Precision determination of α_s from lattice QCD

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Introduction

A fundamental parameter of the SM

QCD

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4g_0^2} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_f \overline{\psi}_f (\gamma_\mu D_\mu + m_{0,f}) \psi_f$$

Renormalization

$$g_0,\, m_{0,f} \rightarrow \alpha_s(\mu),\, \overline{m}_f(\mu),\; f = u, d, s, c, b, t$$

Strong coupling $\alpha_s \equiv \alpha_{\overline{\mathrm{MS}}}(m_Z)$

- Fundamental parameter of the SM
- Its value affects all pQCD processes
- Impacts vacuum stability, unification arguments, etc.

On the other hand: (d'Enterria '18)

- $\delta\alpha_s/\alpha_s \approx 10^{-2} \gg \delta G/G \gg \delta G_F/G_F \gg \delta\alpha_{\mathrm{QED}}/\alpha_{\mathrm{QED}} \approx 10^{-10}$
- Leads to relevant uncertainties, i.e. 3-7%, in key Higgs processes

$$H \rightarrow b\bar{b}, \quad H \rightarrow c\bar{c}, \quad H \leftrightarrow gg, \quad \ldots$$

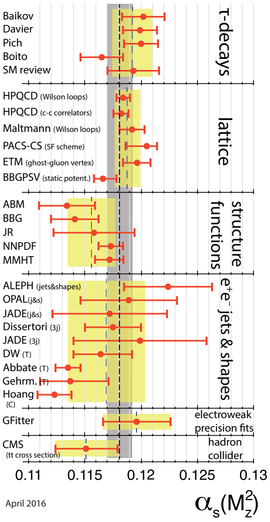
- Limiting factor for future precision determinations e.g. $\overline{m}_t$

Conclusion: Future experiments require **sub-percent** precision!

Introduction

What's the current situation?

(PDG '16)



PDG (2016)

$$\alpha_s = 0.1181 \pm 0.0011$$

- Pre-averages within categories are made which account for the **spread**
- The category averages are then "χ^2-averaged" to get a final estimate

ISSUES

- Many determinations are dominated by **systematic** uncertainties: this limits the attainable precision!
- We need **precise** and **reliable** determinations to really decrease the error!
- This is not how a fundamental parameter should be determined ...

Introduction

Which are the main difficulties?

How do we determine α_s?

(Salam '18)

Given

$$\mathcal{O}_{\rm th}(\alpha_s(\mu),\mu) = \sum_{n=0}^{N} c_n(\mu)\alpha_s^n(\mu) + \mathrm{O}(\alpha_s^{N+1}) + \mathrm{O}\left(\frac{\Lambda^p}{Q^p}\right),$$

we set: $\mathcal{O}_{\rm th} \stackrel{!}{=} \mathcal{O}_{\rm exp}$ and extract $\alpha_s(\mu)$.

Sources of uncertainty

- Experimental errors $\delta\mathcal{O}_{\rm exp}$
- Impact of the unknown c_n, $n > N$; **hard** to quantify within PT (s. later)!
- Size (and form) of **"non-perturbative"** corrections e.g. O(Λ^p/Q^p)
- Uncertainties on other fundamental parameters
- Missing higher-order electroweak terms
- …and new physics contributions?

Eq. view: matching phenomenological couplings

$$\alpha_{\mathcal{O}}(\mu) \equiv \frac{\bar{g}^2_{\mathcal{O}}(\mu)}{4\pi} \equiv \frac{\mathcal{O}_{\rm th}(\alpha_s(\mu),\mu) - c_0}{c_1} = \alpha_s(\mu) + \frac{c_2}{c_1}\alpha_s^2(\mu) + \dots$$

Lattice QCD determinations of α_s

General overview

Lattice QCD in a nutshell

- $a \equiv$ lattice spacing $\Rightarrow a^{-1} \equiv$ **UV-cutoff** $\Rightarrow a \to 0$
- $L \equiv$ space-time extent $\Rightarrow L^{-1} \equiv$ **IR-cutoff** $\Rightarrow L \to \infty$
- Unphysical number of active quark-flavours: here $\boldsymbol{N_f = 3}$!
- Path integrals are evaluated stochastically $\Rightarrow$ statistical errors

