} - k = 0$$

$$N_5\colon \partial^i\, \partial^k\, (\psi_{ik} - g_{ik}\, \psi^n_n) - M^2\, \psi^n_n + f = 0$$

it follows

$$\left(\frac{M^2}{2} + \frac{\Delta}{3}\right) \phi^n_n = \partial^i\, \partial^k\, \eta_{ik} + k$$

$$\left(\frac{2}{3}\, \Delta - M^2\right) \psi^n_n = -\, \partial^i\, \partial^k\, \chi_{ik} + f$$

$$f = 2\, i\, e\, A^k\, \partial^i\, (\psi_{ik} - g_{ik}\, \psi^n_n) + i\, e\, \partial^i\, A^k\, (\psi_{ik} - g_{ik}\, \psi^n_n)$$

$$-\, e^2\, A^i\, A^k\, (\psi_{ik} - g_{ik}\, \psi^n_n) + i\, e\, \alpha^{(0)}\, F^{0k}\, \psi_{0k}$$

$$k = i\, e\, M^2\, A^n\, \psi_{n0} + \partial_n\, p^n + M^2\, u_0$$

$\psi^{\,n}_{\,n}$, $\phi^{\,n}_{\,n}$ can be represented as space integrals

$$\psi^n_n(\vec{x}) = \int \varrho_1(\vec{x}\,,\vec{y})\,[-\,\partial_{y\,i}\,\partial_{y\,k}\,\chi^{i\,k}(\vec{y}) + f(\vec{y})]\;d^3y$$

$$\phi^n_n(\vec{x}) = \int \varrho_2(\vec{x}\,,\vec{y})\,[\partial_{y\,i}\,\partial_{y\,k}\,\eta^{i\,k}(\vec{y}) + k(\vec{y})]\;d^3y$$

ϱ_1 , ϱ_2 are suitably chosen finctions of $\vec{x}$ and $\vec{y}$. In proceeding this way, it appears, that the fields develop a nonlocal character. This difficulty exists also for e = 0, whence the free field is by no means nonlocal. We will not perform the elimination which is suggested in equation (B.12), but rather will in subsection C, transform the subsidiary conditions in such a way, that the latter obtain the same functional structure as for e = 0.

The Hamilton operator has, modulo subsidiary conditions N_k , the form

$$\begin{aligned}
\mathcal{H}' &= {}^{(0)}\phi^{*\mu\nu}\,\partial_0\psi_{\mu\nu} + \partial_0\psi^{*\mu\nu}\,{}^{(0)}\phi_{\mu\nu} + {}^{(0)}\omega^{e}\,\dot{A}_e - C'\,m(N) \\
\mathcal{H}' &= -\,\frac{1}{2}\,\omega^{\nu}\,\omega_{\nu} + \frac{1}{2}\,\partial^k A^{\nu}\,\partial_k A_{\nu} + \phi^{*\,i\,k}\,\phi_{i\,k} - \frac{1}{2}\,\phi^{n}_{n}\,\phi^{m}_{m} \\
&\quad - 2\,[(\partial^i\phi^{*}_{ik})\,\psi^{0\,k} + \mathrm{h.c.}] + 2\,M^2\psi^{*\,0\,i}\,\psi_{0\,i} \\
&\quad - \partial^k\psi^{*\,n}_{n}\,\partial_k\psi^{m}_{m} - \partial^k\psi^{*\,n\,m}\,\partial_k\psi_{n\,m} + 2(\partial^k\psi^{*}_{ki})\,(\partial_m\psi^{m\,i}) \\
&\quad + M^2\,(\psi^{*\,n}_{n}\,\psi^{m}_{m} - \psi^{*\,i\,k}\,\psi_{i\,k}) \\
&\quad + 2\,i\,e\,A^i\,[\phi^{*}_{ik}\,\psi^{0\,k} - \mathrm{h.c.}] + i\,e\,(3-\alpha)\,F^{i\,k}\,\psi^{*}_{0\,k}\,\psi_{0\,i} \\
&\quad + i\,e\,(1-\alpha)\,F^{\,0\,i}(\psi^{*}_{ik}\,\psi^{k}_{n}) + i\,e\,(1+\alpha)\,F_{0\,k}\,[\psi^{*}_{0\,i}\,\psi^{k\,i} - \mathrm{h.c.}] \\
&\quad + i\,e\,A_0\,(\psi^{*}_{ik}\,\phi^{i\,k} - \mathrm{h.c.}) - i\,e\,(1-\alpha)\,F_{0\,k}\,[\psi^{*\,0\,k}\,\psi^{n}_{n} - \mathrm{h.c.}] \\
&\quad - 2\,i\,e\,A^k\,[\psi^{*}_{ki}\,\partial_n\,\psi^{n\,i} - \mathrm{h.c.}] + i\,e\,A^k\,[\psi^{*\,n}_{n}\,\partial_k\psi^{m}_{m} - \mathrm{h.c.}] \\
&\qquad\qquad + i\,e\,A^k\,[\psi^{*\,m\,n}\,\partial_k\psi_{m\,n} - \mathrm{h.c.}] \\
&\quad - \frac{1}{2}\,({}^{(0)}\omega_l - \omega_l)\,({}^{(0)}\omega^l - \omega^l) - e^2\,A^k\,A_k\,\psi^{*\,m}_{m}\,\psi^{m'}_{m'} \\
&\quad - e^2\,A^n\,A_n\,\psi^{*\,i\,k}\,\psi_{i\,k} + 2\,e^2\,A^k\,A^n\,\psi^{*}_{ki}\,\psi^{i}_{n} \quad m(N)
\end{aligned} \qquad \text{(B.13)}$$

We do not continue the translation of my thesis further here, since all basic notions introduced , can clearly be adapted to the case, where gravity is to be included in the list of basic local and quantized interactions. The chosen restrictiom, where gravity is left out, was chosen, to simplify things in higher orders of perturbative expansions, *only*.

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