# Heterotic Thresholds: Universality and the Decompactification Problem

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with I. Florakis & M. Tsulaia work in progress with I. Florakis

INFN



- In the last decades we have witnessed a tremendous progress in the understanding of the structure of supersymmetric theories.
- Semi-realistic vacua incorporating salient features of the MSSM have been constructed
- The low-energy effective supergravity action has been reconstructed incorporating loop (and stringy) effects

- In particular a thorough study of radiative corrections to gauge couplings has been performed, both in heterotic and open strings
- This study has led to many results with applications both in phenomenology and in testing the non-perturbative string dualities

IN THIS TALK I SHALL REVIEW SOME OF THE PROPERTIES OF SUPERSYMMETRIC HETEROTIC THRESHOLDS, I SHALL EXTEND THEM TO THE CASE WHERE SUPERSYMMETRY IS SPONTANEOUSLY BROKEN AND SHALL PRESENT A CHIRAL FOU-DIMENSIONAL MODEL WHICH DOES NOT SUFFER FROM THE "DECOMPACTIFICATION PROBLEM"

### OUTLINE

A very short review of radiative corrections in heterotic vacua

Universality in (non)supersymmetric heterotic vacua

A solution of the decompactification problem

Threshold corrections to gauge couplings in heterotic vacua

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_a \log \frac{M_2^2}{\mu^2} + \Delta_a$$

In String Theory this amounts at computing the one-loop diagram

$$A_{\mu}^{a} = \int_{\mathcal{F}} \frac{d^{2}\tau}{\tau_{2}^{2}} \int d^{2}z \langle \mathcal{V}^{a}(z,\bar{z}) \mathcal{V}^{b}(0) \rangle$$

Threshold corrections to gauge couplings in heterotic vacua

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_a \log \frac{M_2^2}{\mu^2} + \Delta_a$$

The thresholds clearly depend on the amount of supersymmetry present in the vacuum.

Four-dimensional orbifold compactifications with N=1 supersymmetry the thresholds are O(1) numbers, unless the orbifold group contains sectors preserving (individually) N=2 supersymmetry.

In this case, the thresholds depend on the geometric moduli of spectator two-torii.

Threshold corrections to gauge couplings in heterotic vacua

$$-\frac{1}{4g_a^2}F_{\mu\nu}F^{\mu\nu}$$
 is BPS saturated in  $N=2$  theories

In the heterotic string:

$$\{Q, \bar{Q}\} = 2 P_{\mu} \gamma^{\mu} + 2 p_{L}^{i} \Gamma_{i}$$

BPS states:

$$m_{\rm L}^2 = |p_{\rm L}|^2$$
 (NS vacuum)

Threshold corrections to gauge couplings in heterotic vacua

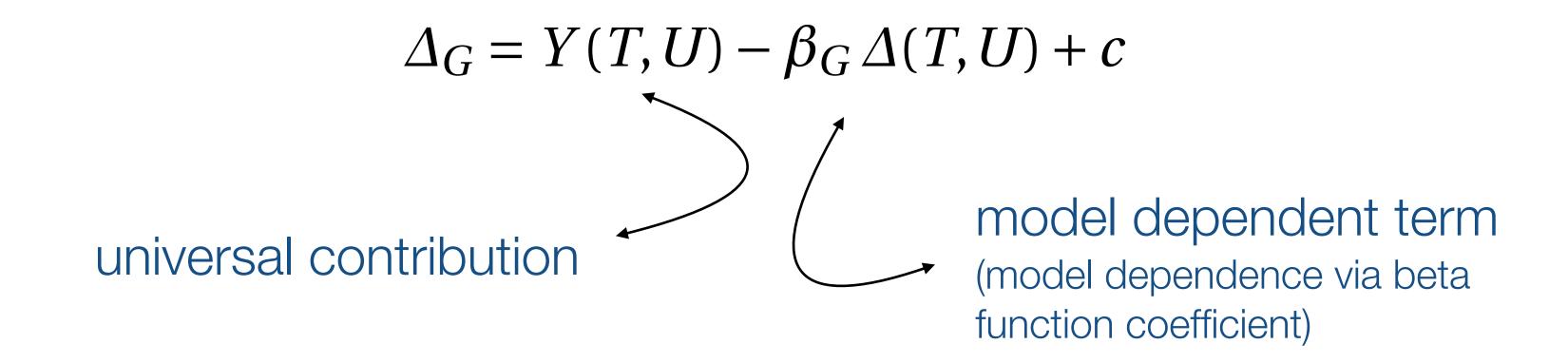
$$-\frac{1}{4g_a^2}F_{\mu\nu}F^{\mu\nu}$$
 is BPS saturated in  $N=2$  theories

$$\sum_{\text{BPS states}} \text{Str} \left( \frac{1}{12} - s^2 \right) \left( Q^2 - \frac{1}{4\pi\tau_2} \right) q^{\frac{1}{4}|p_L|^2} \bar{q}^{\frac{1}{4}|p_R|^2 + \bar{N}_{\text{osc}} - 1}$$