Hadronic renormalization

$$g_0, m_{0,u} = m_{0,d}, m_{0,s} \;\to\; m_p, m_\pi, m_K$$

Once the theory is renormalized $\alpha_s(\mu)$ and $\overline{m}_f(\mu)$ are **predictions!**

Basic idea

Replace $\mathcal{O}_{\rm exp}$ by $\mathcal{O}_{\rm lat}$

Why this seems like a good idea?

- Lots of **freedom** in choosing $\mathcal{O}_{\rm lat}$: pick one which allows for a **small** $\delta\mathcal{O}_{\rm lat}$; not necessarily an experimentally measurable quantity!
- $\mathcal{O}_{\rm lat}$ is defined within QCD **only** and can, in principle, be computed **non-perturbatively** up to arbitrary high-scales μ!
- Does **NOT** rely on any model for hadronization

α_s from the femto-universe

Finite volume schemes and step-scaling

(Lüscher, Weisz, Wolff '86)

Yet not easy ...

Having systematic errors under control means

$$L^{-1} \ll m_\pi \ll m_p \ll \mu \ll a^{-1} \quad \Rightarrow \quad L/a \gg 100 \quad \Rightarrow \quad \textbf{NOT feasable!}$$

Widely different scales **cannot** be resolved simultaneously on a **single** lattice!

Solution: Break the computation into **steps**! How?

1. **Finite-volume (FV) schemes**

$$\bar{g}^2_{\mathcal{O}}(\mu) \quad \text{with} \quad \mu = L^{-1}$$

2. **Step-scaling function (SSF)**

$$\sigma_{\mathcal{O}}(u) = \bar{g}^2_{\mathcal{O}}(2L)\Big|^{u=\bar{g}^2_{\mathcal{O}}(L)}_{\overline{m}(L)=0}, \qquad \sigma_{\mathcal{O}}(u) = \lim_{a/L\to 0} \Sigma_{\mathcal{O}}(a/L,u) \;\Rightarrow\; L/a \gg 1!$$

Iteratively step up/down in scale by factors of 2:

$$\bar{g}^2_{\mathcal{O}}(L_{\rm had}) = u_{\rm had} = u_0, \quad u_k = \sigma(u_{k+1}) = \bar{g}^2_{\mathcal{O}}(2^{-k}L_{\rm had}), \quad k=0,1,\ldots$$

3. **Hadronic matching:** Determine e.g. $m_p L_{\rm had} = {\rm O}(1)$

4. **Perturbative matching:** For $\mu_{\rm PT} = 2^n/L_{\rm had} \gg m_p$, PT can be used to extract $\alpha_s(\mu_{\rm PT})$ given $\alpha_{\mathcal{O}}(\mu_{\rm PT})$.

The Λ-parameter

The fundamental scale of QCD

Definition & properties

$$\frac{\Lambda_{\mathcal{O}}}{\mu} = \ldots \times \exp\left\{ -\int_0^{\bar{g}_{\mathcal{O}}(\mu)} \mathrm{d}g \, \frac{1}{\beta_{\mathcal{O}}(g)} + \ldots \right\}$$

- Non-perturbatively defined if $\bar{g}_{\mathcal{O}}$ (and thus $\beta_{\mathcal{O}}$) is!
- Exact solution of the Callan-Symanzik (CS) eqs.
- Number N_f of (massless) quarks fixed!
- At high-energy we have that

$$\bar{g}_{\mathcal{O}}(\mu) \overset{\mu\to\infty}{\to} 0, \quad \beta_{\mathcal{O}}(g) \overset{g\to 0}{\to} -b_0 g^3 - b_1 g^5 - b_2^{\mathcal{O}} g^7 + \ldots$$

where $b_{0,1}$ are universal, while $b_n^{\mathcal{O}}$, $n \geq 2$, are scheme-dependent

$$b_0 = \left(11 - \tfrac{2}{3}N_\mathrm{f}\right)(4\pi)^{-2}, \quad b_1 = \left(102 - \tfrac{38}{3}N_\mathrm{f}\right)(4\pi)^{-4}, \quad b_2^{\mathcal{O}} = \ldots$$

