CPT VIOLATION:
From MATTER-ANTIMATTER ASYMMETRY in the EARLY UNIVERSE to ENTANGLED QUANTUM STATES

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7th Intl. Conf. on New Frontiers in Physics ICNFP2018, OAC (Kolymbari) 4-12 July 2018
I. Motivation – Insufficient CP Violation induced matter-antimatter asymmetry in Standard Model 
   → must go beyond to reproduce observed baryon asymmetry....

II. Exotic scenarios: CPT Violation in early Universe?
   (a) Lorentz Violating Background (flux) fields & Baryogenesis through Leptogenesis
       → matter-antimatter asymmetry of correct value:
       → evolution from early epochs to present day – current bounds

   (b) Quantum Gravity Decoherence (QGD):
       strong(?) in early Universe → CPT generator ill-defined

III(a). Novel QGD-CPT Violating effects (ω-effect) in entangled states of particles, e.g. neutral mesons - searches in Φ- or B- factories /

III(b). Connection to Baryogenesis

IV. Conclusions-Outlook
Construct microscopic models with strong CPT Violation in Early Universe (due to background fields or quantum gravity), but weak today… Fit with all available data… in particular current stringent constraints → scale back in time. Estimate in this way matter-antimatter asymmetry in Universe. Does it agree with the expected phenomenological value?
IS THIS CPTV ROUTE WORTH FOLLOWING? …. 

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Part I
Motivation:
Matter-Antimatter Asymmetry in the Standard Model
STANDARD MODEL INCOMPATIBLE WITH BARYOGENESIS

- Matter-Antimatter asymmetry in the Universe \( \Rightarrow \) Violation of Baryon \# (B), C & CP

- Tiny CP violation \((O(10^{-3}))\) in Labs: e.g. \( K^0 \bar{K}^0 \)

- But Universe consists only of matter

\[
\frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \quad \text{for } T > 1 \text{ GeV}
\]

Sakharov: Non-equilibrium physics of early Universe, B, C, CP violation

but not quantitatively in SM, still a mystery
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Sakharov: Non-equilibrium physics of early Universe, B, C, CP violation but not quantitatively in SM, still a mystery

Assume CPT invariance
Sakharov’s Conditions for Matter/Antimatter Asymmetry in the Universe

(i) Out of Equilibrium Lepton Asymmetry (Leptogenesis) \[ \Rightarrow \] Baryon Asymmetry via 
B-L conserving (SM) processes

(ii) Directly generated out of equilibrium Baryogenesis

\[ \bar{A} = \text{antiparticle} \quad CP : \quad \bar{X} \leftrightarrow \bar{\ell} + \bar{(\ldots)} \]

Rates \[ \Gamma \neq \bar{\Gamma} \]

\[ X \leftrightarrow \ell + \ldots. \]

C=charge conjugation
P = spatial reflexion \[ \vec{x} \rightarrow -\vec{x} \]
Classical conservations of EW theory: $B, L_e, L_\mu, L_\tau$

Quantum Anomalies:

$$\partial_\mu J_\mu^B = \partial_\mu J_\mu^L = \frac{n_f}{32\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} + U(1) \text{ part}$$

Allowed Processes (change of $B$ by multiples of 3)

$$\text{bosons} \leftrightarrow \text{bosons} + 9q + 3l$$

$L_i - B/3$ Conserved

(three quantities)

**BUT:**

OBSERVED NEUTRINO FLAVOUR OSCILLATIONS

If neutrinos Majorana

L-B conserved (one quantity)

$L=\text{total Lepton \#}$

L not definite
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

Rate of B violation in Early Universe

\[ \Gamma \sim \left\{ \begin{array}{l}
(\alpha_W T)^4 \left( \frac{M_{\text{sph}}}{T} \right)^7 \exp \left( -\frac{M_{\text{sph}}}{T} \right), & T \lesssim M_{\text{sph}}, \\
\alpha_W (\alpha_W T)^4 \log(1/\alpha_W), & T \gtrsim M_{\text{sph}},
\end{array} \right. \]

\[ \alpha_W = \text{SU(2) fine structure } \text{``constant''} \]

Kuzmin, Rubakov, Shaposhnikov
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

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Sphaleron Mass Scale
\( (M_W/\alpha_W) = \text{height of energy barrier separating SU(2) vacua with different topologies} \)

Manton (1983)
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

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\end{cases} \]

Thermal Equilibrium (i.e. \( \Gamma > H \) (Hubble)) for B non conserv. occurs only for:

\[ T_{\text{sph}}(m_H) < T < (\alpha_W)^5 M_{Pl} \sim 10^{12} \text{ GeV} \]

\[ T_{\text{sph}}(m_H) \in [130, 190] \text{ GeV} \]

\[ m_H \in [100, 300] \text{ GeV} \]

BAU could be produced this way only when sphaleron interactions freeze out, i.e.

\[ T \approx T_{\text{sph}} \]
OBSERVED CP VIOLATION UNLIKELY TO EXPLAIN BARYON ASYMMETRY IN THE UNIVERSE

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- \( LHC \) Expts (2012) \( m_H \approx 126 \) GeV
- \( T \sim T_{\text{sph}} \)

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BAU COULD BE PRODUCED @

\[ T \sim T_{\text{sph}} \]

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LHC Expts (2012) \[ m_H \approx 126 \text{ GeV} \]

Kuzmin, Rubakov, Shaposhnikov

Compute CP Violation Effects

Use CKM Matrix for \[ T > T_{\text{sph}} \]
Within the Standard Model, lowest CP Violating structures

\[ d_{CP} = \sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin\delta_{CP} \]
\[ \cdot (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \]

Rubakov, Kuzmin, Shaposhnikov, Gavela, Hernandez, Orloff, Pene

Kobayashi-Maskawa CP Violating phase

Shaposhnikov

\[ \delta_{KM}^CP \sim \frac{D}{T^{12}} \sim 10^{-20} \]

\[ \frac{n_B - \bar{n}_B}{n_B + \bar{n}_B} \sim \frac{n_B - \bar{n}_B}{s} = (8.4 - 8.9) \times 10^{-11} \]

This CP Violation Cannot be the Source of Baryon Asymmetry in The Universe

\[ T \sim T_{sph} \]

\[ T_{sph}(m_H) \in [130, 190] \text{GeV} \]

LHC Expts (2012) \( m_H \approx 126 \text{ GeV} \)
Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.) – to find EXTRA SOURCES OF CP VIOLATION within CPT invariant effective field theories
Beyond the Standard Model

• Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.) – to find EXTRA SOURCES OF CP VIOLATION within CPT invariant effective field theories

• THIS TALK: TRY EXOTIC SCENARIOS WITH (SIMPLIFIED) MODELS OF CPT VIOLATION IN EARLY UNIVERSE ?
Consistency with stringent current constraints must be ensured
Part II
CPT Violation
THEORY
C, P, T are Broken. Why Not CPT?

Lev Okun hep-ph/0210052v1

Point 0: C even, P even, T even $\rightarrow$ CP, PT, TC, CPT even
1: C odd, P even, T even $\rightarrow$ CP odd, PT even, CT odd, CPT odd
2: C even, P odd, T even $\rightarrow$ CP odd, PT odd, CT odd, CPT odd
3: C even, P even, T odd $\rightarrow$ CP even, PT odd, CT odd, CPT odd
4: C odd, P odd, T even $\rightarrow$ CP even, PT odd, CT odd, CPT even

etc

**Mnemonic cube rule:** (C, P, T) : + (-) even (odd)

0(++,+), 1(−,+,+), 2(+,−,+), 3(+,+,−), 4(−,−,+), 5(+,−,−), 6(−,+−), 7(−,−,−)
CPT Theorem

Schwinger 1951
Lüders 1954
J S Bell 1954
Pauli 1955
Res Jost 1958
CPT Invariance Theorem:
A quantum field theory lagrangian is invariant under CPT if it satisfies
(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

Conditions for the Validity of CPT Theorem

\[ P : \vec{x} \rightarrow -\vec{x}, \quad T : t \rightarrow -t(T), \quad C \psi(q_i) = \psi(-q_i) \]

Schwinger, Pauli, Luders, Jost, Bell
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\[ \text{i} \rightarrow \text{f}, \text{T: } \text{f} \rightarrow \text{i} \]

Schwinger, Pauli, Luders, Jost, Bell

e.g. for (Dirac) fermions:

\[ P: \quad \psi(t, \mathbf{x}) \rightarrow e^{i\delta} \gamma^0 \psi(t, -\mathbf{x}) \]

\[ T: \quad \psi_T(t, \mathbf{x}) = i \gamma^1 \gamma^3 \psi^*(\mathbf{t}, -\mathbf{x}) \]

\[ C: \quad \psi_C(t, \mathbf{x}) = i \gamma^2 \gamma^0 \psi^T(t, \mathbf{x}) \]
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Conditions for the Validity of CPT Theorem

Schwinger, Pauli, Luders, Jost, Bell revisited by:
Greenberg, Chaichian, Dolgov, Novikov, Fujikawa, Tureanu ...

(ii)-(iv) Independent reasons for violation
CPT Invariance Theorem:

(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

CPT V well-defined as Operator $\Theta$
does not commute with Hamiltonian
$[\Theta, H] \neq 0$
CPT Invariance Theorem:

(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

(ii)-(iii) CPT V well-defined as Operator Θ does not commute with Hamiltonian

\[ [\Theta, H] \neq 0 \]

(iv) e.g. Quantum Gravity decoherence \( \rightarrow \) CPT operator may not be well defined (Wald)

\( \rightarrow \) consequence for Entangled States (Bernabeu, NEM, Papavassiliou, ...)

Conditions for the Validity of CPT Theorem
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CPT Invariance Theorem:
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Kostelecky, Bluhm, Colladay, Potting, Russell, Lehnert, Mewes, Diaz, Tasson, ...

Standard Model Extension (SME)

(ii)-(iv) Independent reasons for violation

\[ \mathcal{L} \ni \cdots + \overline{\psi}^f \left( i \gamma^\mu \nabla_\mu - m_f \right) \psi^f + a_\mu \overline{\psi}^f \gamma^\mu \psi^f + b_\mu \overline{\psi}^f \gamma^\mu \gamma^5 \psi^f + \cdots \]

\[ [\Theta, H] \neq 0 \]

Lorentz & CPT Violation
Lorentz & CPT Violation
Simplest ideas on CPT Violation (CPTV) do not work for Baryogenesis
Assume CPT Violation was strong in the Early Universe

**ONE POSSIBILITY:**

particle-antiparticle mass differences

\[
[\Theta, H] \neq 0 \quad \Rightarrow \quad m \neq \overline{m}
\]

\[
0 \neq H\Theta|m\rangle - \Theta H|m\rangle = H\Theta|m\rangle - m\Theta|m\rangle
\]

(\(|m\rangle = \text{mass eigenstate}
\Theta |m\rangle = \text{antimatter state})
Equilibrium Distributions different between particle-antiparticles

Can these create the observed matter-antimatter asymmetry?

\[ f(E, \mu) = \frac{1}{\exp[(E - \mu)/T] \pm 1} \]

\[ \delta n = n - \bar{n} = g_{df} \int \frac{d^3p}{(2\pi)^3} \left[ f(E, \mu) - f(\bar{E}, \bar{\mu}) \right] \]

\[ E = \sqrt{p^2 + m^2}, \quad \bar{E} = \sqrt{p^2 + \bar{m}^2} \]

\[ m \neq \bar{m} \]

\[ \delta m = m - \bar{m} \]

Assume dominant contributions to Baryon asymmetry from quarks-antiquarks

\[ m(T) \sim gT \]

Dolgov, Zeldovich Dolgov (2009)

High-T quark mass >> Lepton mass
Equilibrium Distributions different between particle-antiparticles

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Assuming dominant contributions to Baryon asymmetry from quarks-antiquarks

\[ \beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} \left( 18m_u \delta m_u + 15m_d \delta m_d \right) / T^2 \]

\[ n_\gamma = 0.24T^3 \quad \text{photon equilibrium density at temperature } T \]
\[ \beta_T = \frac{n_B}{n_\gamma} = -8.4 \cdot 10^{-3} \left( 18m_u \delta m_u + 15m_d \delta m_d \right) / T^2 \]

\[ n_\gamma = 0.24T^3 \]

Current bound for proton-anti proton mass diff.

\[ \delta m_p < 8 \times 10^{-10} m_e \]
\[ \delta m_p < 7 \cdot 10^{-10} \text{ GeV} \]

\[ \delta m_q \sim \delta m_p \]

Reasonable to take:

\[ \beta^{T=0} = 6 \cdot 10^{-10} \]

\[ \delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p \]

NB: To reproduce the observed

Too small \[ \beta^{T=0} \]
Dolgov (2009)

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**NB:** To reproduce the observed

\[
\beta(T=0) = 6 \cdot 10^{-10}
\]

need

\[
\delta m_q(T = 100 \text{ GeV}) \sim 10^{-5} - 10^{-6} \text{ GeV} \gg \delta m_p
\]

**CPT Violating quark-antiquark Mass difference alone CANNOT REPRODUCE observed BAU**
But CPT Violation (CPTV) is associated with many more effects & parameters to explore in connection to Baryogenesis...
Spontaneous Violation of Lorentz Symmetry

$\mathcal{L} = \frac{1}{2} i \bar{\psi} \Gamma^\nu \partial_\nu \psi - \bar{\psi} M \psi,$

$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu \nu} \sigma_{\mu \nu}$

$\Gamma^\nu \equiv \gamma^\nu + c^{\mu \nu} \gamma_\mu + d^{\mu \nu} \gamma_5 \gamma_\mu + e^\nu + i f^{\nu} \gamma_5 + \frac{1}{2} g^{\lambda \mu \nu} \sigma_{\lambda \mu}$

(LV coefficients are v.e.v. of tensor-valued field quantities)
Microscopic Origin of SME coefficients?