Universal contribution due to dilaton exchange

# A very short review of radiative corrections in supersymmetric vacua Universality of threshold corrections in heterotic vacua

Indeed in a generic heterotic vacuum with N=1 supersymmetry the gauge thresholds take (schematically) the form



# A very short review of radiative corrections in supersymmetric vacua Universality of threshold corrections in heterotic vacua

Therefore, in the difference of gauge thresholds

$$\Delta_G - \Delta_{G'} = (\beta_G - \beta_{G'}) \Delta(T, U)$$

and the power of supersymmetry uniquely fixes the functional dependence on the right-hand side. In fact, ...

# A very short review of radiative corrections in heterotic vacua Universality of threshold corrections in heterotic vacua

In the difference of gauge thresholds, the universal dilaton exchange cancels and

$$\Delta_a - \Delta_b = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \, \Gamma_{2,2}(T,U) \, \left( \frac{c_{-1}}{q} + c_0 + c_1 \, q + \ldots \right)$$
uncharged unphysical tachyon

Universality in the difference of gauge thresholds!

$$\Delta_a - \Delta_b = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \, \Gamma_{2,2}(T, U)$$

The decompactification problem in heterotic vacua

#### Upon computing the modular integral one gets

$$\Delta_G - \Delta_{G'} \propto (\beta_G - \beta_{G'}) \log \left[ T_2 U_2 \left| \eta(T) \eta(U) \right|^4 \right]$$

From this expression one can immediately see the source of the problem

$$\eta(T) = e^{i\pi T/12} \prod_{n=1}^{\infty} \left( 1 - e^{2i\pi nT} \right) \qquad \Rightarrow \qquad \Delta_G - \Delta_{G'} \propto (\beta_G - \beta_{G'}) T_2 + \dots$$

The decompactification problem in heterotic vacua

In the large volume (decompactification) regime the gauge coupling becomes strongly coupled or vanishes

$$\Delta_G - \Delta_{G'} \propto (\beta_G - \beta_{G'}) T_2 + \dots$$

In special cases, the universal contribution can cancel this large volume dependence for a given gauge group, but in general this problem is always present ... (see later!)

# Supersymmetry breaking in String Theory The Scherk-Schwarz reduction

The Scherk-Schwarz mechanisms for the spontaneous breaking of supersymmetry is the (only) viable construction in (closed) superstring theory.

The breaking of supersymmetry is due to different boundary conditions for states in the same supermultiplet, and the scale of supersymmetry breaking is ties to the size of compact dimensions.

In String Theory, it admits an exact CFT description in terms of freely acting orbifolds.

# Supersymmetry breaking in String Theory The Scherk-Schwarz reduction

### Caution!

When breaking supersymmetry in String Theory one has to be very careful with the stability of the vacuum.

Tadpoles are expected to emerge at some order in perturbation theory.

However, often tachyons appear in the classical spectrum, thus invalidating the whole analysis.

# Supersymmetry breaking in String Theory The Scherk-Schwarz reduction

### Caution!

Therefore we shall restrict our analysis to four-dimensional **non-tachyonic** heterotic vacua with spontaneously broken supersymmetry.

The theory is classically stable, and a one-loop dilaton tadpole is generated, calling for a proper redefinition of the (classical) vacuum.

The one-loop corrections to low-energy couplings are not affected, and thus can be **reliably computed**.