- Scheme dependence of $\Lambda_{\mathcal{O}}$ is (almost) trivial,

$$\bar{g}^2_\mathrm{X}(\mu) = \bar{g}^2_\mathrm{Y}(\mu) + c_\mathrm{XY}\,\bar{g}^4_\mathrm{Y}(\mu) + \ldots \quad \Rightarrow \quad \Lambda_\mathrm{X}/\Lambda_\mathrm{Y} = e^{c_\mathrm{XY}/2b_0}$$

NOTE: To extract $\alpha_s(m_Z)$ we need $\Lambda_{\overline{\mathrm{MS}}}^{N_\mathrm{f}=5}$: to go from $N_\mathrm{f} = 3 \to 5$, requires **matching** the two theories (s. later)!

Finite volume couplings

Which properties should they have?

Desired properties

- Non-perturbatively defined in a finite space-time volume!
- Computable in PT (at least 2-loop) with reasonable effort
- Gauge-invariant
- Quark-mass independent (simpler CS eqs. to solve)
- Small statistical errors and small lattice artifacts

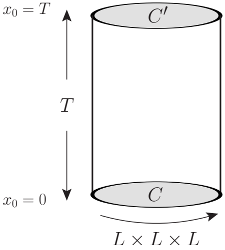
Schrödinger functional (Lüscher et. al '92)

$$\mathcal{Z}[C, C'] = \int_{\mathrm{bc}} DAD\psi D\overline{\psi}\, e^{-S_{\mathrm{QCD}}}$$

$$A_k(x)|_{x_0=0} = C_k, \;\; A_k(x)|_{x_0=T} = C'_k$$

$$P_+\psi(x)|_{x_0=0} = P_-\psi|_{x_0=T} = 0, \;\; \ldots$$

$$P_\pm = \tfrac{1}{2}(1 \pm \gamma_0)$$



- PT in finite volume is feasible w/ these bc's
- Allows computations directly in the chiral limit
- Renormalized once action parameters and fields are renormalized

Finite volume couplings

Our choices

We do not necessarily have to rely on a single coupling ...

SF couplings

(Lüscher et. al. '92)

Given

$$A_k(x)|_{x_0=0} = C_k(\eta,\nu),\ A_k(x)|_{x_0=T} = C'_k(\eta,\nu)$$

$$C_k(\eta,\nu), C'_k(\eta,\nu) \equiv \text{Abelian fields}$$

We can define a **family** of couplings as

$$\bar{g}^2_{\mathrm{SF},\nu}(L) \propto \left.\frac{1}{\partial_\eta \Gamma}\right|_{\eta=0}, \quad \Gamma \equiv -\ln \mathcal{Z}[C, C']$$

i.e. response of the system to a change of bc's.

GF coupling

(Lüscher '10; Fodor et. al. 12'; Fritzsch, Ramos '13)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \quad B_\mu(0,x) = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

Gauge-invariant composite fields of B_μ are **renormalized** quantities for $t > 0$!

$$\bar{g}^2_{\mathrm{GF}}(L) \propto t^2 \langle G^a_{\mu\nu}(t,x) G^a_{\mu\nu}(t,x)\rangle|_{x_0=T/2} \quad \text{with} \quad \sqrt{8t} = 0.3 \times L$$

