Weak values and weak measurement in elementary scattering and reflectivity – a new effect

C Aris Dreismann Institute of Chemistry, TU Berlin

dreismann@chem.tu-berlin.de

7th International Conference on New Frontiers in Physics (ICNFP 2018)

Experimental context:

non-relativistic SCATTERING: incoherent, inelasic

Here: specifically **neutrons** → ***INS*** (also called **IINS**)

Theoretical context:

Weak Values (WV), Weak Measurement, Post-Selection,

Two-State Vector Formalism (TSVF)

Motivation:

This theoretical frame enable us to extract more physical (measurable !) information about the system than just the Born Rule and the Reduction Postulate of standard QM

Outline: *Short introductory remarks*

What is measured: Experimental setup of INS (Time-Of-Flight)

Weak values and INS experiment

New quantities being measurable

Further remarks, „educated guesses“, speculations … technological context

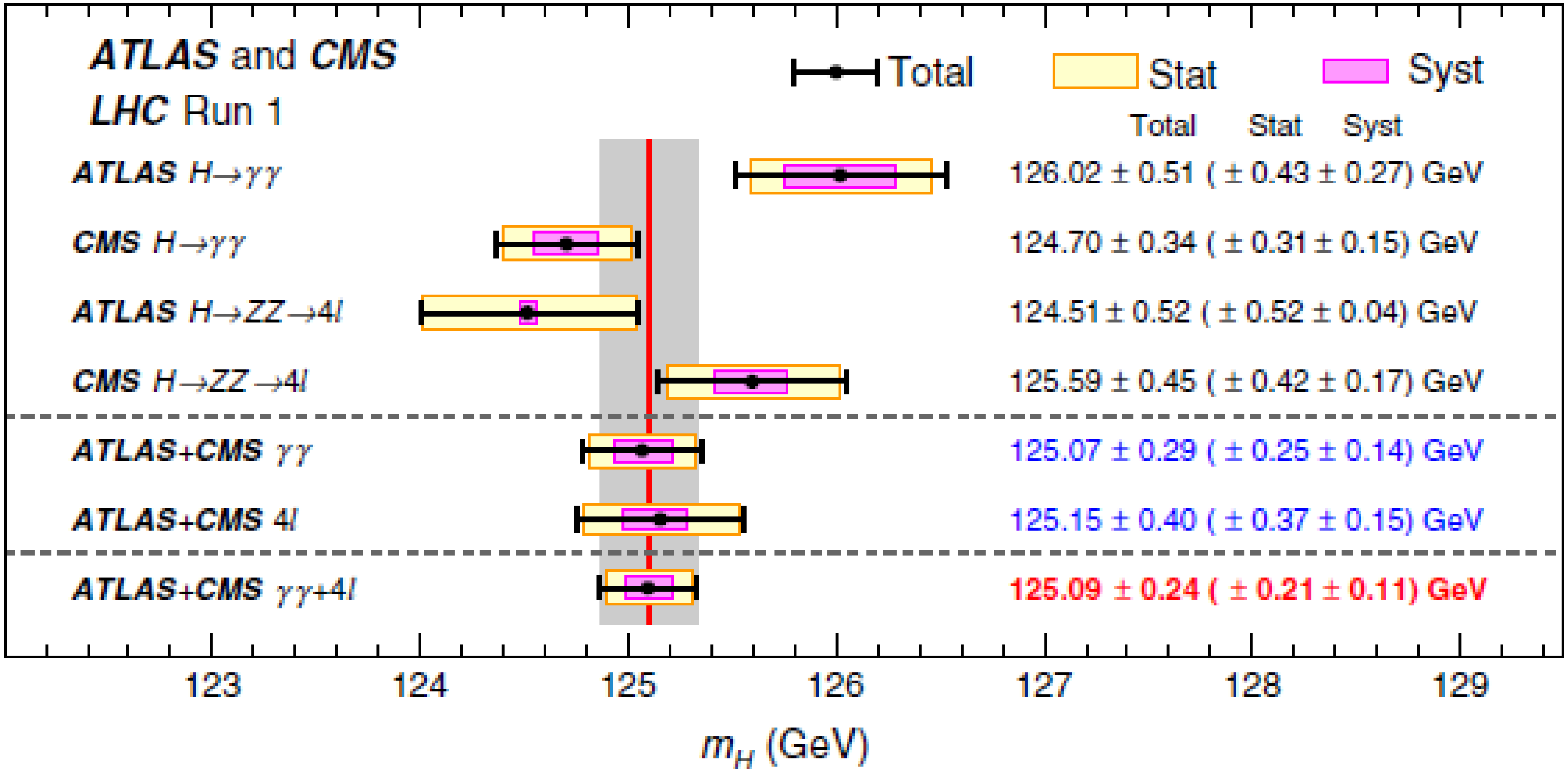
For stimulation: *What we see here?*

Selected for a Viewpoint in *Physics*
PRL 114, 191803 (2015) PHYSICAL REVIEW LETTERS week ending 15 MAY 2015

Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments

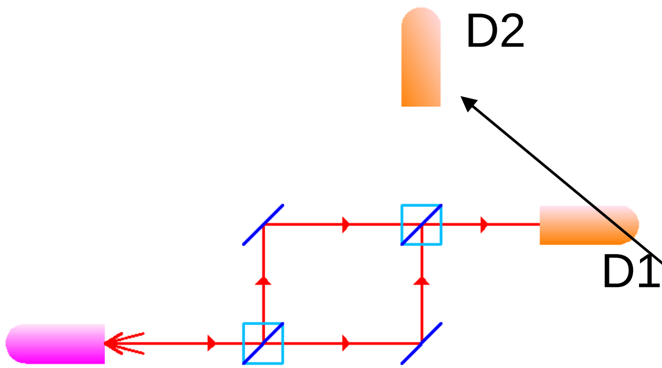
G. Aad *et al.*[*]

(ATLAS Collaboration)[†]
(CMS Collaboration)[‡]

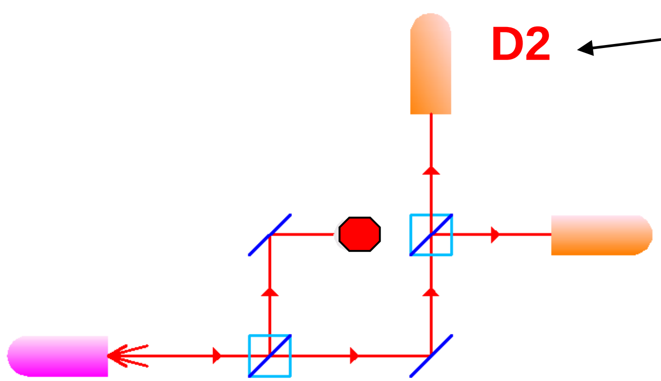


→ ***Elitzur-Vaidman effect:* „Bomb paradox“ *(~1993)***

(Interaction Free Measurement, IFM)



Figs. from Wikipedia.de



!!! **Postselection** of D2 **=** Photon is **measured** in the position of D2:

D2 „klick“ → Bomb „measured“ with certainty but with no energy/momentum exchange!

What are the „costs“ of this information?

„physical meaning“ of wavefunction?

