On photon splitting in Lorentz-violating QED

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Motivation

- Lorentz Violation (LV) is mostly motivated by different approaches to construct quantum theory of gravity
- LV in matter sector is considered in the framework of effective field theory.
- Effects, appeared under hypothesis of LV, is the modification of cross-sections and particle decays.
- One of the effects is the changes of kinematics, allowing several reactions, forbidden in LI case, to occur.
- Well known example is photon decay into electron-positron pair.

The model

Extra LV term, quartic on space derivatives, for a photon, suppressed by the second power of LV mass scale M_{LV}

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \mp \frac{1}{2M_{LV}^2}F_{ij}\Delta^2 F^{ij} + i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi.$$

Come to following DR

$$E^2 = p^2 + \frac{p^4}{M_{LV}^2}.$$

The bound from the absence of photon decay

- $\gamma \to e^+ e^-$ is forbidden in LI, but allowed in LV if the photon energy exceed a certain threshold
- in terms of effective photon mass

$$m^2_{\gamma,eff} \equiv E^2 - p^2 = \frac{p^4}{M^2_{LV}}$$

If "photon mass" $m_{\gamma,eff} \geq 2m_e$ reaction is possible!

$$M_{LV} > \frac{E^2_\gamma}{2m_e}$$

Leads to following constraint on M_{LV}[1]

$$M_{LV,\gamma} > 2.8 \times 10^{12} \text{ GeV}.$$

[1] H. Martínez-Huerta and A. Pérez-Lorenzana, arXiv:1610.00047 [astro-ph.HE].

The photon splitting

- In $m_{\gamma,\text{eff}} < 2m_e$ case, the photon decay $\gamma \to e^+e^-$ is kinematically forbidden
- Photon decay with several photons in the final state $\gamma \to$ nγ, so-called photon splitting, is kinematically allowed whenever the photon dispersion relation is superluminal
- Splitting to two photons $\gamma \to 2\gamma$ do not occur due to the Furry theorem
- Thus the main splitting process is the photon decay to 3 photons $\gamma \to 3\gamma$

Gelmini

The width of photon splitting has been estimated by Gelmini Nussinov and Yaguna [2]. They used notation of effective photon mass with calculations in the artificial rest frame of massive photon, followed by subsequent boost back to laboratory frame

$$\mathcal{L}_{E\text{-}H} = \frac{2\alpha^2}{45 m_e^4}\left[\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right)^2 + 7\left(\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)^2\right] \; .$$

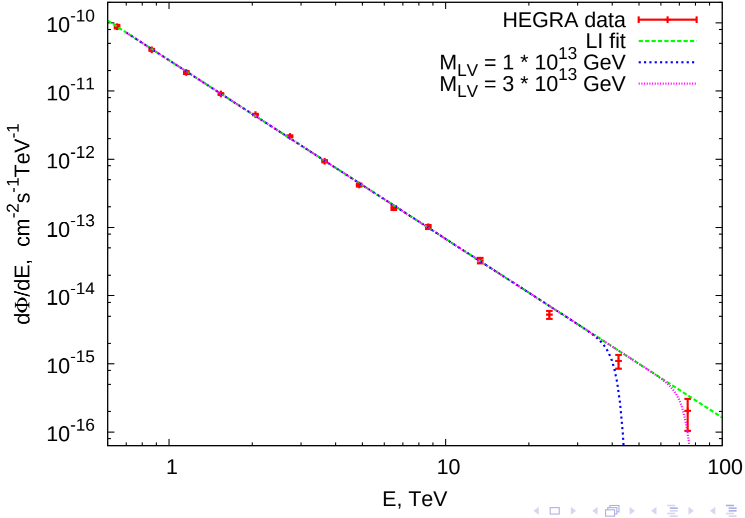
$$\Gamma(\gamma\to 3\gamma) \simeq 0.3\times 10^{-20}\,\frac{E_\gamma^{19}}{m_e^8 M_{LV}^{10}}.$$

[2] G. Gelmini, S. Nussinov and C. E. Yaguna, JCAP **0506** (2005) 012 [hep-ph/0503130].

Experimental constraints

- Using expression for $\Gamma(\gamma \to 3\gamma)$ photon free path distance can be obtained.
- Knowing distance to the source and max energy of incoming photons (the cut-off energy) constraint on M_{LV} can be found.
- 20 TeV from blazar Mrk 501 : $M_{LV} > 8 \cdot 10^{12}$ GeV
- 80 TeV from Crab nebula : $M_{LV} > 4 \cdot 10^{13}$ GeV

HEGRA


HEGRA data
LI fit
M_LV = 1 * 10^13 GeV
M_LV = 3 * 10^13 GeV
dΦ/dE, cm^-2 s^-1 TeV^-1
10^-10
10^-11
10^-12
10^-13
10^-14
10^-15
10^-16
1
10
100
E, TeV

Honest calculations

$$k_1^{\|} = p\frac{1+x+y}{3},\ k_2^{\|} = p\frac{1+x-y}{3},\ k_3^{\|} = p\frac{1-2x}{3}$$

$$k_1^{\|} + k_2^{\|} + k_3^{\|} = p,\ \vec{k}_1^{\perp} + \vec{k}_2^{\perp} + \vec{k}_3^{\perp} = 0$$

Phase Volume:

$$\Phi = \frac{3}{p}\int \frac{dx\,dy\,d^2k_1^{\perp}d^2k_2^{\perp}}{((1+x)^2-y^2)(1-2x)}\delta(E-E_1-E_2-E_3)|M|^2.$$