Several `Geometry-induced` examples:
Microscopic Origin of SME coefficients?

Several "Geometry-induced" examples:
Non-Commutative Geometries LV only \((H_{\mu\nu}, d_{\mu\nu}, \ldots)\)

\[
[x^\mu, x^\nu] = \theta^{\mu\nu} \neq 0
\]
Microscopic Origin of SME coefficients?

Several "Geometry-induced" examples:
- Non-Commutative Geometries LV only
- Axisymmetric Background Geometries of the Early Universe
- Torsionful Geometries (including strings…)

Early Universe T-dependent effects:
- large @ high T, low values today
- for coefficients of SME
Microscopic Origin of SME coefficients?

Several ``Geometry-induced” examples:
- Non-Commutative Geometries
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Early Universe T-dependent effects:
- large @ high T, low values today
- for coefficients of SME
In particular, Space-times with 

CPTV Effects of different Space-Time-Curvature/Spin couplings between fermions/antifermions

Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ g_{\mu\nu} = e^a_{\mu} \eta_{ab} e^b_{\nu} \]

\[ \omega_{bca} = e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a \right). \]

Gravitational covariant derivative including spin connection

\[ \sigma^{ab} = \frac{i}{2} \left[ \gamma^a, \gamma^b \right] \]

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} \left[ (i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \]

\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a \right) \]
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\[ \gamma^a \gamma^b \gamma^c = \eta^{ab} \gamma^c + \eta^{bc} \gamma^a - \eta^{ac} \gamma^b - i \epsilon^{dabc} \gamma_d \gamma^5 \]

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\[ B^d = \epsilon^{abcd} e_b \lambda \left( \partial_a e^\lambda_e + \Gamma^\lambda_{\alpha \mu} e^\alpha_c e^\mu_e \right) \]

Standard Model Extension type Lorentz-violating coupling

(Kostelecky et al.)
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

Gravitational covariant derivative including spin connection:

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right) \]

\[ g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu \]

\[ \omega_{bca} = e_b^\lambda \left( \partial_a e_c^\lambda + \Gamma^\lambda_{\gamma\mu} e_\gamma^c e_\mu^a \right). \]

For homogeneous and isotropic Friedman-Robertson-Walker geometries the resulting \( B^\mu \) vanish.
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\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a \right) \]

Can be constant in a given local frame in Early Universe axisymmetric (Bianchi) cosmologies or near rotating Black holes,
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

**Gravitational covariant derivative including spin connection**

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right) \]

\[ g_{\mu\nu} = e^a_{\mu} \eta_{ab} e^b_{\nu} \]

\[ \omega_{bca} = e_b\lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a \right) \]

If torsion then \( \Gamma_{\mu\nu} \neq \Gamma_{\nu\mu} \) **antisymmetric** part is the contorsion tensor, contributes
Dirac Lagrangian (for concreteness, it can be extended to Majorana neutrinos)

\[ \mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right) \]

Gravitational covariant derivative including spin connection

\[ D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \]

\[ g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu \]

\[ \omega_{bca} = e_b^\lambda \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma\mu} e^\gamma_c e^\mu_a \right). \]

Fractured cosmological backgrounds lead to constant \( B^0 \) in FRW frame

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g} \bar{\psi} \left[ (i \gamma^a \partial_a - m) + \gamma^a \gamma^5 B_a \right] \psi, \]

\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\alpha\mu} e^\alpha_c e^\mu_a \right) \]
Part IIa
CPT Violation in a String-Inspired Model of the Early Universe
A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

John Ellis, NEM & Sarkar, arXiv:1304.5433
De Cesare, NEM & Sarkar arXiv:1412.7077
Bossingham, NEM & Sarkar, arXiv:1712.03312

Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symmetric rank 2 tensor (graviton)
spin 1 antisymmetric rank 2 tensor
A non-trivial example of Torsion: String Theories with Antisymmetric Tensor Backgrounds

Massless Gravitational multiplet of (closed) strings: spin 0 scalar (dilaton)
spin 2 traceless symmetric rank 2 tensor (graviton)
spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD \( B_{\mu\nu} = -B_{\nu\mu} \)

Effective field theories (low energy scale \( E \ll M_s \)) "gauge" invariant
\[
B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta(x)_\nu
\]

Depend only on field strength: \( H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} \)

Bianchi identity: \[ \partial \left[ \sigma H_{\mu\nu\rho} \right] = 0 \rightarrow d \star H = 0 \]
ROLE OF H-FIELD AS TORSION

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

\[ S^{(4)} = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \]

\[ = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} \bar{R} \right) \]

\[ \kappa^2 = 8\pi G \]

4-DIM PART

generalised curvature

Contorsion
ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

\[ S^{(4)} = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \]

\[ = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} \overline{\Gamma} \right) \]

\[ \kappa^2 = 8\pi G \]

IN 4-DIM DEFINE DUAL OF H AS:

\[ -3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho} \]

\[ b(x) = \text{Pseudoscalar (Kalb-Ramond (KR) axion)} \]
FERMIONS COUPLE TO H–TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

\[ S_\psi = \frac{i}{2} \int d^4 x \sqrt{-g} \left( \bar{\psi} \gamma^\mu \overline{D}_\mu \psi - (\overline{D}_\mu \bar{\psi}) \gamma^\mu \psi \right) \]

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

\[ \overline{D}_a = \partial_a - \frac{i}{4} \bar{\omega}_{bca} \sigma^{bc} \]

\[ \bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu} \]

contorsion

\[ K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right) \]
Fermions couple to H–torsion via gravitational covariant derivative.

\[ S_\psi = \frac{i}{2} \int d^4 x \sqrt{-g} \left( \bar{\psi} \gamma^\mu \overline{\text{D}_\mu} \psi - (\overline{\text{D}_\mu} \psi) \gamma^\mu \psi \right) \]

Torsionful connection, first-order formalism

\[ \overline{\text{D}}_a = \partial_a - \frac{i}{4} \bar{\omega}_{bca} \sigma^{bc} \]

\[ \bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu} \]

\[ K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right) \]
FERMIIONS COUPLE TO H–TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

\[ S_\psi = \frac{i}{2} \int d^4x \sqrt{-g} \left( \bar{\psi} \gamma^\mu \overline{\mathcal{D}}_\mu \psi - (\overline{D}_\mu \bar{\psi}) \gamma^\mu \psi \right) \]

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\[ \overline{D}_a = \partial_a - \frac{i}{4} \bar{\omega}_{bca} \sigma^{bc} \]

\[ \bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu} \]

contorsion

\[ K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right) \]

Non-trivial contributions to \( B^\mu \)

\[ B^d = \epsilon^{abcd} e_{b\lambda} \left( \partial_a e_\lambda^\alpha + \Gamma_\alpha^{\lambda}_{\lambda\mu} e_\lambda^\alpha e_\mu^\alpha \right) \]

\[ \bar{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^\mu_{\nu\rho} \neq \bar{\Gamma}^\mu_{\rho\nu} \]

\( H_{cab} \)
FERMIONS COUPLE TO H–TORSION VIA GRAVITATIONAL COVARIANT DERIVATIVE

\[ S_\psi = \frac{i}{2} \int d^4 x \sqrt{-g} \left( \overline{\psi} \gamma^\mu \overline{D}_\mu \psi - (\overline{D}_\mu \overline{\psi}) \gamma^\mu \psi \right) \]

TORSIONFUL CONNECTION, FIRST-ORDER FORMALISM

\[ S_\psi \ni \int d^4 x B_\alpha \overline{\psi} \gamma^a \gamma^5 \psi \]

\[ B^d \sim \epsilon^{abcd} H_{bca} \]

Non-trivial contributions to \( B^\mu \)

\[ B^d = \epsilon^{abcd} e_b^\lambda \left( \partial_a e_c^\lambda + \Gamma_\alpha^\lambda e_c^\alpha e_a^\mu \right) \]

\[ K_{abc} = \frac{1}{2} \left( T_{cab} - T_{abc} - T_{bca} \right) \]

\[ \bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu} \]

contorsion

\[ H_{cab} \]

\[ \Gamma_{\nu\rho}^\mu - \Gamma_{\nu\rho}^\mu + \frac{\kappa}{\sqrt{3}} H_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho \nu}^\mu \]
When $\frac{db}{dt} = \text{constant} \rightarrow \text{Torsion is constant}$

Covariant Torsion tensor

$$\overline{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^{-2\Phi} H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} + T^\lambda_{\mu\nu}$$

$$T_{ijk} \sim \epsilon_{ijk} \dot{b}$$

$$S_\psi \ni \int d^4x B_a \overline{\psi} \gamma^a \gamma^5 \psi$$

constant $B^0 \propto \dot{b}$
When $\frac{db}{dt} = \text{constant} \rightarrow$ Torsion is constant

Covariant Torsion tensor

$$\overline{\Gamma}_\mu^{\lambda \nu} = \Gamma^{\lambda}_{\nu \mu} + e^{-2\Phi} H^{\lambda}_{\nu \mu} \equiv \Gamma^{\lambda}_{\nu \mu} + ...$$

In string theory a constant $B^0$ background is guaranteed by exact solutions with linear in FRW time $b = (\text{const } t)$

Antoniadis, Bachas, Ellis, Nanopoulos

$$T_{ijk} \sim \epsilon_{ijk} \dot{b}$$

Constant

$$S_\psi \ni \int d^4 x B_a \overline{\psi} \gamma^a \gamma^5 \psi$$

constant $B^0 \propto \dot{b}$
When \( \frac{db}{dt} = \text{constant} \rightarrow \text{Torsion is constant} \)

Covariant Torsion tensor

\[
\bar{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} + e^{-2\Phi} H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} + T^\lambda_{\mu\nu}
\]

\( T_{ijk} \sim \epsilon_{ijk} \dot{b} \)  

\[
S_\psi \supset \int d^4x B_a \bar{\psi} \gamma^a \gamma^5 \psi
\]

\[
\mathcal{L} = \frac{1}{2} i \bar{\psi} \gamma^\nu \nabla_\nu \psi - \bar{\psi} M \psi, \quad M \equiv m + b_\mu \gamma^5 \gamma^\mu
\]

Standard Model Extension type with CPT and Lorentz Violating background \( b^0 = B^0 \)
NB: Perturbatively we may also have such a constant $B^0$ background in the presence of Lorentz-violating condensates of fermion axial current temporal component $\langle 0 \mid J^{05} \mid 0 \rangle \neq 0$ at the high-density, high-temperature Early Universe epochs.

Lagrangian:

$$\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} (\partial_\mu b)^2 - \Omega + \sum_i \left[ \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i + \frac{\kappa}{3\sqrt{6}} \partial_\mu b \bar{\psi}_i \gamma^\mu \gamma^5 \psi_i \right] + \ldots \right]$$

$i =$ Standard Model fermionic species

$\mathcal{O}((\partial b)^4)$

higher derivative terms in strings
NB:
Perturbatively we may also have such a constant $B^0$ background in the presence of Lorentz-violating condensates of fermion axial current temporal component $\langle 0 | J^{05} | 0 \rangle \neq 0$ at the high-density, high-temperature Early Universe epochs.

Eqs of motion for pseudoscalar:

$$\partial^\mu \left( \sqrt{-g} \left[ \epsilon_{\mu\nu\rho\sigma} (\partial^\sigma \bar{b} - \tilde{c} J^5{}^\sigma) + O((\partial \bar{b})^3) \right] \right) = 0$$

$$\dot{\bar{b}} = \tilde{c} \langle J^5_0 \rangle = \tilde{c} \langle \psi_i^\dagger \gamma^5 \psi_i \rangle = \text{constant} \neq 0$$

Condensate may be subsequently destroyed at a temperature $T_c$, $\langle 0 | J^{05} | 0 \rangle \to 0$ by relevant operators so eventually in an expanding FRW Universe for $T < T_c$.