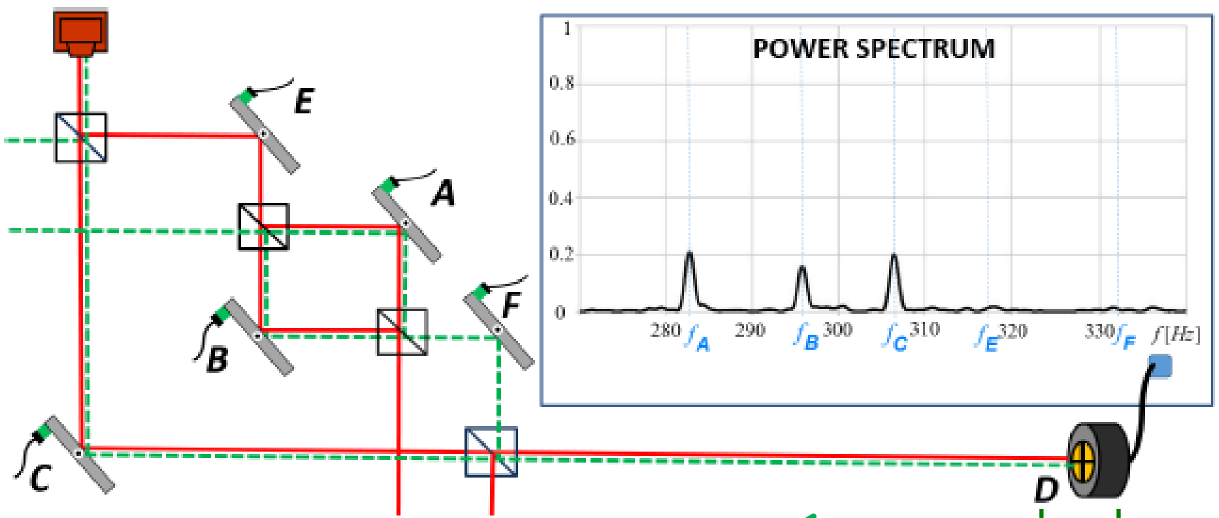
PRL 111, 240402 (2013) PHYSICAL REVIEW LETTERS week ending 13 DECEMBER 2013

Asking Photons Where They Have Been

A. Danan, D. Farfurnik, S. Bar-Ad, and L. Vaidman

Illustration of TSVF

„forward in time“



„backward in time“

188 | NATURE | VOL 474 | 9 JUNE 2011

Direct measurement of the quantum wavefunction

Jeff S. Lundeen[1], Brandon Sutherland[1], Aabid Patel[1], Corey Stewart[1] & Charles Bamber[1]

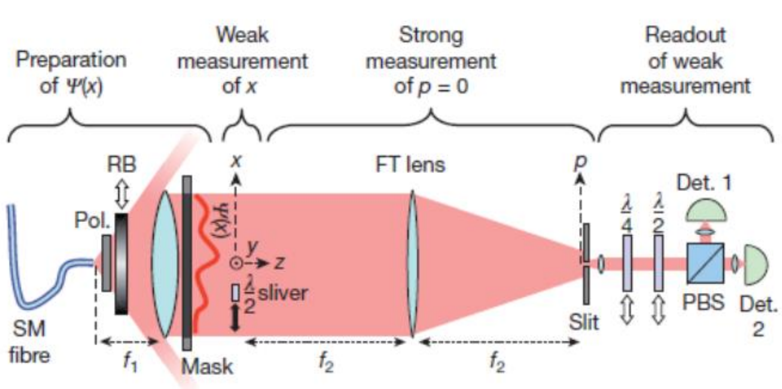


Figure 1 | Direct measurement of the photon transverse wavefunction. To begin with photons having identical wavefunctions, we transmit them through

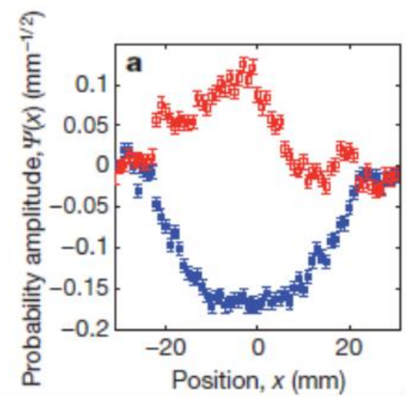


Figure 2 | The measured single-photon wavefunction, $\Psi(x)$, and its modulus squared and phase. a, Re$\Psi(x)$ (solid blue squares) and Im$\Psi(x)$ (open red squares) measured for the truncated Gaussian wavefunction.

Coherent vs. *Incoherent* Scattering

From: ***The Feynman Lectures on Physics, Volume III***

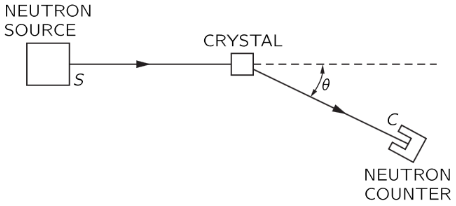


Fig. 3–5. Measuring the scattering of neutrons by a crystal.

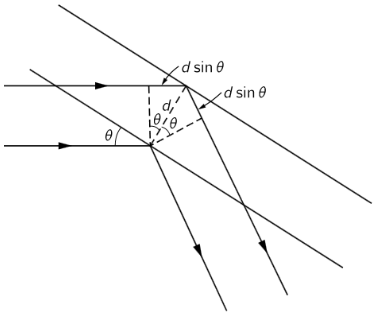
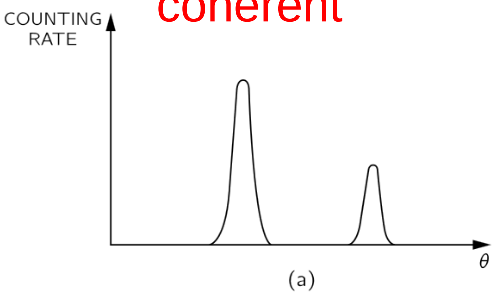


Fig. 2–4. Scattering of waves by crystal planes.

