Work drives time evolution

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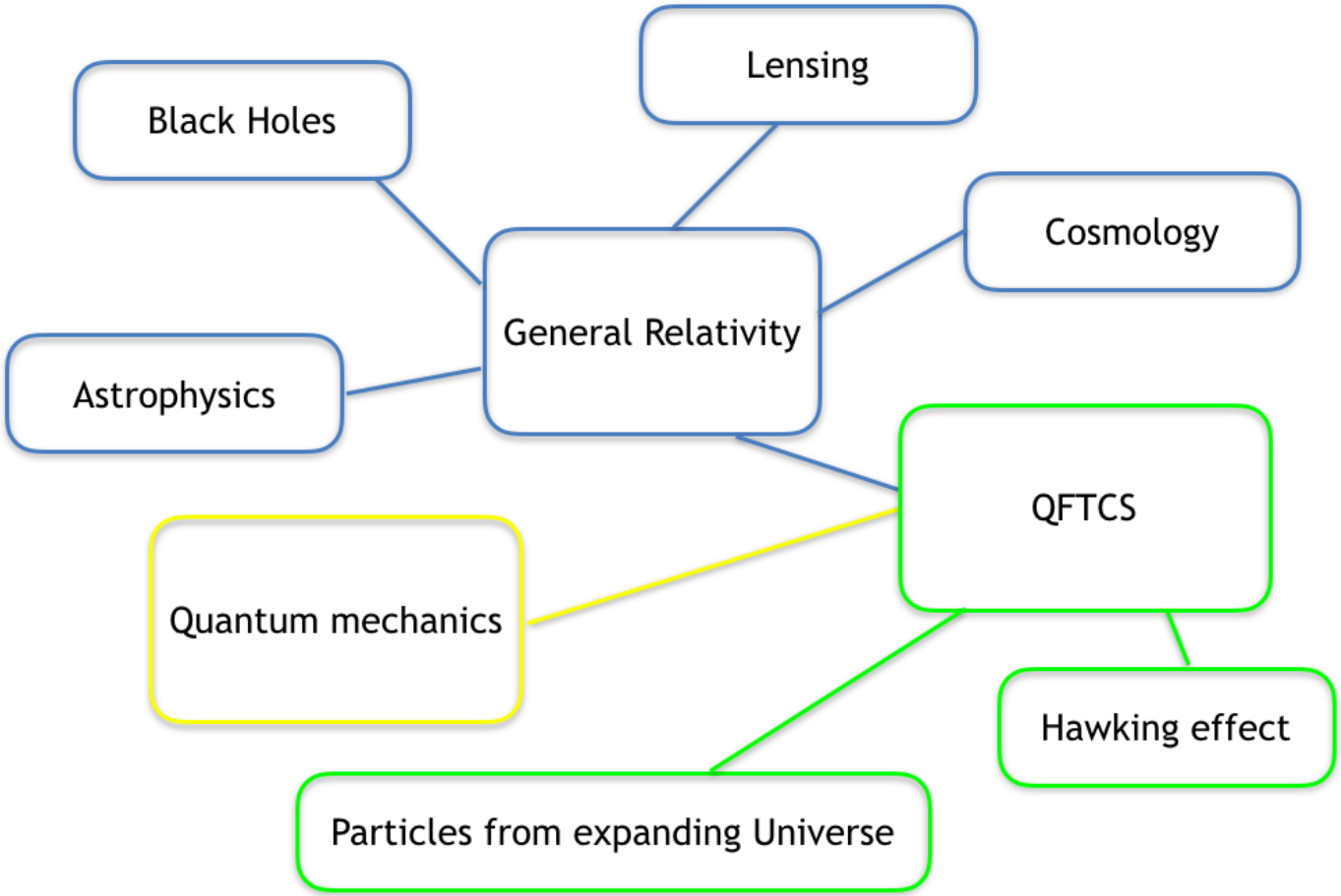
General relativity

General relativity is a successful theory for classical gravity.