Weak torsion today, compatible with stringent experimental limits.
$B^0$: (string) theory underwent a phase transition @ $T \approx T_d = 10^5$ GeV, from $B^0 = \text{const} = 1$ MeV to:

(i) either $B^0 = 0$

(ii) or $B^0$ small today but non zero

\[ B^0 \sim \dot{B} \sim 1/a^3(t) \sim T^3 \]

\[
B_0 = c_0 T^3 \quad \text{with} \quad c_0 = 10^{-42} \text{meV}^{-2}
\]

\[
B_{0\text{today}} = \mathcal{O}(10^{-44}) \text{meV}
\]

Quite safe from stringent Experimental Bounds

\[
|B^0| < 10^{-2} \text{eV} \quad B_i \equiv b_i < 10^{-31} \text{GeV}
\]
If Fermions are **DIRAC** (e.g. quarks, electrons)

**DISPERSION RELATIONS OF FERMIONS ARE **DIFFERENT** FROM THOSE OF ANTI-FERMIONS IN SUCH GEOMETRIES**

_CPTV Dispersion relations (B₀ = b₀)_

\[
E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}| B_0}
\]

\[
\overline{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}| B_0}
\]

but (bare) **masses** are equal between **particle/anti-particle** sectors

**Abundances** of fermions in Early Universe, then, **different** from those of antifermions, if B₀ is **non-trivial**, **ALREADY IN THERMAL EQUILIBRIUM**
If Fermions are **DIRAC** (e.g. quarks, electrons)

**DISPERSION RELATIONS OF FERMIONS ARE DIFFERENT FROM THOSE OF ANTI-FERMIONS IN SUCH GEOMETRIES**

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\]

but (bare) **masses** are equal between **particle/anti-particle** sectors

**Abundances** of fermions in Early Universe, then, **different** from those of antifermions, if \( B_0 \) is **non-trivial**, **ALREADY IN THERMAL EQUILIBRIUM**

\[
n - \overline{n} = \frac{g}{(2\pi)^3} \int d^3p \left( \frac{1}{1 + eE/T} - \frac{1}{1 + e\overline{E}/T} \right) \neq 0
\]

\( E \neq \overline{E} \)
If Fermions are **DIRAC** (e.g. quarks, electrons)

**DISPERSION RELATIONS OF FERMIONS ARE DIFFERENT FROM THOSE OF ANTI-FERMIONS IN SUCH GEOMETRIES**

*CPTV Dispersion relations* \((B_0 = b_0)\)

\[
E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}| B_0}
\]

\[
\bar{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}| B_0}
\]

but (bare) **masses** are equal between **particle/anti-particle** sectors

**Abundances** of fermions in Early Universe, then, **different** from those of antifermions, if \(B_0\) is **non-trivial**, **ALREADY IN THERMAL EQUILIBRIUM**

But for **Majorana fermions** (their own antiparticles, situation is different... **cf below**...
Right-Handed Heavy Majorana Neutrinos

Mechanism
For Torsion-Background-Induced tree-level

Leptogenesis $\rightarrow$ Baryogenesis

Through B-L conserving Sphaleron processes
In the standard model
SM Extension with N extra right-handed neutrinos

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \]
SM Extension with N extra right-handed neutrinos

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I^c N_I + \text{h.c.} \]

Yukawa couplings
Matrix (N=2 or 3)

\[ F = \tilde{K}_L f_d \tilde{K}_R^\dagger \]
SM Extension with N extra right-handed neutrinos

$\nu$MSM

\[ L = L_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - F_{\alpha I} \bar{L}_\alpha N_I \phi - \frac{M_I}{2} \bar{N}_I N_I + h.c. \]

Majorana masses to (2 or 3) active neutrinos via seesaw

Yukawa couplings

Matrix (N=2 or 3)

Minkowski, Yanagida, Mohapatra, Senjanovic, Sechter, Valle …

\[ m_\nu = -M^D \frac{1}{M_I} [M^D]^T. \]

\[ M_D = F_{\alpha I} \nu \quad M_D \ll M_I \]

\[ \nu = \langle \phi \rangle \sim 175 \text{ GeV} \]
Early Universe
T > 10^5 GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_I \rightarrow H\nu, \bar{H}\bar{\nu} \]
Early Universe
$T > 10^5$ GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$
CPTV Thermal

\[ \mathcal{L} = i\bar{N} \gamma N - \frac{m}{2} (\bar{N} N^c + \bar{N} N) - \bar{N} \gamma^5 N - Y_k \bar{L}_k \phi N + h.c. \]

Early Universe
\[ T > 10^5 \text{ GeV} \]

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

One generation of massive neutrinos \( N \) suffices for generating CPTV Leptogenesis;

\[ N_I \rightarrow H\nu, \bar{H}\bar{\nu} \]
Early Universe
$T > 10^5 \text{ GeV}$

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$$

One generation of massive neutrinos $N$
suffices for generating CPTV Leptogenesis;
mass $m$ free
to be fixed
Early Universe
T > 10^5 GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_1 \rightarrow H\nu, \bar{H}\bar{\nu} \]

Produce Lepton asymmetry

CPTV Thermal

CPT Violation

Constant H-torsion
(antisymmetric tensor field strength in string models)
CPTV Thermal

CPT Violation

Early Universe $T > 10^5 \text{GeV}$

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$N_I \rightarrow H\nu, \bar{H}\bar{\nu}$

Produce Lepton asymmetry

Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)

CPT Violation

Leptogenesis

Fukugita, Yanagida,
CPTV Thermal

\[ \mathcal{L} = iN \bar{\phi} N - \frac{m}{2} \left( \bar{N}^c N + \bar{N} N^c \right) - \bar{N} \gamma^\mu N - Y_k \bar{L}_k \bar{\phi} N + h.c. \]

Early Universe
T > 10^5 GeV

CPT Violation

Constant H-torsion

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_I \rightarrow H \nu, \bar{H} \bar{\nu} \]

Produce Lepton asymmetry
CPTV Thermal

\[ \mathcal{L} = i\bar{N}\gamma^\mu\partial_\mu N - \frac{m}{2}(\bar{N}^c N + \bar{N}N^c) - \bar{N}\gamma^5 N - Y_k \bar{L}_k \phi N + h.c. \]

Early Universe
\[ T > 10^5 \text{ GeV} \]

CPT Violation

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_I \rightarrow H\nu, \overline{H}\nu \]

Constant H-torsion
\[ B^0 \neq 0 \text{ background} \]

Solving system of Boltzmann eqs

\[ \frac{\Delta L}{n_\gamma} \approx 10^{-10}, \]

Produce Lepton asymmetry
Decoupling Temperature $T_D$: decay process out of equilibrium @ which Lepton asymmetry is evaluated

\[ \Gamma \simeq H = 1, 66 T_D^2 N^{1/2} m_p^{-1} \]

assume standard cosmology
d.o.f.

\[ T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{N^{1/4}} \sqrt{\frac{m_p (\Omega^2 + B_0^2)}{\Omega}} \]

for one generation of RH heavy neutrino

\[ \Omega = \sqrt{B_0^2 + m_N^2} . \]

**Estimate:** Total Lepton number asymmetry

\[ \left( N \rightarrow \ell^- \phi^+, \nu \phi^0 \right) - \left( N \rightarrow \ell^+ \phi^-, \bar{\nu} \phi^0 \right) \]

via solving the appropriate system of **Boltzmann equations**:
CPTV Thermal

\[ \mathcal{L} = i \overline{N} \gamma^\mu \gamma_5 N - \frac{m}{2} (\overline{N}^c N + \overline{N} N^c) - \overline{N} B \gamma^5 N - Y_k \overline{L}_k \phi N + h.c. \]

Early Universe

\[ T > 10^5 \text{ GeV} \]

CPT Violation

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_I \rightarrow H \nu, \overline{H} \nu \]

CPT Violation

\[ \frac{\Delta L}{n_\gamma} \sim 10^{-10}, \]

\[ \frac{B_0}{m} \sim 10^{-8} \]

Produce Lepton asymmetry

\[ Y_k \sim 10^{-5} \]

\[ m \geq 100 \text{ TeV} \rightarrow \]

\[ B^0 \sim 1 \text{ MeV} \]

\[ T_D \sim m \sim 100 \text{ TeV} \]

Constant H-torsion

\[ B^0 \neq 0 \text{ background} \]
CPTV Thermal

\[ \mathcal{L} = i \bar{N} \gamma^\mu \partial_\mu N - \frac{m}{2} (\bar{N}^c N + \bar{N} N^c) - \bar{N} \gamma^5 \gamma^\mu B \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\nu \gamma^\rho \gamma^\sigma N + \text{h.c.} \]

Early Universe
\[ T > 10^5 \text{ GeV} \]

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_I \rightarrow H\nu, \bar{H}\bar{\nu} \]

CPT Violation

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Constant H-torsion
\[ B^0 \neq 0 \text{ background} \]
CPTV Thermal

Early Universe
T > 10^5 GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

Equilibrated electroweak
B+L violating sphaleron interactions

Environmental
Conditions Dependent

Produce Lepton asymmetry

B-L conserved

Observed Baryon Asymmetry
In the Universe (BAU)

Fukugita, Yanagida,

\[ \mathcal{L} = i\bar{N} \not{\!} \! \! \phi N - \frac{m}{2} (\bar{N}^c N + \bar{N} N^c) - \bar{N} \not{\!} \! \! B \gamma^5 N - Y_k \bar{L}_k \bar{\phi} N + h.c. \]
CPTV Thermal

\[ \mathcal{L} = i\bar{N}\gamma^\mu N - \frac{m}{2}(\bar{N}^c N + \bar{N}N^c) - \bar{N}\gamma^5 N - Y_k\bar{L}_k\phi N + h.c. \]

Early Universe

\( T > 10^5 \) GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

\[ N_L \rightarrow H\nu, \bar{H}\bar{\nu} \]

CPT Violation

Constant H-torsion

\( B^0 \neq 0 \) background

Equilibrated electroweak

B+L violating sphaleron interactions

Lepton number asymmetry

Environmental Conditions Dependent

\[ \frac{\Delta L}{n_\gamma} \approx 10^{-10}, \]

\[ \frac{B_0}{m} \approx 10^{-8} \]

Produce Lepton asymmetry

\[ Y_k \sim 10^{-5} \]

\[ m \geq 100 \text{TeV} \rightarrow \]

\[ B^0 \sim 1 \text{MeV} \]

\[ T_D \approx m \sim 100 \text{ TeV} \]

Observed Baryon Asymmetry

In the Universe (BAU)

Estimate BAU by fixing CPTV background parameters

In some models this means fine tuning ....
$B^0$: (string) theory underwent a **phase transition** @ $T \approx T_d = 10^5$ GeV, from $B^0 = \text{const} = 1$ MeV to:

(i) **either** $B^0 = 0$

(ii) **or** $B^0$ small today but non-zero

$$B^0 \sim \dot{b} \sim 1/a^3(t) \sim T^3$$

$$B_0 = c_0 T^3 \quad c_0 = 10^{-42} \text{meV}^{-2}$$

$$B_{0\text{ today}} = \mathcal{O}(10^{-44}) \text{meV}$$

Quite safe from stringent Experimental Bounds

$$|B^0| < 10^{-2} \text{eV} \quad B_i \equiv b_i < 10^{-31} \text{GeV}$$
CPTV Thermal

Early Universe
$T > 10^5 \text{ GeV}$

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

Equilibrated electroweak
B+L violating sphaleron interactions

Environmental
Conditions Dependent

CPT Violation

Constant H-torsion
$B^0 \neq 0$ background

$\Delta L \approx 10^{-10}$

$Y_k \approx 10^{-5}$

$m \geq 100 \text{ TeV} \rightarrow$

$B^0 \approx 1 \text{ MeV}$

$T_D \approx m \approx 100 \text{ TeV}$

Estimate BAU by fixing CPTV background parameters
In some models this means fine tuning ....
CPTV Thermal

Early Universe
T > 10^5 GeV

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

Equilibrated electroweak
B+L violating sphaleron interactions

Environmental
Conditions Dependent

Produce Lepton asymmetry

\[ \Delta L \approx 10^{-10}, \]

\[ \frac{B_0}{m} \approx 10^{-8}, \]

B-L conserved

Observed Baryon Asymmetry
In the Universe (BAU)

\[ T_D \approx m \approx 100 \text{ TeV} \]

Estimate BAU by fixing CPTV background parameters
In some models this means fine tuning ....

e.g. May Require
Fine tuning of Vacuum energy

常\[\mathcal{L} = i\bar{N}
\bar{\phi} N - \frac{m^2}{2} (\bar{N} c N + \bar{N} N c) - \bar{N} \gamma^5 N - \bar{N} \gamma^5 N - Y_k \bar{L}_k \bar{\phi} N + h.c.\]
Early Universe
\( T > 10^5 \text{ GeV} \)

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

Lepton number \( N \rightarrow H \nu, \bar{H} \bar{\nu} \)

Equilibrated electroweak
B+L violating sphaleron interactions

Environmental Conditions Dependent

Produce Lepton asymmetry

B-L conserved

Constant H-torsion
\( B^0 \neq 0 \) background

\( \frac{\Delta L}{n_\gamma} \approx 10^{-10} \)

\( \frac{B_0}{m} \approx 10^{-8} \)

\( Y_k \sim 10^{-5} \)

\( m \geq 100 \text{ TeV} \rightarrow \)

\( B^0 \sim 1 \text{ MeV} \)

\( T_D \approx m \sim 100 \text{ TeV} \)

e.g. May Require
Fine tuning of
Vacuum energy

Estimate BAU by fixing CPTV background parameters
In some models this means fine tuning ....
ROLE OF H-FIELD AS TORSION – AXION FIELD

EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT

\[
S^{(4)} = \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} \right)
\]

\[
= \int d^4 x \sqrt{-g} \left( \frac{1}{2\kappa^2} \overline{R} \right)
\]

\[\kappa^2 = 8\pi G\]

IN 4-DIM DEFINE DUAL OF H AS :

\[-3 \sqrt{2} \partial_\sigma b = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}\]

\[b(x) = \text{Pseudoscalar (Kalb-Ramond (KR)) axion)}\]

e.g. May Require Fine tuning of Vacuum energy
**ROLE OF H-FIELD AS TORSION – AXION FIELD**

**EFFECTIVE GRAVITATIONAL ACTION IN STRING LOW-ENERGY LIMIT**

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\[ b(x) = \text{Pseudoscalar (Kalb-Ramond (KR) axion)} \]

- E.g. May Require Fine tuning of Vacuum energy

- Constant if \[ \frac{1}{2} \partial^\mu b \partial_\mu b \] need to be cancelled by bulk contrib. in brane models
Part II(b) 
Quantum-Gravity induced Decoherence & intrinsic CPT Violation
CPT Invariance Theorem:
(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

(ii)-(iv) Independent reasons for violation

J.A. Wheeler

10^{-35} m

e.g. QUANTUM SPACE-TIME FOAM AT PLANCK SCALES
CPT Invariance Theorem:
(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

Hawking, Ellis, Hagelin, Nanopoulos, Srednicki, Banks, Peskin, Strominger, Lopez, NEM, Barenboim…

(ii)-(iv) Independent reasons for violation

Quantum Gravity Induced Decoherence
Evolution of Pure QM States to Mixed At Low Energies

Low Energy CPT Operator Not Well Defined

10^{-35} m
**CPT Invariance Theorem:**

(i) Flat space-times
(ii) Lorentz invariance
(iii) Locality
(iv) Unitarity

**Conditions for the Validity of CPT Theorem**

- Hawking, Ellis, Hagelin, Nanopoulos
- Srednicki, Banks, Peskin, Strominger, Lopez, NEM, Barenboim...