#### The Scherk-Schwarz reduction

### The vacuum

$$\frac{T^4}{\mathbb{Z}_N} \times \frac{T^2}{\mathbb{Z}_2}$$

- The  $\mathbb{Z}_N$  rotates the two complex  $T^4$  coordinates by opposite phases
- K3 singular limit (*N*=2,3,4,6)
- Would yield a four-dimensional theory with two supersymmetries

- $\bullet$  The  $\mathbb{Z}_2$  implements the Itoyama-Taylor construction
- It is a freely-acting (spontaneous)
   breaking of the E<sub>8</sub> x E<sub>8</sub> theory down to the non-supersymmetric O(16) x
   O(16) construction
- The spectrum is tachyon-free

#### The Scherk-Schwarz reduction

### The vacuum

$$\frac{T^4}{\mathbb{Z}_N} \times \frac{T^2}{\mathbb{Z}_2}$$

$$z_1 \rightarrow e^{2i\pi/N} z_1$$

$$z_2 \rightarrow e^{-2i\pi/N} z_2$$

$$(-1)^{F_{\rm st}+F_1+F_2}\delta$$

$$\delta: z_3 \to z_3 + \frac{\lambda_1}{2} + \frac{\lambda_2}{2} U$$

#### The Scherk-Schwarz reduction

# The partition function

$$\mathcal{Z} = \frac{1}{2} \sum_{H,G=0}^{1} \frac{1}{N} \sum_{h,g=0}^{N-1} \left[ \frac{1}{2} \sum_{a,b=0}^{1} (-)^{a+b} \vartheta \begin{bmatrix} a/2 \\ b/2 \end{bmatrix}^{2} \vartheta \begin{bmatrix} a/2+h/N \\ b/2+g/N \end{bmatrix} \vartheta \begin{bmatrix} a/2-h/N \\ b/2-g/N \end{bmatrix} \right]$$

$$\times \left[ \frac{1}{2} \sum_{k,\ell=0}^{1} \bar{\vartheta} \begin{bmatrix} k/2 \\ \ell/2 \end{bmatrix}^{6} \bar{\vartheta} \begin{bmatrix} k/2+h/N \\ \ell/2+g/N \end{bmatrix} \bar{\vartheta} \begin{bmatrix} k/2-h/N \\ \ell/2-g/N \end{bmatrix} \right] \left[ \frac{1}{2} \sum_{r,s=0}^{1} \bar{\vartheta} \begin{bmatrix} r/2 \\ s/2 \end{bmatrix}^{8} \right]$$

$$\times \frac{1}{\eta^{12} \bar{\eta}^{24}} (-)^{H(b+\ell+s)+G(a+k+r)+HG} \Gamma_{2,2} \begin{bmatrix} H \\ G \end{bmatrix} \Lambda^{K3} \begin{bmatrix} h \\ g \end{bmatrix} .$$

#### The Scherk-Schwarz reduction

# The light spectrum

In the gravity sector:

$$g_{\mu\nu}$$
,  $B_{\mu\nu}$ ,  $\varphi$ ,  $T$ ,  $U$ , ...

In the gauge sector:

$$SO(12) \times SO(16)$$

Gravitini have got the mass

$$m_{3/2}^2 = \frac{|U|^2}{T_2 U_2} \sim R_5^{-2}$$

The Scherk-Schwarz reduction

# The light spectrum

Extra massless states at special points of moduli space

$$O_4 O_4 \bar{V}_{12} \bar{O}_4 \bar{V}_{16} \times \frac{1}{2} \left( \Gamma_{2,2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \Gamma_{2,2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

$$\sim (12, 16)$$

charged with respect to gauge group

$$m_{\text{lightest}}^2 = \frac{\left|\frac{1}{2}T - U\right|^2}{T_2 U_2} \sim \left(R_5 - \frac{2}{R_5}\right)^2$$

Remember these states.

They will play an important role later on

## Radiative corrections in theories with spontaneously broken supersymmetry

One-loop gauge thresholds for SO(12) and SO(16)

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2}{g_s^2} + \beta_a\,\log\frac{M_2^2}{\mu^2} + \Delta_a$$
 [C.A., I. Florakis, M. Tsulaia, 2014, 2015]

In String Theory this amounts (again) at computing the one-loop diagram

$$A_{\mu}^{a} = \int_{\mathcal{F}} \frac{d^{2}\tau}{\tau_{2}^{2}} \int d^{2}z \langle \mathcal{V}^{a}(z,\bar{z}) \mathcal{V}^{b}(0) \rangle$$

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

$$-\frac{1}{4g_a^2}F_{\mu\nu}F^{\mu\nu}$$

 $-rac{1}{4\sigma^2}F_{\mu\nu}F^{\mu\nu}$  is no-longer BPS saturated

all states run in the loop

The freely acted  $T^2$  is no-longer spectator

$$\Delta_{a} = \int_{\mathcal{F}} \frac{d^{2}\tau}{\tau_{2}^{2}} \sum_{H,G=0,1} \Gamma_{2,2} \begin{bmatrix} H \\ G \end{bmatrix} (T,U) \Phi_{a} \begin{bmatrix} H \\ G \end{bmatrix} (\tau,\bar{\tau})$$

### Radiative corrections in theories with spontaneously broken supersymmetry

### One-loop gauge thresholds for SO(12) and SO(16)

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

# Example: N=2

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

# Example: N=2

Similar expressions for the SO(12) group.

