Quark Number Susceptibilites and Equation of State in QCD at finite μ_B

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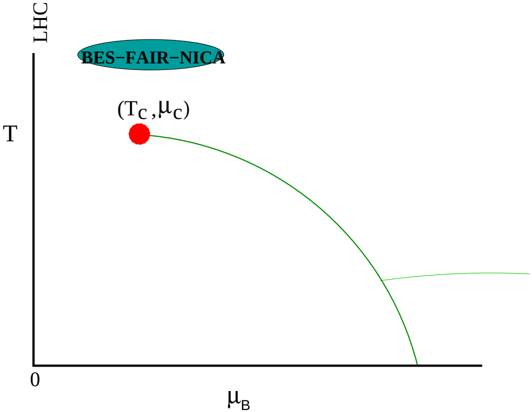
Introduction



The physics of (relatively) low energy heavy ion collisions will involve nonperturbative QCD at finite μ_B.
Direct numerical evaluation of thermodynamic quantities not possible: $S(\mu_B, T)$ not real, so cannot put e^{-S} in measure.

Taylor Expansion in μ_B

Calculate observables at small μ_B by expanding in μ_B:

$$P(\mu_B, T) = P(0, T) + \sum_{n=2,4,6,\ldots} \frac{\mu_B^n}{n!} \chi_{nB}$$

where the generalized baryon number susceptibilities χ_{nB} are obtained from suitable derivatives of P.

Gavai & Gupta 2003; Allton et al. 2003

One can write the series as an expansion for baryon susceptibility

$$\chi_B \equiv \chi_{2B} = \frac{\partial^2 P}{\partial \mu_B^2}$$

The convergence of the above series has been used to estimate the location of the QCD critical point in (μ_B, T) plane.

Gavai & Gupta 2005

Quark number susceptibilities

One can introduce a chemical potential for each flavor of quark; e.g., for two flavors,

$$\Delta P(\mu_B, T) = P(\mu_B, T) - P(0, T) = \sum_{n_u n_d} \chi_{n_u n_d} \frac{\mu_u^{n_u}}{n_u!} \frac{\mu_d^{n_d}}{n_d!}$$

The generalized Quark number susceptibilities (QNS)

$$\chi_{n_u\, n_d} = \frac{\partial^{n_u + n_d} P}{\partial \mu_u^{n_u}\, \partial \mu_d^{n_d}}$$

can be easily connected to, e.g., susceptibilities of baryon number and charge quantum number.
For three flavors, one has also μ_s.

Quark number susceptibilities

What do susceptibilities tell us?

- $\chi_{n_u\,n_d}$ have been widely studied in the lattice for understanding properties of QGP, and for calculations at finite μ_q.

  Gottlieb et al, PRL 59 ('87) 2247.
  Gavai & Gupta, PRD 68 ('03) 034506; Allton et al., *ibid*, 014507.

- χ_{20} is connected to fluctuation of baryon number, and shows a rise at T_c.
- Correlation observables like χ_{11} go to very small values in the QGP phase.
- Similarly, higher order susceptibilities show very characteristics properties at T_c, associated with the deconfining or chiral symmetry restoring nature of the transition.

  see, e.g., Ejiri, Karsch, Redlich (2006); Redlich, talk on 5th July.

Quark number susceptibilities

- χ_{nB} and their ratios can be connected to event-by-event fluctuations of baryon numbers: have been used for phenomenology of heavy ion collisions, in particular, to study freezeout.

  Gavai & Gupta, PLB (2011). Gupta et al., Science (2012).

- This assumes that fluctuation of net proton number mimics that of baryon number.
- Alternately, susceptibilities of charge and other conserved quantum numbers can be used.

  Bazavov et al., PRL (2012); Borsanyi et al. (2013)

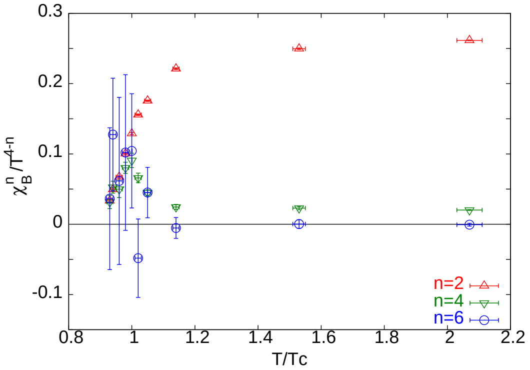
- χ_{nB} coefficients of expansion in μ_B: their ratios can be used for an estimation of location of critical point.