The strategy in a picture

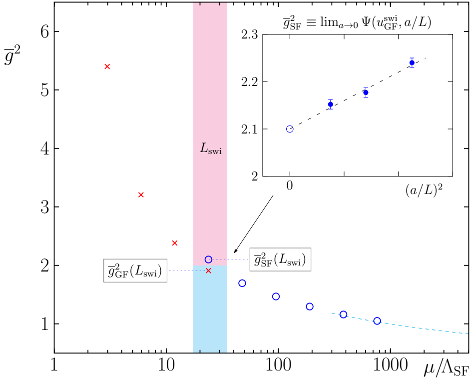
Going from high to low-energy

SF couplings

- β-function known to NNLO i.e. $b_2^{\mathcal{O}}$
- $\mathrm{var}(\overline{g}^2_{\mathrm{SF}})\,/\,\overline{g}^4_{\mathrm{SF}} \propto \overline{g}^2_{\mathrm{SF}}$: becomes more precise at **high-energy**
- $\mathrm{var}(\overline{g}^2_{\mathrm{SF}})$ is typically **large** and increases for $a/L \to 0$
- **Small** lattice artifacts though: smallish L/a are sufficient

GF coupling

- β-function: only the universal part is known i.e. b_0, b_1
- $\mathrm{var}(\overline{g}^2_{\mathrm{GF}})\,/\,\overline{g}^4_{\mathrm{GF}} \propto \mathrm{const.}$
- $\mathrm{var}(\overline{g}^2_{\mathrm{GF}})$ is typically **small** and essentially constant as $a/L \to 0$
- Largish lattice artifacts: requires **large** lattice resolutions L/a

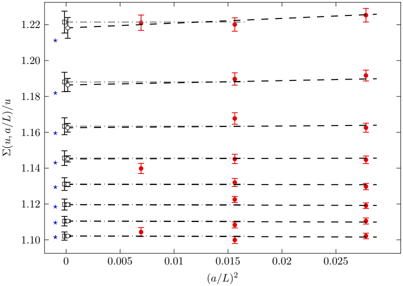


(Fritzsch '13)

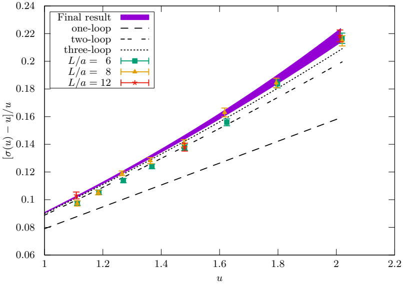
Step I: Step-scaling function SF coupling ($\nu = 0$)

Non-perturbative running from $4\,\mathrm{GeV}$ to $128\,\mathrm{GeV}$ (MDB, Fritzsch, Korzec, Ramos, Sint, Sommer '16 '18)

Continuum extrapolation



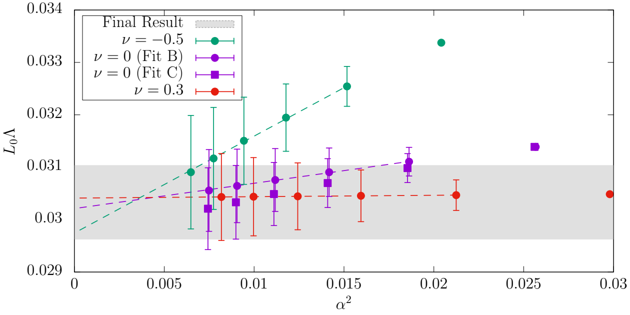
Continuum limit result



Step II: determination of $L_0\Lambda_{\overline{\rm MS}}^{N_f=3}$

…and a test of perturbation theory at high-energy

(MDB, Fritzsch, Korzec, Ramos, Sint, Sommer '16 '18)