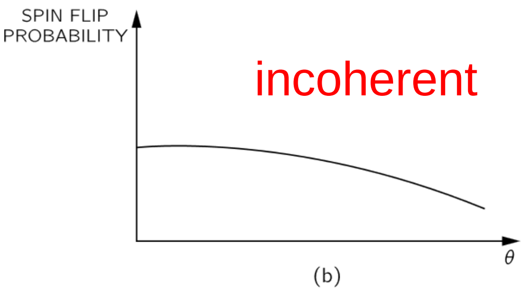
Bragg‘s Law

$2d\sin\theta = n\lambda \quad (n=1,2,\ldots).$

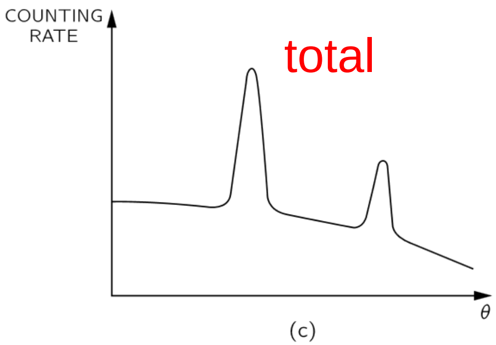
coherent



incoherent



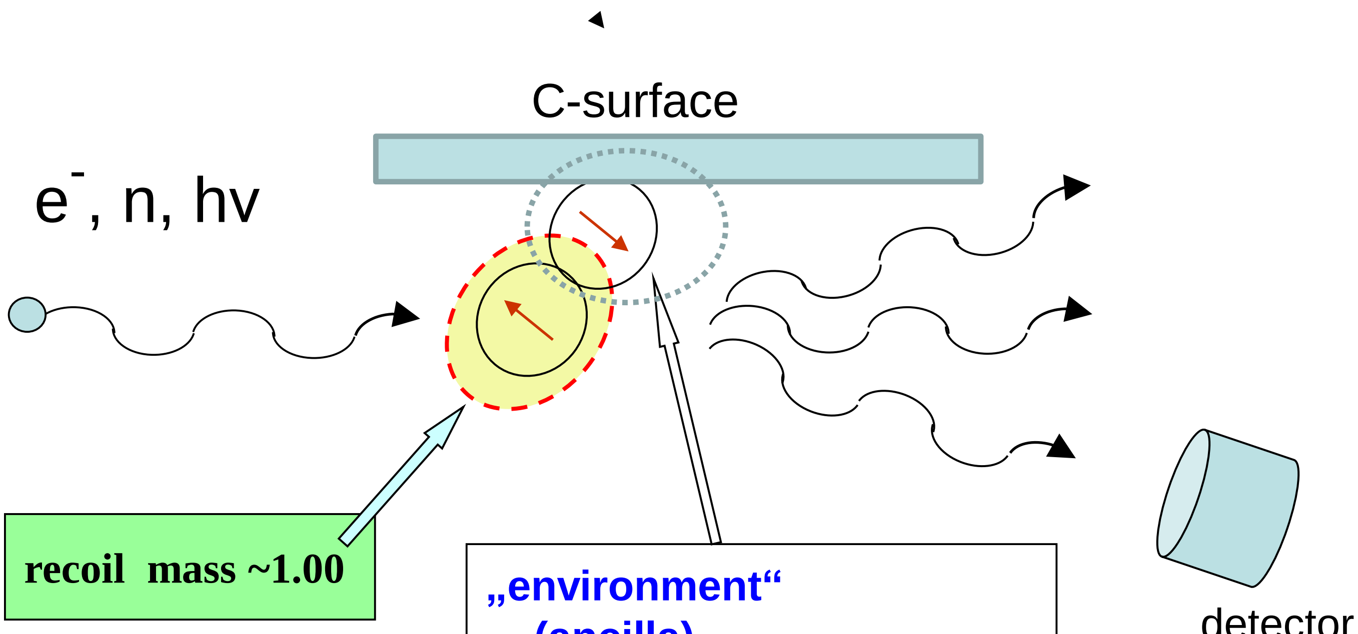
total



…..You may argue, “I don’t care which atom is up.” Perhaps you don’t, but nature knows; ….

Compton-like (i.e. incoherent) scattering - impulsive limit

E.g. H_2 molecules in C-nanostructures

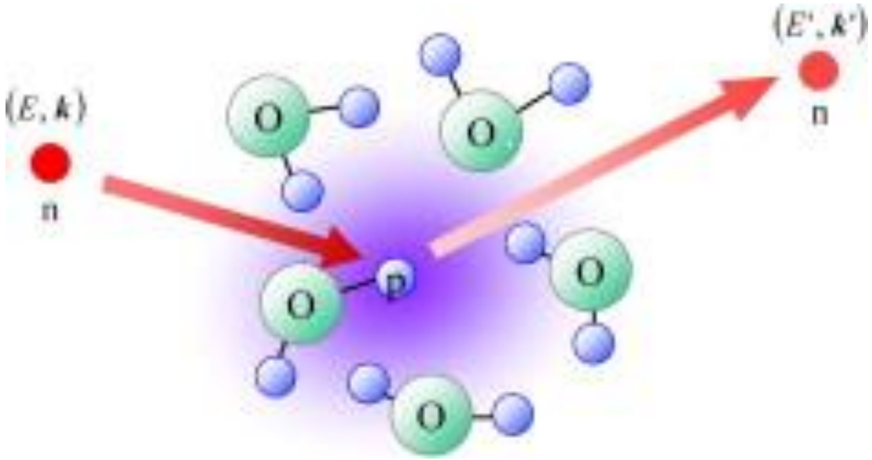


„environment“
(ancilla)
rest atoms, C-nanotube,
molecule-surface interactions,
phonons, excitons, etc. etc

scattering system:

a **part** of the **H_2**

molecule **!**



Conventional theory

Basic formula of neutron scattering theory (van Hove, 1954)

Textbook: **G L Squires,** Introduction to the Theory of Thermal Neutron Scattering (Mineola, Dover 1996)

$$\left(\frac{d^2\sigma}{d\Omega d\omega}\right)_{\nu\to\nu'} = \frac{k_1}{k_0}\left(\frac{m}{2\pi\hbar^2}\right)^2 \left|\langle \nu' \mathbf{k}_1 | V(\mathbf{r}) | \nu \mathbf{k}_0 \rangle\right|^2 \delta(E_\nu - E_{\nu'} + E_0 - E_1)$$

First-order pert. Theory,
δ-function: E-conservation

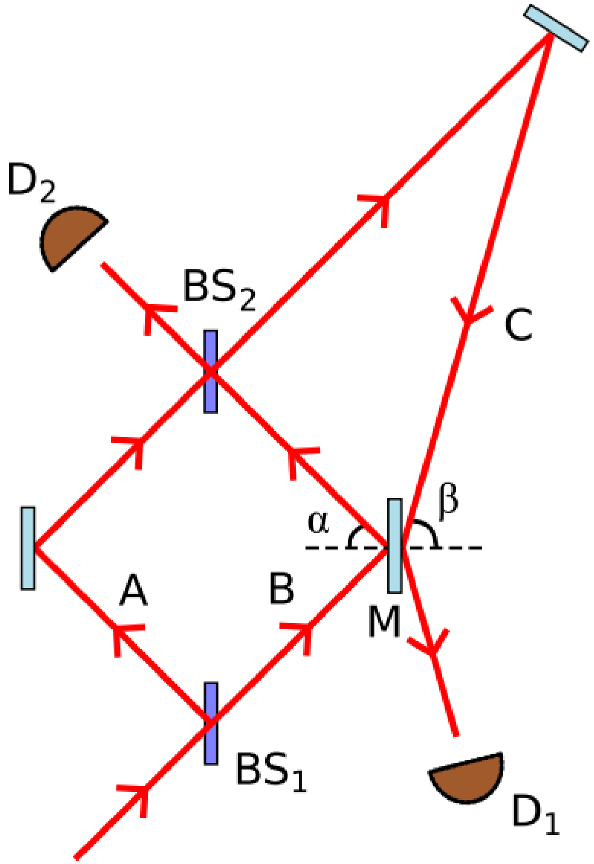
All dynamical quantities refer to the scattering system.
The quantum degrees-of-freedom of neutron „disappeared“ here, i.e here the neutron „is“ classical.

Claim**:** post-selection and weak values yield novel effects …

The classical limit of quantum optics: not what it seems at first sight

Yakir Aharonov[1,2], Alonso Botero[3], Shmuel Nussinov[1], Sandu Popescu[4], Jeff Tollaksen[2], and Lev Vaidman

New J Phys 15 **(2013)** *093006*

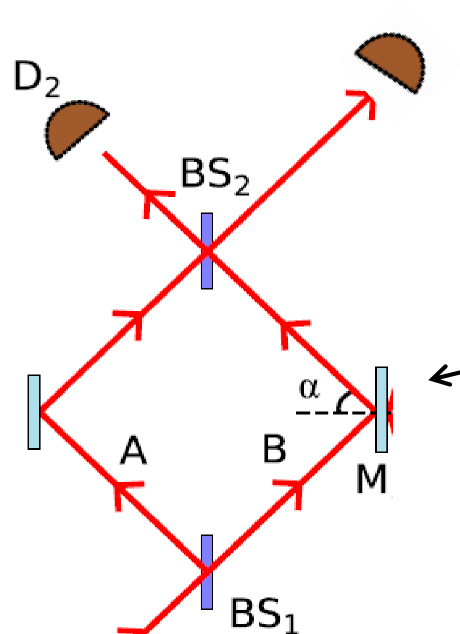


……

"An essential point to emphasize is that this whole discussion is not just restricted to interpretations. Quite the opposite. Allowing us to have a better intuition is essential for finding new and interesting quantum effects and may lead to new experiments and potential practical applications." *(p. 2)*