Moving forward

From general relativity...

General relativity dogma: **all** energy gravitates: $E = mc^2$.

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...through quantum field theory in curved spacetime...

QFTCS has a problem: renormalise background effects and energy gravitates but does not back-react.

$$G_{\mu\nu} = \frac{8\,\pi\,G_N}{c^4}\,T_{\mu\nu}, \qquad \nabla_\mu \nabla^\mu \hat{\Phi} = 0.$$

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...towards a fundamental question

Q: Is it true that **all** energy gravitates?

In particular

Q: Is it true that **energy that cannot be extracted** gravitates?

A novel proposal

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Proposal: only **extractible work** (i.e., energy) contributes to gravity

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$$G_{\mu\nu} = \frac{8\,\pi\,G_N}{c^4}\,\left[\langle T_{\mu\nu}\rangle_{\hat{\rho}} - \langle T_{\mu\nu}\rangle_{\hat{\rho}_p}\right].$$

We have introduced

- The **global** state $\hat{\rho}$ of fields and matter;
- The "unique" corresponding passive state $\hat{\rho}_p$.

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- The "unique" corresponding passive state $\hat{\rho}_p$.

Corresponding passive state $\hat{\rho}_p$ is given by

$$\hat{\rho}_p := \hat{U}_p \hat{\rho} \hat{U}_p^\dagger$$

with the **same** entropy.

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Changing Heisenberg *Annals of Physics* 394, 155 – 161 (2018)

Time evolution must be adapted to this theory of gravitation. We propose to modify Heisenberg equation to

$$\frac{d}{dt}\hat{A} = \frac{i}{\hbar}\left[\hat{H}, \hat{A}\right] - \frac{i}{\hbar}\left[\hat{U}_p^\dagger \hat{H} \hat{U}_p, \hat{A}\right] + \frac{\partial \hat{A}}{\partial t}.$$

note that $\hat{U}_p$ is time independent. It is defined through the initial state.

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Formal solution for the time evolution operator

$$\hat{U} = \overleftarrow{\mathcal{T}} \exp\left[-\frac{i}{\hbar}\int_0^t dt' \left(\hat{H} - \hat{U}_p^\dagger \hat{H} \hat{U}_p\right)\right].$$

General formulas

We have proposed a new time evolution operator.

- Restrict to pure states with unique vacuum $|0\rangle$;
- Initial state $|\Psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|\chi\rangle$;
- We have $\langle 0|\chi\rangle = 0$.

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Notice that, the operator $\hat{U}_p$ has **always** the expression

$$U_p = \cos\theta\,(|0\rangle\langle 0| + |\chi\rangle\langle\chi|) + \sin\theta\,|0\rangle\langle\chi| - \sin\theta\,|\chi\rangle\langle 0| + \mathbb{1} - |0\rangle\langle 0| - |\chi\rangle\langle\chi|$$

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and we also have $E_0 := H\,|0\rangle$, $E_\chi := \langle\chi|H|\chi\rangle$ and $\Delta E := E_\chi - E_0$ and

$$\begin{aligned}\dot{A} = &-\frac{i}{\hbar}\left((2\,(1-\cos\theta)\,E_\chi - \sin\theta^2\,\Delta E)\,[|\chi\rangle\langle\chi|, A] + \sin^2\Delta E\,[|0\rangle\langle 0|, A]\right.\\ &+ \sin\theta\,(E_\chi - \cos\theta\,\Delta E)\,[|\chi\rangle\langle 0|, A] + \sin\theta\,(E_\chi - \cos\theta\,\Delta E)\,[|0\rangle\langle\chi|, A]\\ &- (1-\cos\theta)\,[H|\chi\rangle\langle\chi|, A] - (1-\cos\theta)\,[|\chi\rangle\langle\chi|H, A]\\ &\left.- \sin\theta\,[|0\rangle\langle\chi|H, A] - \sin\theta\,[H|\chi\rangle\langle 0|, A]\right).\end{aligned}$$

Single mode results I

General pure state

We start by the initial pure state

$$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|\chi\rangle,$$

where $H\,|\chi\rangle = E_\chi\,|\chi\rangle$.