In the difference of gauge thresholds the dilaton exchange cancels out and one is left with

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} \sim \int_{\mathcal{F}_0(2)} \frac{d^2 \tau}{\tau_2^2} \, \Gamma_{2,2} \left[ \begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right] \, \left[ \vartheta_2^4 \, |\vartheta_3^4 + \vartheta_4^4|^2 + \vartheta_4^4 \, |\vartheta_2^4 - \vartheta_4^4|^2 - \vartheta_3^4 \, |\vartheta_2^4 + \vartheta_3^4|^2 \right] \, \frac{\vartheta_2^4}{\eta^{12}} \, \frac{\bar{\vartheta}_3^4 \bar{\vartheta}_4^4}{\bar{\eta}^{12}}$$

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

# Example: N=2

Similar expressions for the SO(12) group.

In the difference of gauge thresholds the dilaton exchange cancels out and one is left with

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} \sim 12 \int_{\mathcal{F}_0(2)} \frac{d^2 \tau}{\tau_2^2} \Gamma_{2,2}[^0_1] \left( O_8^2 V_8 + 3V_8^3 \right) \left( \bar{O}_8^2 \bar{V}_8 - \bar{V}_8^3 \right)$$

in terms of the SO(2n) characters

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

# Example: N=2

$$\Delta_{\text{SO}(16)} - \Delta_{\text{SO}(12)} \sim 12 \int_{\mathcal{F}_0(2)} \frac{d^2 \tau}{\tau_2^2} \Gamma_{2,2}[^0_1] \left( O_8^2 V_8 + 3V_8^3 \right) \left( \bar{O}_8^2 \bar{V}_8 - \bar{V}_8^3 \right)$$

Still, this expressions looks very non-holomorphic ... however

$$\bar{O}_8^2 \, \bar{V}_8 - \bar{V}_8^3 = 8$$
 (one of the MSDS identities)

[Florakis, Kounnas, 2009]

so that

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

Example: N=2

$$\Delta_{\text{SO(16)}} - \Delta_{\text{SO(12)}} = \frac{1}{6} \int_{\mathcal{F}_0(2)} \frac{d^2 \tau}{\tau_2^2} \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left( 8 - \frac{\vartheta_2^{12}}{\eta^{12}} \right)$$

The integrand is purely holomorphic, and can be "easily" computed

Similar expressions for the other K3 orbifolds.

# Radiative corrections in theories with spontaneously broken supersymmetry

One-loop gauge thresholds for SO(12) and SO(16)

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

$$\Delta_{SO(16)} - \Delta_{SO(12)} = \alpha \log \left[ T_2 U_2 |\eta(T)\eta(U)|^4 \right] + \beta \log \left[ T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4 \right]$$
$$+ \gamma \log \left[ |\hat{\jmath}_2(T/2) - \hat{\jmath}_2(U)|^4 |j_2(U) - 24|^4 \right]$$

Universality in the difference of gauge thresholds!

(even for non-supersymmetric vacua)

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

$$\Delta_{SO(16)} - \Delta_{SO(12)} = \alpha \log \left[ T_2 U_2 |\eta(T)\eta(U)|^4 \right] + \beta \log \left[ T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4 \right]$$
$$+ \gamma \log \left[ |\hat{\jmath}_2(T/2) - \hat{\jmath}_2(U)|^4 |j_2(U) - 24|^4 \right]$$

The coefficients are O(1) numbers related to the difference of beta functions

This expression is invariant under  $\Gamma^0(2)_T \times \Gamma_0(2)_U$ , i.e. the left-over duality group.

