


Superpositions and loops, in space and time (an old experiment and some new ideas)

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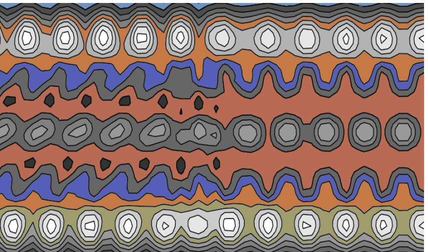
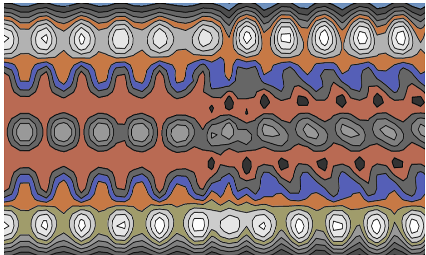
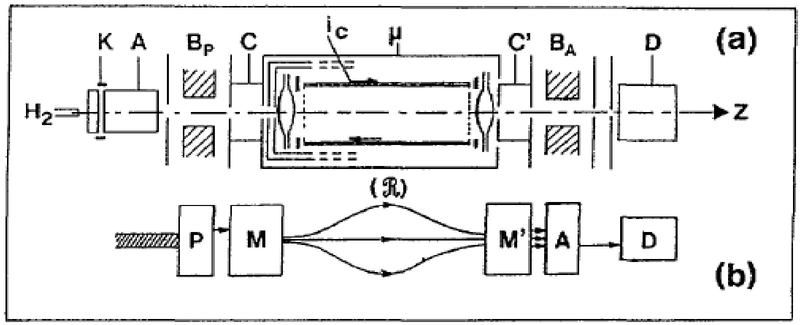
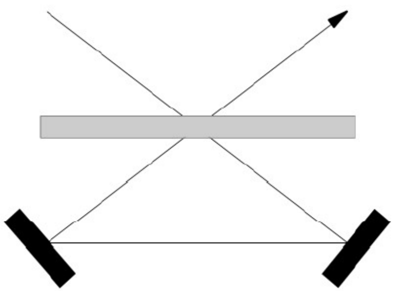



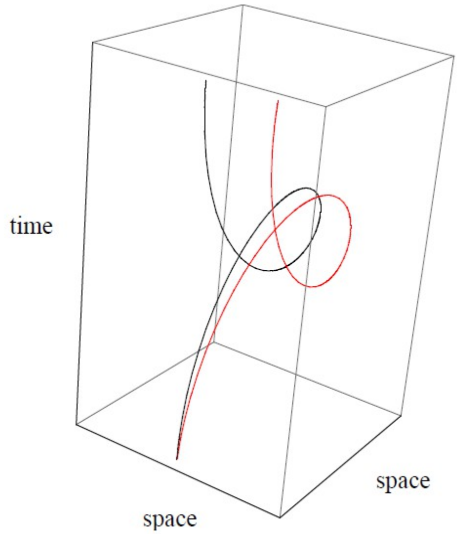
Plan

1. An old atom-interferometer experiment (Paris, 1991-92) revisited



2. Looped (in space or time) quantum dynamics





Part 1: On superposition in time

J Phys II 1 (1991) 425-436 AVRIL 1991, PAGE 425

A longitudinal Stern-Gerlach interferometer : the « beaded » atom

Ch. Miniatura, F. Perales (*), G. Vassilev, J. Reinhardt, J Robert and J. Baudon

Laboratoire de Physique des Lasers (**), Université Paris-Nord, Avenue J. B. Clément, 93430 Villetaneuse, France

Experiment

J. Phys. II France 2 (1992) 601–614 APRIL 1992, PAGE 601

Atomic quantum phase studies with a longitudinal Stern-Gerlach interferometer

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ord,

J. Phys. B: At. Mol. Opt. Phys. 25 (1992) L319–L322. Printed in the UK

LETTER TO THE EDITOR

Spontaneous emission from an extended wavepacket

K Rzążewski†§ and W Żakowicz‡

† MPI für Quantenoptik, W-8046 Garching, Federal Republic of Germany
‡ Instytut Fizyki PAN, Al. Lotników 32/46, 02 668 Warsaw, Poland

Theory

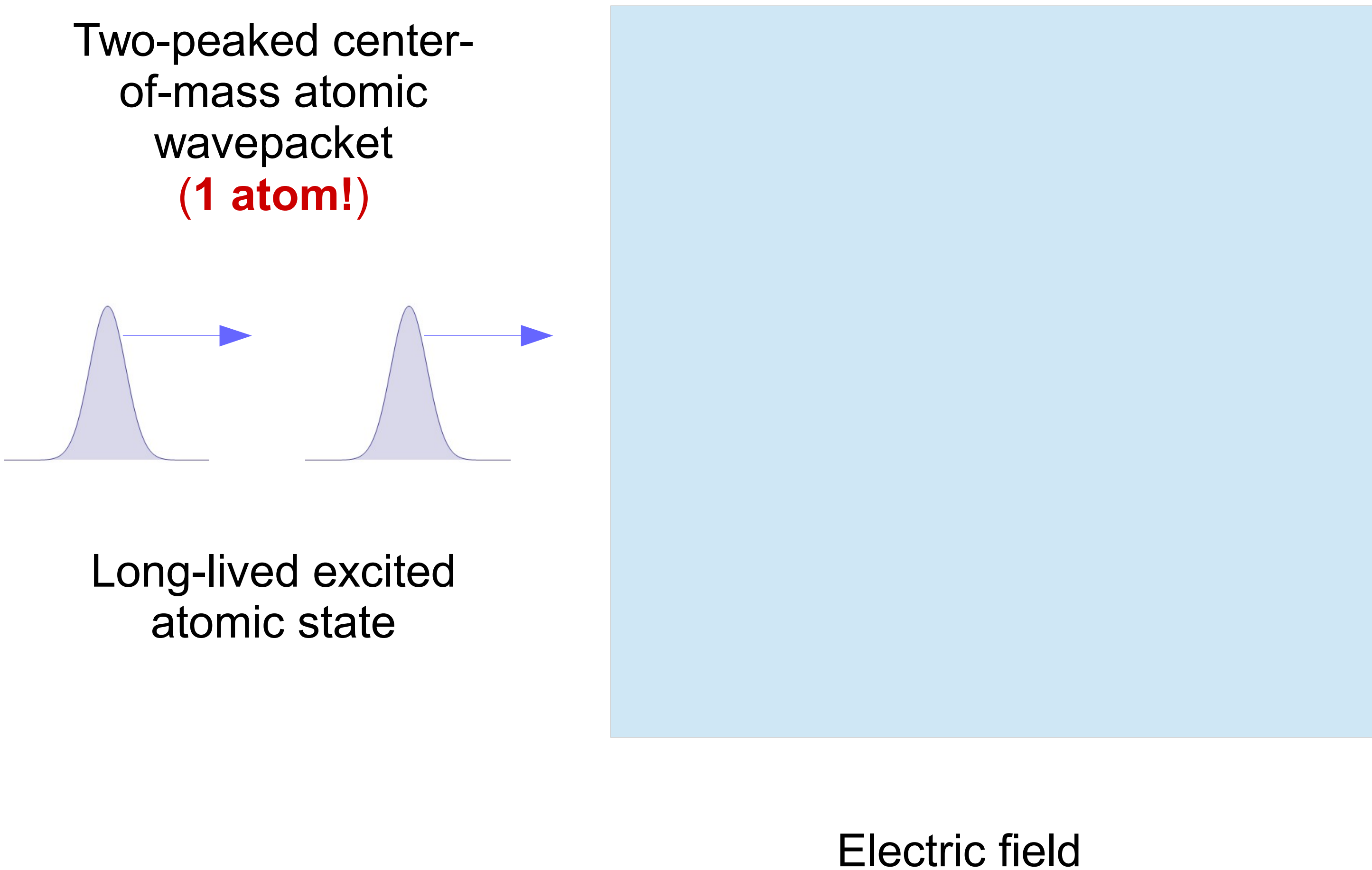
PHYSICAL REVIEW A VOLUME 53, NUMBER 5 MAY 1996

Spontaneous emission from an extended wave packet: Field correlations

O. Steuernagel and H. Paul

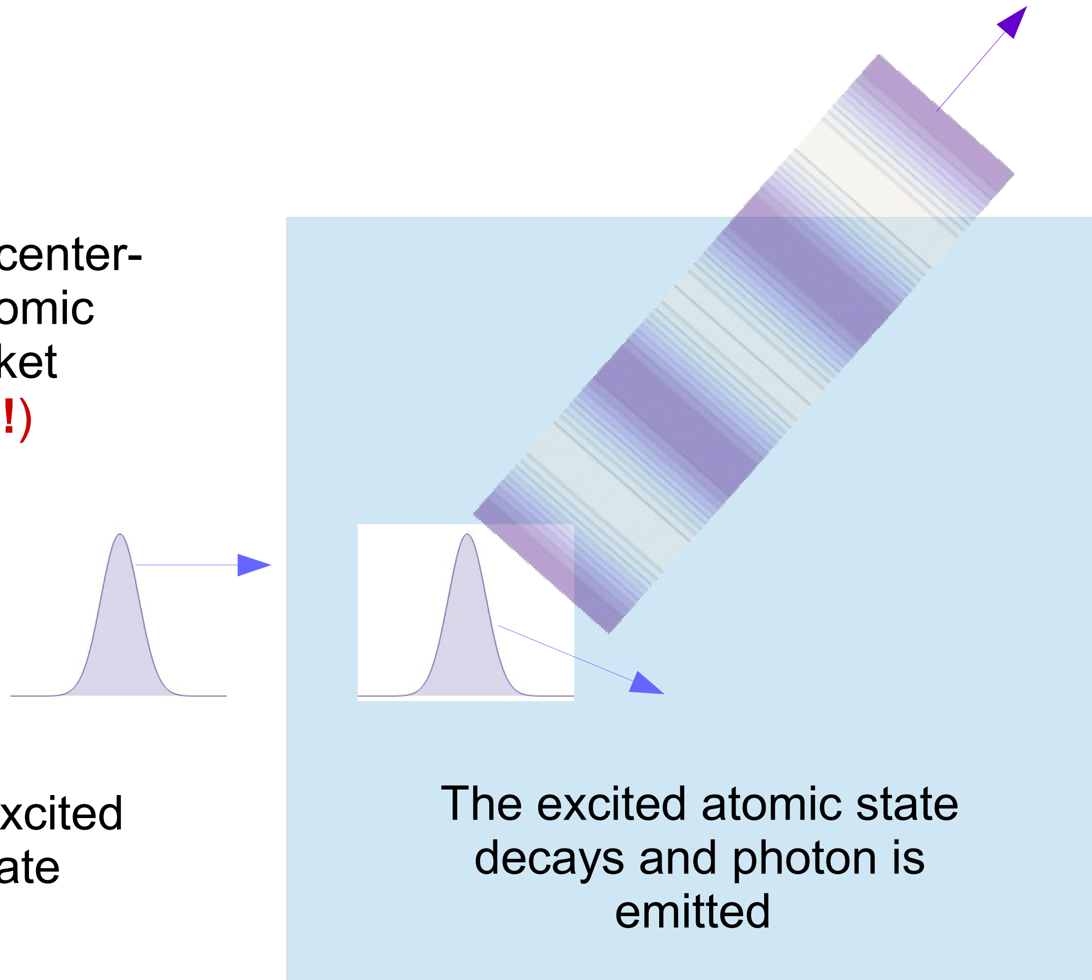
Arbeitsgruppe 'Nichtklassische Strahlung'' der Max-Planck-Gesellschaft an der Humboldt-Universität zu Berlin, Rudower Chaussee 5, 12489 Berlin, Germany

The idea of the experiment



The idea of the experiment

Two-peaked center-of-mass atomic wavepacket
(1 atom!)