**Quadratic Gravity Induced Decoherence**

Evolution of pure QM states to mixed at low energies

Low energy **CPT Operator NOT well defined**

- 10^{-35} m
- R. Wald
Decoherence implies that asymptotic density matrix of low-energy matter:

\[ \rho = \text{Tr} |\psi\rangle \langle \psi| \]

\[ \rho_{\text{out}} = \$ \rho_{\text{in}} \]

\[ \$ \neq S S^\dagger \]

\[ S = e^{i \int H \, dt} \]

QG may induce quantum decoherence of propagating matter and intrinsic CPT Violation in the sense that the CPT operator \( \Theta \) is not well-defined.

\[ \Theta \rho_{\text{in}} = \overline{\rho}_{\text{out}} \]

If \( \Theta \) well-defined can show that

\[ \$^{-1} = \Theta^{-1} \$ \Theta^{-1} \]

exists!

\[ \text{INCOMPATIBLE WITH DECOHERENCE!} \]

Hence \( \Theta \) ill-defined at low-energies in QG foam models.

Wald (79)
Decoherence implies that asymptotic density matrix of low-energy matter:

$$\rho = \text{Tr} \left| \psi \right\rangle \left\langle \psi \right|$$

$$\left| i \right\rangle = N \left[ \left| M_0(\vec{k}) \right\rangle \left| \overline{M}_0(-\vec{k}) \right\rangle - \left| \overline{M}_0(\vec{k}) \right\rangle \left| M_0(-\vec{k}) \right\rangle \right]$$

$$+ \omega \left( \left| M_0(\vec{k}) \right\rangle \left| \overline{M}_0(-\vec{k}) \right\rangle + \left| \overline{M}_0(\vec{k}) \right\rangle \left| M_0(-\vec{k}) \right\rangle \right)$$

$$\omega = \left| \omega \right| e^{i\Omega}$$

May induce quantum decoherence of propagating matter and intrinsic CPT Violation in the sense that the CPT operator $\Theta$ is not well-defined $\rightarrow$ beyond Local Effective Field theory.

Bernabeu, NEM, Papavassiliou (04),…

Hence $\Theta$ ill-defined at low-energies in QG foam models $\rightarrow$ may affect EPR

Wald (79)
NB: Including conventional CPTV ($\theta$) in the Hamiltonian

$$H|B_H\rangle = \mu_H|B_H\rangle, \quad |B_H\rangle = p_H|B_d^0\rangle + q_H|\bar{B}_d^0\rangle,$$

$$H|B_L\rangle = \mu_L|B_L\rangle, \quad |B_L\rangle = p_L|B_d^0\rangle - q_L|\bar{B}_d^0\rangle.$$  

H (L) = (High (Low) mass states

$$\Psi_0 \propto |B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle$$

$$+ \omega \left\{ \theta [|B_H\rangle|B_L\rangle + |B_L\rangle|B_H\rangle] + (1 - \theta) \frac{p_L}{p_H} |B_H\rangle|B_H\rangle - (1 + \theta) \frac{p_H}{p_L} |B_L\rangle|B_L\rangle \right\}$$

$\omega$-effect

CPTV in Hamiltonian

$$\theta = \frac{H_{22} - H_{11}}{\mu_H - \mu_L}$$
Decoherence implies that asymptotic density matrix of low-energy matter:

$$\rho = \text{Tr} |\psi\rangle\langle\psi|$$

$$|i\rangle = N \left[ |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle - |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle + \omega \left( |M_0(\vec{k})\rangle |\overline{M}_0(-\vec{k})\rangle + |\overline{M}_0(\vec{k})\rangle |M_0(-\vec{k})\rangle \right]$$

May induce **quantum decoherence** of propagating matter and **intrinsic CPT Violation** in the sense that the CPT operator $\Theta$ is **not well-defined** → beyond Local Effective Field theory

Bernabeu, NEM, Papavassiliou (04),…

**Hence $\Theta$ ill-defined at low-energies in QG foam models → may affect EPR**
Decoherence implies that asymptotic density matrix of low-energy matter:

\[ \rho = \text{Tr} \left| \psi \right\rangle \langle \psi \left| \right. \]

\[ | i \rangle = \mathcal{N} \left[ | M_0(\vec{k}) \rangle | \overline{M}_0(-\vec{k}) \rangle - | \overline{M}_0(\vec{k}) \rangle | M_0(-\vec{k}) \rangle \right. \]
\[ + \omega \left( | M_0(\vec{k}) \rangle | \overline{M}_0(-\vec{k}) \rangle + | \overline{M}_0(\vec{k}) \rangle | M_0(-\vec{k}) \rangle \right) \]

\[ \omega = | \omega | e^{i\Omega} \]

May induce quantum decoherence of propagating matter and intrinsic CPT Violation in the sense that the CPT operator \( \Theta \) is not well-defined \( \Rightarrow \) beyond Local Effective Field theory

Thus \( \Theta \) ill-defined at low-energies in QG foam models \( \Rightarrow \) may affect EPR

May contaminate initially antisymmetric neutral meson \( M \) state by symmetric parts (\( \omega \)-effect)

\[ \text{NB: Decoherence & CPTV} \]

Wald (79)

Bernabeu, NEM, Papavassiliou (04),…
If CPT ill-defined → tiny effect (if due to Quantum Gravity decoherence) → concept of antiparticle still well-defined, but...

(i) observable effects in entangled (neutral) meson-states ω-effect
• Neutral mesons **no longer indistinguishable** particles, initial entangled state:

\[
|i> = \mathcal{N} \left( |K_S(\bar{k}), K_L(-\bar{k}) > - |K_L(\bar{k}), K_S(-\bar{k}) > \right) \\
+ \omega \left( |K_S(\bar{k}), K_S(-\bar{k}) > - |K_L(\bar{k}), K_L(-\bar{k}) > \right)
\]

\[\omega = |\omega| e^{i\Omega}\]

\[|\omega|^2 \sim \frac{\zeta^2 k^4}{M_{QG}^2 (m_1 - m_2)^2}, \Delta k = \zeta k \quad \text{(particle momentum transfer)}\]

If QCD effects, sub-structure in neutral mesons ignored, and D-foam acts as if they were structureless particles, then for \(M_{QG} \sim 10^{18} \text{ GeV}\) the estimate for \(\omega\): \(|\omega| \sim 10^{-4} |\zeta|\), for \(1 > |\zeta| > 10^{-2}\) (natural) Not far from sensitivity of upgraded meson factories (e.g. KLOE2)

If CPT ILL-DEFINED (e.g. flavour violating (FV) D-particle Foam)

\(\Phi K S K_L\)

\(K_S K_S, K_L K_L\)

\(K_S K_L\)
Neutral mesons no longer indistinguishable particles, initial entangled state:

\[ |i> = \mathcal{N} \left[ (|K_S(\bar{k}), K_L(-\bar{k}) > - |K_L(\bar{k}), K_S(-\bar{k}) >) + \omega (|K_S(\bar{k}), K_S(-\bar{k}) > - |K_L(\bar{k}), K_L(-\bar{k}) >) \right] \]

\[ \omega = |\omega| e^{i\Omega} \]

\[ |\omega|^2 \sim \frac{\zeta^2 k^4}{M_{QG}^2 (m_1 - m_2)^2}, \Delta k = \zeta k (\text{particle momentum transfer}) \]

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\[ |\omega| \sim 10^{-4} |\zeta|, \text{ for } 1 > |\zeta| > 10^{-2} \text{ (natural)} \]
Not far from sensitivity of upgraded meson factories (e.g. KLOE2)

In a concrete model of space-time foam:

If CPT ILL-DEFINED (e.g. flavour violating (FV) D-particle Foam)
Part III(a)  
ω-effect searches  
in  
Entangled Neutral Meson Systems
Current Measurement Status of $\omega$-effect

Neutral Kaons

Prospects KLOE-2  $\text{Re}(\omega), \text{Im}(\omega) \to 2 \times 10^{-5}$

KLOE result: PLB 642(2006) 315

\[ \Re \omega = \left( -1.6^{+3.0}_{-2.1}^{\text{STAT}} \pm 0.4_{\text{SYST}} \right) \times 10^{-4} \]
\[ \Im \omega = \left( -1.7^{+3.3}_{-3.0}^{\text{STAT}} \pm 1.2_{\text{SYST}} \right) \times 10^{-4} \]
\[ |\omega| < 1.0 \times 10^{-3} \text{ at } 95\% \text{ C.L.} \]
Consider the $\phi$ decay amplitude: final state $X$ at $t_1$ and $Y$ at time $t_2$ ($t = 0$ at the moment of $\phi$ decay)

The "intensity" $I(\Delta t)$: ($\Delta t = t_1 - t_2$) is an observable

$$I(\Delta t) \equiv \frac{1}{2} \int_{|\Delta t|}^{\infty} dt |A(X,Y)|^2$$

if $\omega$-effect present
\[ I(\Delta t) \equiv \frac{1}{2} \int_{|\Delta t|}^{\infty} dt \left| A(\pi^+\pi^-, \pi^+\pi^-) \right|^2 = \left| \langle \pi^+\pi^- | K_S \rangle \right|^4 |\mathcal{N}|^2 |\eta_{+-}|^2 \left[ I_1 + I_2 + I_{12} \right] \]

\[ I_1(\Delta t) = \frac{e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} - 2e^{-(\Gamma_S+\Gamma_L)\Delta t/2} \cos(\Delta M \Delta t)}{\Gamma_L + \Gamma_S} \]

\[ I_2(\Delta t) = \frac{\frac{\omega}{|\eta_{+-}|^2}}{2\Gamma_S} e^{-\Gamma_S \Delta t} \]

\[ I_{12}(\Delta t) = -\frac{4}{4(\Delta M)^2 + (3\Gamma_S + \Gamma_L)^2} \frac{|\omega|}{|\eta_{+-}|} \times \]

\[ 2\Delta M \left( e^{-\Gamma_S \Delta t} \sin(\phi_{+-} - \Omega) - e^{-(\Gamma_S+\Gamma_L)\Delta t/2} \sin(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \]

\[-(3\Gamma_S + \Gamma_L) \left( e^{-\Gamma_S \Delta t} \cos(\phi_{+-} - \Omega) - e^{-(\Gamma_S+\Gamma_L)\Delta t/2} \cos(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \]

\[ \Delta M = M_S - M_L \text{ and } \eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}}. \]

NB: sensitivities up to \(|\omega| \sim 10^{-6}\) in \(\phi\) factories, due to enhancement by \(|\eta_{+-}| \sim 10^{-3}\) factor.
\[ I(\Delta t) \equiv \frac{1}{2} \int_{|\Delta t|}^{\infty} dt \left| A(\pi^+\pi^-, \pi^+\pi^-) \right|^2 = \left| \langle \pi^+\pi^- | K_S \rangle \right|^4 |\mathcal{N}|^2 |\eta_{+-}|^2 \left[ I_1 + I_2 + I_{12} \right] \]

\[ I_1(\Delta t) = \frac{e^{-\Gamma_S \Delta t} + e^{-\Gamma_L \Delta t} - 2e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \cos(\Delta M \Delta t)}{\Gamma_L + \Gamma_S} \]

\[ I_2(\Delta t) = \frac{|\omega|^2}{|\eta_{+-}|^2} \frac{e^{-\Gamma_S \Delta t}}{2\Gamma_S} \]

\[ I_{12}(\Delta t) = -\frac{4}{4(\Delta M)^2 + (3\Gamma_S + \Gamma_L)^2} \left( |\omega| \frac{|\omega|}{|\eta_{+-}|} \times \right) \]

\[ \left[ 2\Delta M \left( e^{-\Gamma_S \Delta t} \sin(\phi_{+-} - \Omega) - e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \sin(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \right. \]

\[ \left. - (3\Gamma_S + \Gamma_L) \left( e^{-\Gamma_S \Delta t} \cos(\phi_{+-} - \Omega) - e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \cos(\phi_{+-} - \Omega + \Delta M \Delta t) \right) \right] \]