  Gavai & Gupta 2005

Caveat: Only 3-4 terms in series, higher coefficients noisier.

Technical details

- Our calculation: $N_f = 2$ QCD, temperature range 0.9 – 2.1 T_c.
- Lattices with $a = 1/8T$. Cutoff effect estimated by comparing with earlier calculations at $a = 1/4T$ and $1/6T$.
- Degenerate u, d quarks, $m_\pi \sim 220$ MeV.
- T_c determined from peak of the susceptibility of the Polyakov loop, and its physical value set using w_0 (Wilson flow).
- Other temperatures set with a combination of w_0 and two-loop running.
- Broad features of susceptibility measurements agree with results on susceptibilities with other discretizations.



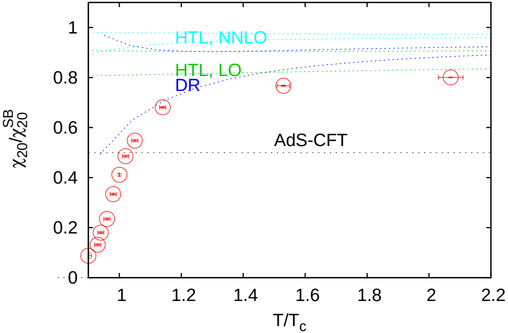
From these terms one gets an estimate

$$\frac{\mu_E}{T_E} = 1.85 \pm 0.04, \qquad \frac{T_E}{T_c} = 0.94 \pm 0.01$$

The errors are purely statistical.

Datta, Gavai, Gupta, PRD 95 (2017) 054512.

χ_{20}

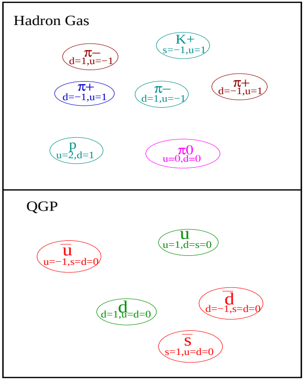


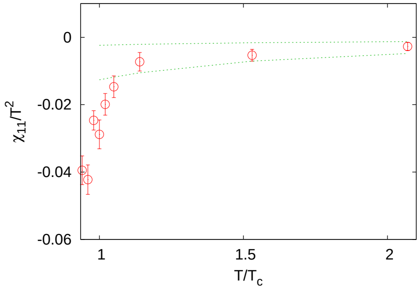
Cutoff effect small near T_c, but substantial at 2 T_c (due to one diagram which has large cutoff effect in free theory).
The $N_\tau = 8$ results approach SB limit from below.

Datta, Gavai, Gupta, PoS LATTICE2013 (2014) 202; PRD, 2017.
LO HTL, DR: Andersen,Mogliacci,Strickland,Su,Vuorinen, 1307.8098.
NLO HTL: Haque, Mustafa, Strickland, 1302.3228.

Quantities like χ_{ud} probe correlations between quantum numbers: very different between bound state phase and free quarks.

Koch, Majumdar & Randrup, PRL 95(05) 182301

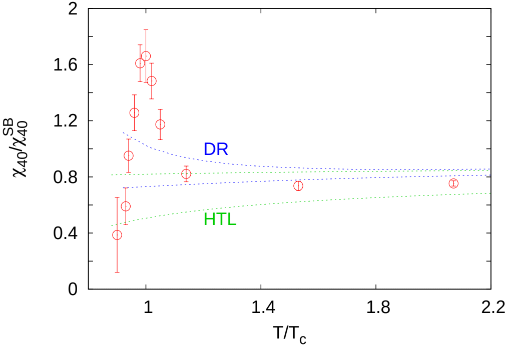




DR: Andersen, et al., 1307.8098.

Leading contribution at $g^6 \log g$: convergence of pert. theory?

χ_{40} and perturbation theory



χ_{40} shows a peak near T_c, where cutoff effect is pronounced.