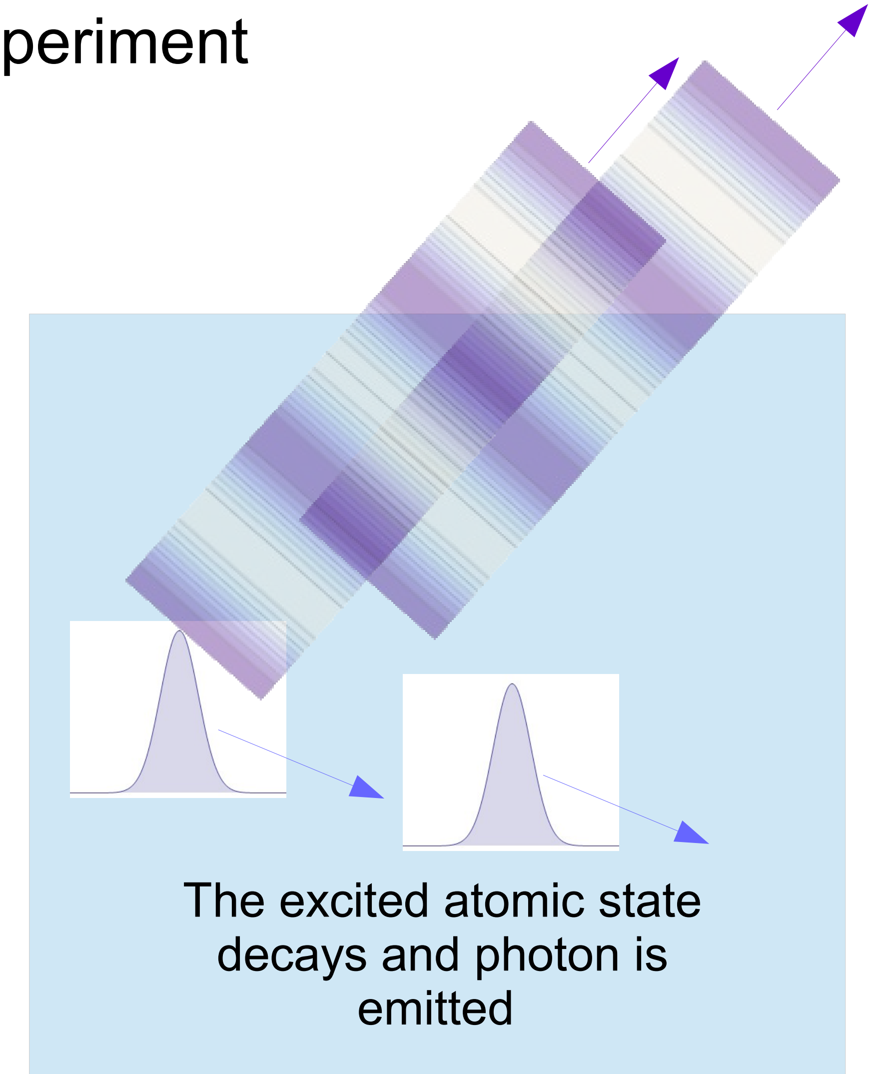
Long-lived excited atomic state

Electric field

The idea of the experiment

Two-peaked center-of-mass atomic wavepacket
(1 atom!)

Long-lived excited atomic state



Electric field

The idea of the experiment

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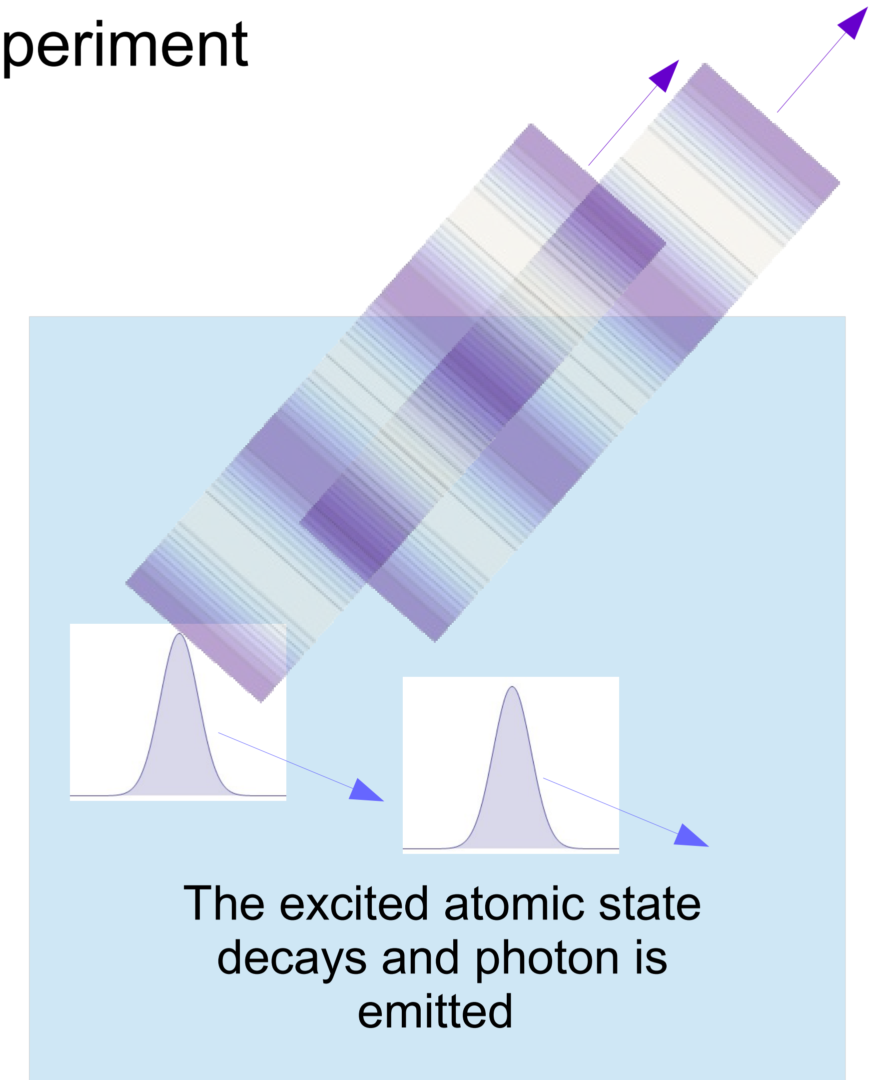

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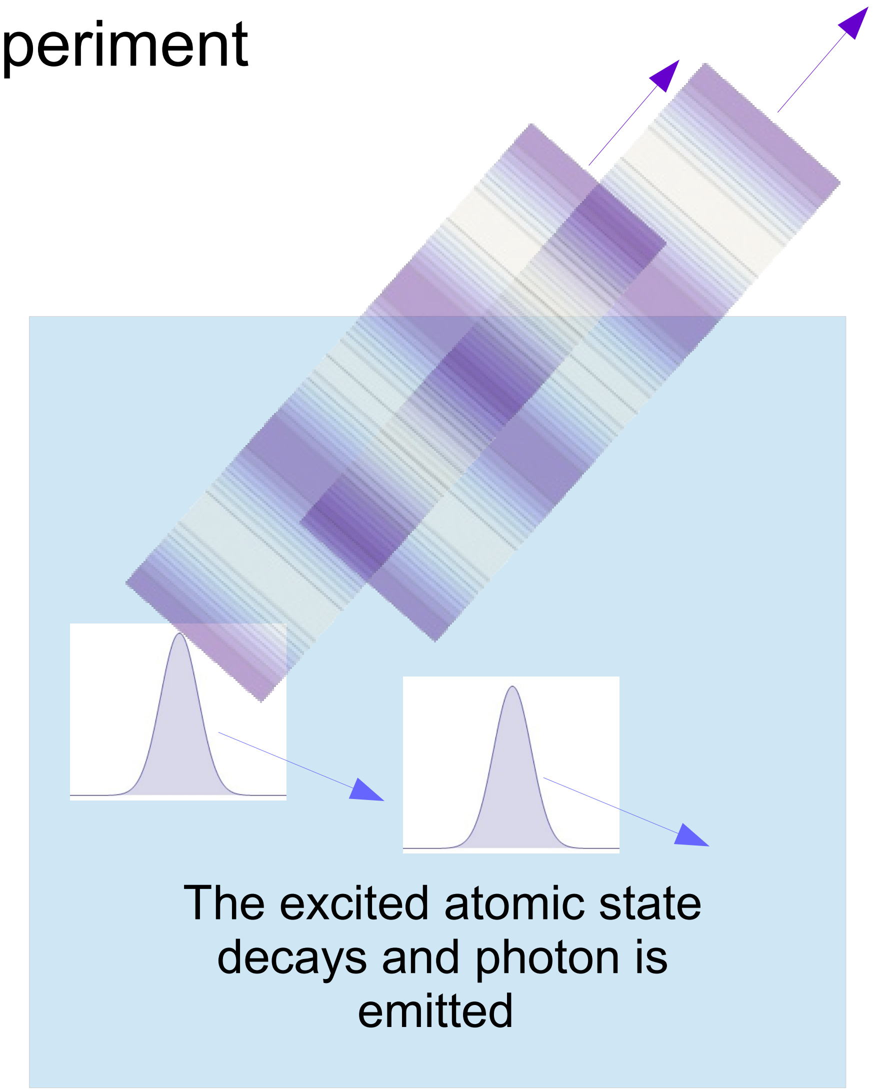
But this is a 1-atom wavepacket!

The idea of the experiment

Two-peaked center-of-mass atomic wavepacket
(1 atom!)