\( \Delta M = M_S - M_L \) and \( \eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}}. \)

NB: sensitivities up to \( |\omega| \sim 10^{-6} \) in \( \phi \) factories, due to enhancement by \( |\eta_{+-}| \sim 10^{-3} \) factor.
Current Measurement Status of $\omega$-effect

Neutral B-mesons

Equal Sign Dilepton Asymmetry

($\text{Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087}$)

$$-0.0084 \leq Re(\omega) \leq 0.0100 \quad 95\% \text{C.L}$$
Current Measurement Status of $\omega$-effect

$A_{sl} = \frac{I(\ell^+, \ell^+, \Delta t) - I(\ell^-, \ell^-, \Delta t)}{I(\ell^+, \ell^+, \Delta t) + I(\ell^-, \ell^-, \Delta t)} \bigg|_{\omega=0} = 4 \frac{\Re(\varepsilon)}{1 + |\varepsilon|^2} + \mathcal{O}((\Re\varepsilon)^2)$

$\omega = |\omega|e^{i\Omega} \neq 0$

$\Delta t_{peak} = \frac{1}{\Gamma}1.12|\omega|$

$A_{sl}(\Delta t_{peak}) = 0.77 \cos(\Omega)$

Equal Sign Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

$-0.0084 \leq \Re(\omega) \leq 0.0100 \quad 95\% \text{C.L}$

Neutral B-mesons
Current Measurement Status of $\omega$-effect

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$$\omega = |\omega|e^{i\Omega} \neq 0$$

$$\Delta m \Delta t \approx 2\pi \ (\Delta t \approx 8.2\Gamma^{-1})$$

Equal Sign Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

$$-0.0084 \leq \text{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L}$$

Neutral B-mesons
Current Measurement Status of $\omega$-effect

\[ I(f, g; t) = \int_0^\infty dt_0 \left| \langle f, t_0; g, t + t_0 | T | \Psi_0 \rangle \right|^2 = \frac{\langle \Gamma_f \rangle \langle \Gamma_g \rangle}{\Gamma} e^{-\Gamma t} \left\{ \mathcal{C}_h^{\omega}[f, g] + \mathcal{C}_c^{\omega}[f, g] \cos(\Delta M t) + \mathcal{J}_c^{\omega}[f, g] \sin(\Delta M t) \right\} \]

Observables

\[ C[f, g] = \frac{\mathcal{C}_c^{\omega}[f, g]}{\mathcal{C}_h^{\omega}[f, g]} \quad \text{and} \quad S[f, g] = \frac{\mathcal{J}_c^{\omega}[f, g]}{\mathcal{C}_h^{\omega}[f, g]} \]

\[ C[\ell^\pm, g] - C[g, \ell^\pm] = \frac{1}{1 + (x/2)^2} \left\{ [x S_g + 2(C_g^2 - 1) - x C_g S_g] \text{Re}(\omega) + x R_g [C_g \pm 1] \text{Im}(\omega) \right\} \]

\[ x = \frac{\Delta M}{\Gamma} \simeq 0.77 \]

Equal Sign Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

$-0.0084 \leq \text{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L}$

Novel signal from $(f,g) \leftrightarrow (g,f)$
(Bernabeu, Botella, NEM, Nebot (2018))

<table>
<thead>
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<th>Im($\theta$)</th>
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Current Measurement Status of $\omega$-effect

\[ I(f, g; t) = \int_0^\infty dt_0 \langle f, t_0; g, t + t_0|T|\Psi_0\rangle^2 = \frac{\langle \Gamma_f \rangle \langle \Gamma_g \rangle}{\Gamma} e^{-\Gamma t} \left\{ C_h[f, g] + C_c[f, g] \cos(\Delta M t) + \mathcal{J}_c[f, g] \sin(\Delta M t) \right\} \]

\[ C[f, g] = \frac{C_c[f, g]}{C_h[f, g]} \quad \text{and} \quad S[f, g] = \frac{\mathcal{J}_c[f, g]}{C_h[f, g]}, \]

\[ S[\ell^\pm, g] + S[g, \ell^\pm] = \frac{1}{1 + (x/2)^2} \left\{ x C_g + x (1 - S_g^2) + 2 C_g S_g \right\} \operatorname{Re}(\omega) \]

\[ + R_g [x S_g + 2] \operatorname{Im}(\omega) \]

\[ x = \frac{\Delta M}{\Gamma} \simeq 0.77 \]

**Observables**

**Equal Sign** Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

\[ -0.0084 \leq \operatorname{Re}(\omega) \leq 0.0100 \quad 95\% \text{C.L.} \]

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Current Measurement Status of $\omega$-effect

Neutral B-mesons

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Correlations among $\omega$ & $\theta$

(a) $\text{Re}(\omega)$ vs. $\text{Re}(\theta)$.

(b) $\text{Re}(\omega)$ vs. $\text{Im}(\theta)$.

(c) $\text{Im}(\omega)$ vs. $\text{Re}(\theta)$.

(d) $\text{Im}(\omega)$ vs. $\text{Im}(\theta)$.
**Current Measurement Status of ω-effect**

Neutral Kaons

Prospects KLOE-2 Re(ω), Im(ω) $\rightarrow 2 \times 10^{-5}$

**KLOE result:**

- $\text{Re}(\omega) = \left(-1.6^{+3.0}_{-2.1}^{\text{STAT}} \pm 0.4^{\text{Syst}}\right) \times 10^{-4}$
- $\text{Im}(\omega) = \left(-1.7^{+3.3}_{-3.0}^{\text{STAT}} \pm 1.2^{\text{Syst}}\right) \times 10^{-4}$
- $|\omega| < 1.0 \times 10^{-3}$ at 95% C.L.

**Prospects**

Equal Sign Dilepton Asymmetry

(Alvarez, Bernabeu, Nebot, JHEP 0611 (2006) 087)

- Novel signal from $(f,g) \leftrightarrow (g,f)$
  (Bernabeu, Botella, NEM, Nebot EPJC 77 (2017) 865)

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Part III(b)
Connection to Baryogenesis
Connection to Baryogenesis
Effective metric due to LV foam medium

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu , \quad h_{0i} = (u_{i\parallel}^a \sigma_a) \]

Stochastic media

\[ \ll u_{i\parallel} \gg = 0 , \quad \ll u_{i\parallel} u_{j\parallel} \gg = \sigma^2 \delta_{ij} . \]
Connection to Baryogenesis

Effective metric due to LV foam medium

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu , \quad h_{0i} = (u_0^a \parallel \sigma_a) \]

Stochastic media

\[ \ll u_i \parallel \gg = 0 , \quad \ll u_i \parallel u_j \gg = \sigma^2 \delta_{ij} \]

Energy differences between matter-antimatter

\[ \ll E \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

\[ \ll \bar{E} \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} + \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

situation equivalent to an induced chemical potential

\[ \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \equiv \mu_{\text{CPTV}} > 0 \]
Connection to Baryogenesis

Effective metric due to LV foam medium

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu , \quad h_{0i} = (u_i^a \sigma_a) \]

Stochastic media

\[ \ll u_i \ll = 0 , \quad \ll u_i \parallel u_j \ll = \sigma^2 \delta_{ij} . \]

Energy differences between matter-antimatter

\[ \ll E \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} - \frac{1}{2 g_s} M_s \sigma^2 \]
\[ \ll \overline{E} \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} + \frac{1}{2 g_s} M_s \sigma^2 \]

Baryon asymmetry in Universe

\[ \Delta n_q = n - \overline{n} = g_{d.o.f} \int \frac{d^3 p}{(2\pi)^3} [f(E, \mu) - f(\overline{E}, \overline{\mu})] \]
\[ \sim \frac{T^2}{2} \left( \frac{M_s \sigma^2}{g_s} \right) \]
Connection to Baryogenesis

Effective metric due to LV foam medium

\[ ds^2 = g_{\mu \nu} dx^\mu dx^\nu = (\eta_{\mu \nu} + h_{\mu \nu}) dx^\mu dx^\nu, \quad h_{0i} = (u_i^a \sigma_a) \]

Stochastic media

\[ \ll u_i \gg = 0, \quad \ll u_i \parallel u_j \gg = \sigma^2 \delta_{ij}. \]

Energy differences between matter-antimatter

\[ \ll E \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} - \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]
\[ \ll \bar{E} \gg = \sqrt{p^2 + m^2} + \frac{p^2 \sigma^2}{2 \sqrt{p^2 + m^2}} + \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

Baryon asymmetry in Universe @ $T_D$

\[ s \sim \frac{2\pi^2}{45} g_*(T) T^3 \]

\[ \Delta n(T < T_D) = \frac{\Delta n_q}{s} \sim \frac{M_s}{g_s} \frac{45 \sigma^2}{g_*(T_D) T_D} \]
Construct microscopic models with strong CPT Violation in Early Universe (due to background fields or quantum gravity), but weak today… Fit with all available data… in particular current stringent constraints \( \Rightarrow \) scale back in time Estimate in this way matter-antimatter asymmetry in Universe Does it agree with the expected phenomenological value?
Evolution of $\sigma^2$ with the cosmic temperature $T$ can lead to interesting phenomenology on Baryogenesis models through such QG effects by tests of QGD CPTV $\omega$-effect in current era:

Scaling with Temperature $T$

$$k \sim T$$

$$\sigma^2 = g_s^2 \frac{T^2}{M_s^2} \Delta$$

$$\langle r^2 \rangle = \Delta$$

$$\langle r \rangle = 0$$

T-dependence of $\Delta$ depends on details of QG model
Evolution of $\sigma^2$ with the cosmic temperature $T$ can lead to interesting phenomenology on Baryogenesis models through such QG effects by tests of QGD CPTV $\omega$-effect in current era:

**Scaling with Temperature $T$**

$$k \sim T$$

$$\sigma^2 = g_s^2 \Delta \frac{T^2}{M_s^2} \Delta$$

$$\langle r^2 \rangle = \Delta$$

$$\langle r \rangle = 0$$

**D-particles in brane models if dark-matter like:**

$$\Delta \propto n_{d-particles-density} \sim T^3$$
Neutral Kaons in $\Phi$-factories

$$\text{Re}(\omega) = (-1.6^{+3.0}_{-2.1} \text{stat} \pm 0.4 \text{syst}) \times 10^{-4}$$

$$\text{Im}(\omega) = (-1.7^{+3.3}_{-3.0} \text{stat} \pm 1.2 \text{syst}) \times 10^{-4}$$

$$|m_1 - m_2| \sim 10^{-15} \text{ GeV} \quad k \sim 1 \text{ GeV}$$

$$|\omega|^2 \sim \frac{k^2}{|m_1 - m_2|^2} \Delta$$

$$\langle r^2 \rangle = \Delta \quad \langle r \rangle = 0$$

$$0 < \Delta < 1$$

$\Delta_{\text{today}} < 10^{-38}$
Neutral Kaons in $\Phi$-factories

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D-particle induced Baryogenesis

Scaling with Temperature $T$ in D-particle foam model

$D$-particles = dark-matter like

$\Delta \propto n_{d-particles-density} \sim T^3$

e.g. for quarks $T_D \approx 100 \text{ GeV}$

$\Delta n(T = 100 \text{ GeV}) \sim 10^{-10}$

$$\frac{\Delta n(T < T_D)}{s} \sim \frac{M_s}{g_s} \frac{45 \sigma^2}{g_*(T_D) T_D}$$

$$\sigma^2 = g_s^2 \frac{T^2}{M_s^2} \Delta$$

$$\Delta n \sim \frac{g_s}{M_s} T \Delta$$

$$\Delta_{100 \text{ GeV}} \sim \frac{M_s}{g_s} \frac{1}{\text{GeV}} \times 10^{26}$$

$$\sim \left( \frac{T = 100 \text{ GeV}}{T_{\text{CMB}}^0 = 0.2 \text{ meV}} \right)^3 \sim 10^{44}$$

$$\frac{M_s}{g_s} \sim 10^{18} \text{ GeV}$$

Consistent string model!
Neutral Kaons in Φ-factories

\[ \text{Re}(ω) = (-1.6^{+3.0}_{-2.1 \text{ stat}} \pm 0.4_{\text{syst}}) \times 10^{-4} \]
\[ \text{Im}(ω) = (-1.7^{+3.3}_{-3.0 \text{ stat}} \pm 1.2_{\text{syst}}) \times 10^{-4} \]

\[ |m_1 - m_2| \sim 10^{-15} \text{ GeV} \quad k \sim 1 \text{ GeV} \]

[\text{Neutral Kaons in Φ-factories}]

\[ |ω|^2 \sim \frac{k^2}{|m_1 - m_2|^2} \Delta \]

\[ \langle r^2 \rangle \sim T^3 \]

Scaling with Temperature T in D-particle foam model

D-particles = dark-matter in Consistent string model

\[ \Delta \text{today} < 10^{-38} \]

\[ \sigma \Delta \Delta \Delta \]

\[ \Delta \sim \frac{g_s}{M_s} \frac{45 \sigma^2}{g_s(T_D) T_D} \]

[\text{D-particle induced Baryogenesis}]

\[ \Delta n(T < T_D) = \Delta \]

\[ \Delta n(T = 100 \text{ GeV}) \sim 10^{-10} \]

\[ \frac{M_{s}}{g_{s}} \sim 10^{18} \text{ GeV} \]

[\text{Consistent string model}]

\[ \frac{\Delta 100 \text{ GeV}}{\Delta \text{today}} \sim \frac{M_s}{g_s} \frac{1}{\text{GeV}} \times 10^{26} \]

\[ \sim \left( \frac{T = 100 \text{ GeV}}{T_{CMB}^0 = 0.2 \text{ meV}} \right)^3 \sim 10^{44} \]

[\text{NB: RHN leptogenesis also consistent}]
V. CONCLUSIONS

- CPT Violation (CPTV) due to (strong) quantum fluctuations in space-time at early eras or LV early Universe Geometries (with background flux fields) is a possibility and may explain the observed matter-antimatter asymmetry in the cosmos, consistently with current-era bounds of CPTV.