……

Here: postselection, WV, and „anomalous" momentum transfer



$$|\mathrm{in}\rangle \to ir|R\rangle + t|T\rangle$$

$$r^2 + t^2 = 1, \text{ and } \underline{r > t.}$$

Consider the photons (particles) measured with (***postselected*** by) detector $\mathbf{D_2}$:

M: „small“ mirror (*quantum* object)

What **direction** has the **momentum exchanged** with the mirror **M?**

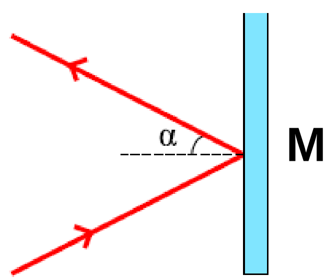
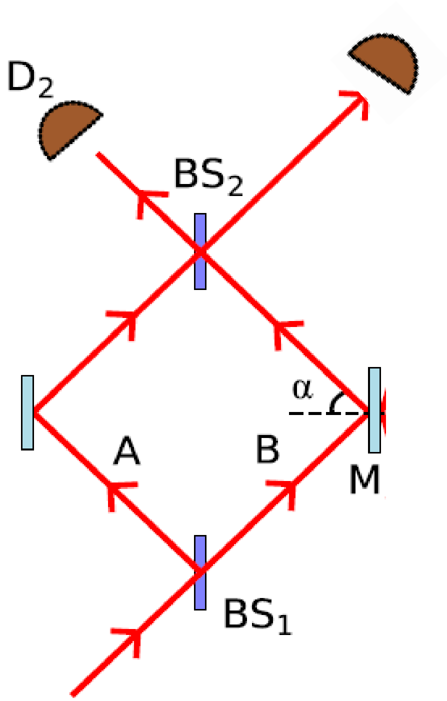


Figure 1. Light reflected on a mirror.

replace:
M → atom
?



when a single photon impinges from the left on BS_1 the effect of the beamsplitter is to produce inside the interferometer the state $|\Psi\rangle = ir|A\rangle + t|B\rangle$

(… postselection …)

As one can readily check, a photon in the quantum state $|\Phi_2\rangle = -ir|A\rangle + t|B\rangle$ emerges towards detector D_2. **!!!**

Thus, when a single photon enters the interferometer by impinging on the left side of the beamsplitter BS_1, the probabilities to be found in the arms A and B are r^2 and t^2 respectively. The probability of emerging towards D_2 is $|\langle\Phi_2|\Psi\rangle|^2 = (r^2 - t^2)^2 = 1 - 4r^2t^2$.

if $\phi(p)$ is the initial quantum state of the mirror and by $|\Psi\rangle$ the quantum state of the photon after the input beamsplitter BS1, but before reaching the mirror, the reflection on the mirror results in

$$|\Psi\rangle\phi(p) = (ir|A\rangle + t|B\rangle)\phi(p) \to ir|A\rangle\phi(p) + t|B\rangle\phi(p - \delta). \tag{6}$$

the kick given by the photon

$$\delta = 2\hbar\omega\cos\alpha$$ **(with c=1)**

momentum centred at **± 0** momentum centred at **+δ** we can approximate the state (6) of the photon and mirror just before the photon reaches the output beamsplitter by

$$|\Psi\rangle\phi(p) \approx ir|A\rangle\phi(p) + t|B\rangle\left(\phi(p) - \frac{d\phi(p)}{dp}\delta\right)$$
$$= |\Psi\rangle\phi(p) - t|B\rangle\frac{d\phi(p)}{dp}\delta. \qquad (7)$$

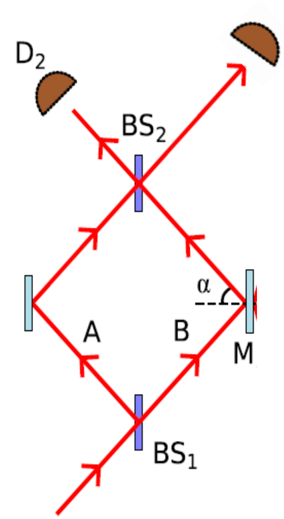
Suppose now that the photon emerges in the beam directed towards D_2. The state of the mirror is then given (up to normalisation) by projecting the joint state onto the state of the photon corresponding to this beam,

$$\langle\Phi_2|\left(|\Psi\rangle\phi(p) - t|B\rangle\frac{d\phi(p)}{dp}\delta\right)$$
$$= \langle\Phi_2|\Psi\rangle\left(\phi(p) - \frac{t\langle\Phi_2|B\rangle}{\langle\Phi_2|\Psi\rangle}\frac{d\phi(p)}{dp}\delta\right)$$
$$= \langle\Phi_2|\Psi\rangle\left(\phi(p) - \frac{\langle\Phi_2|P_B|\Psi\rangle}{\langle\Phi_2|\Psi\rangle}\frac{d\phi(p)}{dp}\delta\right)$$
$$= \langle\Phi_2|\Psi\rangle\phi(p - P_B^w\delta).$$

with P_B^w being in this case the weak value of P_B between $|\Psi\rangle$ and $|\Phi_2\rangle$,

$$P_B^w = \frac{\langle\Phi_2|P_B|\Psi\rangle}{\langle\Phi_2|\Psi\rangle} = \frac{(ir\langle A| + t\langle B|)P_B(ir|A\rangle + t|B\rangle)}{(ir\langle A| + t\langle B|)(ir|A\rangle + t|B\rangle)}$$
$$= -\frac{t^2}{r^2 - t^2}. \qquad (12)$$

Negative for r > t !!!



Hence the momentum kick received by the mirror due to a photon emerging towards D_2 is

$$\delta p_M = P_B^w\delta = -\frac{t^2}{r^2 - t^2}2\hbar\omega\cos\alpha. \qquad (13)$$

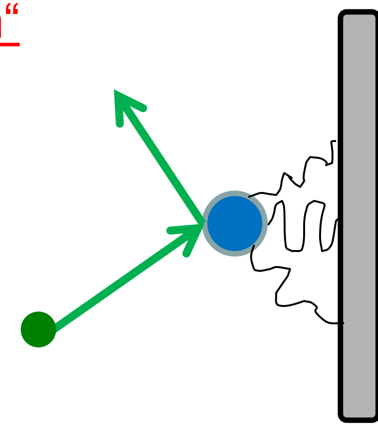
Astonishingly, although the photons

collide with the mirror only from the inside of the interferometer, they do not push the mirror outwards; rather they somehow succeed to pull it in! This is realized by a superposition of giving the mirror zero momentum and positive momentum - the superposition results in the mirror gaining negative momentum.

This is a novel effect predicted by WM and WV

!