Long-lived excited atomic state

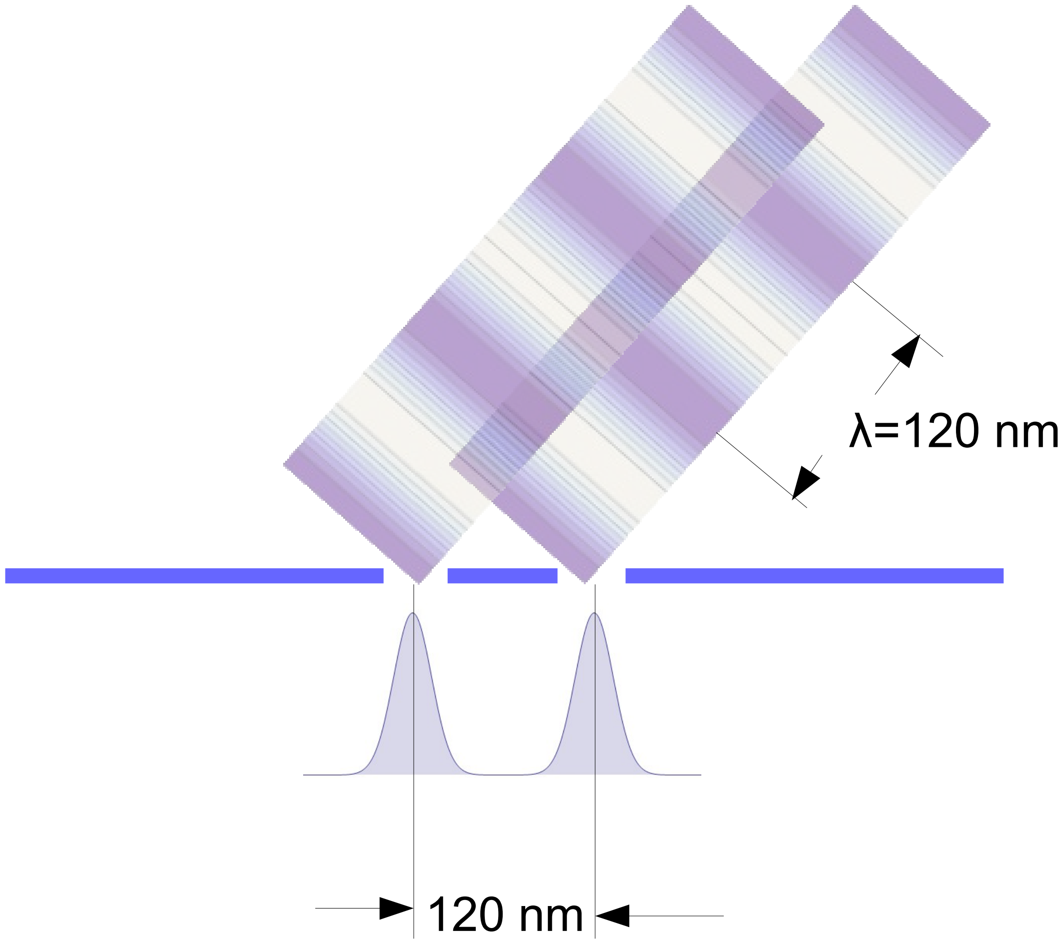


Electric field

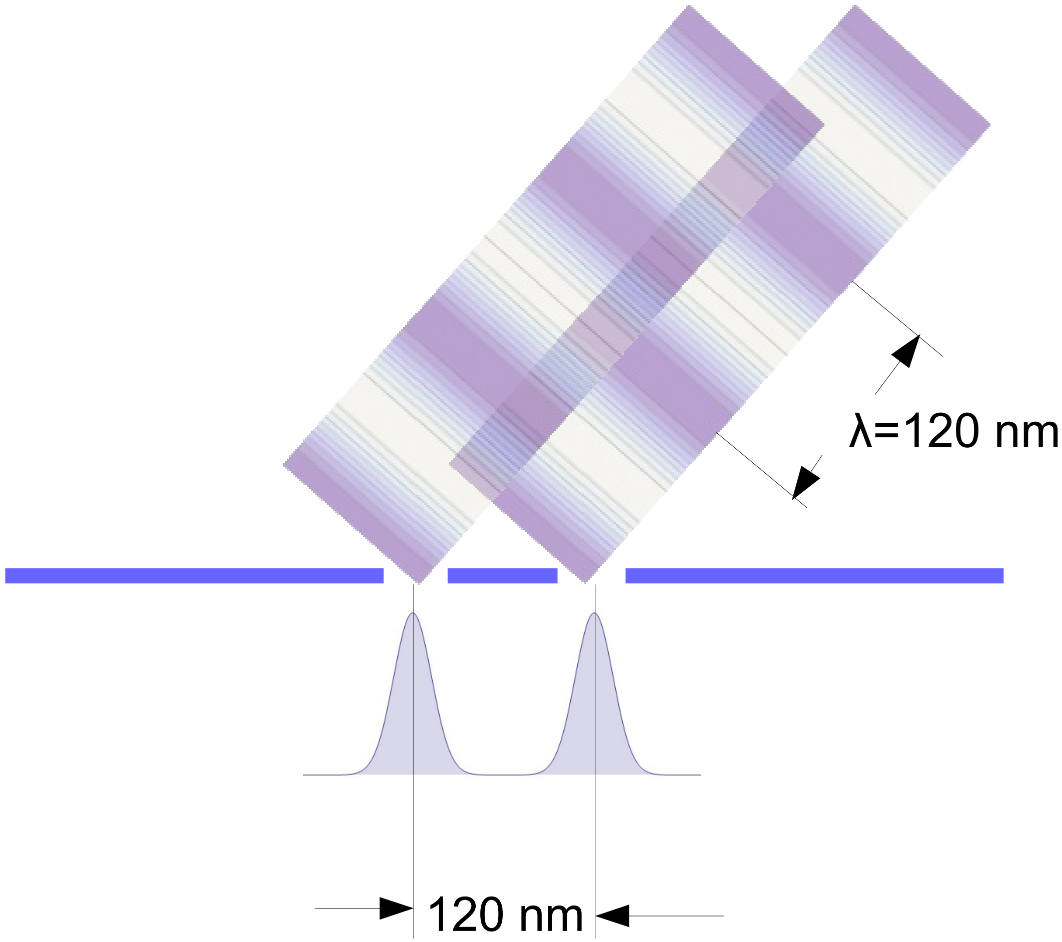
But this is a 1-atom wavepacket!

Will photons interfere?

Analogy to the standard Young double slit experiment



Analogy to the standard Young double slit experiment is misleading!



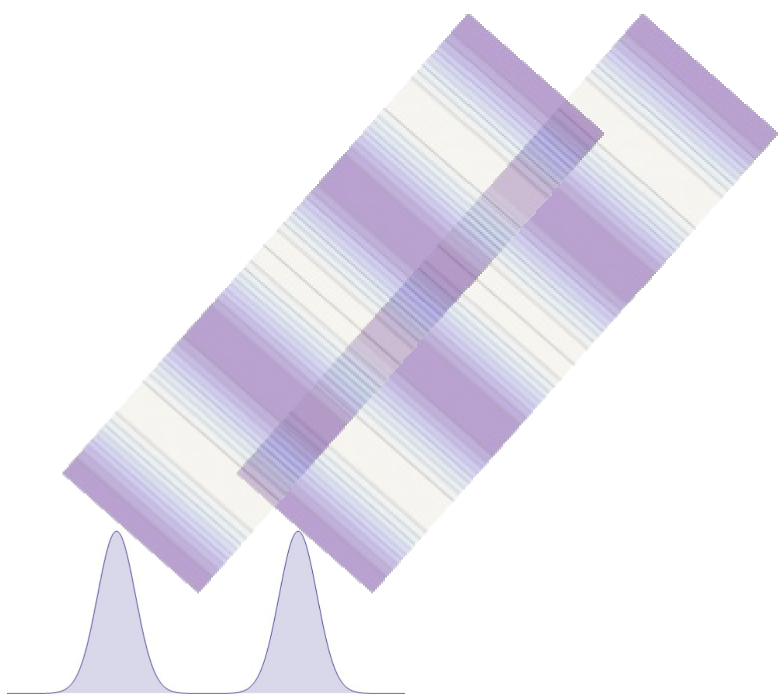
Rather EPR than Young!
A „which-way” experiment

Non-naive prediction (momentum conservation):

After spontaneous emission an entangled state is formed

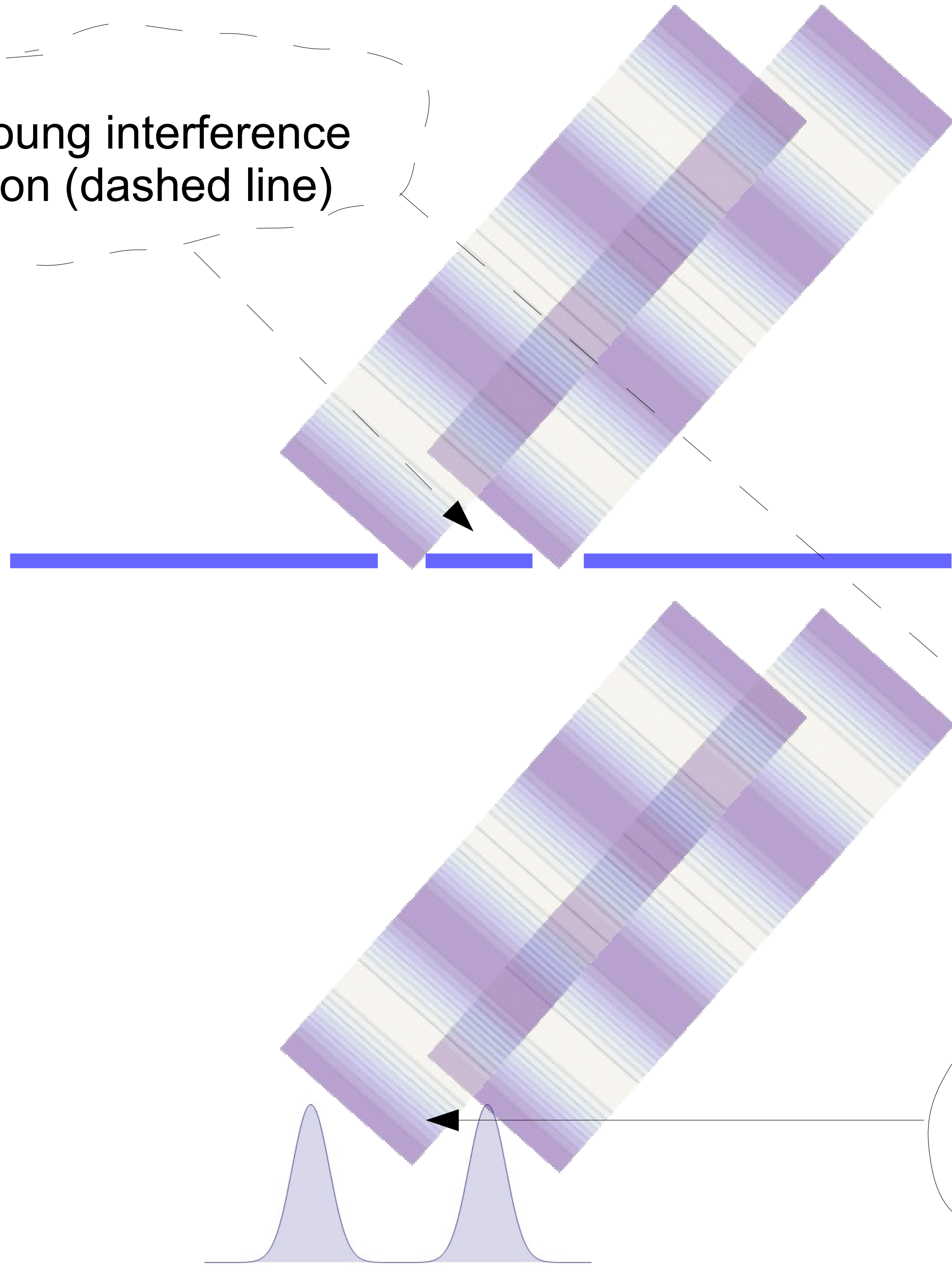
$$\int d\boldsymbol{P}\,\alpha_t(\boldsymbol{P})|+,\boldsymbol{P}\rangle\otimes|0\rangle+\int d\boldsymbol{P}d\boldsymbol{p}\,\beta_t(\boldsymbol{P}-\boldsymbol{p},\boldsymbol{p})|-,\boldsymbol{P}-\boldsymbol{p}\rangle\otimes|\boldsymbol{p}\rangle$$