DR: Andersen,Mogliacci,Strickland,Su,Vuorinen, 1307.8098.
NLO HTL: Haque, Mustafa, Strickland, 1302.3228.

Physics at finite μ_B

To get equation of state or susceptibilities at finite μ_B, we need to sum the series.

But a straight summing of the series will fail near a critical point.

We explore a Pade resummation of the series.

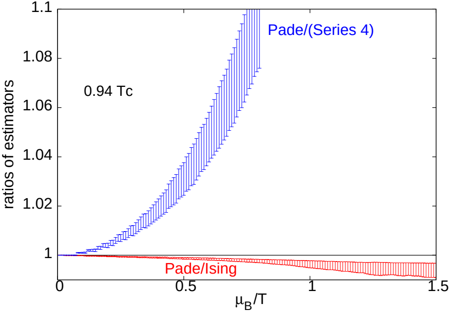
Gavai & Gupta, PLB 2011

$$\frac{\chi_{2B}(\mu_B, T_E)}{T_E^2} \sim |\mu_B^2 - \mu_B^{E^2}|^{-\psi} + \text{regular}$$

$$\Rightarrow \quad m_1 = \frac{\partial \log(\chi_{2B}/T^2)}{\partial \mu_B / T} = \frac{\chi_B^3/T}{\chi_{2B}/T^2} \sim \frac{\psi}{|\mu_B^2 - \mu_B^{E^2}|} + \text{regular}$$

Convert series expansion to one of m_1; integrate to get χ_{2B}, equation of state.

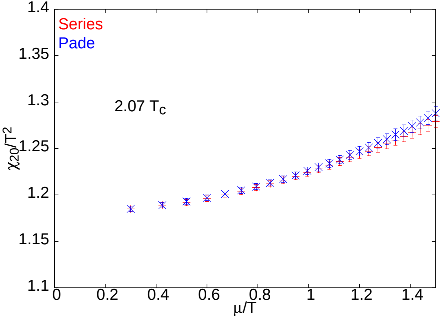
Pade vs series sum



Datta, Gavai, Gupta, PRD 95 (2017) 054512.

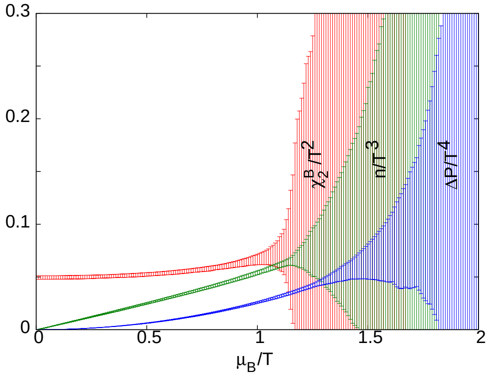
- For temperatures close to but below T_c, the Pade and series summation start deviating already at small μ_B.
- μ_E from Pade analysis consistent with radius of convergence estimate.
- Pade analysis consistent with one where Ising behavior is assumed.

Pade vs series sum



For temperatures far away from critical region, Pade and series summation give mutually consistent results.