- One framework for early universe CPTV: Standard Model Extension (SME).

- A string-inspired model of the Early Universe entailing CPT and Lorentz Violation due to Kalb-Ramond-axion-modified background geometries – Consistent phenomenology in current era.
  
  ...to explore further, in connection with Early Universe Cosmology – CMB polarization etc.

- Quantum Gravity Flcts in early Universe might be strong → Decoherence of quantum matter
  
  CPT operator in effective theory ill-defined
  
  → affects EPR entangled states → current bounds from meson factories → explore link with early Universe in concrete models of space-time foam from string/brane theory.
CPT Violation (CPTV) due to (strong) quantum fluctuations in space-time at early eras or early Universe Geometries (with background flux fields) is a possibility and may explain the observed matter-antimatter asymmetry in the cosmos, consistently with current-era bounds of CPTV.

One framework for early universe CPTV: Standard Model Extension (SME)

A string-inspired model of the Early Universe entailing CPT and Lorentz Violation due to Kalb-Ramond axion-modified background geometries — Consistent phenomenology in current era...

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Quantum Gravity Fluctuations in early Universe might be strong → Decoherence of quantum matter CPT operator in effective theory ill-defined → affects EPR entangled states → current bounds from meson factories → explore link with early Universe in concrete models of space-time foam from string/brane theory.
SPARES
In string theory a constant $B^0$ background is guaranteed by exact conformal Field theory with linear in FRW time $b = (\text{const} \ t)$

Antoniadis, Bachas, Ellis, Nanopoulos

**Strings in Cosmological backgrounds**

$$ ds^2 = g^E_{\mu \nu}(x) dx^\mu dx^\nu = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j $$

$$ a(t) = t $$

$$ \Phi = -\ln a(t) + \phi_0 $$

$$ H_{\mu \nu \rho} = e^{2\Phi} \epsilon_{\mu \nu \rho \sigma} \partial^\sigma b(x) $$

$$ b(x) = \sqrt{2} e^{-\phi_0} \sqrt{Q^2} \frac{M_s}{\sqrt{n}} t $$

Central charge of underlying world-sheet conformal field theory $n \in \mathbb{Z}^+$

``internal'' dims

central charge

Kac-Moody algebra level
Proper Treatment through solving Boltzmann Eqs.

Boltzmann equation in presence of CPTV & LV Background $B_0$

RHN Helicity specific $\lambda_r$:

$$\frac{dn_r}{dt} + 3Hn_r - \frac{g}{2\pi^2} 2\lambda_r \frac{B_0}{T} T^3 \int du u f(E(B_0 = 0), u)$$

$$= \frac{g}{8\pi^3} \int \frac{d^3 p}{E(B_0 \neq 0)} C[f] + \mathcal{O}(B_0^2)$$

Summing over RHN Helicities $\Sigma_r \lambda_r = 0$ (for small $B_0/T \ll 1$):

$$\frac{dn_N}{dt} + 3Hn_N = \frac{g}{8\pi^3} \int \frac{d^3 p}{E} \tilde{C}[f] + \mathcal{O}(B_0^2)$$
Proper Treatment through solving Boltzmann Eqs.

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RHN Helicity specific $\lambda_r$ :

$$\frac{dn_r}{dt} + 3Hn_r - \frac{g}{2\pi^2} 2\lambda_r \frac{B_0}{T} T^3 \int du \, u \, f(E(B_0 = 0), u)$$

$$= \frac{g}{8\pi^3} \int \frac{d^3p}{E(B_0 \neq 0)} C[f] + O(B_0^2)$$

Summing over RHN Helicities $\sum_r \lambda_r = 0$ (for small $B_0/T \ll 1$) :

$$\frac{dn_N}{dt} + 3Hn_N = \frac{g}{8\pi^3} \int \frac{d^3p}{E} \tilde{C}[f] + O(B_0^2)$$
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T. Bossingham, N.E.M., Sarkar

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$$\frac{dn_N}{dt} + 3 H n_N = \frac{g}{8 \pi^3} \int \frac{d^3 p}{E} \tilde{C}[f] + \mathcal{O}(B_0^2)$$

But still modified due to $B_0$–Dependence of Energy-Momentum dispersion $E(p, B_0)$

$$E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}| B_0}$$

$$\tilde{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}| B_0}$$
Proper Treatment through solving Boltzmann Eqs.

T. Bossingham, N.E.M., Sarkar

Boltzmann equation in presence of CPTV & LV Background $B_0$

RHN Helicity specific $\lambda_r$:

\[
\frac{dn_r}{dt} + 3H n_r = \frac{g}{2\pi^2} 2\lambda_r \frac{B_0}{T} T^3 \int du \ u \ f(E(B_0 = 0), u)
\]

\[
= \frac{g}{8\pi^3} \int \frac{d^3p}{E(E(B_0 \neq 0))} C[f] + \mathcal{O}(B_0^2)
\]

Summing over RHN Helicities $\sum_r \lambda_r = 0$ (for small $B_0/T << 1$):

\[
\frac{dn_N}{dt} + 3H n_N = \frac{g}{8\pi^3} \int \frac{d^3p}{E} \tilde{C}[f] + \mathcal{O}(B_0^2)
\]

\[
E = \sqrt{\vec{p}^2 + m^2 + B_0^2 + 2|\vec{p}|B_0}
\]

\[
\overline{E} = \sqrt{\vec{p}^2 + m^2 + B_0^2 - 2|\vec{p}|B_0}
\]

But still modified due to $B_0$—Dependence of Energy-Momentum dispersion $E(p, B_0)$
Proper Treatment through solving Boltzmann Eqs.

\[ Y_x = \frac{n_x}{s} \]

\[ z = \frac{m_N}{T} \]

\( Y^{eq} \): Thermal Equilibrium Distributions (at high T)

Standard Cosmology at early eras (radiation era)

\[ T \sim a^{-1} \Rightarrow s \sim T^3, \quad a \sim t^{1/2} \]

Hubble rate

\[ H \sim T^2 / 2 = \frac{m_N^2}{2z^2}. \]

Averaged (over helicities) heavy (right-handed) neutrino abundance

\[ \bar{Y}_N = \frac{Y_{N^(-)} + Y_{N^+}}{2} \]

Lepton Asymmetry:

\[ \mathcal{L} = \bar{Y}_{l^-} - \bar{Y}_{l^+} \]

\[ \bar{Y}_l = \frac{Y_{l^(-)} + Y_{l^+}}{2} = \frac{Y_{N^(-)} + Y_{N^+}}{2} = \bar{Y}_N \]
Proper Treatment through solving Boltzmann Eqs.

\[ Y_x = n_x / s \]

\[ z = m_N / T \]

\( Y^{eq} \): Thermal Equilibrium Distributions (at high T)

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Averaged (over helicities) heavy (right-handed) neutrino abundance

\[ \bar{Y}_N \equiv \frac{Y_N^{(-)} + Y_N^{(+)}}{2}, \]

Lepton Asymmetry:

\[ \mathcal{L} = \bar{Y}_l^- - \bar{Y}_l^+ \]

\[ \Delta L^{TOT} / s = Y_l^{(-)} / Y_l^{(-)} + Y_l^{(+)} = \mathcal{L} / 2\bar{Y}_N \]
Heavy (right-handed) neutrino abundance

\[
\frac{dY_N^{(\lambda)}}{dz} - \lambda I = - \left\{ \gamma_{eq,(\lambda)}^{(\lambda)}(N \rightarrow l^-h^+) \frac{Y_N^{(\lambda)}}{Y_{N_{eq}}^{(\lambda)}} - \gamma_{eq,(\lambda)}^{(\lambda)}(l^-h^+ \rightarrow N) \frac{Y_{l^-}^{(\lambda)}}{Y_{l^-_{eq}}^{(\lambda)}} \frac{Y_{h^+}}{Y_{h^+_{eq}}} \right\} \\
+ \left\{ \gamma_{eq,(\lambda)}^{(\lambda)}(N \rightarrow l^+h^-) \frac{Y_N^{(\lambda)}}{Y_{N_{eq}}^{(\lambda)}} - \gamma_{eq,(\lambda)}^{(\lambda)}(l^+h^- \rightarrow N) \frac{Y_{l^+}^{(\lambda)}}{Y_{l^+_{eq}}^{(\lambda)}} \frac{Y_{h^-}}{Y_{h^-_{eq}}} \right\} \\
+ \left\{ \gamma_{eq,(\lambda)}^{(\lambda)}(N \rightarrow \nu h^0) \frac{Y_N^{(\lambda)}}{Y_{N_{eq}}^{(\lambda)}} - \gamma_{eq,(\lambda)}^{(\lambda)}(\nu h^0 \rightarrow N) \frac{Y_{\nu}^{(\lambda)}}{Y_{\nu_{eq}}^{(\lambda)}} \frac{Y_{h^0}}{Y_{h^0_{eq}}} \right\} \\
+ \left\{ \gamma_{eq,(\lambda)}^{(\lambda)}(N \rightarrow \bar{\nu} h^0) \frac{Y_N^{(\lambda)}}{Y_{N_{eq}}^{(\lambda)}} - \gamma_{eq,(\lambda)}^{(\lambda)}(\bar{\nu} h^0 \rightarrow N) \frac{Y_{\bar{\nu}}^{(\lambda)}}{Y_{\bar{\nu}_{eq}}^{(\lambda)}} \frac{Y_{h^0}}{Y_{h^0_{eq}}} \right\} \right\}.
\]
Proper Treatment through solving Boltzmann Eqs.

Heavy (right-handed) neutrino abundance

\[ zH_s \frac{dY_N^{(\lambda)}}{dz} - \lambda I = - \left\{ \gamma_{eq,(-)}^{(\lambda)} (N \leftrightarrow l^--h^+) \frac{Y_N^{(\lambda)}}{Y_{eq}^{(\lambda)}} - \gamma_{eq,(-)}^{(\lambda)} (l^--h^+ \rightarrow N) \frac{Y_{l^-}^{(\lambda)}}{Y_{eq}^{(\lambda)}} \right\} \]

\[ \gamma_{eq,(-)}^{(\lambda)} (N \leftrightarrow l^--h^+) = \gamma_{eq,(-)}^{(\lambda)} (N \leftrightarrow \nu h^0) \]

\[ \gamma_{eq,(+)}^{(\lambda)} (N \leftrightarrow l^+h^-) = \gamma_{eq,(+)}^{(\lambda)} (N \leftrightarrow \bar{\nu} h^0) \]

\[ + \left\{ \gamma_{eq,(\lambda)}^{(\lambda)} (N \rightarrow \bar{\nu} h^0) \frac{Y_N^{(\lambda)}}{Y_{eq}^{(\lambda)}} - \gamma_{eq,(\lambda)}^{(\lambda)} (\bar{\nu} h^0 \rightarrow N) \frac{Y_{\bar{\nu}}^{(\lambda)}}{Y_{eq}^{(\lambda)}} \right\} \]
Proper Treatment through solving Boltzmann Eqs.