WM, Short Remarks – „two-body collision“



von Neumann (strong, impulsive) measurement (1932)

$$\boxed{\hat{H}_{\mathrm{vN}} = -g(t)\,\hat{q} \otimes \hat{A}.}$$

meas. device (n) | system

the device-momentum is shifted by a measured Â-eigenvalue $\boldsymbol{a_i}$:

$$\langle \hat{p} \rangle_f - \langle \hat{p} \rangle_i = +g\,\boldsymbol{a_i} \qquad \text{where} \quad g = \int g(t)dt$$

Aharonov et al. (1988): „Weak Measurement“:

Device (apparatus) measures:

$$\langle \hat{p} \rangle_f - \langle \hat{p} \rangle_i = +g\,\mathrm{Re}[A_w] \qquad \text{where} \quad \hat{A}_w \equiv \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

is the „weak value“ of A defined for the selected initial and final states ψ_i and ψ_f

original publication Aharonov Y, Albert DZ, Vaidman L. How the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100. *Physical Review Letters* 1988; **60**(14): 1351–1354. doi:10.1103/PhysRevLett.60.1351

Quantum momentum-transfer deficit

interaction (von Neumann-type) Hamiltonian

$$\boxed{\hat{H}_{\rm int}(t) = +\lambda\,\delta(t)\,\hat{q} \otimes (\hat{P} - \hbar K\,\hat{I}_A)}$$

Represents a WV-correction to the conventional momentum-transfer operator

Assuming momentum conservation in the two-body collision, it holds

$$\boxed{-\hbar K_n = \hbar K_A \equiv \hbar K,} \qquad (40)$$

where $\hbar K_A$ is the momentum transfer *on the atom* due to the collision. (We choose K_A with positive sign, following

The scattering atom is assumed at rest in its initial state before collision, $\langle \hat{P} \rangle_i = 0$. After the collision, one conventionally expects that

$$\hbar K_A = +\hbar K = \langle \hat{P} \rangle_f = \langle \hat{P} \rangle_f - \langle \hat{P} \rangle_i \qquad (41)$$

and correspondingly for the neutron momentum

$$\hbar K_n = -\hbar K = \langle \hat{p} \rangle_f - \langle \hat{p} \rangle_i. \qquad (42)$$

weak value of atomic operator

$$\boxed{(\hat{P} - \hbar K\,\hat{I}_A)_w = P_w - \hbar K}$$

Represents a WV-correction to the conventional momentum transfer

a Special Case:

Initial and final states have the same width in momentum space.

initial atomic wave function $\Xi(P)_i$ can often be approximated by a Gaussian G_A centered at zero momentum,

$$\Xi(P)_i \approx G_A(P).$$

The atomic final state is centered at the transferred momentum:

$$\Xi(P)_f = \Xi(P - \hbar K_A)_i$$

The weak value of the atomic momentum operator is now as follows:

$$\begin{aligned} P_w &= \frac{\langle \Xi_f | \hat{P} | \Xi_i \rangle}{\langle \Xi_f | \Xi_i \rangle} \\ &= \frac{\int dP\, \Xi(P - \hbar K_A)_i\, P\, \Xi(P)_i}{\int dP\, \Xi(P - \hbar K_A)_i\, \Xi(P)_i} \\ &= +\frac{\hbar K_A}{2} = +\frac{\hbar K}{2}. \end{aligned} \qquad (57)$$

applying the above formulas yields:

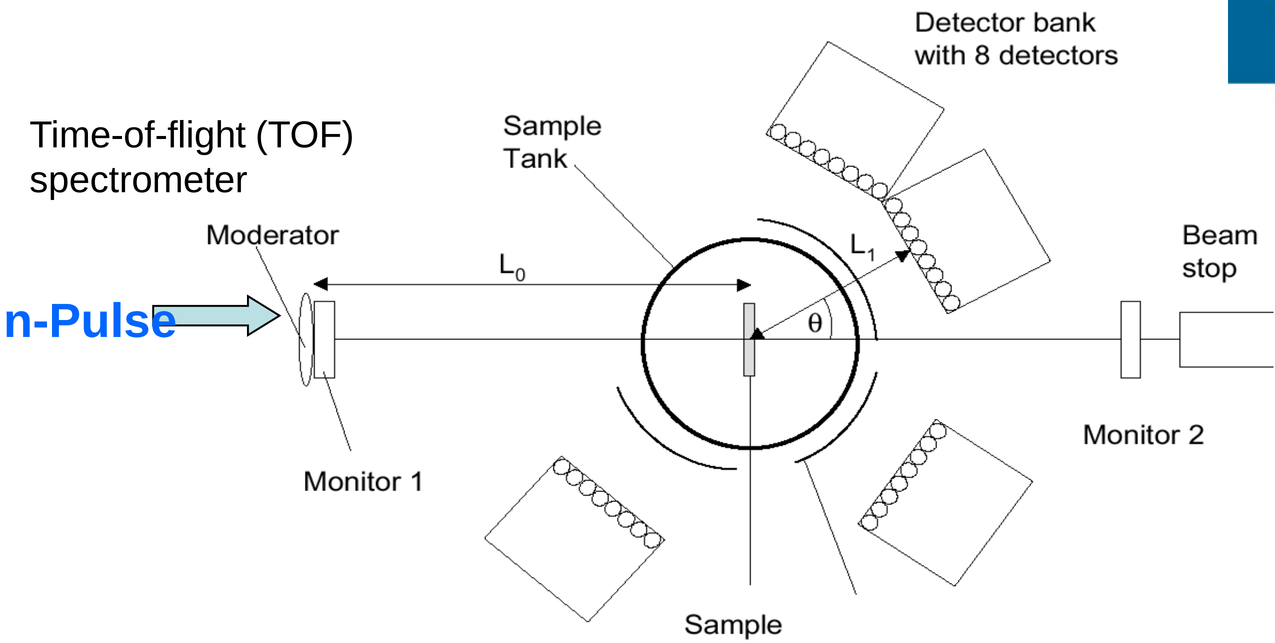
$$\boxed{\langle \hat{p} \rangle_f - \langle \hat{p} \rangle_i = -\lambda\,(\hat{P} - \hbar K\,\hat{I}_A)_w = +\lambda\,\frac{\hbar K}{2}.}$$

I.e., *the apparatus measures* ***only half of the conventionally expected momentum transfer !***

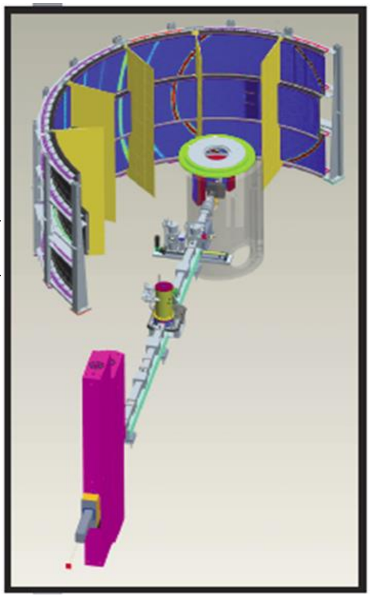
EXPERIMENT: Inelastic incoherence Neutron Scattering (INS)

Schematic:

Time-of-flight (TOF) spectrometer



OAK RIDGE National Laboratory | HIGH FLUX ISOTOPE REACTOR | SPALLATION NEUTRON SOURCE



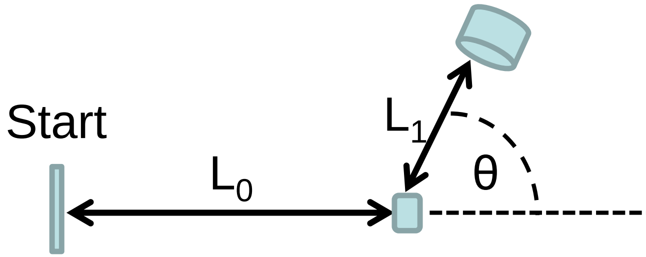
TOF (time of flight):

$$\boxed{t = \frac{L_0}{v_0} + \frac{L_1}{v_1} + t_0}$$

Time-of-Flight (TOF) spectrometer
Sample-Detectors: **3.0 – 3.4m**
Detectors (pixels) **~110.000**

CALIBRATION parameters: **L_0 , L_1, scatt. angle θ, t_0, v_0**

TOF-Measurement:



v_0 is FIXED (in case under consideration)
$E_0 = mv_0^2/2 = (\hbar k_0)^2/2m$: Kinetic energy
$mv_0 = \hbar k_0$: Momentum