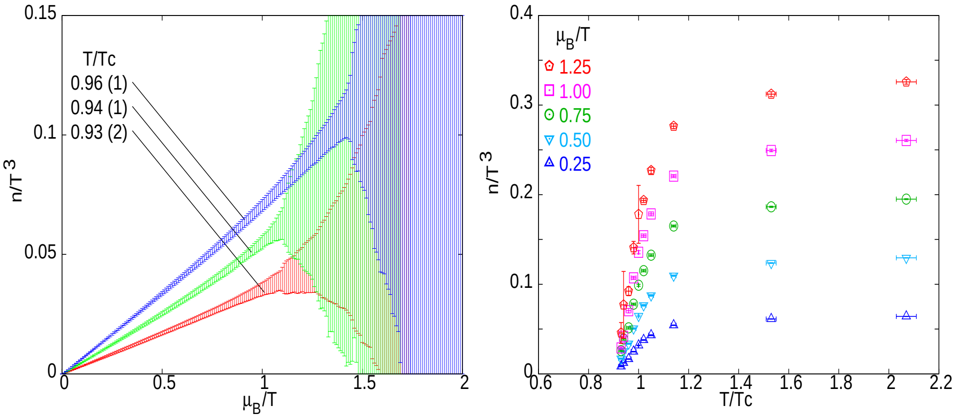
Uncertainty in Equation of state



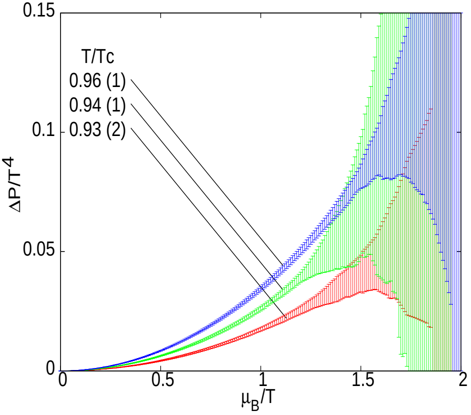
Critical behavior leads to very large errors in the Pade resummation. The error is estimated by a bootstrap analysis. For quantities with less singular behavior, the effect starts showing at higher μ_B.

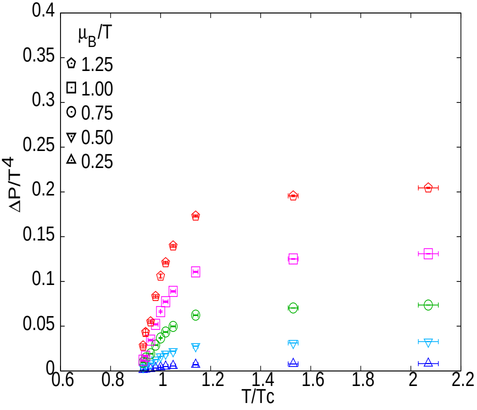
Gupta, Karthik, Majumdar (PRD, 2014); Datta, Gavai, Gupta (PRD, 2017)

Equation of state



Equation of state





Freezeout and m_1

Susceptibility ratios can be connected to ratios of baryon number cumulants.

$$m_1 = \frac{\chi_{3B}/T}{\chi_{2B}/T^2}, \qquad m_2 = \frac{\chi_{4B}}{\chi_{2B}/T^2}$$

can be compared with cumulants $S\sigma, \kappa\sigma^2$ to extract T and μ_B.
Can be used to find the freezeout curve.

Gavai and Gupta 2011; Gupta et al. 2012; Bazavov et al. 2012; Borsanyi et al. 2013

Difference between different observables: indication of how close to equilibrium the system was.
It has also been used by other groups; charge and stranegeness cumulants have also been used.

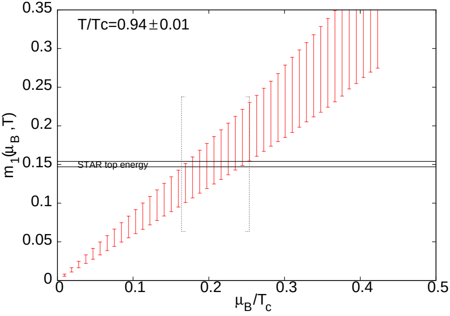
Bazavov et al., 2012; Borsanyi et al., 2013.

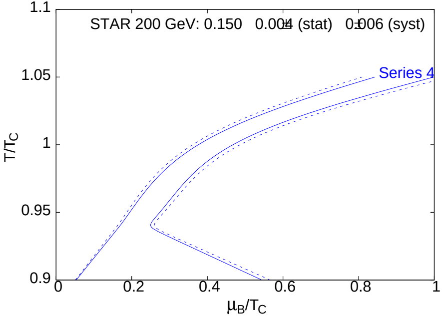
The enhanced error makes prognosis of such a determination much worse.

Freezeout region from m_1

What can we learn about freezeout curve from m_1?

STAR, Au-Au 200 GeV: $m_1 = 0.150 \pm 0.004 \pm 0.006$.