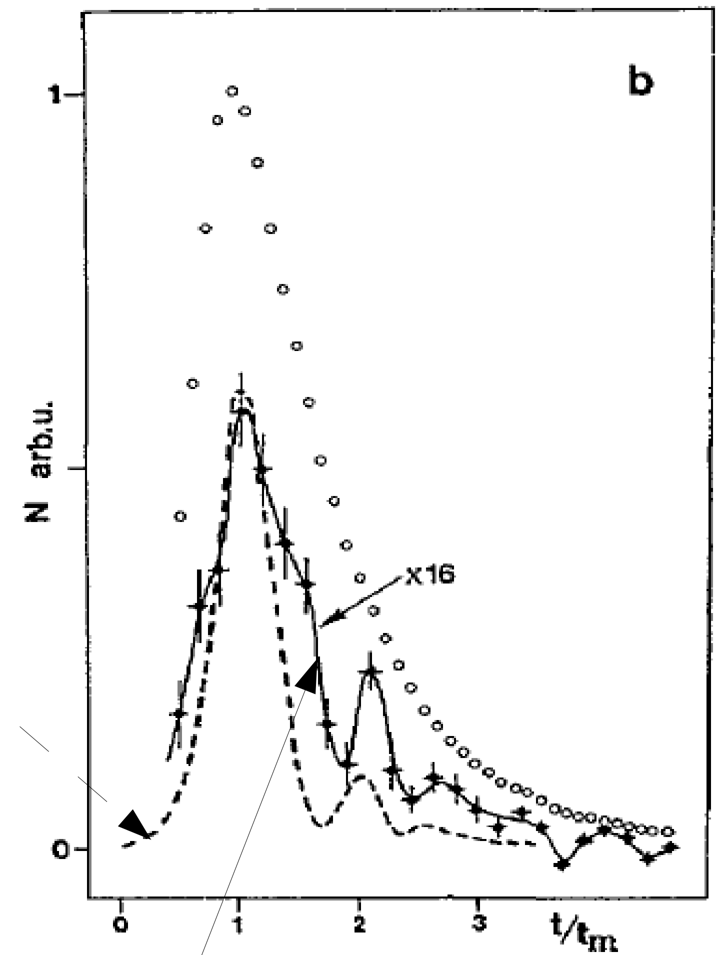
Photons become an incoherent mixture of states emitted by the two atomic peaks. The atomic phase-shift is unobservable in photon statistics.



A surprise!

Naive Young interference prediction (dashed line)





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Disagree!

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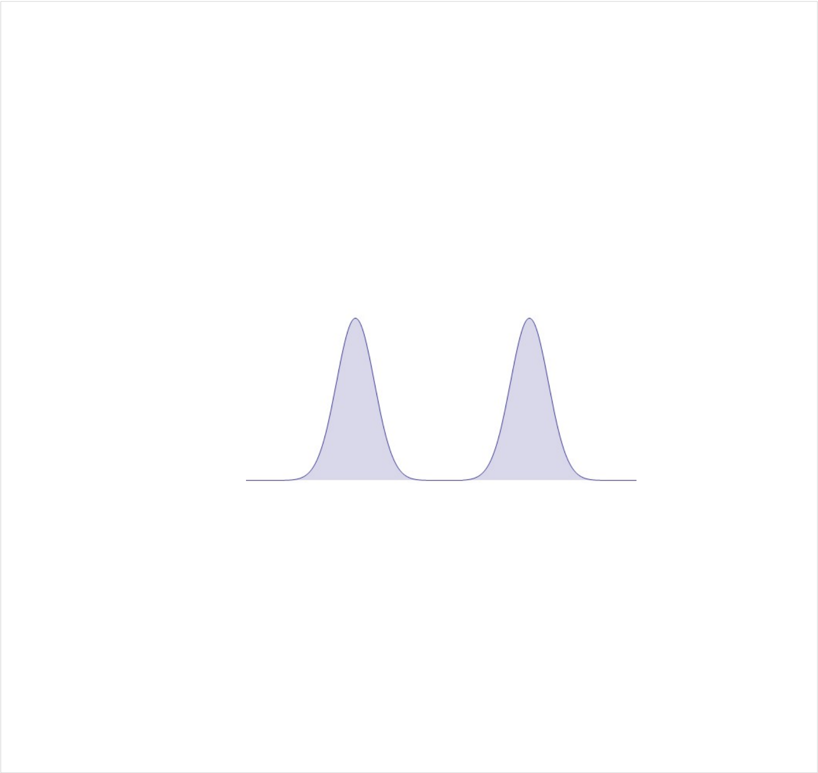
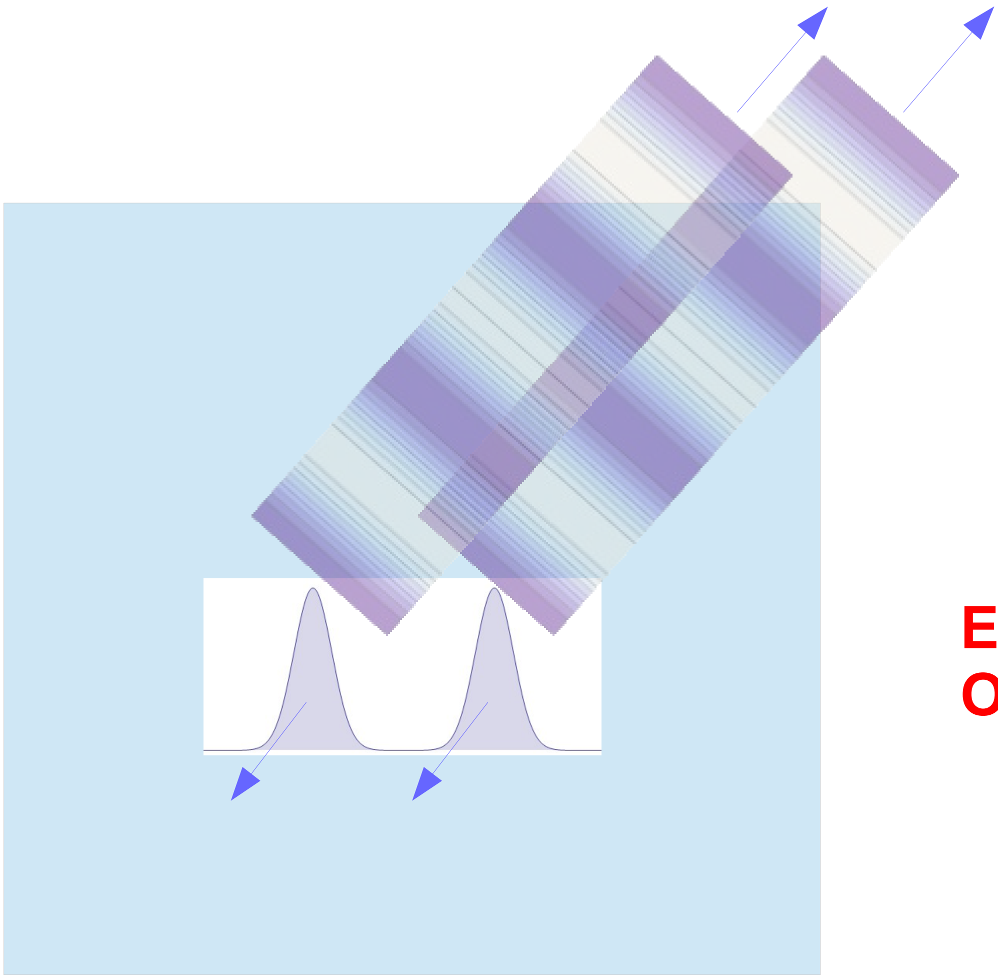