Heavy (right-handed) neutrino abundance

\[
\frac{dY_{N}^{(\lambda)}}{dz} - \lambda I = -\left\{ \left[ \gamma^{eq,(\lambda)}_{e}(N \rightarrow l^{-}h^{+}) \frac{Y_{N}^{(\lambda)}}{Y_{N}^{(\lambda),eq}} - \gamma^{eq,(\lambda)}_{\nu}(l^{-}h^{+} \rightarrow N) \frac{Y_{l^{-}}^{(\lambda)}}{Y_{l^{-}}^{(\lambda),eq}} \frac{Y_{h^{+}}^{eq}}{Y_{h^{+}}^{eq}} \right] \left[ \gamma^{eq,(\lambda)}_{e}(N \leftrightarrow l^{-}h^{+}) \frac{Y_{h^{-}}^{eq}}{Y_{h^{-}}^{eq}} - \gamma^{eq,(\lambda)}_{\nu}(\nu h^{0} \rightarrow N) \frac{Y_{\nu}^{(\lambda)}}{Y_{\nu}^{(\lambda),eq}} \frac{Y_{h^{0}}^{eq}}{Y_{h^{0}}^{eq}} \right]
\right. \\
+ \left. \left[ \gamma^{eq,(\lambda)}_{e}(N \rightarrow \bar{\nu}h^{0}) \frac{Y_{N}^{(\lambda)}}{Y_{N}^{(\lambda),eq}} - \gamma^{eq,(\lambda)}_{\nu}(\nu h^{0} \rightarrow N) \frac{Y_{\nu}^{(\lambda)}}{Y_{\nu}^{(\lambda),eq}} \frac{Y_{h^{0}}^{eq}}{Y_{h^{0}}^{eq}} \right] \right\}.
\]

Lepton Asymmetry:

\[
2zHS \frac{dL}{dz} + 4I = -2\left[ \gamma^{eq,(\lambda)}_{e}(N \leftrightarrow l^{-}h^{+}) \left\{ \frac{Y_{l^{-}}^{(-)}}{Y_{l^{-}}^{(-),eq}} - \frac{Y_{N}^{(-)}}{Y_{N}^{(-),eq}} \right\}
\right.
\\
- \left. \gamma^{eq,(\lambda)}_{e}(N \leftrightarrow l^{+}h^{-}) \left\{ \frac{Y_{l^{+}}^{(+)}}{Y_{l^{+}}^{(+),eq}} - \frac{Y_{N}^{(+)}}{Y_{N}^{(+),eq}} \right\} \right].
\]
\[
\frac{\Delta L^{TOT,\text{complete}}}{s} \sim (0.016 - 0.019) \frac{B_0}{m_N}
\]

at freezeout temperature

\[T = T_D : \quad m_N/T_D \sim (1.44 - 1.77)\]
\[ \frac{\Delta L^{TOT,\text{complete}}}{s} \sim (0.016 - 0.019) \frac{B_0}{m_N} \]

at freezeout temperature

\[ T = T_D : \quad m_N / T_D \sim (1.44 - 1.77) \]

\[ \Gamma \sim H = 1, 66 T^2 N^{1/2} m_p^{-1} \]

\[ T_D \sim 6.2 \cdot 10^{-2} \frac{|Y|}{N^{1/4}} \sqrt{\frac{m_p (\Omega^2 + B_0^2)}{\Omega}} \]

\[ \Omega = \sqrt{B_0^2 + m_N^2} \cdot Y_k \sim 10^{-5} \]
\[
\frac{\Delta L^{TOT,\text{complete}}}{s} \approx (0.016 - 0.019) \frac{B_0}{m_N}
\]

at freezeout temperature

\[T = T_D: \quad m_N / T_D \approx (1.44 - 1.77)\]

\[
\frac{\Delta L^{TOT,\text{complete}}}{s} \approx \mathcal{O}(8 \times 10^{-11})
\]
\[
\frac{\Delta L^{TOT,\text{complete}}}{s} \sim (0.016 - 0.019) \frac{B_0}{m_N}
\]

at freezeout temperature

\[T = T_D : \quad m_N/T_D \sim (1.44 - 1.77)\]

\[
\frac{\Delta L^{TOT,\text{complete}}}{s} \sim \mathcal{O}(8 \times 10^{-11})
\]

\[
\frac{B_0}{m_N} \sim 5.0 \times 10^{-9} - 4.2 \times 10^{-9},
\]

at freezeout temperature
Proof of III-defined CPT in decoherent situations

A THEOREM BY R. WALD (1979): If $S \neq SS^\dagger$, then CPT is violated, at least in its strong form.

PROOF: Suppose CPT is conserved, then there exists unitary, invertible operator $\Theta$ acting on density matrices $\rho = \text{Tr} |\psi><\psi|$

$\rho_{out} = S\rho_{in} \rightarrow \Theta \rho_{in} = S \Theta^{-1} \rho_{out} \rightarrow \rho_{in} = \Theta^{-1} S \Theta^{-1} \rho_{out}$.

But $\rho_{out} = S\rho_{in}$, hence: $\rho_{in} = \Theta^{-1} S \Theta^{-1} S \rho_{in}$

BUT THIS IMPLIES THAT $S$ HAS AN INVERSE- $\Theta^{-1} S \Theta^{-1}$, IMPOSSIBLE (information loss), hence CPT MUST BE VIOLATED (at least in its strong form).

NB1: IT ALSO IMPLIES: $\Theta = S \Theta^{-1} S$ (fundamental relation for a full CPT invariance).

NB2: My preferred way of CPTV by Quantum Gravity Introduces fundamental arrow of time/microscopic time irreversibility...

NB3: Effective theories decoherence, i.e. (low-energy ) experimenters do not have access to all d.o.f. of quantum gravity (e.g. back-reaction effects...)
Proof of ill-defined CPT in decoherent situations

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PROOF: Suppose CPT is conserved, then there exists unitary, invertible operator $\Theta$

acting on density matrices $\rho = \text{Tr} |\psi\rangle\langle\psi|$

$\Theta \rho_{\text{in}} = \rho_{\text{out}}$

$\rho_{\text{out}} = S \rho_{\text{in}} \rightarrow \Theta \rho_{\text{in}} = \Theta^{-1} \rho_{\text{out}} \rightarrow \rho_{\text{in}} = \Theta^{-1} S \Theta^{-1} \rho_{\text{out}}$

But $\rho_{\text{out}} = \rho_{\text{in}}$, hence $\rho_{\text{in}} = \Theta^{-1} S \Theta^{-1} \rho_{\text{in}}$

BUT THIS IMPLIES THAT $S$ HAS AN INVERSE $\Theta^{-1} S \Theta^{-1}$,

(information loss), hence CPT MUST BE VIOLATED (at least in its strong form)

NB: CPT is antiunitary (due to T) when acting on $|\psi\rangle$

but unitary when acting on $|\psi\rangle\langle\psi|$

NB1: IT ALSO IMPLIES: $\Theta = S \Theta^{-1} S$ (fundamental relation for a full CPT invariance).

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PROOF: Suppose CPT is conserved, then there exists unitary, invertible operator $\Theta$ acting on density matrices $\rho = \text{Tr} |\psi><\psi|$

$$\rho_{out} = S \rho_{in} \rightarrow \Theta \rho_{in} = S \Theta^{-1} \rho_{out} \rightarrow \rho_{in} = \Theta^{-1}$$. $$\Theta^{-1} \rho_{out}$$.

But $\bar{\rho}_{out} = S \bar{\rho}_{in}$, hence $\bar{\rho}_{in} = \Theta^{-1} \Theta^{-1} S \bar{\rho}_{in}$.

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NB3: Effective theories decoherence, i.e. (low-energy ) experimenters do not have access to all d.o.f. of quantum gravity (e.g. back-reaction effects...)
But....nature may be tricky: **WEAK FORM OF CPT INVARINCE** might exist, such that the fundamental “arrow of time” does not show up in any experimental measurements (scattering experiments).

Probabilities for transition from $\psi =$ initial pure state to $\phi =$ final state

$$P(\psi \rightarrow \phi) = P(\theta^{-1}\phi \rightarrow \theta\psi)$$

where $\theta: \mathcal{H}_{in} \rightarrow \mathcal{H}_{out}$, $\mathcal{H} =$ Hilbert state space,

$\Theta \rho = \theta \rho \theta^\dagger$, $\theta^\dagger = -\theta^{-1}$ (anti-unitary).

In terms of superscattering matrix $\$: $\$^\dagger = \Theta^{-1}\$\Theta^{-1}$

Here, $\Theta$ is well defined on pure states, but $\$ has no inverse, hence $\$^\dagger \neq \$^{-1}$ (full CPT invariance: $\$ = SS^\dagger$, $\$^\dagger = \$^{-1}$).
CPT symmetry without CPT invariance?

But....nature may be tricky: WEAK FORM OF CPT INVARIANCE might exist such that the fundamental "symmetry" of time measurement proceeds.

Supporting evidence for Weak CPT from Black-hole thermodynamics: Although white holes do not exist (strong CPT violation), nevertheless the CPT reverse of the most probable way of forming a black hole is the most probable way a black hole will evaporate: the states resulting from black hole evaporation are precisely the CPT reverse of the initial states which collapse to form a black hole.

\[ \Theta \Psi = \Phi \]

In terms of initial and final state densities, we have

\[ \Psi^\dagger = \Theta^{-1} \Psi \Theta^{-1} \]

Here, \( \Theta \) is well defined on pure states, but \( \Psi \) has no inverse, hence \( \Psi^\dagger \neq \Psi^{-1} \) (full CPT invariance: \( \Psi = SS^\dagger, \Psi^\dagger = \Psi^{-1} \)).
CPT symmetry without CPT invariance?

But....nature may be tricky: WEAK FORM OF CPT INVARIANCE might exist, such that the fundamental "arrow of time"
measure $\phi = \text{finite}.

$\Theta \rho = \text{initial states which collapse to form a black hole.}

In term

$$\dagger = \Theta^{-1} \rho \Theta^{-1}$$

Here, $\Theta$ is well defined on pure states, but $\rho$ has no inverse, hence $\dagger \neq \rho^{-1}$ (full CPT invariance: $\rho = SS^\dagger$, $\dagger = \rho^{-1}$).

In principle this question can be settled experimentally.
The Model
D-particle recoil and entangled Meson States

Bernabeu, Sarkar, NEM
D-particle recoil and entangled Meson States

Recoil of massive Defect distorts neighboring space-time on brane world
D-particle recoil and entangled Meson States

Recoil of massive Defect distorts neighboring space-time on brane world

Propagation of matter string in curved background interaction with metric
D-particle recoil and entangled Meson States

Recoil of massive Defect distorts neighboring space-time on brane world

Gravitationally dressed quantum states viewed as quantum mechanically perturbed

propagation of matter string in curved background interaction with metric
D-particle Recoil & the $\omega$-effect

For neutral mesons: Consider Klein-Gordon in curved background due to recoil-induced metric distortions with two “flavours” corresponding to mass eigenstates, e.g. for Kaons $K_L, K_S$.

Logarithmic conformal field theory describes the impulse at stage (II)

\[ \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \]

\[ (g^{\alpha\beta} D_\alpha D_\beta - m^2) \Phi = 0 \]

mass matrix \( m = \frac{1}{3} (m_1 + m_2) 1 + \frac{1}{3} (m_1 - m_2) \sigma_3 \)

\[ \langle r_\mu \rangle = 0, \quad \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu} \]
D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct "gravitationally dressed" states from

\[ |k, \uparrow \rangle^{(i)}, |k, \downarrow \rangle^{(i)}, i = 1, 2 \]
D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct “gravitationally dressed” states from

\[
|k^{(i)}, \downarrow^{(i)}\rangle_{QG} = |k^{(i)}, \downarrow^{(i)}\rangle + |k^{(i)}, \uparrow^{(i)}\rangle \alpha^{(i)}
\]

\[
\alpha^{(i)} = \frac{\langle \uparrow, k^{(i)} | \hat{H}_I | k^{(i)}, \downarrow^{(i)} \rangle}{E_2 - E_1}
\]

\[
\hat{H} = -\left(r_1 \sigma_1 + r_2 \sigma_2\right) \hat{k}
\]

\[
(g^{ab} \nabla_a \nabla_b - m^2) \Phi = 0
\]

**FLAVOUR FLIP**

Perturbation due to recoil distortion of space-time

\[
g_{0i} \propto \Delta k_i / M_P \otimes (\text{flavour} - \text{flip})
\]

\[
\Delta k_i = r_i k, \langle \langle r_i \rangle \rangle = 0, \langle \langle r_i r_j \rangle \rangle = \Delta \delta_{ij}
\]

Bernabeu, Sarkar, NEM
D-particle recoil and entangled Meson States

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\[ |k^{(i)}, \uparrow^{(i)}\rangle, |k^{(i)}, \downarrow^{(i)}\rangle, \ i = 1, 2 \]

\[ \alpha^{(i)} = \frac{\langle \uparrow, k^{(i)} | \hat{H} | k^{(i)}, \downarrow^{(i)}\rangle}{E_2 - E_1} \]

\[ \hat{H} = -\left(r_1 \sigma_1 + r_2 \sigma_2\right) \hat{k} \]

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\left(g^{ab} \nabla_a \nabla_b - m^2\right) \Phi = 0
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Apply non-degenerate perturbation theory to construct "gravitationally dressed" states from

\[ |k, \uparrow\rangle^{(i)}, |k, \downarrow\rangle^{(i)}, i = 1, 2 \]

Similarly for the dressed state

\[ |\downarrow\rangle \leftrightarrow |\uparrow\rangle \text{ and } \alpha \rightarrow \beta \]

\[ \alpha^{(i)} = \frac{\langle i \mid \uparrow, k^{(i)} \rangle \hat{H}_1 |k^{(i)}, \downarrow\rangle^{(i)}}{E_2 - E_1} \]

\[ \beta^{(i)} = \frac{\langle i \mid \downarrow, k^{(i)} \rangle \hat{H}_1 |k^{(i)}, \uparrow\rangle^{(i)}}{E_1 - E_2} \]

\[ \begin{align*}
|k, \uparrow\rangle_{QG}^{(1)} &- |k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} |k, \uparrow\rangle_{QG}^{(2)} = \\
|k, \uparrow\rangle_{QG}^{(1)} |k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} |k, \uparrow\rangle_{QG}^{(2)} &+ |k, \downarrow\rangle_{QG}^{(1)} |k, \uparrow\rangle_{QG}^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\
+ \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle_{QG}^{(1)} |k, \uparrow\rangle_{QG}^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \downarrow\rangle_{QG}^{(1)} |k, \uparrow\rangle_{QG}^{(2)}
\end{align*} \]
D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct “gravitationally dressed” states from

\[ |k^{(i)}, \uparrow\rangle_{QG}^{(i)} = |k^{(i)}, \downarrow\rangle_{QG}^{(i)} + |k^{(i)}, \uparrow\rangle^{(i)} \alpha^{(i)} \]

\[ \alpha^{(i)} = \frac{\langle \uparrow, k^{(i)} | \hat{H}_I | k^{(i)}, \downarrow\rangle^{(i)}}{E_2 - E_1} \]