Datta, Gavai, Gupta, PRD 95, 054512 (2017)

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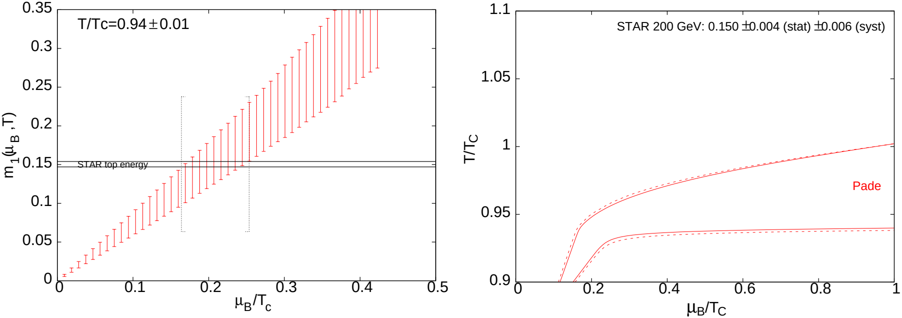


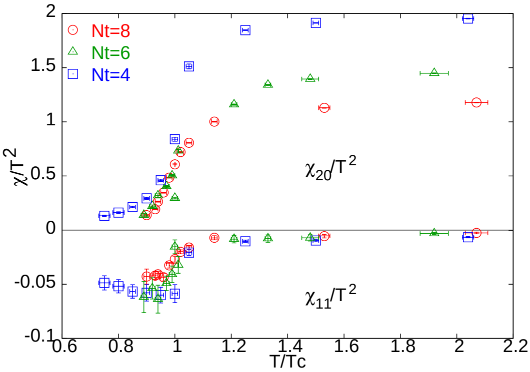
Accurate determination of freezeout parameters from baryon number cumulants difficult.
Freezeout temperature is likely to be below T_c.

Datta, Gavai, Gupta, PRD 95, 054512 (2017)

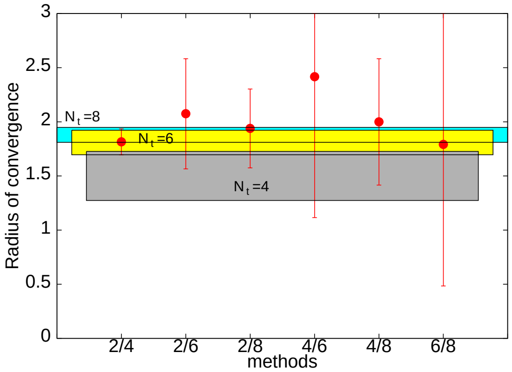
Summary

- We present results of QCD equation of state at finite μ_B using an expansion in μ_B.
- In particular, we stress the need of using resummation rather than simple series summation for temperatures close to T_c.
- Such resummation leads to rapidly increasing error in susceptibility ratio m_1 due to critical slowing down, dramatically decreasing its efficacy in determining the freezeout curve.
- The susceptibilities at $T = 0$ give us important insights about the nature of the plasma phase. In particular, the lowest order susceptibilities approach the perturbative limit soon after T_c. χ_{2B} is substantially different from AdS/CFT results.
- Similarly, correlations like χ_{ud} puts strong constraints on the presence of multiquark clusters in the QGP phase.

Cutoff effect in χ_{2B}



Estimate for critical point

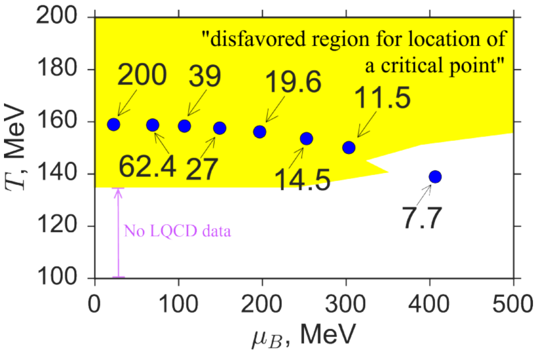


$$\frac{\mu_E}{T_E} = 1.85 \pm 0.04, \qquad \frac{T_E}{T_c} = 0.94 \pm 0.01$$

Taking $T_c = 170$ MeV this translates to

$$\mu_E = 286 - 305 \text{ MeV}, \qquad T_E \approx 158 - 161 \text{ MeV}$$

HotQCD exclusion space for critical point



HotQCD 1701.04325; taken from Pisarski, talk on 5th July