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

$$\Delta_{SO(16)} - \Delta_{SO(12)} = \alpha \log \left[ T_2 U_2 |\eta(T)\eta(U)|^4 \right] + \beta \log \left[ T_2 U_2 |\vartheta_4(T)\vartheta_2(U)|^4 \right]$$
$$+ \gamma \log \left[ |\hat{\jmath}_2(T/2) - \hat{\jmath}_2(U)|^4 |j_2(U) - 24|^4 \right]$$

The term in the second line develops logarithmic singularities at T=2U and at their  $\Gamma_0(2)$  images. Extra massless states appear there!

[C.A., I. Florakis, M. Tsulaia, 2014, 2015]

# Back to the (universal) expression

$$\Delta_{\text{SO(16)}} - \Delta_{\text{SO(12)}} = \frac{1}{6} \int_{\mathcal{F}_0(2)} \frac{d^2 \tau}{\tau_2^2} \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left( 8 - \frac{\vartheta_2^{12}}{\eta^{12}} \right)$$

The integrand is purely holomorphic.

Is this amplitude BPS saturated?

If so, BPS states originate from the right-moving sector!

This is consistent with the MSDS identity, that seems to imply a hidden spectral flow in the right-moving sector of the heterotic string!

# Solving the decompactification problem in heterotic vacua Preliminary observations

[C.A., I. Florakis, to appear]

Remember the origin of the problem ...

$$\Delta_a - \Delta_b = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \, \Gamma_{2,2}(T, U)$$

In the lattice unfolding approach to the evaluation of the integral, the linear volume dependence comes from the vanishing orbit

To solve the problem one should change the integrand so that the vanishing orbit is missing and one is left (at most) with logarithmic dependence

# Solving the decompactification problem in heterotic vacua Preliminary observations

[C.A., I. Florakis, to appear]

This clearly happens if supersymmetry is (partially) broken à la Scherk-Schwarz, or the orbifold action is freely acting. In this case the non-universal term reads

$$\Delta \propto \int_{\mathcal{F}_0(N)} d\mu \, \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T, U)$$

and indeed it does not grow linearly with the volume, since the lattice shift implies that the vanishing orbit is missing.

# Solving the decompactification problem in heterotic vacua Preliminary observations

[C.A., I. Florakis, to appear]

Well ... is this the end of the story? After all, model with spontaneous (partial) breaking of supersymmetry have been studied since the 90's!

The main problem is, however, that this spontaneous breaking of supersymmetry is often in conflict with four-dimensional chirality.

Take for instance the  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orbifold, much studied in the literature. In this case, chiral fermions emerge **only** from the twisted sector.

In the freely acting variant, twisted sectors are *shorter* and only enjoy N=2 supersymmetry, thus being **non-chiral**.

#### A chiral model without decompactification problem

[C.A., I. Florakis, to appear]

Therefore, if we want to get a chiral spectrum and, at the same time, avoid the decompactification problem, the freely-acting orbifold should admit a chiral untwisted sector

Is this possible?