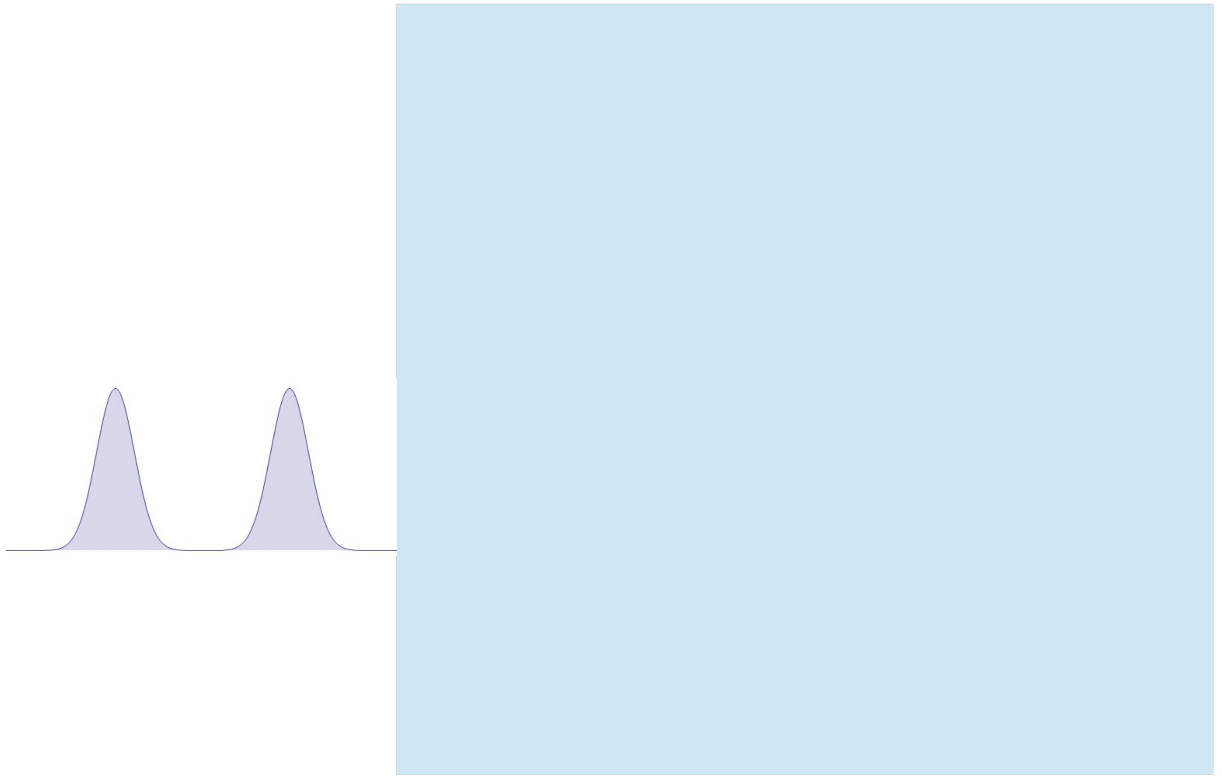
Electric field OFF for *t* < 0

Theorists analyze this Gedankenexperiment



Electric field ON for $t \geq 0$

But the actual experiment is different



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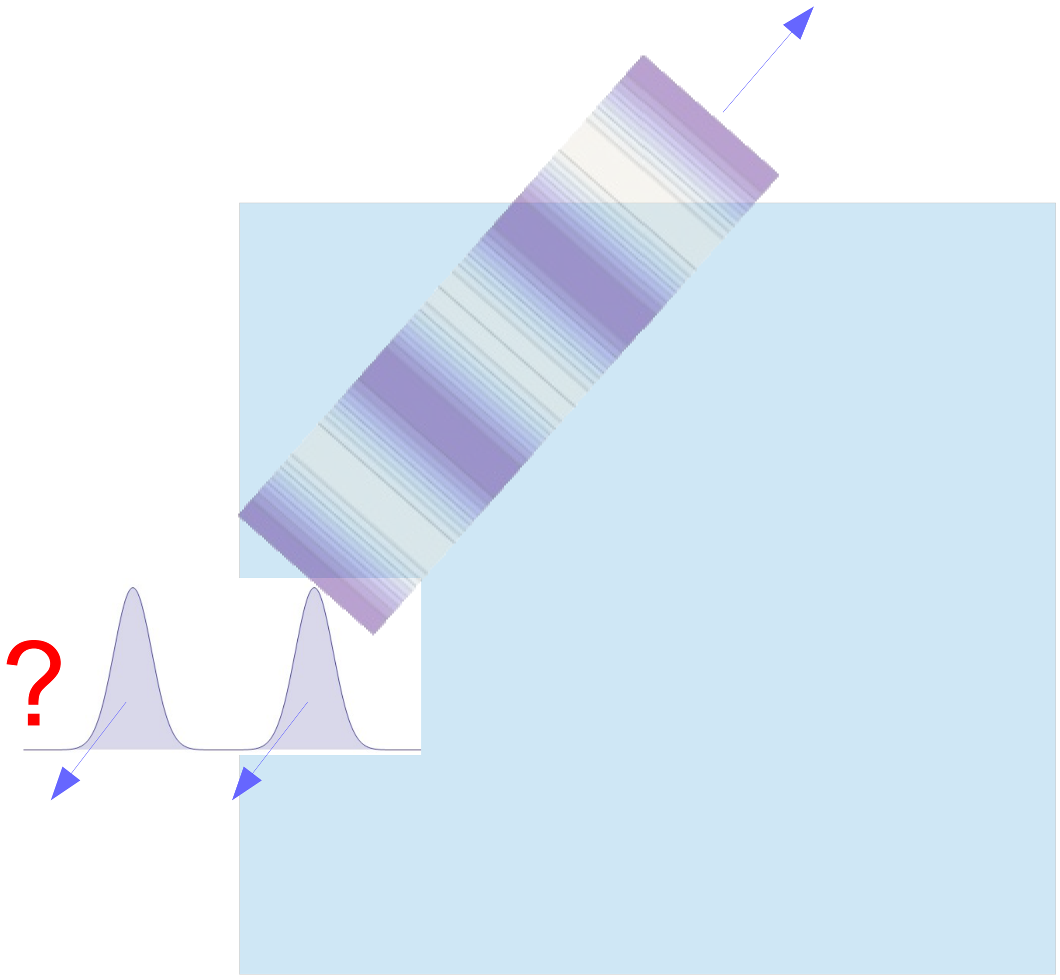

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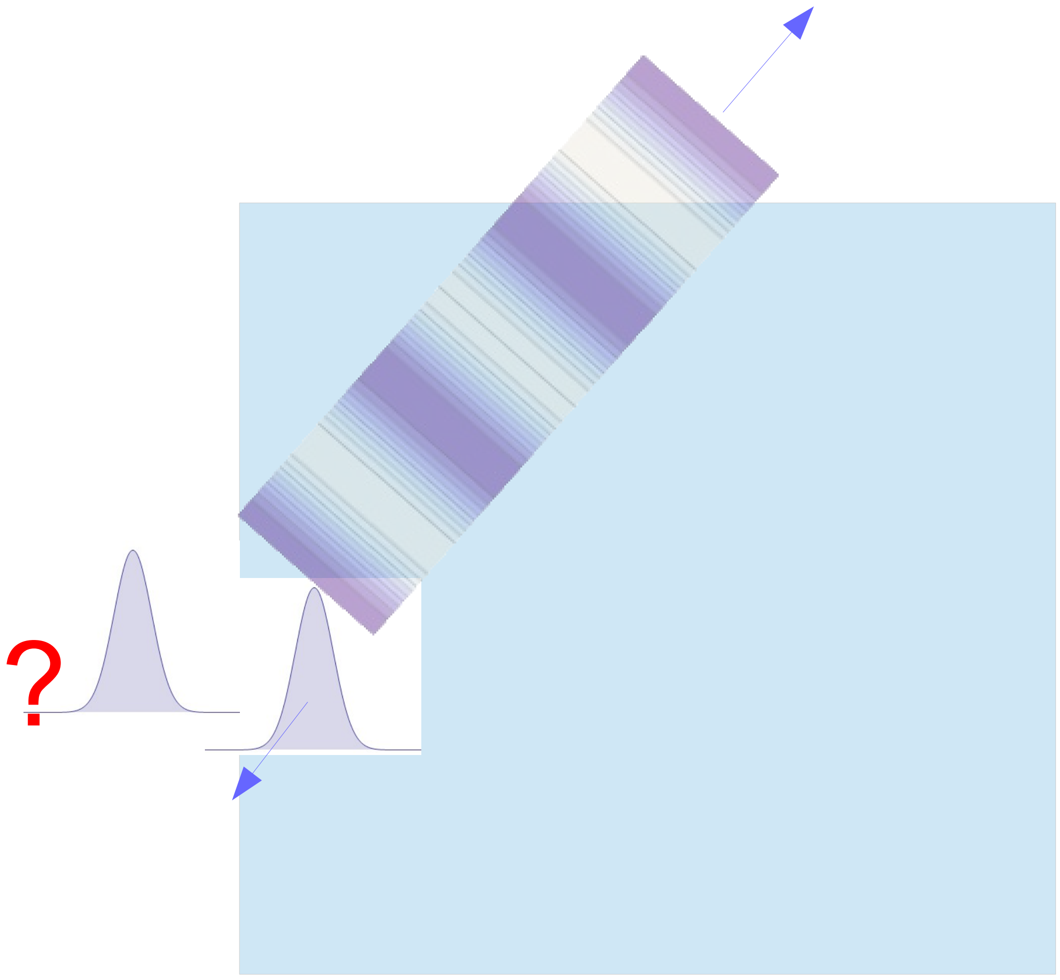
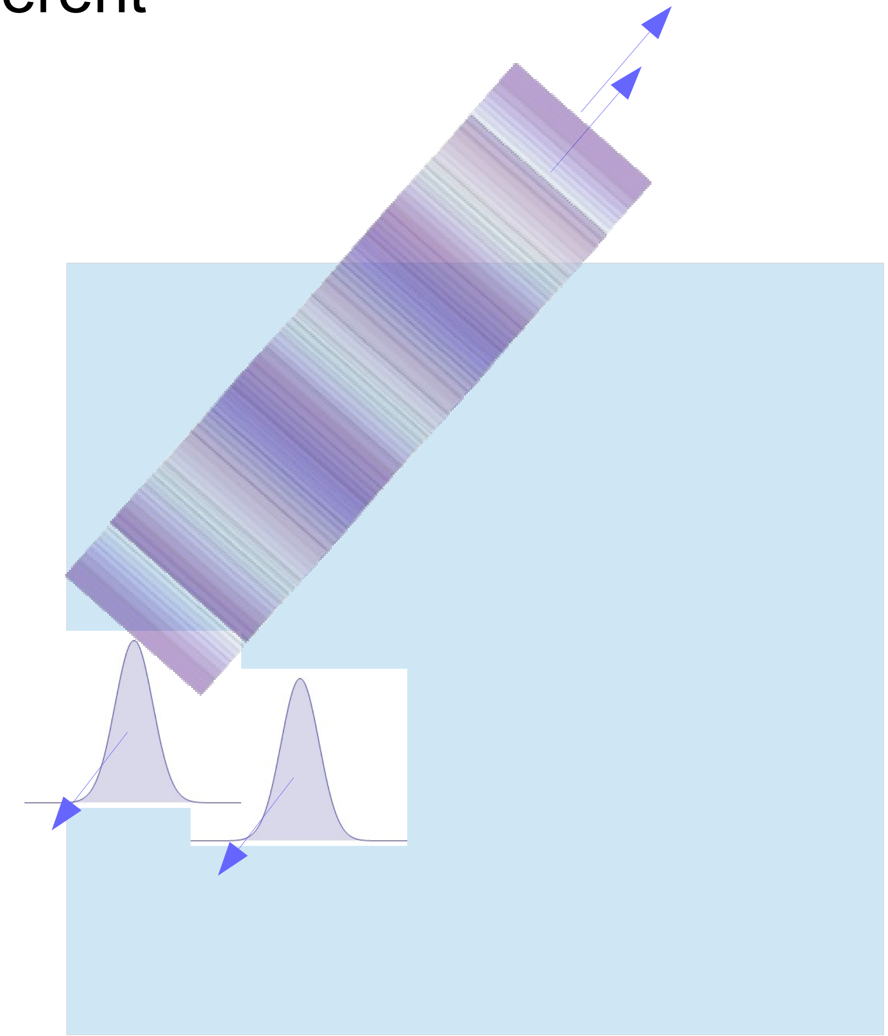


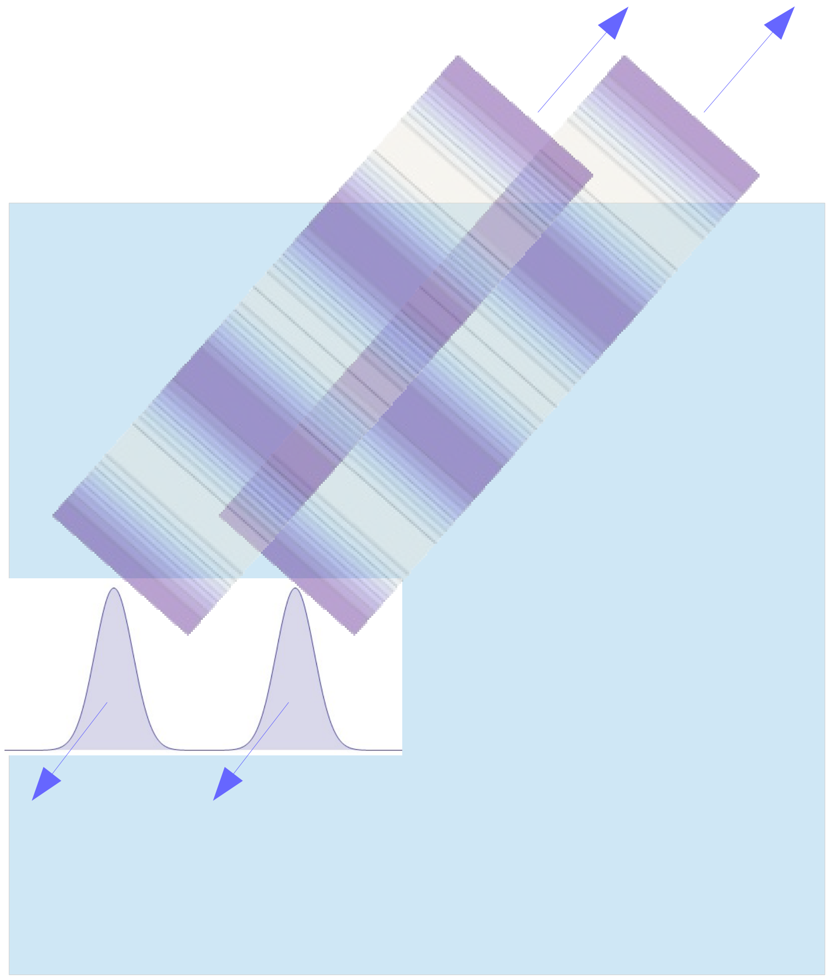
But the actual experiment is different

Superposition in time?