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Lengthy algebra shows us that

$$p_{|0\rangle}(t) = \cos^2\theta - \cos^2\theta\,\sin^2\left(\sin\theta\,\frac{\Delta E}{\hbar}\,t\right)$$
$$p_{|\chi\rangle}(t) = \sin^2\theta + \cos^2\theta\,\sin^2\left(\sin\theta\,\frac{\Delta E}{\hbar}\,t\right).$$

This is not very realistic since there is an initial coherent superposition of eigenstates of the Hamiltonian with different energy.

Single mode results II

One mode pure state (from now on consider free Hamiltonian)

We start by the initial pure state of mode $\hat{a}$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |n\rangle\right],$$

where $\hat{H}_0 |n\rangle = n\hbar\omega |n\rangle$ and $\hat{H}_0 |0\rangle = E_0 |0\rangle$.

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$$p_{|0\rangle}(t) = \frac{1}{2}\left[1 - \sin^2\left(\frac{(n\,\hbar\omega - E_0)\,t}{\sqrt{2}\,\hbar}\right)\right]$$
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Two mode separable pure state

We start by the initial pure state of modes $\hat{a}$ and $\hat{b}$

$$|\psi\rangle = \frac{1}{2}(|0\rangle + |n\rangle) \otimes (|0\rangle + |m\rangle) = \frac{1}{2}[|00\rangle + |0m\rangle + |n0\rangle + |nm\rangle],$$

where $\hat{H}_0 |nm\rangle = \hbar(n\,\omega_a + m\,\omega_b)\,|nm\rangle$ and $\hat{H}_0 |0\rangle = E_0 |0\rangle$.

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$$p_{|00\rangle}(t) = \frac{1}{4}\left[1 - \sin^2\left(n\,\frac{\omega_a\,t}{\sqrt{2}}\right)\right]\left[1 - \sin^2\left(m\,\frac{\omega_b\,t}{\sqrt{2}}\right)\right]$$

$$p_{|n0\rangle}(t) = \frac{1}{4}\left[1 + \sin^2\left(n\,\frac{\omega_a\,t}{\sqrt{2}}\right)\right]\left[1 - \sin^2\left(m\,\frac{\omega_b\,t}{\sqrt{2}}\right)\right]$$

$$p_{|0m\rangle}(t) = \frac{1}{4}\left[1 - \sin^2\left(n\,\frac{\omega_a\,t}{\sqrt{2}}\right)\right]\left[1 + \sin^2\left(m\,\frac{\omega_b\,t}{\sqrt{2}}\right)\right]$$

$$p_{|nm\rangle}(t) = \frac{1}{4}\left[1 + \sin^2\left(n\,\frac{\omega_a\,t}{\sqrt{2}}\right)\right]\left[1 + \sin^2\left(m\,\frac{\omega_b\,t}{\sqrt{2}}\right)\right].$$

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$$|\psi\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |nm\rangle],$$

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Again,

$$p_{|00\rangle}(t) = \frac{1}{2}\left[1 - \sin^2\left(\frac{(n\omega_a + m\omega_b)t}{\sqrt{2}}\right)\right]$$
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M00N states

We start by the initial pure state of mode $\hat{a}$

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Physical consideration

We need to require

$$\Delta E = 0$$

for physical superpositions.

Two mode results III

In this case we obtain

$$p_{|M0\rangle}(t) = \cos^2\phi$$
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compatible with what expected for standard N00N states where $M = N$, $\omega_a = \omega_b = \omega$ and $\phi = \pi/4$.