TOF (**time of flight**): $\boxed{t = \frac{L_0}{v_0} + \frac{L_1}{v_1} + t_0}$

TOF <u>measured</u>, then one calculates:

→ **energy transfer** E_0-E_1= **E** = $\hbar\omega = \frac{(\hbar k_0)^2}{2m} - \frac{(\hbar k_1)^2}{2m}$

→ **momentum transfe** $\hbar\mathbf{q} = \hbar\mathbf{k}_0 - \hbar\mathbf{k}_1$

$$q = \sqrt{k_0^2 + k_1^2 - 2k_0 k_1 \cos\theta}$$

importtant:

<u>each measured TOF-point → only one E- and only one q-value !</u>

Detector „sees“ (selects) one **trajectory** (line) in the two-dim. ***q-E*** plane

<u>Elastic Scattering, crystallography:</u> **energy transfer =0** and $|k_0| = |k_1|$

Incoherent Inelastic NEUTRON Scattering (INS)

OAK RIDGE National Laboratory | HIGH FLUX ISOTOPE REACTOR | SPALLATION NEUTRON SOURCE

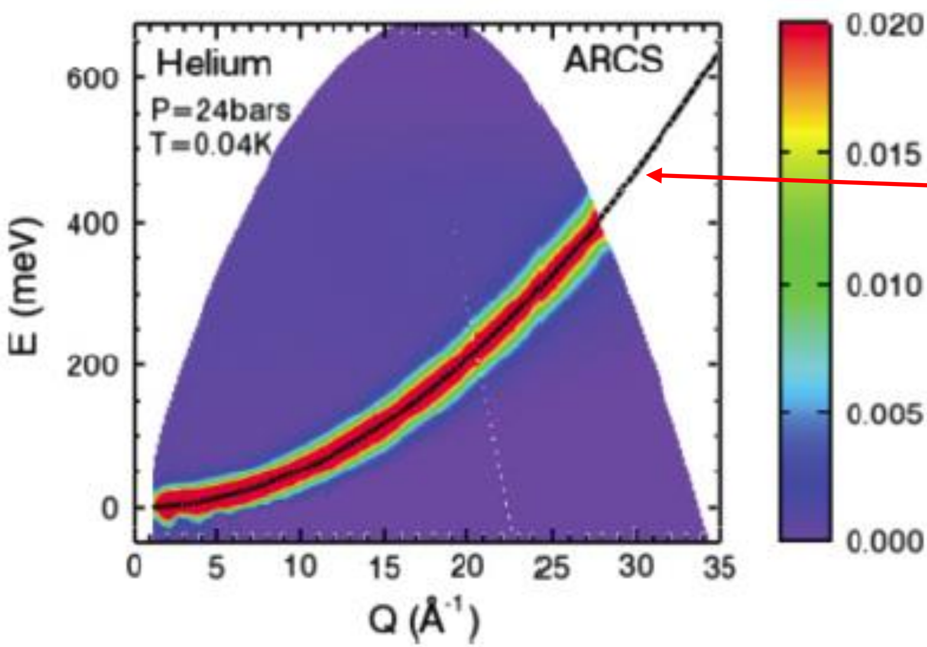
INSTRUMENT Fact Sheet — BEAM LINE 18 — SPALLATION NEUTRON SOURCE

ARCS – WIDE ANGULAR-RANGE CHOPPER SPECTROMETER

Example: PHYSICAL REVIEW B 85, 140505(R) (2012)

Bose-Einstein condensation in liquid ^{4}He near the liquid-solid transition line

DIALLO, AZUAH, ABERNATHY, ROTA, BORONAT, AND GLYDE



$$E_r = \hbar^2 Q^2 / 2m$$

Measured effective mass

***m* = 4.00 amu,**

as conventionally expected

FIG. 1. (Color online) Observed scattering intensity $S(Q,\omega)$ as a function of energy transfer $E = \hbar\omega$ and momentum transfer $\hbar Q$ from liquid ^{4}He at $p = 24$ bars and $T = 40$ mK. The signal from the empty Al container has been subtracted. The black dashed line is the calculated ^{4}He recoil line $E_r = \hbar^2 Q^2/2m$, shown as a guide to the eye.

CARBON 58 (2013) 46–58

ELSEVIER

Available at www.sciencedirect.com

SciVerse ScienceDirect

journal homepage: www.elsevier.com/locate/carbon

OAK RIDGE National Laboratory | HIGH FLUX ISOTOPE REACTOR | SPALLATION NEUTRON SOURCE

INSTRUMENT Fact Sheet — BEAM LINE 18 — SPALLATION NEUTRON SOURCE

ARCS – WIDE ANGULAR-RANGE CHOPPER SPECTROMETER

Quantum excitation spectrum of hydrogen adsorbed in nanoporous carbons observed by inelastic neutron scattering

Raina J. Olsen [a,b,*], *Matthew Beckner* [a], *Matthew B. Stone* [c], *Peter Pfeifer* [a], *Carlos Wexler* [a], *Haskell Taub* [a]

[a] *Department of Physics and Astronomy, University of Missouri, Columbia, MO 65211, USA*
[b] *Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*
[c] *Quantum Condensed Matter Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*

Inelastic neutron scattering spectra have been collected over a wide range of momentum transfer from H_2 adsorbed in several high-porosity carbon substrates.

FIGURE 2. Sample with oriented adsorption planes used for INS experiment.

pores: ~ 5-30 Å

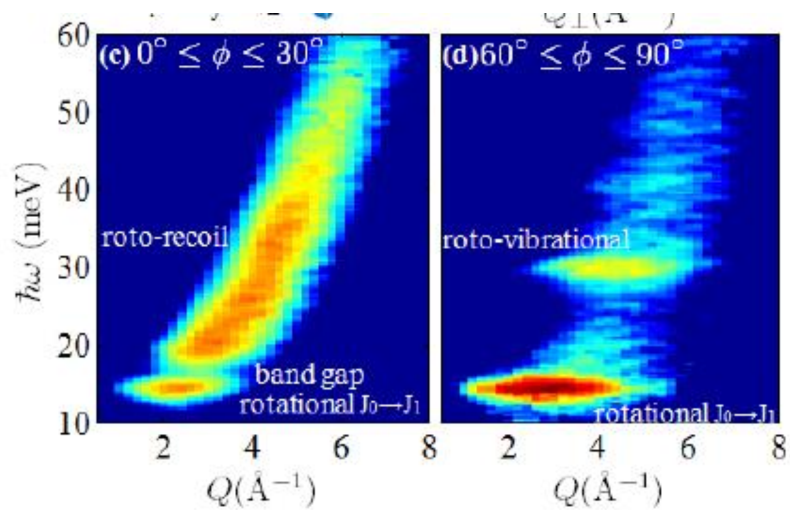


FIGURE 3. (a) Experimental geometry for IINS measurements of an oriented carbon sample, with energy transfer (ħω) and momentum transfer relative to the adsorption plane (Q) defined. (b) Spectrum of the main rotational peak as a function of Q|| and Q⊥. (c) Spectrum with the momentum transfer tending to be parallel or (d) perpendicular to the adsorption plane. Intensity is plotted on a log scale, temperature is 15 K, and the sample contains hydrogen at 25% coverage.