Similarly for the dressed state

\[ |\downarrow\rangle \leftrightarrow |\uparrow\rangle \text{ and } \alpha \rightarrow \beta \]

\[ |k^{(i)}, \uparrow\rangle_{QG}^{(i)} - |k^{(i)}, \downarrow\rangle_{QG}^{(i)} - |k^{(i)}, \uparrow\rangle_{QG}^{(i)} = |k^{(i)}, \uparrow\rangle_{QG}^{(i)} - |k^{(i)}, \downarrow\rangle_{QG}^{(i)} - |k^{(i)}, \uparrow\rangle_{QG}^{(i)} + |k^{(i)}, \uparrow\rangle_{QG}^{(i)} \]

\[ + |k^{(i)}, \downarrow\rangle_{QG}^{(i)} - |k^{(i)}, \downarrow\rangle_{QG}^{(i)} (\beta^{(1)} - \beta^{(2)}) + |k^{(i)}, \uparrow\rangle_{QG}^{(i)} - |k^{(i)}, \uparrow\rangle_{QG}^{(i)} (\alpha^{(2)} - \alpha^{(1)}) \]

\[ + \beta^{(1)}\alpha^{(2)} |k^{(i)}, \downarrow\rangle_{QG}^{(i)} - |k^{(i)}, \downarrow\rangle_{QG}^{(i)} - \alpha^{(1)}\beta^{(2)} |k^{(i)}, \downarrow\rangle_{QG}^{(i)} |k^{(i)}, \uparrow\rangle_{QG}^{(i)} - |k^{(i)}, \downarrow\rangle_{QG}^{(i)} \]

\[ w\text{-effect} \]
D-particle recoil and entangled Meson States

- Apply non-degenerate perturbation theory to construct “gravitationally dressed” states from:
  \[ |k, \uparrow \rangle^{(i)}, |k, \downarrow \rangle^{(i)}, i = 1, 2 \]

  \[
  |k^{(i)}, \downarrow \rangle^{(i)}_{QG} = |k^{(i)}, \downarrow \rangle^{(i)} + |k^{(i)}, \uparrow \rangle^{(i)} \alpha^{(i)}
  \]

Similarly for \( |k^{(i)}, \uparrow \rangle^{(i)} \) the dressed state

  \[
  \beta^{(i)} = \frac{\langle i, k^{(i)} | \hat{H}_I | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2}
  \]

\[ |k, \uparrow \rangle^{(1)}_{QG} |k, \downarrow \rangle^{(2)}_{QG} - |k, \downarrow \rangle^{(1)}_{QG} |k, \uparrow \rangle^{(2)}_{QG} =
\[
|k, \uparrow \rangle^{(1)}_{QG} |k, \downarrow \rangle^{(2)}_{QG} - |k, \downarrow \rangle^{(1)}_{QG} |k, \uparrow \rangle^{(2)}_{QG} =
\]

\[ + |k, \downarrow \rangle^{(1)} |k, \uparrow \rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) - |k, \uparrow \rangle^{(1)} |k, \downarrow \rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)})
\]

\[ + \beta^{(1)} \alpha^{(2)} |k, \downarrow \rangle^{(1)} |k, \uparrow \rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \downarrow \rangle^{(1)} |k, \uparrow \rangle^{(2)}
\]

\[ w\text{-effect} \]
RHN Leptogenesis in D-particle foam context

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + iN_{\bar{R}} \phi N_{R} - \frac{M_{N}}{2} (N_{\bar{R}} \cdot N_{R} + N_{R} \cdot N_{\bar{R}}) - Y_{k} \overline{L}_{k} \tilde{\phi} N_{R} + \text{h.c.}
\]

\[
\mathcal{N} \equiv N_{R} + N_{\bar{R}}
\]

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + i \frac{1}{2} \overline{\mathcal{N}} \phi \mathcal{N} - \frac{M_{N}}{2} \overline{\mathcal{N}} \mathcal{N} - Y_{1} \overline{L}_{1} \phi \frac{1}{2} (1 + \gamma_{5}) \mathcal{N} - Y_{1}^{*} \overline{\mathcal{N}} \frac{1}{2} (1 - \gamma_{5}) \phi^{\dagger} L_{1}
\]

\[
\mathcal{L}_{D-\text{foam/RHN-matter}} = \mu_{\text{CPTV}} N_{R}^{\dagger} \overline{N}_{R}
\]

\[
\mu_{\text{CPTV}} = \frac{1}{2} \frac{M_{s}}{g_{s}} \sigma^{2}
\]

\[
\mathcal{L}_{D-\text{foam/RHN-matter}} = \frac{1}{2} B_{\mu} \overline{\mathcal{N}} \gamma^{\mu} \gamma^{5} \mathcal{N} , \quad B_{\mu} = \delta_{\mu 0} \mu_{\text{CPTV}} = \frac{1}{2} \frac{M_{s}}{g_{s}} \sigma^{2} > 0
\]

\[
E_{r}^{2} = M_{N}^{2} + (B_{0} + \lambda_{r} |\vec{p}|)^{2}
\]

\[
\lambda_{r} = \pm 1, \quad r = +, -
\]

\[
\Delta = \frac{g_{s}}{2 M_{s}} T^{2} \Delta
\]

\[
B_{0} \sim \frac{g_{s}}{2 M_{s}} T^{2} \Delta
\]
RHN Leptogenesis in D-particle foam context

\[ \mathcal{L} = L_{SM} + iN_R \bar{\phi} N_R - \frac{M_N}{2} (N_R^c N_R + N_R N_R^c) - Y_k \bar{L}_k \tilde{\phi} N_R + \text{h.c.} \]

\[ \mathcal{N} \equiv N_R + N_R^c \]

\[ \mathcal{L} = L_{SM} + i \frac{1}{2} \bar{\mathcal{N}} \bar{\phi} \mathcal{N} - \frac{M_N}{2} \bar{\mathcal{N}} \mathcal{N} - Y_1 \bar{L}_1 \tilde{\phi} \frac{1}{2} (1 + \gamma_5) \mathcal{N} - Y_1^* \bar{\mathcal{N}} \frac{1}{2} (1 - \gamma_5) \phi \dagger L_1 \]

\[ \mathcal{L}_{D\text{-foam}/RHN\text{-matter}} = \mu_{\text{CPTV}} N_R^\dagger N_R \]

\[ \mu_{\text{CPTV}} = \frac{1}{2 g_s} \frac{M_s}{\sigma^2} \]

\[ \mathcal{L}_{D\text{-foam}/RHN\text{-matter}} = \frac{1}{2} B_\mu \bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N}, \quad B_\mu = \delta_{\mu 0} \mu_{\text{CPTV}} = \frac{1}{2 g_s} \frac{M_s}{\sigma^2} > 0 \]

non constant \( B_0 \) in general, Leptogenesis understanding pending or not relevant in this context

depends on \( \Delta \)

\[ B_0 \sim \frac{g_s}{2 M_s} T^2 \Delta \]
RHN Leptogenesis in D-particle foam context

\[ \mathcal{L} = \mathcal{L}_{SM} + i\bar{N}_R \phi N_R - \frac{M_N}{2} (\bar{N}_R^c N_R + \bar{N}_R N_R^c) - Y_k \bar{L}_k \tilde{\phi} N_R + \text{h.c.} \]

\[ \mathcal{N} = N_R + N_R^c \]

\[ \mathcal{L} = \mathcal{L}_{SM} + i \frac{1}{2} \bar{N} \phi \mathcal{N} - \frac{M_N}{2} \bar{N} \mathcal{N} - Y_1 \bar{L}_1 \tilde{\phi} \frac{1}{2} (1 + \gamma_5) \mathcal{N} - Y_1^* \bar{N} \frac{1}{2} (1 - \gamma_5) \tilde{\phi}^\dagger L_1 \]

\[ \mathcal{L}_{D-foam/RHN-matter} = \mu_{\text{CPTV}} N_R^\dagger N_R \]

\[ \mu_{\text{CPTV}} = \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

\[ \mathcal{L}_{D-foam/RHN-matter} = \frac{1}{2} B_\mu \bar{N} \gamma^\mu \gamma^5 \mathcal{N}, \quad B_\mu = \delta_{\mu 0} \mu_{\text{CPTV}} = \frac{1}{2} \frac{M_s}{g_s} \sigma^2 \]

BUT may be approx. constant \( B_0 \) during leptogenesis (RHN freezeout) period

\[ B_0 \sim \frac{g_s}{2 M_s} T^2 \Delta \]
RHN Leptogenesis as for constant $B_0$

$$\frac{B_0}{M_N} \simeq 10^{-8}$$

for phenomenologically acceptable baryogenesis through leptogenesis

Assume $\Delta \sim$ constant during RHN freezeout period ($T \sim T_D$) due to constant density of D-particles on brane world, due to influx from bulk

$$\frac{\dot{B}}{M_P B_0} \simeq - \frac{H}{M_P} \ll 1$$

BUT approx. constant $B_0$

If D-particles on brane world dark-matter like $\rightarrow \Delta \sim T^3 \ (T < T_D)$

$$B_0 \sim \frac{g_s}{2 M_s} T^2 \Delta$$
RHN Leptogenesis as for constant $B_0$

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Assume $\Delta \sim$ constant during RHN freezeout period ($T \sim T_D$)

$$\frac{\dot{B}}{M_P B_0} \simeq - \frac{H}{M_P} \ll 1$$

$$\Delta(T \sim T_D \simeq M_N) \sim 2 \times 10^{-8} \frac{M_s}{g_s M_N} < 1$$

$$\Rightarrow \frac{M_s}{g_s} < 5 \times 10^{-9} M_N,$$

$$M_N \sim 10^2 \text{ TeV} \Rightarrow \frac{M_s}{g_s} < 10^{13} \text{ GeV}$$

BUT approx. constant $B_0$

$$B_0 \sim \frac{g_s}{2 M_s} T^2 \Delta$$
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Assume $\Delta \sim$ constant during RHN freezeout period ($T \sim T_D$)

$$\frac{\dot{B}}{M_P B_0} \simeq -\frac{H}{M_P} \ll 1$$

$$\Delta(T \sim T_D \simeq M_N \simeq 10^2 \text{ TeV}) \frac{\Delta_{\text{today}}}{\Delta_{\text{today}}} \sim \left(\frac{T_D}{T_{\text{CMB}}}\right)^3 \sim 1.13 \times 10^{53}$$

BUT approx. constant $B_0$

If D-particles on brane world dark-matter like $\rightarrow \Delta \sim T^3$ ($T < T_D$)

$$B_0 \sim \frac{g_s}{2 M_s} T^2 \Delta$$
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Assume $\Delta \sim \text{constant during RHN freezeout period (} T \sim T_D)$

$$\frac{\dot{B}}{M_P B_0} \simeq -\frac{H}{M_P} \ll 1$$

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Neutral Kaons in $\Phi$-factories

$$\Delta_{\text{today}} < 10^{-38}$$

$$\Delta(T \sim T_D \simeq M_N \simeq 10^2) \sim 1.13 \times 10^{53} \Delta_0$$

But $\Delta(T = M_N \simeq 10^2 \text{ TeV}) < 1 \Rightarrow \Delta_0 < 8 \times 10^{-53}$
RHN Leptogenesis as for constant $B_0$

$$\frac{B_0}{M_N} \simeq 10^{-8}$$

for phenomenologically acceptable baryogenesis through leptogenesis

Assume $\Delta \sim \text{constant}$ during RHN freezeout period ($T \sim T_D$)

$$\frac{\dot{B}}{M_P B_0} \sim - \frac{H}{M_P} \ll 1$$

$$\frac{\Delta(T \sim T_D \simeq M_N \simeq 10^2 \text{ TeV})}{\Delta_{\text{today}}} \sim \left(\frac{T_D}{T_{\text{CMB}}}\right)^3$$

$$\sim 1.13 \times 10^{53}$$

Neutral Kaons in $\Phi$-factories

$$\Delta_{\text{today}} < 10^{-38}$$

$$\Delta(T \sim T_D \simeq M_N \simeq 10^2) \sim 1.13 \times 10^{53} \Delta_0$$

But $\Delta(T = M_N \simeq 10^2 \text{ TeV}) < 1 \Rightarrow \Delta_0 < 8 \times 10^{-53}$

Far below current sensitivities!
RHN Leptogenesis as for constant $B_0$

$$\frac{B_0}{M_N} \simeq 10^{-8}$$

for phenomenologically acceptable baryogenesis through leptogenesis

Assume $\Delta \sim \text{constant during RHN freezeout period} (T \sim T_D)$

$$\dot{B} \sim \frac{H}{M_{N} P} \sim \frac{\Delta(T \sim T_D)}{T_D^{3}}$$

But of course this is one crude model of QG, we do not have a concrete theory yet.
→ hence cannot tell ....

$$\Delta(T \sim T_D \sim M_N \simeq 10^2) \sim 1.13 \times 10^{53} \Delta_0$$

But $\Delta(T = M_N \simeq 10^2 \text{TeV}) < 1 \Rightarrow \Delta_0 < 8 \times 10^{-53}$

Far below current sensitivities!