$$T^6/\mathbb{Z}_3 \times \mathbb{Z}_3'$$

$$U = e^{2i\pi/6}$$

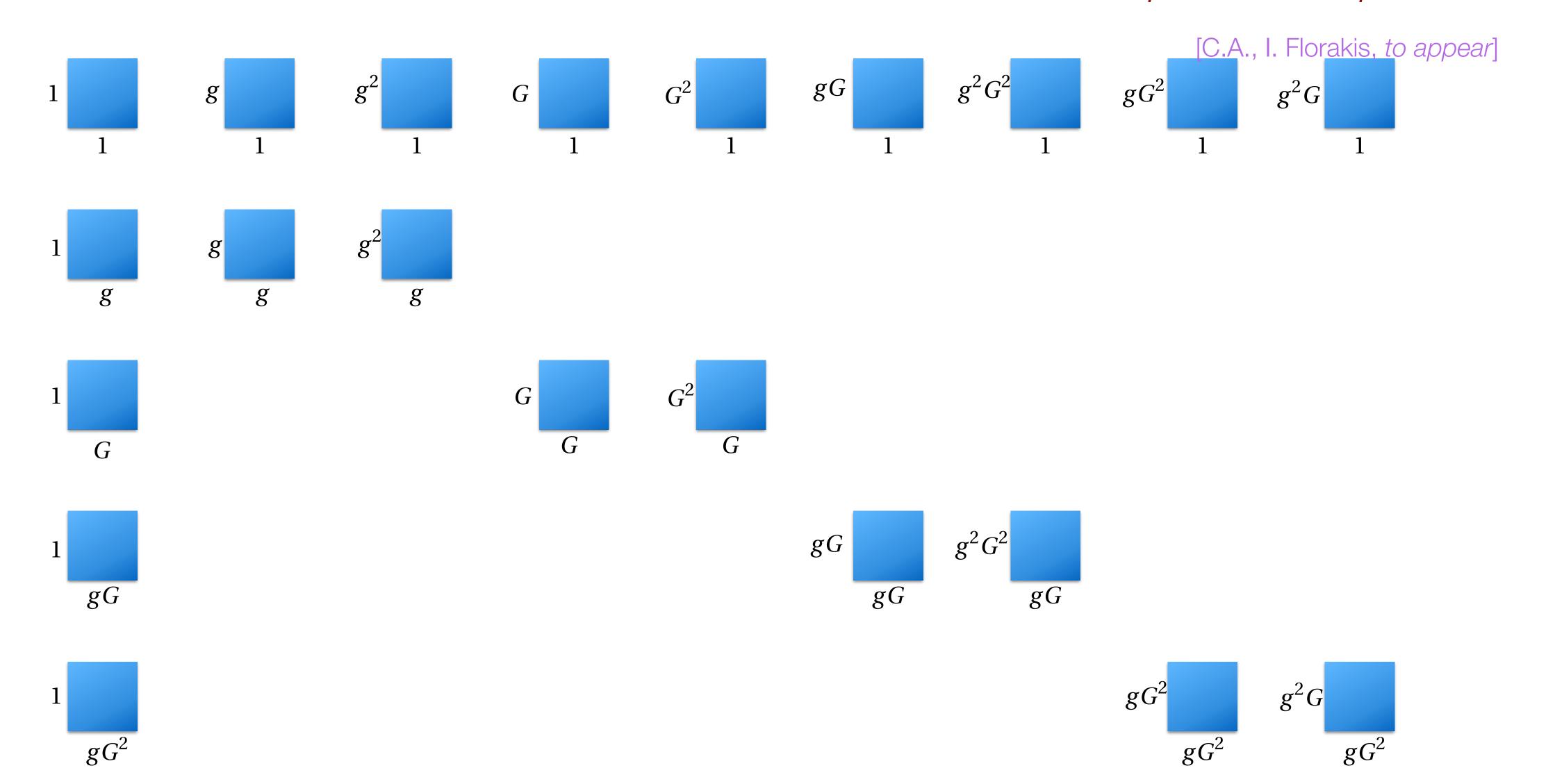
$$g = (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$$

$$G=(\frac{1}{3}+\delta_3,-\frac{1}{3}+\delta_3,\delta_3)$$
 The freely acting K3

$$\delta_3: Z \to Z + \frac{1}{3}(1+U)$$

An order-3 shift compatible with the structure of Z3 fixed points

#### A chiral model without decompactification problem



A chiral model without decompactification problem

[C.A., I. Florakis, to appear]

The gauge group is

$$G = E_6 \times E_8 \times U(1)^2$$

and the untwisted sector comprises chiral multiplets in the (27,1) chiral representation

#### A chiral model without decompactification problem

[C.A., I. Florakis, to appear]

Threshold corrections can be computed (almost explicitly) using the methods developed by C.A. I. Florakis and B. Pioline, and read

$$\Delta_G = Y(T, U) - \beta_G \Delta(T, U) + c$$

$$\Delta = \int_{\mathscr{F}_0(3)} d\mu \, \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T, U) = \frac{3}{2} \int_{\mathscr{F}} d\mu \, \Gamma_{2,2} (\frac{1}{3} \, T, \frac{1}{3} (1 + U)) - \frac{1}{2} \int_{\mathscr{F}} d\mu \, \Gamma_{2,2} (T, U)$$

#### A chiral model without decompactification problem

[C.A., I. Florakis, to appear]

Threshold corrections can be computed (almost explicitly) using the methods developed by C.A. I. Florakis and B. Pioline, and read

$$\Delta_G = Y(T, U) - \beta_G \Delta(T, U) + c$$

$$\Delta = -\log \left[ T_2 U_2 \left| \frac{\eta^3(T/3)}{\eta(T)} \frac{\eta^3(U/3)}{\eta(U)} \right|^2 \right] + \text{cost}$$

#### A chiral model without decompactification problem

[C.A., I. Florakis, to appear]