- We define the reference scale L_0 through the condition

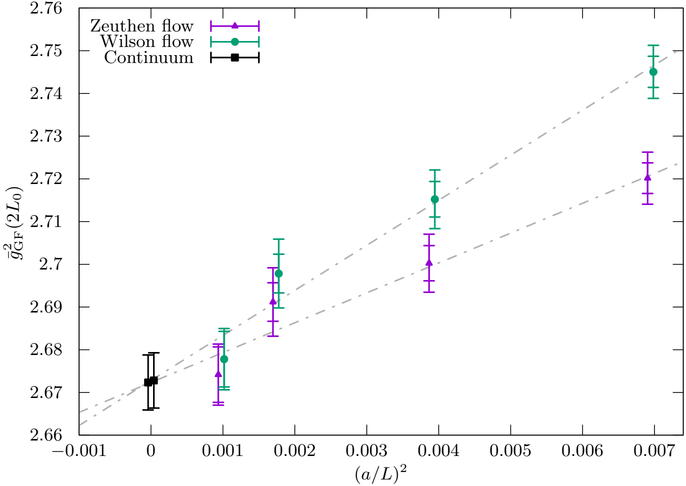
$$\bar{g}^2_{\rm SF,\nu=0}(L_0) = 2.012 \quad \Rightarrow \quad \alpha_{\rm SF,\nu=0}(L_0) \approx 0.16$$

- …and use $\beta^{3-\rm loop}_{\rm SF,\nu}$ to evaluate the relation $L_0\Lambda_{\rm SF,\nu} \leftrightarrow \bar{g}_{\rm SF,\nu}(2^{-n}L_0)$
  - $\Rightarrow$ once converted to the same scheme, $\Lambda \equiv \Lambda_{\rm SF,\nu=0}$, the results for different ν and n should agree, up to O(α^2) corrections
- O(α^2)-corrections become irrelevant once $\boldsymbol{\alpha \approx 0.1}$. There we can quote:

$$L_0\Lambda^{N_f=3} = 0.0303(8) \quad \Rightarrow \quad L_0\Lambda^{N_f=3}_{\overline{\rm MS}} = 0.0791(21)$$

Step III: matching the couplings

(MDB, Fritzsch, Korzec, Ramos, Sint, Sommer '16)



$$\bar{g}^2_{\rm SF,\nu=0}(L_0) = 2.012 \quad \Rightarrow \quad \bar{g}^2_{\rm GF}(2L_0) = 2.6723(64)$$

Step IV: running to low-energy

(MDB, Fritzsch, Korzec, Ramos, Sint, Sommer '16)

How do we do it?

Goal:

L_0 must be expressed in physical units in order to obtain $\Lambda_{\overline{\mathrm{MS}}}^{N_\mathrm{f}=3}$

Step scaling function

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, a/L), \qquad \Sigma(u, a/L) = \bar{g}^2_{\mathrm{GF}}(2L)\Big|^{u=\bar{g}^2_{\mathrm{GF}}(L)}_{\overline{m}(L)=0}$$

β-function

$$\log 2 = -\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{\mathrm{d}g}{\beta(g)}, \qquad \beta(\bar{g}_{\mathrm{GF}}) = \mu \frac{\partial \bar{g}_{\mathrm{GF}}(\mu)}{\partial \mu} = -L \frac{\partial \bar{g}_{\mathrm{GF}}(L)}{\partial L}$$

Ratios of scales

$$\frac{L_2}{L_1} = \exp\left\{ -\int_{\bar{g}_{\mathrm{GF}}(L_1)}^{\bar{g}_{\mathrm{GF}}(L_2)} \frac{\mathrm{d}g}{\beta(g)} \right\}$$

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Ratios of scales

$$\frac{L_{\mathrm{had}}}{2L_0} = \exp\left\{ -\int_{\bar{g}_{\mathrm{GF}}(2L_0)}^{\bar{g}_{\mathrm{GF}}(L_{\mathrm{had}})} \frac{\mathrm{d}g}{\beta(g)} \right\} = 10.93(21)$$