But the actual experiment is different

?????



We don't have good intuitions...
This is not a standard entangled-state problem, so...

...compute!

An attempt of quantum optical calculation (1995)

NONSIMULTANEOUS SPONTANEOUS EMISSION

FROM AN ATOMIC WAVEPACKET

Marek Czachor[1] and Li You[2]

[1]RLE, MIT, Cambridge, MA 02139, USA
[2]ITAMP, Harvard-Smithsonian CfA, Cambridge, MA 02138, USA

Coherence and Quantum Optics VII
Edited by Eberly, Mandel, and Wolf. Plenum Press. New York, 1996

International Journal of Theoretical Physics, Vol. 38, No. 1, 1999

Spatially Sequential Turn-On of Spontaneous Emission from an Atomic Wave Packet

Marek Czachor[1] and Li You[2]

It didn't clarify the situation: too many approximations, no control over unitarity of evolution etc. (we gave up)

A better strategy:

- We first should understand if the effect is not excluded on general grounds
- Stripe it of all inessential details
- A hint: In EPR-type situations almost anything can be reduced to a 2-qubit problem
- Look for a 2-qubit analogue of the Paris experiment
- Solve it exactly

A 2-qubit exactly solvable toy model

Atomic position

$$|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Ctrl-NOT

Hamiltonian

$$H = \underbrace{b\,\mathbb{I} \otimes \sigma_x}_{\text{free evolution in zones 1 and 0}} + \underbrace{a\,\sigma_x \otimes |1\rangle\langle 1|}_{\text{interaction in zone 1}}$$

$$|11\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |10\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |00\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Atom-photon/vacuum basis

$$|\Psi(0)\rangle = |0\rangle \otimes \left(\alpha|1\rangle + \beta e^{i\phi}|0\rangle\right) = \begin{pmatrix} 0 \\ 0 \\ \alpha \\ \beta e^{i\phi} \end{pmatrix}$$

Initial condition

The question: Does the probability of detecting a „photon”

$$\langle\Psi(t)|\Big(|1\rangle\langle 1|\otimes 1\Big)|\Psi(t)\rangle \;=\; |\Psi_{11}(t)|^2+|\Psi_{10}(t)|^2$$

depend on the phase of the „atomic” initial condition?

$$|\Psi(0)\rangle = |0\rangle\otimes\Big(\alpha|1\rangle+\beta e^{i\phi}|0\rangle\Big) = \begin{pmatrix} 0 \\ 0 \\ \alpha \\ \beta e^{i\phi} \end{pmatrix}$$

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$$|\Psi(0)\rangle = |0\rangle\otimes\Big(\alpha|1\rangle+\beta e^{i\phi}|0\rangle\Big) = \begin{pmatrix} 0\\ 0\\ \alpha\\ \beta e^{i\phi}\end{pmatrix}$$

A naive „non-naive” prediction is something like

Interaction zone 1

$$|\Psi(t)\rangle = \alpha_t|0\rangle\otimes|1\rangle+\gamma_t|1\rangle\otimes|1\rangle + e^{i\phi}\beta_t|0\rangle\otimes|0\rangle = \begin{pmatrix} \gamma_t\\ 0\\ \alpha_t\\ \beta_t e^{i\phi}\end{pmatrix}$$

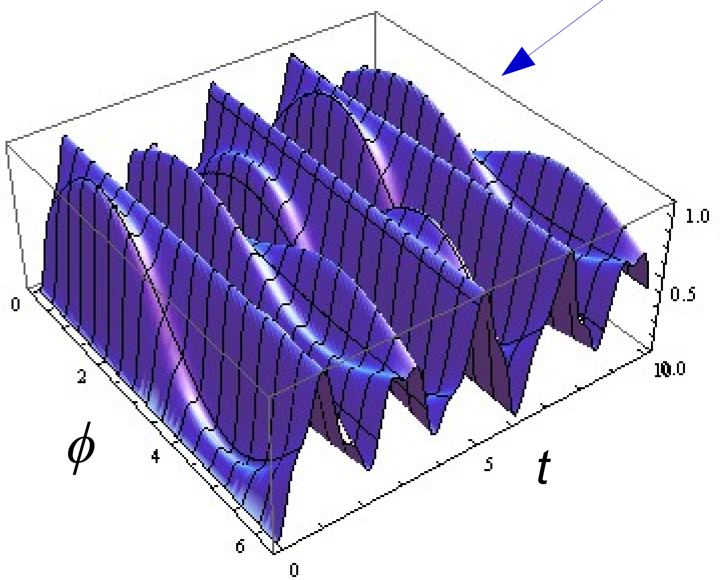
For a=3, b=2 the eigenvalues of *H* are ±1, ±4

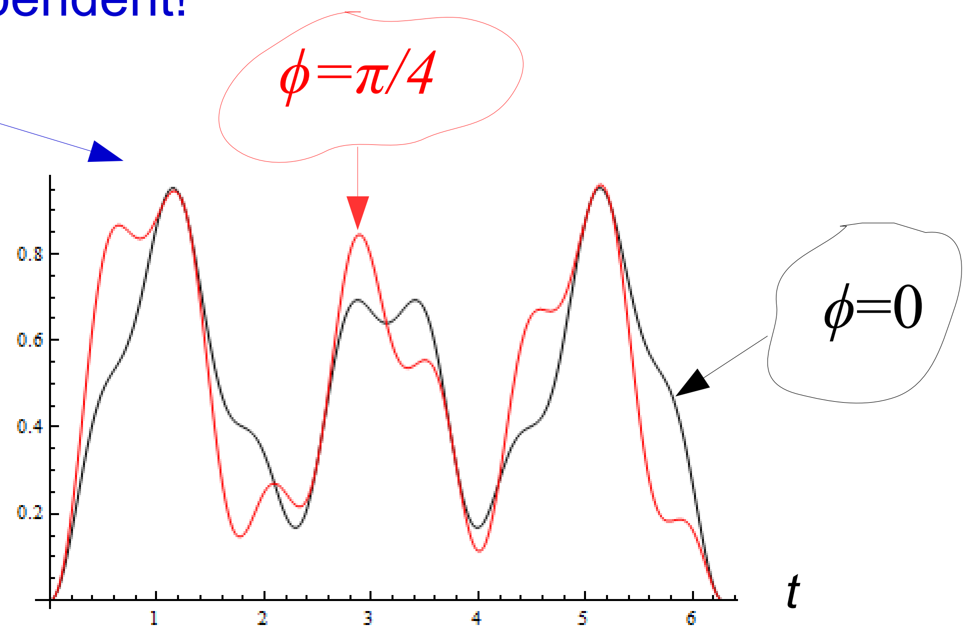
$$
\begin{aligned}
H &= 2\,\mathbb{I}\otimes\sigma_x + 3\sigma_x\otimes|1\rangle\langle 1| \\
&= \begin{pmatrix} 0 & 2 & 3 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}
\end{aligned}
$$

$$
i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle
$$

$$|\Psi(0)\rangle = |0\rangle \otimes \left(\alpha|1\rangle + \beta e^{i\phi}|0\rangle\right) = \begin{pmatrix} 0 \\ 0 \\ \alpha \\ \beta e^{i\phi} \end{pmatrix}$$

Probability of „1-photon" is atomic-phase dependent!





$$|\Psi(t)\rangle = \frac{1}{5}\begin{pmatrix} 2e^{i\phi}\beta(\cos 4t - \cos t) + i\alpha(\sin t - 4\sin 4t) \\ 2\alpha(\cos 4t - \cos t) + ie^{i\phi}\beta(4\sin t - \sin 4t) \\ \alpha(\cos t + 4\cos 4t) - 2ie^{i\phi}\beta(\sin t + \sin 4t) \\ e^{i\phi}\beta(4\cos t + \cos 4t) - 2i\alpha(\sin t + \sin 4t) \end{pmatrix}$$

Modification #1: Both zones coupled in the same way

$$H \;=\; \underbrace{a\,\sigma_x \otimes |0\rangle\langle 0|}_{\text{interaction in zone 0}} \;+\; \underbrace{a\,\sigma_x \otimes |1\rangle\langle 1|}_{\text{interaction in zone 1}} \;+\; \underbrace{b\,1 \otimes \sigma_x}_{\text{free evolution in zones 1 and 0}}$$

$$H \;=\; a\,\sigma_x \otimes 1 + b\,1 \otimes \sigma_x$$

$$U(t) \;=\; e^{-ia\sigma_x t} \otimes e^{-ib\sigma_x t}$$

$$U(t)|0\rangle \otimes (\alpha|1\rangle + \beta e^{i\phi}|0\rangle) \;=\; e^{-ia\sigma_x t}|0\rangle \otimes e^{-ib\sigma_x t}(\alpha|1\rangle + \beta e^{i\phi}|0\rangle)$$

$$\langle\Psi|\Big(|1\rangle\langle 1| \otimes 1\Big)|\Psi\rangle \;=\; \langle 0|e^{ia\sigma_x t}|1\rangle\langle 1|e^{-ia\sigma_x t}|0\rangle = \sin^2 at$$

The effect disappears! (independent evolutions)

Modification #2: Free evolution eliminated

$$H = a\,\sigma_x \otimes |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|\Psi(t)\rangle = \begin{pmatrix} -i\alpha \sin at \\ 0 \\ \alpha \cos at \\ \beta e^{i\phi} \end{pmatrix}$$

$$\langle \Psi | \Big(|1\rangle\langle 1| \otimes 1 \Big) | \Psi \rangle \;=\; \alpha^2 \sin^2 at$$

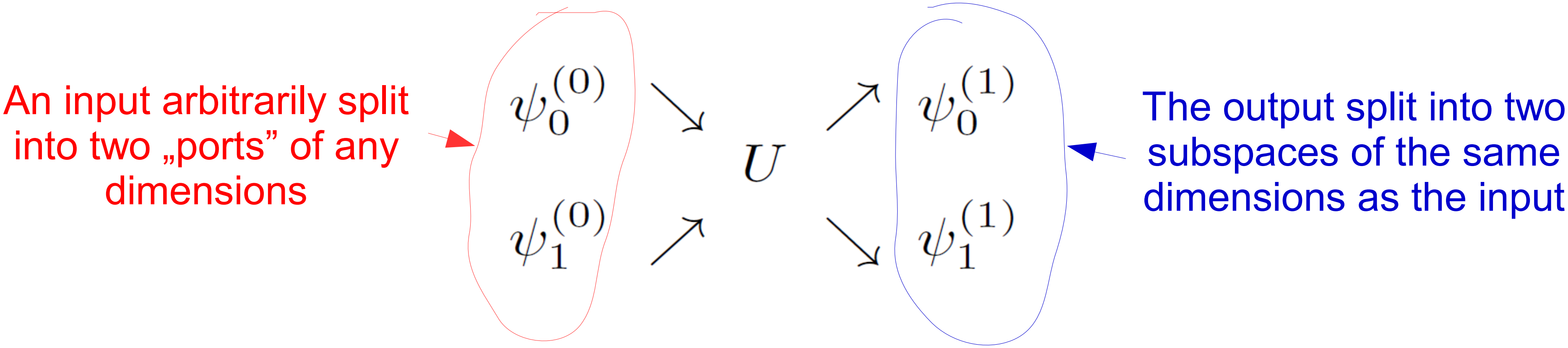
Again the effect disappears!