#### Similarly

$$Y = \frac{1}{144} \int_{\mathcal{F}_0(3)} d\mu \, \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T, U) \left[ \frac{\hat{E}_2(3\bar{E}_4^2\bar{X}_3 - 2\bar{E}_4\bar{E}_6)}{2\bar{\Delta}} + \frac{2\bar{E}_4^3 - 3\bar{X}_3\bar{E}_4\bar{E}_6}{2\bar{\Delta}} + 1152 \right]$$

$$= -24 \log |j(T) - j(U)|^4 - 24^2 \log \left[ T_2 U_2 |\eta(T) \eta(U)|^4 \right]$$

+exponentially suppressed terms

## **CONCLUSIONS**

Surprisingly enough, radiative corrections may be still under control even when supersymmetry is broken (universality)

It is possible to evade the decompactification problem in chiral four-dimensional heterotic vacua

# THANK YOU

$$T^6/\mathbb{Z}_3 \times \mathbb{Z}_3'$$
  $U = e^{2i\pi/6}$ 

$$g = (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$$

$$G = (\frac{1}{3} + \delta_3, -\frac{1}{3} + \delta_3, \delta_3)$$

$$\delta_3: Z \to Z + \frac{1}{3}(1+U)$$

$$\Delta_G = Y(T, U) - \beta_G \Delta(T, U) + c$$

$$G = E_6 \times E_8 \times U(1)^2$$

$$\Delta = \int_{\mathscr{F}_0(3)} d\mu \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T, U)$$

$$\Delta = \int_{\mathcal{F}_0(3)} d\mu \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T,U) = \frac{3}{2} \int_{\mathcal{F}} d\mu \Gamma_{2,2} (\frac{1}{3}T, \frac{1}{3}(1+U)) - \frac{1}{2} \int_{\mathcal{F}} d\mu \Gamma_{2,2} (T,U)$$

$$\Delta = -\log \left[ T_2 U_2 \left| \frac{\eta^3(T/3)}{\eta(T)} \frac{\eta^3(U/3)}{\eta(U)} \right|^2 \right] + \text{cost}$$

$$Y = \frac{1}{144} \int_{\mathscr{F}_0(3)} d\mu \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (T, U) \left[ \frac{\hat{E}_2(3\bar{E}_4^2\bar{X}_3 - 2\bar{E}_4\bar{E}_6)}{2\bar{\Delta}} + \frac{2\bar{E}_4^3 - 3\bar{X}_3\bar{E}_4\bar{E}_6}{2\bar{\Delta}} + 1152 \right]$$

$$\frac{\bar{E}_2(3\bar{E}_4^2\bar{X}_3 - 2\bar{E}_4\bar{E}_6)}{2\bar{\Delta}} + \frac{2\bar{E}_4^3 - 3\bar{X}_3\bar{E}_4\bar{E}_6}{2\bar{\Delta}} + 1152 = \Phi^{(3)} + 2016$$

$$\Phi^{(3)} = 24 \,\mathscr{F}_{\infty}^{(3)}(1,1,0) - 4 \,\mathscr{F}_{\infty}^{(3)}(2,1,0) - 48 \,\mathscr{F}_{0}^{(3)}(1,2,0) + 8 \,\mathscr{F}_{0}^{(3)}(2,2,0) - 72 \,\mathscr{F}_{0}^{(3)}(1,1,0) + 12 \,\mathscr{F}_{0}^{(3)}(2,1,0)$$

$$Y = 14 \int_{\mathcal{F}_0(3)} d\mu \Gamma_{2,2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \frac{1}{2 \cdot 144} \int_{\mathcal{F}} d\mu \Gamma_{2,2} \varphi + \frac{3}{2 \cdot 144} \int_{\mathcal{F}_0(3)} d\mu \Gamma_{2,2} \Phi^{(3)}$$

$$\varphi = -4\mathscr{F}(2,1,0) + 24\mathscr{F}(1,1,0)$$

$$\int_{\mathscr{F}} d\mu \, \Gamma_{2,2} \, \varphi = -4 \, I^{(0)}(2) - 4 (I^{(+)}(2) + \text{c.c.})$$