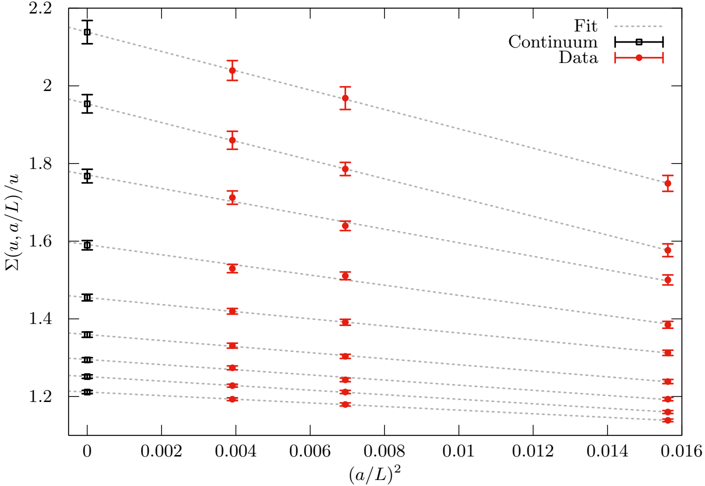
$$\bar{g}^2_{\mathrm{GF}}(L_{\mathrm{had}}) = 11.31 \quad \Rightarrow \quad L_{\mathrm{had}} \approx 200\,\mathrm{MeV}$$

NOTE: L_{had} can safely be connected to hadronic physics (**Step V**)!

Step IV: running to low-energy

Taking the continuum limit

(MDB, Fritzsch, Korzec, Ramos, Sint, Sommer '16)

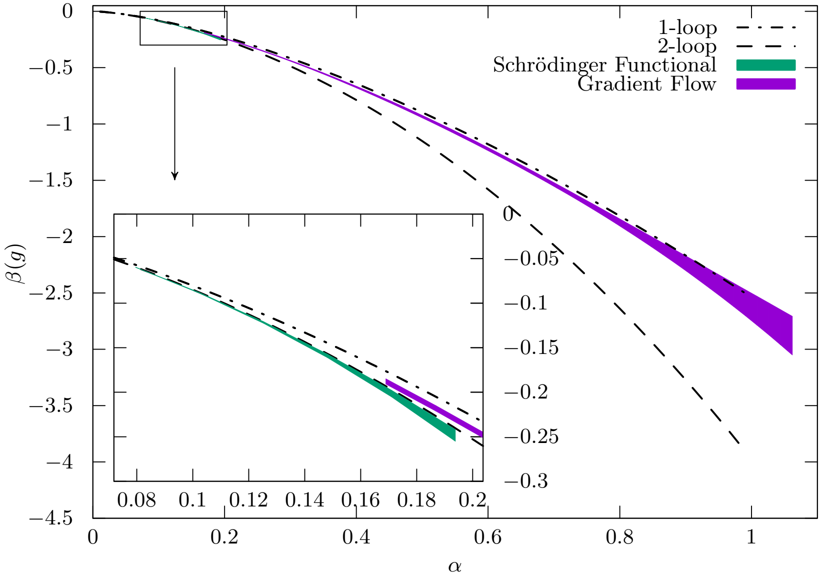


- sizable discretization effects → **careful** extrapolations are needed!
- continuum results are nonetheless very **precise**!

Step IV: jogging to low-energy

The non-perturbative β-function(s)

$\alpha \equiv g^2/(4\pi)$



Step V: matching to hadronic physics

Setting the scale

The story so far:

$$L_{\rm had}\Lambda_{\overline{\rm MS}}^{N_{\rm f}=3} = L_0\Lambda_{\overline{\rm MS}}^{N_{\rm f}=3} \times \frac{L_{\rm had}}{L_0} = 1.729(57) \;\Leftarrow\; L_{\rm had} = ???\,{\rm fm}$$

A (relative) scale:

(Lüscher '10)

$$t_0^2\,\langle\,{\rm tr}\{G_{\mu\nu}(t_0,x)G_{\mu\nu}(t_0,x)\}\rangle = 0.3$$

- ✓ simply and accurately measurable in simulations
- ✓ gluonic quantity → very mild m_π dependence
- ✗ not directly measurable in experiments

Strategy:

(Bruno, Korzec, Schaefer '16)

$$\left.\begin{array}{c}\text{CLS effort + PDG}\\ m_\pi, m_K, f_\pi, f_K\end{array}\right\} \Rightarrow \lim_{\substack{m_{\pi,K}\to m_{\pi,K}^{\rm phys}\\ a\to 0}} \sqrt{t_0}\cdot f_{\pi K}\,, \qquad f_{\pi K} = \tfrac{2}{3}(f_K + \tfrac{1}{2}f_\pi)$$

Result: $\sqrt{8\,t_0} = 0.413(4)\,{\rm fm}$ → very **precise** relative scale!