Copyright: CARBON

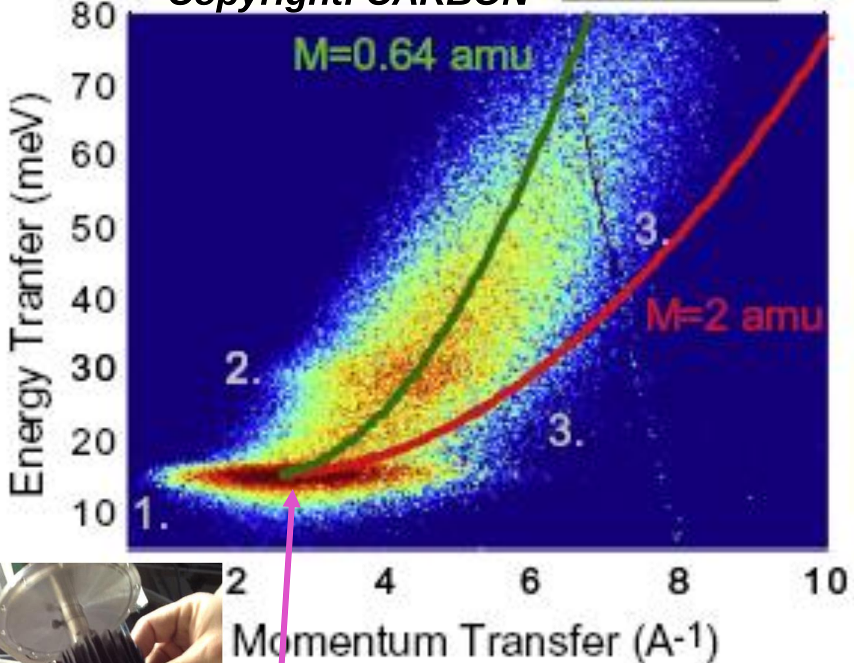


FIGURE 2. Sample with oriented adsorption planes used for INS experiment.

→Neutron collides with a „particle“ (H_2)
which **recoils (= translation)**
as having mass

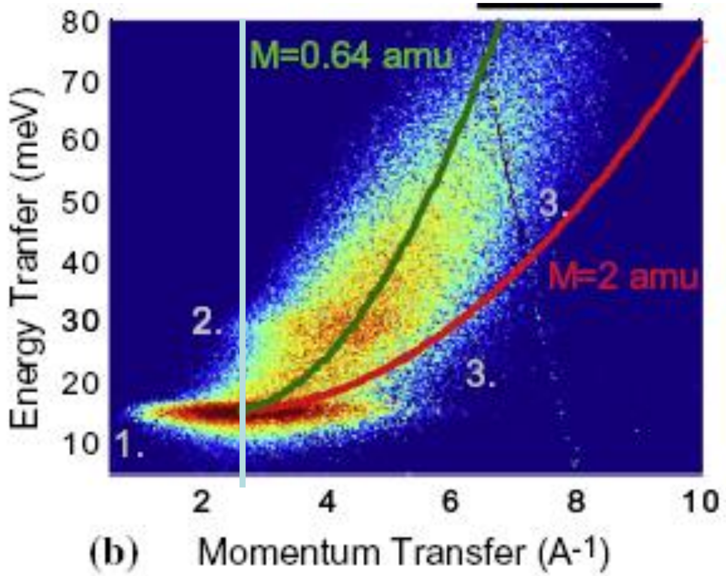
M

$\boldsymbol{M_{trans}=0.64}$ **amu** !!!

This effect has NO conventional interpretation

Remark:
H_2, n, C-surface are <u>quantum</u> objects

Neutron-H_2 collision
excites molecular **rotation**,
and scattering particle has
mass ***M ≈ 1 amu,***
as conventionally expected



At any fixed E_{trans}-transfer:
this measurement determines width of final-state wavefunction.

width of $\Xi(P)_f$ is „somewhat smaller“ than that of $\Xi(P)_i$

In conv. theory: Final state is a **plane wave**
i.e. it has $\Delta p_f = 0$!

For more details, see:

Weak Measurement and Two-State-Vector Formalism: Deficit of Momentum Transfer in Scattering Processes

Chariton Aris Chatzidimitriou-Dreismann

Institute of Chemistry (C2), Faculty II, Technical University of Berlin, D-10623 Berlin, Germany
E-mail: dreismann@chem.tu-berlin.de

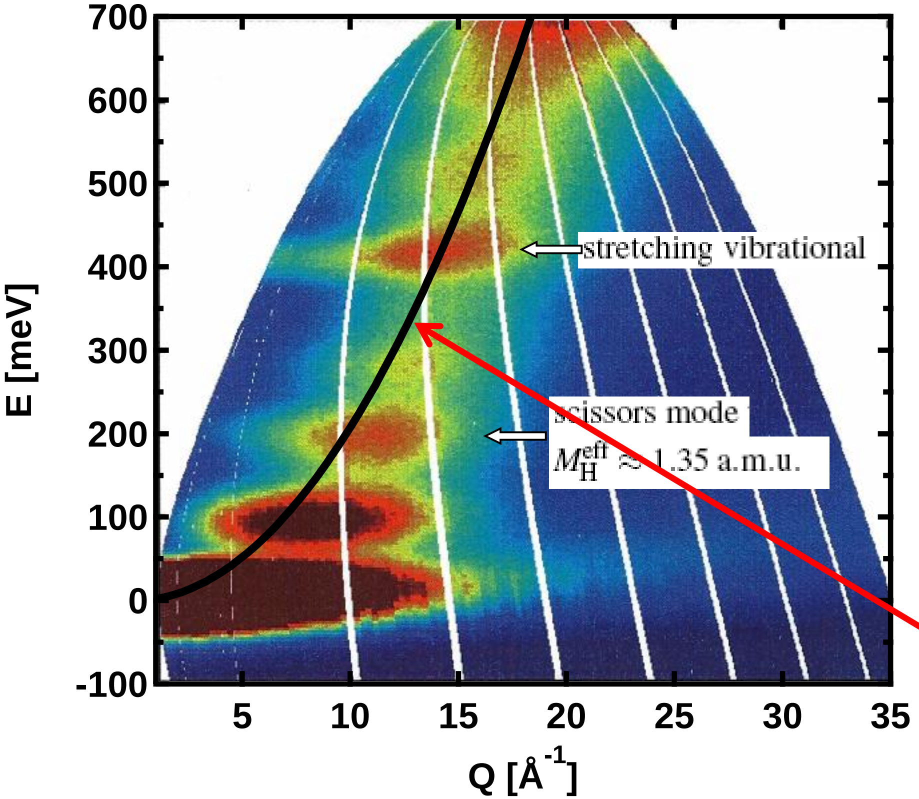
Editors: *Eliahu Cohen & Tomer Shushi*

Article history: Submitted on July 31, 2015; Accepted on October 20, 2016; Published on October 28, 2016.

Quanta | DOI: 10.12743/quanta.v5i1.48

WATER molecule, INS off protons

Example: *H_2O-ice in Al-cell, 20K, E_0=750 meV, MARI-spectrometer*



Z. Naturforsch. **69a**, 287 – 296 (2014)

INS from protons (hydrogen atoms; with mass M_H) of a molecule. Due to the similarity of neutron and proton masses, one may put $m_n = M_H$, and then the kinetic equations (8) and (9) yield the simple result $k_0^2 - k_1^2 = Q^2$. Thus one obtains for the neutron energy transfer (6) for n–H scattering:

$$E = E_0 - E_1 = \frac{\hbar^2 Q^2}{2m_n}. \quad (12)$$

....