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Consider a N00N state where $\delta E := N\hbar(\omega_b - \omega_a)$ and $\epsilon := \frac{\delta E}{\hbar\omega_a} \ll 1$. Then

$$p_{|N0\rangle_a}(t) = \cos^2\phi - 4\epsilon F(\phi)\sin^2\left(\frac{N\omega_a t}{2}\right)$$
$$p_{|0N\rangle_b}(t) = \sin^2\phi + 4\epsilon F(\phi)\sin^2\left(\frac{N\omega_a t}{2}\right).$$

where $F(\phi) := \sin^4\phi\cos^2\phi(1 + 2\cos^2\phi)$.

Experimental proposal

Mach-Zehnder-like interferometer in the gravitational field of the Earth

Frequencies ω_0 and $\omega_b = \omega(L) = (1 - \frac{r_S}{2}\frac{L}{r_E^2})\,\omega_0$ with $\frac{L}{r_E} \ll 1$ and $\frac{r_S}{r_E} \ll 1$.

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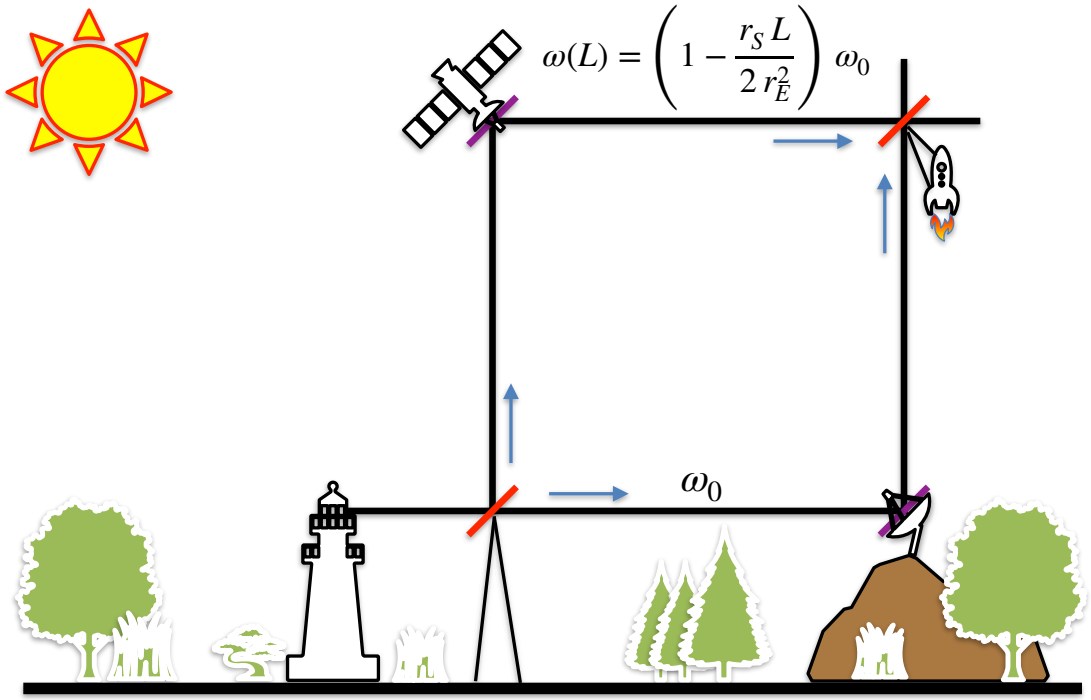

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Given that $r_S = 10^{-2}$m and $r_E = 6.371 \times 10^6$m for the Earth, assuming that $L = 10^5$m gives us that $\frac{r_S\,L}{r_E^2} \sim 4.2 \times 10^{-11} \ll 1$. This allows us to provide a rough estimate of the order of magnitude of the expected effects.

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Then:

- Applications include single mode cases and two mode cases;
- Predictions are very different from the standard Heisenberg case;
- Populations oscillate depending on measure of quantum coherence $\mathcal{C}$;
- Oscillation frequency $\omega_{osc} = \sqrt{1 - \sqrt{1 - \mathcal{C}^2}}\, \frac{\Delta E}{\sqrt{2}\,\hbar}$;
- More work to come...

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$\epsilon\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}.$

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