Step V: matching to hadronic physics

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The story so far:

$$L_{\text{had}}\Lambda_{\overline{\text{MS}}}^{N_\text{f}=3} = L_0\Lambda_{\overline{\text{MS}}}^{N_\text{f}=3} \times \frac{L_{\text{had}}}{L_0} = 1.729(57) \;\Leftarrow\; L_{\text{had}} = ???\,\text{fm}$$

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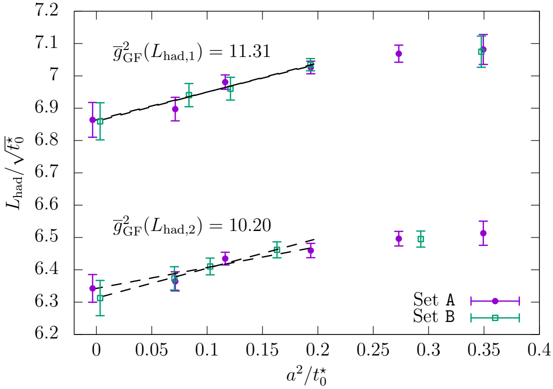
Result: $\sqrt{8\,t_0^*} = 0.413(5)\,\text{fm}$, with $m_\pi^* = m_K^* \approx 400\,\text{MeV}$

Step V: matching to hadronic physics

The Λ-parameter in physical units (Bruno, MDB, Fritzsch, Korzec, Ramos, Schaefer, Simma, Sint, Sommer '17)

Continuum extrapolation

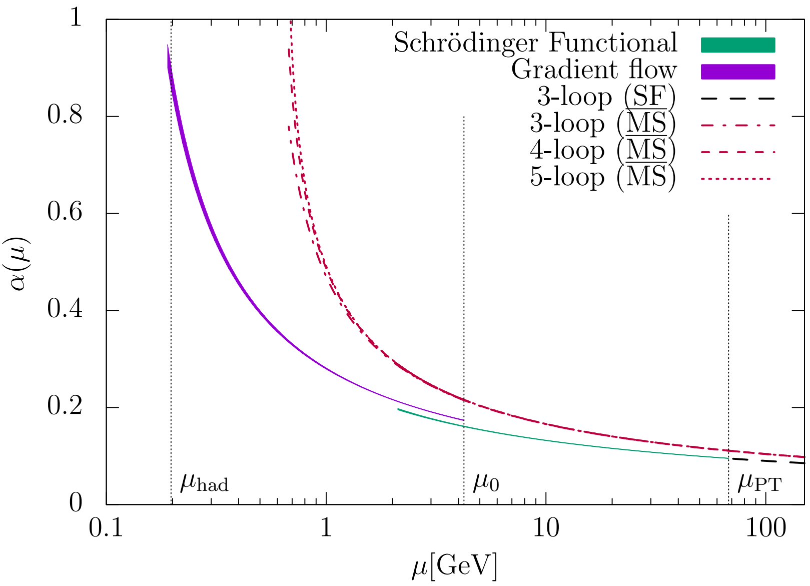


Λ-parameter

$$\Lambda_{\overline{\rm MS}}^{N_{\rm f}=3} = L_{\rm had}\Lambda_{\overline{\rm MS}}^{N_{\rm f}=3} \times \frac{\sqrt{t_0^*}}{L_{\rm had}} \times \frac{1}{\sqrt{t_0^*}} = 341(12)\,\mathrm{MeV} \ \sim 3.5\%$$