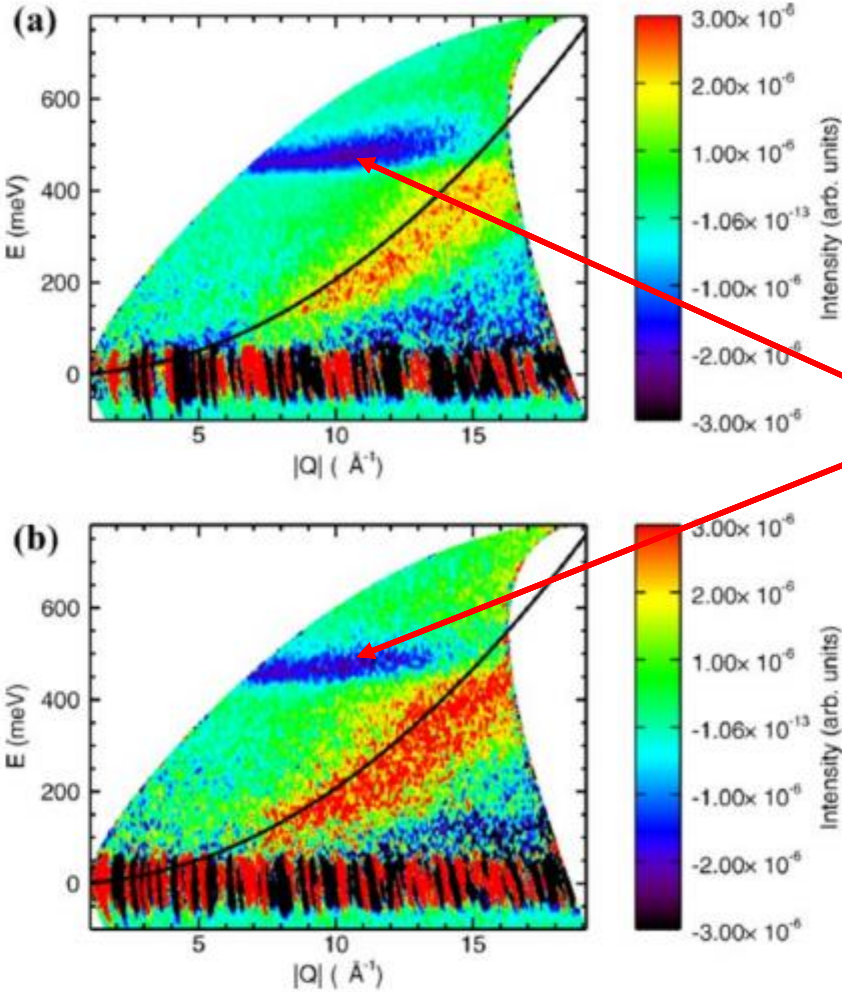
Rutherford Appleton Laboratory, UK) [32]. The inserted black (parabola) line is the calculated recoil line of a free hydrogen atom (being at rest before collision)

$$E_r^{\text{free}} = \frac{\hbar^2 Q^2}{2M_H}, \quad (13)$$

(M_H: hydrogen atom mass) shown as a guide to the eye.

increased effective mass: conventional effect --- no surprise

SEQUOIA | BL-17



Water molecules in nanotubes

A. I. Kolesnikov, L. M. Anovitz, E. Mamontov, A. Podlesnyak, G. Ehlers, *J. Phys. Chem. B* **2014**, *118*, 13414.

Scattering from single H – stretching vibrational mode

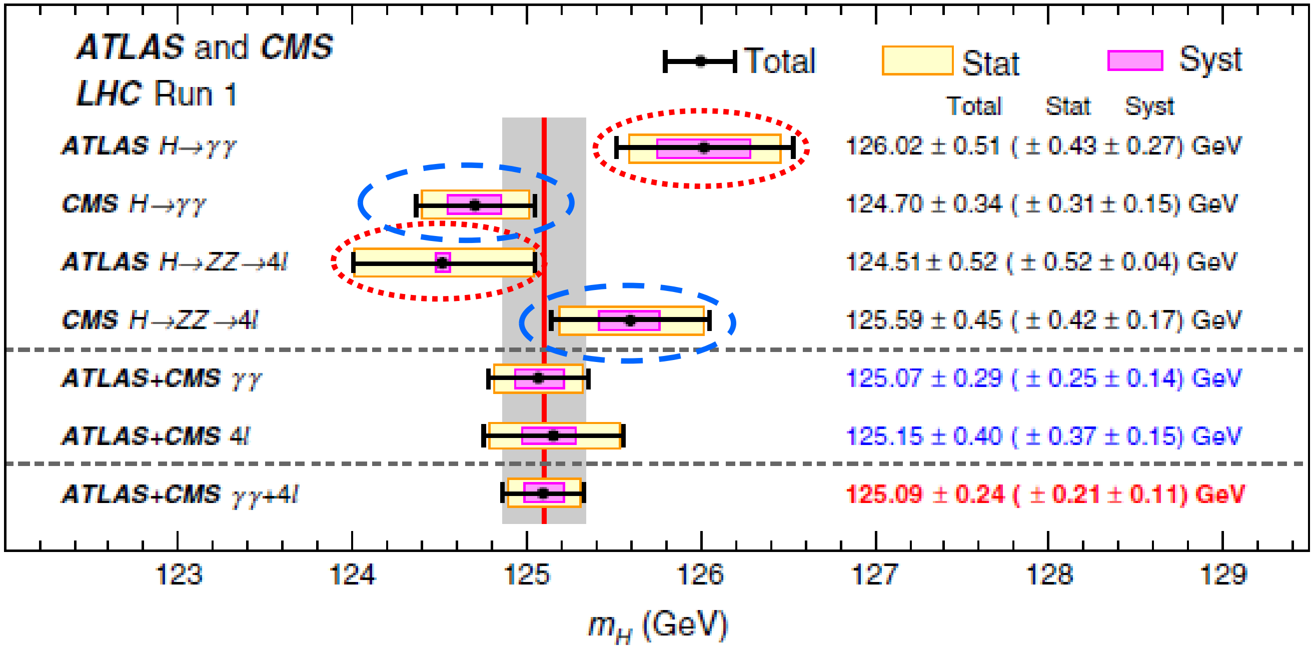
$$\boxed{M_{\mathrm{H}}^{\mathrm{eff}} \approx 0.55 \pm 0.1\ \mathrm{a.m.u.}}$$

Figure 7. Difference of the measured *S*(*Q*, *E*) intensity maps of oriented single H_2O molecules in sub-nano-channels of (a) beryl and (b) cordierite, at $T=6$ K with $E_0=800$ meV, for *Q*-perpendicular-to-*c* and *Q*-parallel-to-*c* orientations (see the text), recorded on SEQUOIA (SNS, Oak Ridge).[86] The black line (parabola) shows the calculated recoil trajectory of a free proton. Reprinted with permission from Ref. [86]; copyright 2014 American Chemical Society.

A speculative consideration:

PRL 114, 191803 (2015) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 15 MAY 2015

Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments



Do the two detectors ATLAS and CMS „make“ different post-selections?
What is precisely, directly measured?
What (and how) is it„deduced“?

Summary of main new results:

1. *New effect of INS*
2. *Neutron **must** be treated as a quantum object (contrary to textbook wisdom)*
3. *Experimental determination of (minimal) **number of qubits** „affected“ in scatt. experiments (neutron, electron X-rays, …)*
4. *Experimental determination of **width** of **final-state** wavefunction of a scattering particle (atom, molecule, nucleus, …)*
5. *Indicated applicability to „broader“ scientific & technological fields (e.g., H-mobility in fuel cells, H storage materials, ionic mobility in batteries…)*

Concluding remarks:

The theoretical formalisms of
weak values, weak measurement and two-state-vector formalism (Aharonov et al.)
not only shed new light on interpretational issues concerning fundamental quantum theory but they also offer a new guide for our intuition to predict, plan, and also to carry out new experiments and reveal novel quantum effects.

Thank you !