We introduce dimensionless variables β_1, β_2:

$$k_1^{\perp} = \frac{2p^2}{3M}\cdot\beta_1, \qquad k_2^{\perp} = \frac{2p^2}{3M}\cdot\beta_2.$$

Phase volume:

$$\Phi = \frac{8\pi}{9}\frac{p^4}{M^2}\int \frac{dx\, dy\, d\beta_1 d\beta_2}{((1+x)^2 - y^2)} \frac{1}{\sin\varphi_2|_{podstanovka}} |M|^2.$$

$$\cos\varphi_2 = \frac{2/3 f(x,y)(1-2x) - \frac{2-x+y}{1+x+y}\beta_1^2 - \frac{2-x-y}{1+x-y}\beta_2^2}{2\beta_1\beta_2}, \qquad |\cos\varphi_2| < 1$$

$$(p, k_{1,2}) = \frac{E_p^4}{9M^2}\left[(1+x\pm y) + \frac{6\beta_{1,2}^2}{1+x\pm y} + \frac{1}{6}(1+x\pm y)^3\right]$$

$$(p, k_3) = \frac{E_p^4}{M^2} - (p, k_1) - (p, k_2)$$

$$(k_1, k_2) = \frac{2E_p^4}{9M^2}[\frac{\beta_1^2}{1+x+y} + \frac{\beta_2^2}{1+x-y} + \frac{(1+x)^4 - y^4}{18} - \frac{2}{3}f(x,y)(1-2x)]$$

$$(k_{1,2}, k_3) = \frac{1}{2M^2}(E_p^4 \pm E_2^4 \mp E_1^4 - E_3^4) - (p, k_{1,2})$$

where

$$f(x,y) = 1 - \frac{3x^2}{4} + \frac{x^3}{4} - (1+x^2)y^2$$

Squared amplitude

Preliminary!

$$|M|^2 = |M|^2_{LI} + \delta|M|^2$$

$$|M|^2_{LI} = 24k_1^2k_2^2k_3^2p^2 + 888(k_1, k_2)^2(k_3, p)^2 +$$
$$136k_3^2p^2(k_1, k_2)^2 + 720k_3^2(k_1, k_2)(k_1, p)(k_2, p) +$$
$$+ 2672(k_1, k_2)(k_1, p)(k_2, k_3)(k_3, p) + 2672(k_1, k_2)(k_1, k_3)(k_2, p)(k_3, p) +$$
$$720p^2(k_1, k_2)(k_1, k_3)(k_2, k_3) +$$
$$+ 888(k_1, k_3)^2(k_2, p)^2 + 136k_1^2k_2^2(k_3, p)^2 +$$
$$2672(k_1, k_3)(k_1, p)(k_2, k_3)(k_2, p) + 136k_1^2k_3^2(k_2, p)^2 + 136k_2^2k_3^2(k_1, p)^2 +$$
$$+ 720k_2^2(k_1, k_3)(k_1, p)(k_3, p) +$$
$$720k_1^2(k_2, k_3)(k_2, p)(k_3, p) + 136k_1^2p^2(k_2, k_3)^2 + 136k_2^2p^2(k_1, k_3)^2$$

$$
\begin{aligned}
\delta|M|^2 = \frac{222}{M^2}(|\vec{k_1}|^2 + |\vec{k_2}|^2 + |\vec{k_3}|^2 + |\vec{p}|^2)(k_1,k_2)^2(k_3,p)^2 + \\
\frac{720}{M^2}(|\vec{k_3}|^2 k_{3}{}_0^2 (k_1,k_2)(k_1,p)(k_2,p)) + \\
\frac{2672}{M^2}(|\vec{k_1}|^2 + |\vec{k_2}|^2 + |\vec{k_3}|^2 + |\vec{p}|^2)(k_1,k_2)(k_1,p)(k_2,k_3)(k_3,p) + \\
\frac{2672}{M^2}(|\vec{k_1}|^2 + |\vec{k_2}|^2 + |\vec{k_3}|^2 + |\vec{p}|^2)(k_1,k_2)(k_1,k_3)(k_2,p)(k_3,p) + \\
\frac{720}{M^2}(|\vec{p}|^2 p_0^2)(k_1,k_2)(k_1,k_3)(k_2,k_3) + \\
\frac{222}{M^2}(|\vec{k_1}|^2 + |\vec{k_2}|^2 + |\vec{k_3}|^2 + |\vec{p}|^2)(k_1,k_3)^2(k_2,p)^2 + \\
\frac{2672}{M^2}(|\vec{k_1}|^2 + |\vec{k_2}|^2 + |\vec{k_3}|^2 + |\vec{p}|^2)(k_1,k_3)(k_1,p)(k_2,k_3)(k_2,p) + \\
\frac{720}{M^2}(|\vec{k_2}|^2 k_{2}{}_0^2)(k_1,k_3)(k_1,p)(k_3,p) + \\
\frac{720}{M^2}(|\vec{k_1}|^2 k_{1}{}_0^2)(k_2,k_3)(k_2,p)(k_3,p)
\end{aligned}
$$

Summary

- $\Phi \sim \frac{E_p^4}{M^2}\frac{1}{m_e^8}(\frac{E_p^4}{M^2})^4 = \frac{E_p^{20}}{m_e^8 M^{10}}$, $\Gamma \sim E_p^{-1}\Phi$ - the same order as in Gelmini estimation
- x=0 y=0 (three equal momenta photons) case is dominating
- Preliminary We determined the coefficient in expression for phase volume

$$20 \times \left(\frac{8\pi}{9}\right)\left(\frac{\alpha^2}{90}\right)^4$$