Preliminary moral

- It seems the 1991-92 experiments discovered a new type of entanglement-related quantum effect
- The effect seems to be a consequence of a „superposed-in-time initial condition”, although standard QM is enough
- The analysis we gave in 1995 was probably basically correct

Part 2: On loops in space and time

Consider a general unitary map (a scattering, a unitary evolution, a beam splitter...)



Consider a general unitary map (a scattering, a unitary evolution, a beam splitter...)

An input arbitrarily split into two „ports" of any dimensions

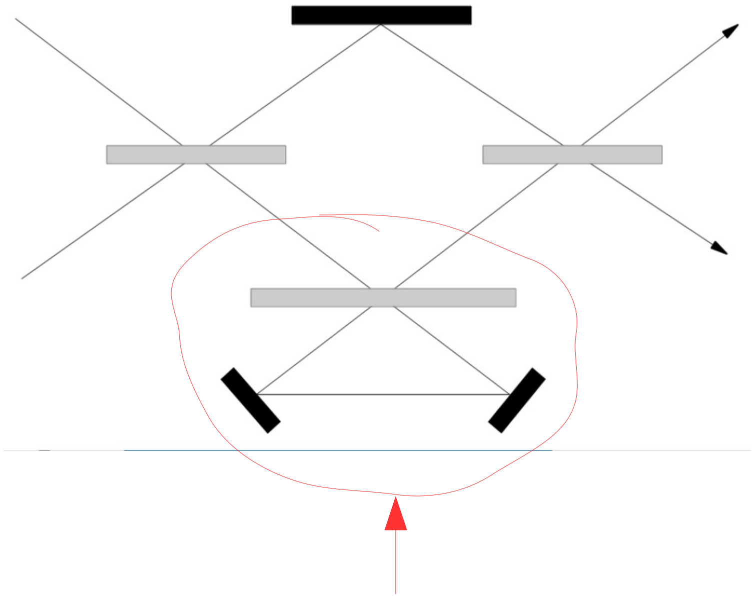
$$\begin{array}{ccccc} \psi_0^{(0)} & \searrow & & \nearrow & \psi_0^{(1)} \\ & & U & & \\ \psi_1^{(0)} & \nearrow & & \searrow & \psi_1^{(1)} \end{array}$$

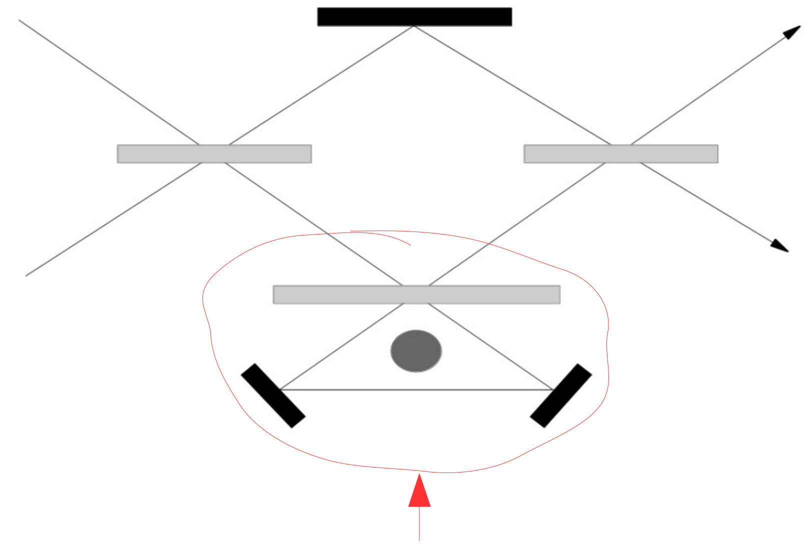
The output split into two subspaces of the same dimensions as the input

The problem: How to formally model a feedback loop where a part of the output is fed again into an input

$$\begin{array}{ccccccc} \psi_0^{(0)} & \searrow & & & & \nearrow & \psi_0^{(1)} \\ & & & U & & & \\ & \nearrow & & & & \searrow & \\ & \longleftarrow & & V & & \longleftarrow & \end{array}$$

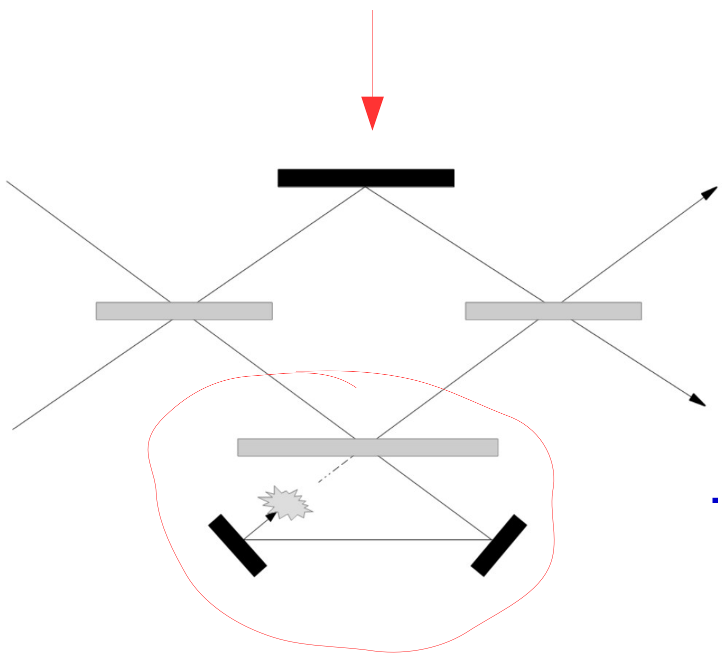
Is it a linear map? If so, what is the formula? What is the dependence on *U* or *V*? Is it useful?

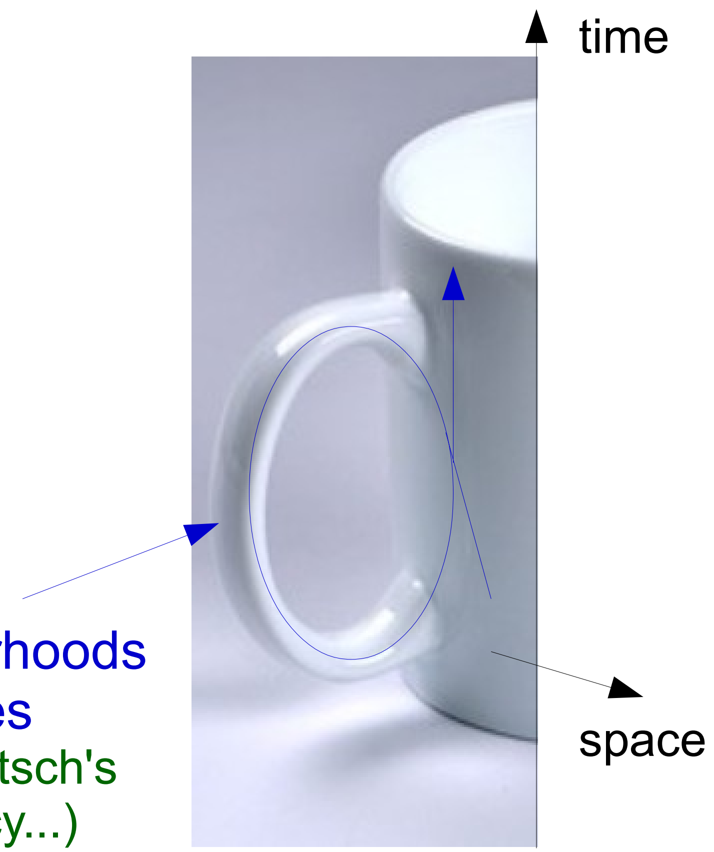




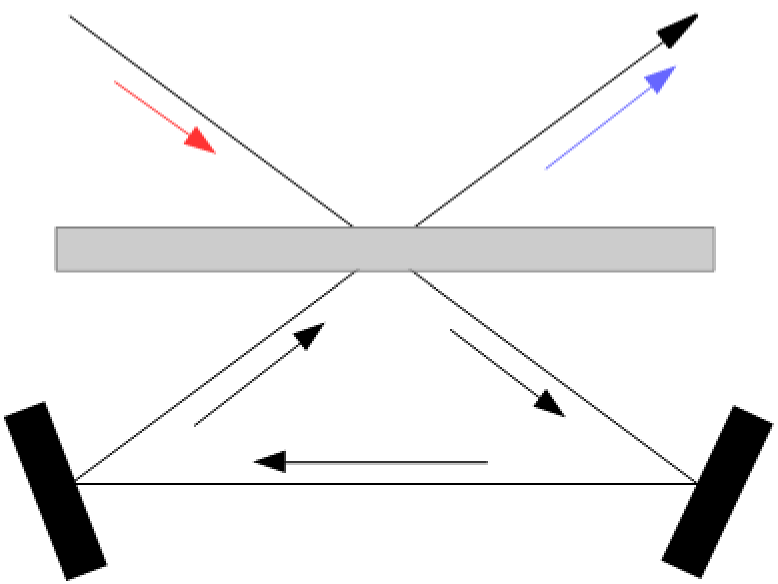
...or Aharonov-Bohm configurations...

Variants of Elitzur-Vaidman...

