Does entanglement arise from random global fluctuations?

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Outline

- Deriving quantum mechanics
- The action S in classical and quantum mechanics
- Two extra axioms for quantum mechanics
- Deriving the uncertainty relations
- Entanglement as a global fluctuation?

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The conventional axioms of quantum mechanics are physically opaque. So-called “maximally nonlocal” or “PR-box” correlations violate Tsirelson’s bound but do not allow superluminal signalling. But Tsirelson’s bound is recovered in the classical limit of quantum mechanics!

M. Navascués and H. Wunderlich, A glance beyond the quantum model. *Proc. R. Soc*. A **2010**, 466, 881–890.

D. R., PR-box correlations have no classical limit, in *Quantum Theory: A Two-Time Success Story* [Yakir Aharonov Festschrift], eds. D. C. Struppa and J. M. Tollaksen (Milan: Springer), 2013, pp. 205-211.

D.R., Stronger-than-quantum bipartite correlations violate relativistic causality in the classical limit, arXiv:1408.3125.

Gisin, N. Quantum correlations in Newtonian space and time: Faster than light communication or nonlocality, in (The Frontiers Collection) *Quantum [Un]Speakables II: Half a Century of Bell’s Theorem*, eds. R. Bertlmann and A. Zeilinger, (Berlin: Springer), 2017; pp. 321–330.

A. Budiyono and D. R., Quantum mechanics as classical statistical mechanics with an ontic extension and an epistemic restriction. *Nat. Commun.* **8**, 1306 (2017).

Could there be parallel derivations of classical and quantum mechanics? Where do these theories part ways? To recover quantum mechanics, all we need is a *global fluctuating parameter* $\xi(t)$ with $\langle \xi(t) \rangle = 0$ and $\langle \xi^2(t) \rangle = \hbar^2$. We also require an *epistemic constraint.*

Roughly speaking, the global fluctuating parameter leads to *entanglement*, and the epistemic constraint leads to the *uncertainty principle*. Together, they allow us to derive quantum mechanics (without spin, and for quadratic Hamiltonians).

J-D. Bancal, S. Pironio, A. Acín, Y-C. Liang, V. Scarani and N. Gisin, "Quantum non-locality based on finite-speed causal influences leads to superluminal signalling", *Nature Physics* **8**, 867 (2012):

Even superluminal signalling *that we cannot* access contradicts relativity, unless (in some inertial reference frame) all the correlations are created *instantaneously*. The work of Bancal et al. shows that only *global* fluctuations could conceivably generate entanglement; and Agung Budiyono showed that, indeed, they do!

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The action S in classical mechanics is a function of N positions $q_1,\ldots,q_N$ and yields N conjugate momenta $p_1,\ldots,p_N$ via

$$p_i(q,t) \;=\; \partial S(q,t)/\partial q_i \quad .$$

Its explicit time dependence is (Hamilton-Jacobi)

$$\partial S(q,t)/\partial t \;=\; -H(p,q) \quad ,$$

and therefore its total time dependence is

$$\frac{dS(q,t)}{dt} = \sum_{i=1}^{N} \frac{\partial S(q,t)}{\partial q_i}\frac{dq_i}{dt} + \frac{\partial S(q,t)}{\partial t}$$

$$= \sum_{i=1}^{N} p_i \dot{q}_i - H(p,q) = L \qquad .$$

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In this sense, the Hamilton-Jacobi equation is the classical limit of Schrödinger's equation:

$$i\frac{\partial}{\partial t} e^{iS(q,t)} = H(p,q) e^{iS(q,t)} \qquad .$$

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We obtain classical mechanics – i.e. the Hamilton-Jacobi equation, which governs the time evolution of $S(q,t)$ – by postulating the principle of least action, or alternatively – if $H(p,q)$ is at most quadratic in p – by postulating conservation of average energy and probability current.

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Can we obtain quantum mechanics – i.e. the Schrödinger equation – by an analogous approach?

Classical mechanics contains a probability distribution $\rho(q,t)$ over the position degrees of freedom q_i at time t. But then every q determines a corresponding p via $p_i(q,t) = \partial S(q,t)/\partial q_i$ and so this $\rho(q,t)$ cannot be consistent with quantum mechanics.

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Let us replace the classical definition $p_i(q,t) = \partial S(q,t)/\partial q_i$ with the following quantum version:

$$p_i(q,t) = \frac{\partial}{\partial q_i} S(q,t) + \boxed{\frac{\xi}{2\rho}\frac{\partial \rho}{\partial q_i}} \quad ,$$

where ξ is a *randomly fluctuating global* variable having $\langle \xi \rangle = 0$ and $\langle \xi^2 \rangle = \hbar^2$.

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The central claim of this paper is that we obtain quantum mechanics – i.e. the Schrödinger equation, for $H(p,q)$ at most quadratic in p – by postulating conservation of average energy and probability current and

$$p_i(q,t) = \frac{\partial}{\partial q_i} S(q,t) + \boxed{\frac{\xi}{2\rho}\frac{\partial \rho}{\partial q_i}} \quad .$$

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$$p_i(q,t) = \frac{\partial}{\partial q_i} S(q,t) + \boxed{\frac{\xi}{2\rho}\frac{\partial \rho}{\partial q_i}} \quad .$$

The new, additional term in $p_i(q,t)$ implies the Heisenberg uncertainty principle. For example, if $\Delta p_i = 0$ then $\partial\rho/\partial q_i = 0$, so there is no information about q_i – the distribution of q_i is flat – and Δq diverges. Conversely if $\rho(q,t)$ is a δ-function in q_i (so $\Delta q = 0$), then Δp diverges.