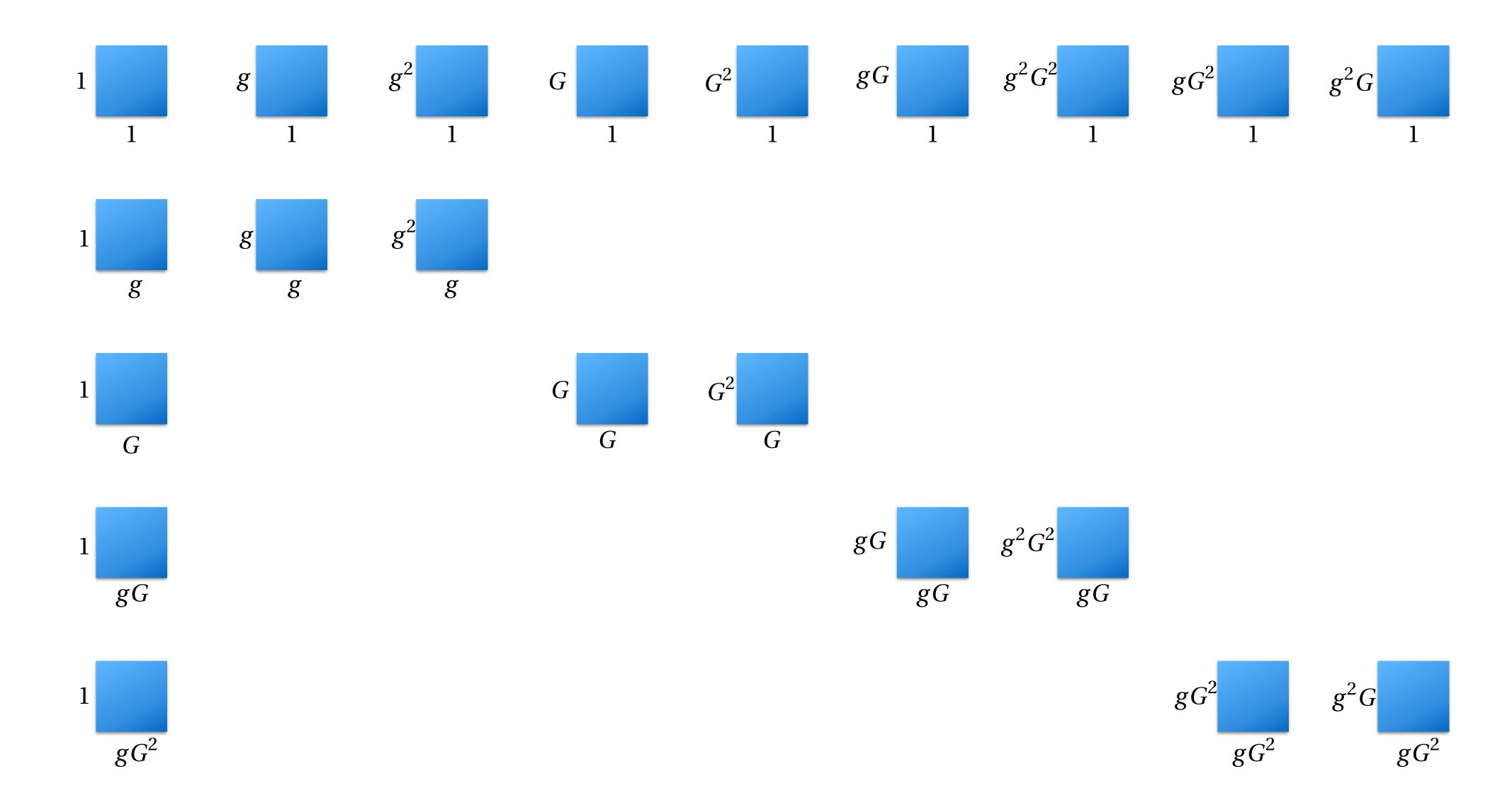
$$-24 \, \log \left| j(T) - j(U) \right|^4 - 24^2 \, \log \left[ T_2 U_2 \, \left| \eta(T) \, \eta(U) \right|^4 \right]$$

$$I^{(0)}(s) = 2^{2s} \sqrt{4\pi} \Gamma(s - \frac{1}{2}) T_2^{1-s} E(s, 0; U)$$

$$I^{(+)}(s) =$$
 exponentially suppressed

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)$$

$$j(\tau) = q^{-1} + O(q)$$



Very little progress in the understanding of non-supersymmetric string vacua

## STILL, SUPERSYMMETRY IS ELUSIVE AT THE LHC ENERGIES

- Is supersymmetric spontaneously broken at very high scales?
- Is supersymmetry hardly broken at the string scale?
- Is supersymmetry at all realised in Nature?