The non-perturbative running in $N_\mathrm{f} = 3$ QCD

(Bruno, MDB, Fritzsch, Korzec, Ramos, Schaefer, Simma, Sint, Sommer '17)



Conclusions & Outlook

Conclusions

- The lattice is well-suited for the determination of α_s: errors can **systematically** be improved!
- The result for $\Lambda_{\overline{\rm MS}}^{N_f=3}$ does not rely on PT below O(100) GeV!
- The effect of the charm and bottom quarks may be included using **perturbation theory**, given $\overline{m}^c_{\overline{\rm MS}}(\overline{m}^c_{\overline{\rm MS}})$ and $\overline{m}^b_{\overline{\rm MS}}(\overline{m}^b_{\overline{\rm MS}})$ (PDG),

$$\Lambda_{\overline{\rm MS}}^{N_f=3} \to \Lambda_{\overline{\rm MS}}^{N_f=4} \to \Lambda_{\overline{\rm MS}}^{N_f=5} = 215(10)(3)\,\mathrm{MeV}$$

cf. (Bruno et. al. Lattice '15)

- Having $\Lambda_{\overline{\rm MS}}^{N_f=5}$ we quote,

$$\alpha_{\overline{\rm MS}}(m_Z) = 0.1185(8)(3)$$

(Bruno, MDB, Fritzsch, Korzec, Ramos, Schaefer, Simma, Sint, Sommer '17)

$$\alpha_{\overline{\rm MS}}^{\text{FLAG '16}}(m_Z) = 0.1182(12), \quad \alpha_{\overline{\rm MS}}^{\text{PDG '16}}(m_Z)|_{\text{w/o LQCD}} = 0.1174(16)$$

Outlook

- Include the **charm quark** non-perturbatively: not too hard, it will be done!
- A reduction of the error from 0.7% to 0.5% is feasible: our dominant error is **statistical**!
- Further reducing the error becomes much more challenging: QED effects, …

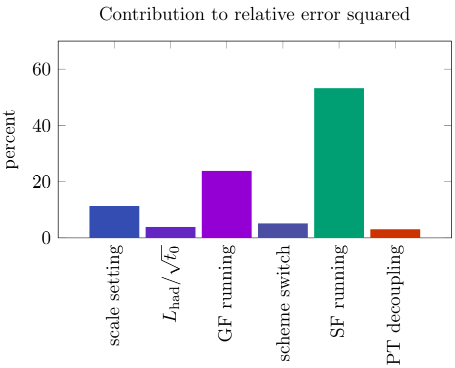

Thanks

BACKUP

Final error budget

Different sources of error

(Korzec Lattice '17)



Perturbative decoupling

How do we include the missing quarks? (Korzec Lattice '17)

Decoupling:

$$\bar{g}^{N_\mathrm{f}}(\mu) = \bar{g}^{N_\mathrm{f}+1}(\mu) \times \xi\left(g^{N_\mathrm{f}}(\mu), \overline{m}_h/\mu\right) + \mathrm{O}(\overline{m}_h^{-2})$$

Equivalently a relation for $\Lambda^{N_\mathrm{f}}/\Lambda^{N_\mathrm{f+1}}$ (Bruno et. al. Lattice '15)

- $O(\overline{m}_h^{-2})$ are very small already for $\overline{m}_h = \overline{m}_c$ (Bruno et. al. '15)
- ξ known in perturbation theory to 4-loops (Chetyrkin, Kühn, C. Sturm '06; Schröder, Steinhauser '06)
- Perturbation theory looks surprisingly well-behaved already at $\mu = \overline{m}_c$!

n (loops)	$\alpha_{\overline{\mathrm{MS}}}^{(N_\mathrm{f}=5)}$	$\alpha_n - \alpha_{n-1}$
1	0.11699	
2	0.11827	0.00128
3	0.11846	0.00019
4	0.11852	0.00006

Conclusion

Within PT a conservative error is: $\alpha_4 - \alpha_2 \approx 0.0003$