Theorem 1: $\Delta q_i \, \Delta p_i \geq \hbar/2$, where $(\Delta q_i)^2 = \left\langle \left(q_i - \langle q_i \rangle \right)^2 \right\rangle$

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Proof of Theorem 1: from $1 = \int dq \;\; \rho$ we get

$$-1 = \int dq \;\; (q_i - \langle q_i \rangle)\sqrt{\rho} \;\; \frac{1}{\sqrt{\rho}} \partial_{q_i} \rho \quad .$$

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Cauchy-Schwartz law: $\langle a | a \rangle \cdot \langle b | b \rangle \geq \left| \langle a | b \rangle \right|^2$, hence

$$1 \leq \left[\int dq \;\; \rho (q_i - \langle q_i \rangle)^2 \right] \cdot \left[\int dq \frac{1}{\rho} (\partial_{q_i} \rho)^2 \right]$$

or

$$\frac{\hbar^2}{4} \leq (\Delta q_i)^2 \cdot \left[\int dq \;\; \rho \left(\frac{\hbar}{2} \frac{\partial_{q_i} \rho}{\rho} \right)^2 \right]$$

We have $\frac{\hbar^2}{4} \leq (\Delta q_i)^2 \cdot \left[\int dq \;\; \rho \left(\frac{\hbar}{2} \frac{\partial_{q_i} \rho}{\rho} \right)^2 \right]$, and from

$p_i = \partial_{q_i} S + \xi \left(\partial_{q_i} \rho \right) / 2\rho$ we have

$$(\Delta p_i)^2 = \int dq \, d\xi \, \mu(\xi) \rho \left(\frac{\xi}{2} \frac{\partial_{q_i} \rho}{\rho} + \partial_{q_i} S - \langle p_i \rangle \right)^2$$

$$\geq \int dq \, d\xi \, \mu(\xi) \rho \left(\frac{\xi}{2} \frac{\partial_{q_i} \rho}{\rho} \right)^2$$

$$= \int dq \, \rho \left(\frac{\hbar}{2} \frac{\partial_{q_i} \rho}{\rho} \right)^2$$

Hence $\hbar/2 \leq (\Delta q_i)\,(\Delta p_i)$.

Theorem 2: Expectation values of $\mathsf{O}(p,q)$ determined via ρ equal the quantum expectation values of corresponding Hermitian operators $\hat{\mathrm{O}}(p,q)$ obtained by applying the Dirac quantization scheme in the state $\left|\psi(t)\right\rangle$, where

$$\psi(q,t) \equiv \sqrt{\rho(q,t)}\; e^{iS(q,t)/\hbar} \quad ;$$

that is,

$$\left\langle \psi(t) \middle| \hat{O}(p,q) \middle| \psi(t) \right\rangle = \left\langle \mathsf{O}(p,q) \right\rangle$$

Theorem 3 (Schrödinger equation):

$$i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle \quad ,$$

where $\hat{H}$ is the Hermitian operator obtained from $H(p,q)$ by the Dirac quantization scheme.

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Entanglement: Note, from the definition of $\psi(q,t)$,

$$\psi(q,t) \equiv \sqrt{\rho(q,t)}\; e^{iS(q,t)/\hbar}$$

that if two systems indexed by i and j never interact, then their overall wave function factorizes, i.e. it is *separable*. Conversely, if there is any (local) interaction connecting the two systems, their overall wave function will be *entangled* rather than separable. This entanglement arises only because of the random global fluctuation ξ, which would leave the overall wave function separable if it did not correlate independent degrees of freedom.

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Entanglement of (say) particles 1 and 2 requires $S \neq S_1 + S_2$.

Model: assume that the results of a measurement of a_i by Alice and b_j by Bob (where $i,j = 0,1$) are $a_i = \mathrm{sgn}[\xi(t)] = (-1)^{ij} b_j$. Then the correlation C_{ij} of their measurements is $C_{ij} = (-1)^{ij}$, i.e. we get PR-box correlations.

$a_1 = -1$

$b_1 = 1$

Bob

Alice

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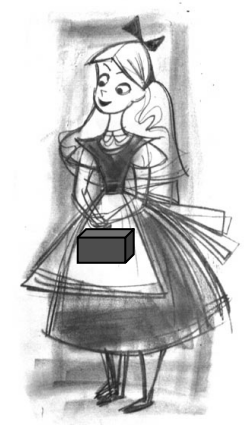
$b_0 = 1$

$a_1 = 1$

Bob



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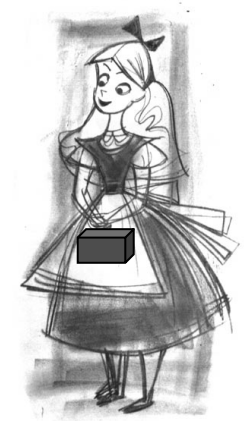

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Thus the global fluctuations imply epistemic restrictions: If Alice can measure a_0 *and* a_1 and Bob can measure b_0 *and* b_1, they get inconsistent results. And if Alice can measure *only* a_0 *or* a_1 but Bob can measure *both* b_0 *and* b_1, there is no logical contradiction but the no-signalling constraint is violated. Hence Alice and Bob can each measure only one observable out of two.