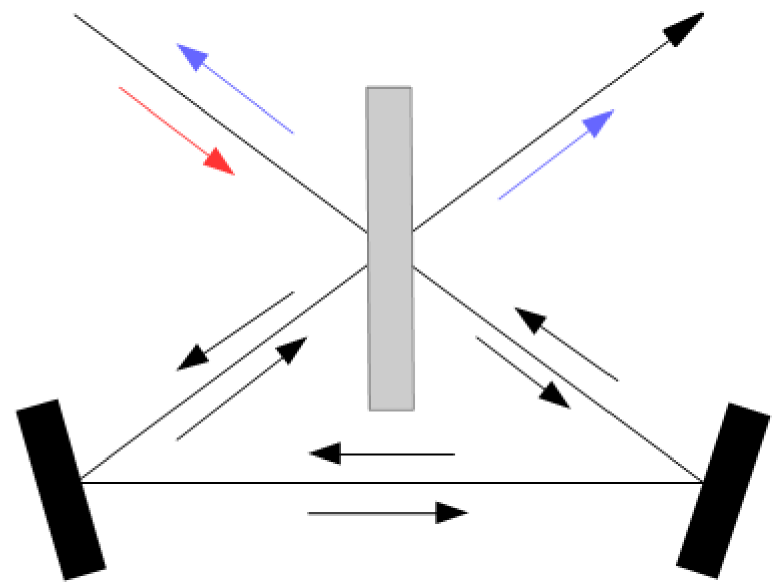




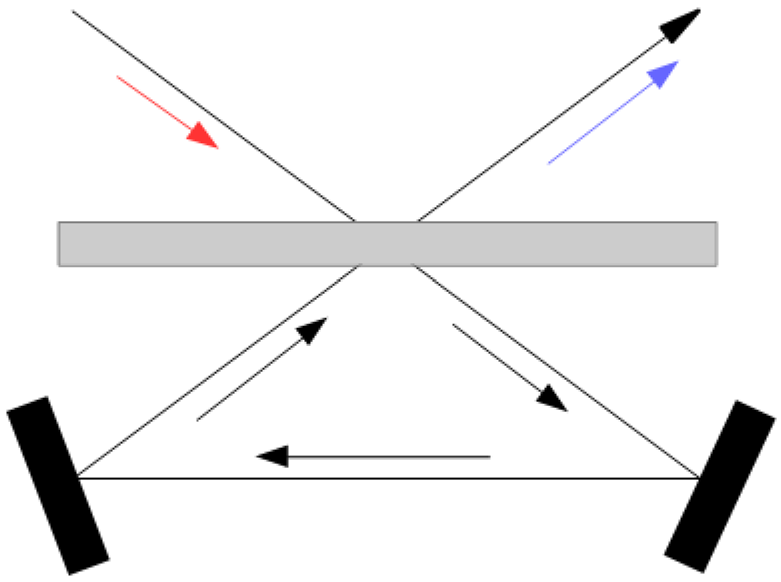
..or dynamics in neighborhoods
of timelike wormholes
(grandfather paradox, Deutsch's
nonlinear self-consistency...)



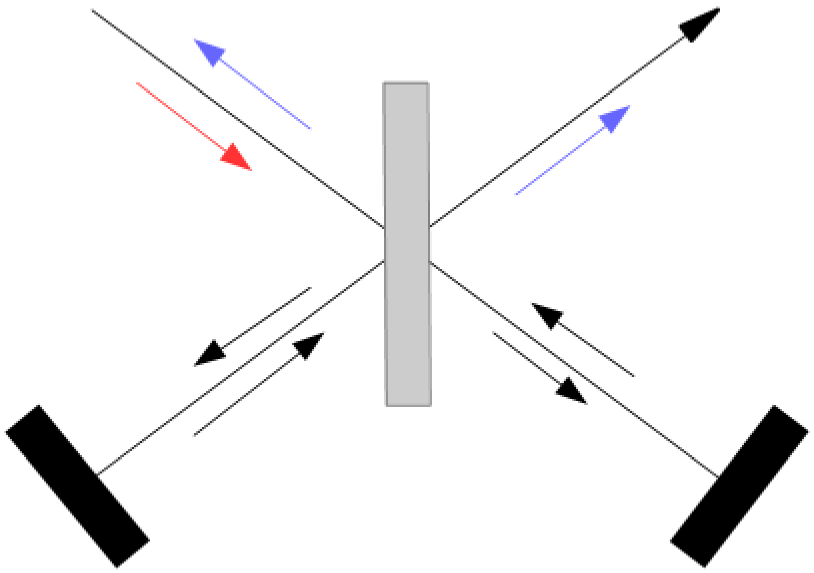
Loop



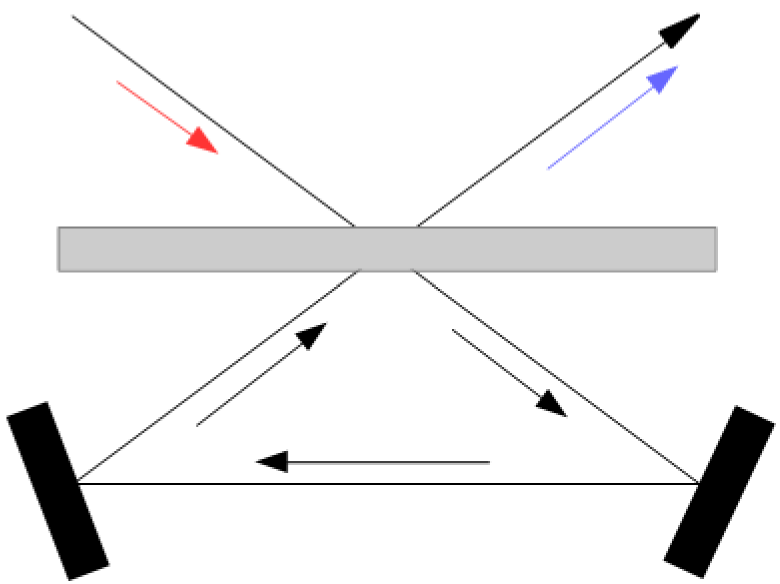
Sagnac interferometer
(no loop)



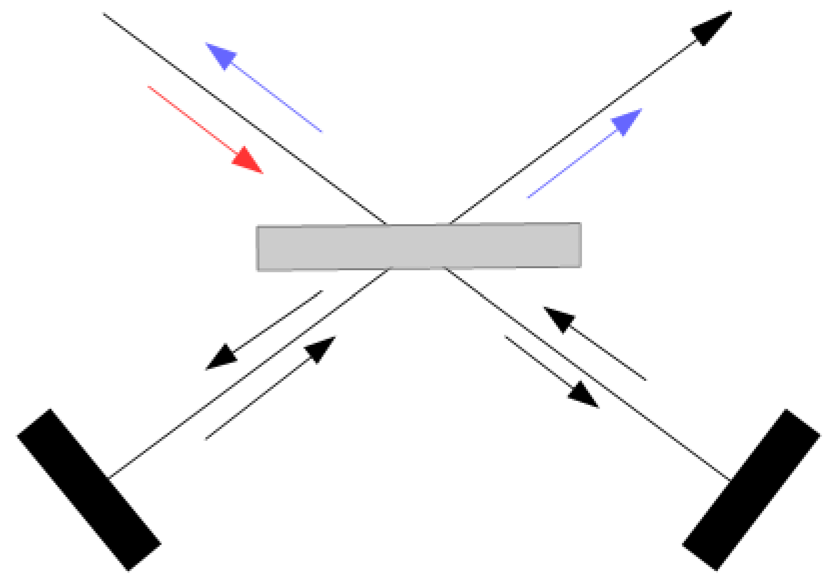
Loop



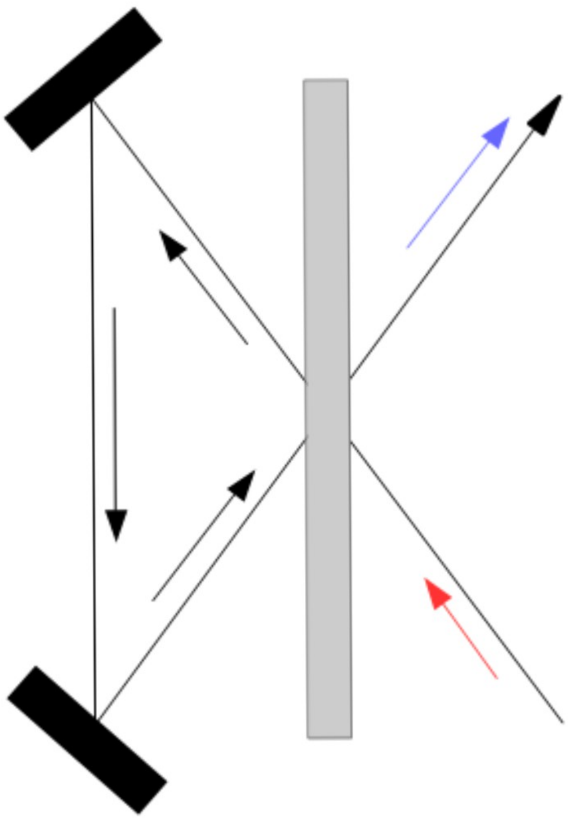
Michelson interferometer
(no loop)

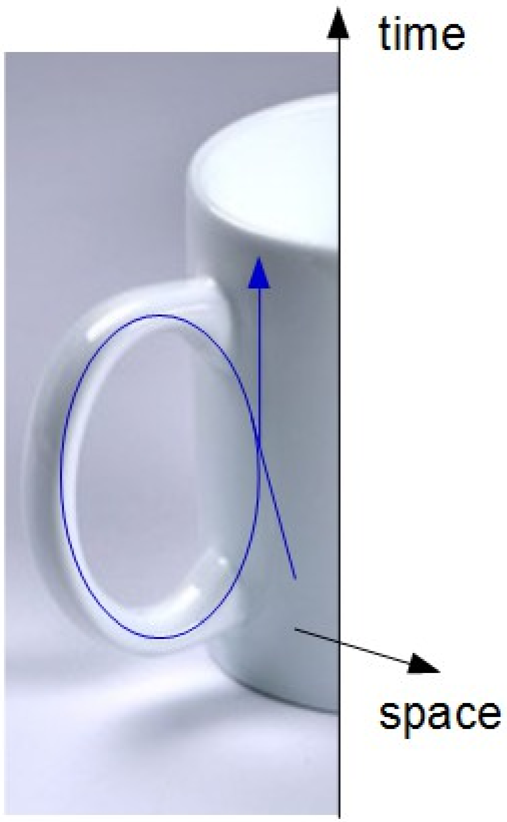


Loop



Michelson-type interferometer with a loop




time
space

$$
\begin{array}{ccccc}
\psi_0^{(0)} & \searrow & & \nearrow & \psi_0^{(1)} \\
& & U & & \\
& \nearrow & & \searrow & \\
\longleftarrow & & V & & \longleftarrow
\end{array}
$$

$$
\begin{array}{ccccc}
\psi_0^{(0)} & \searrow & & \nearrow & \psi_0^{(1)} \\
& & U & & \\
& \nearrow & & \searrow & \psi_1^{(1)}
\end{array}
+
\begin{array}{ccccc}
& \searrow & & \nearrow & \psi_0^{(2)} \\
& & U & & \\
V\psi_1^{(1)} & \nearrow & & \searrow & \psi_1^{(2)}
\end{array}
+
\begin{array}{ccccc}
& \searrow & & \nearrow & \psi_0^{(3)} \\
& & U & & \\
V\psi_1^{(2)} & \nearrow & & \searrow & \psi_1^{(3)}
\end{array}
+ \ldots
$$

Theorem: Let $\psi_0 = \sum_{j=1}^{\infty} \psi_0^{(j)}$. Then $||\psi_0|| = ||\psi_0^{(0)}||$

for any U and V. Any looped beam splitter is fully reflecting!

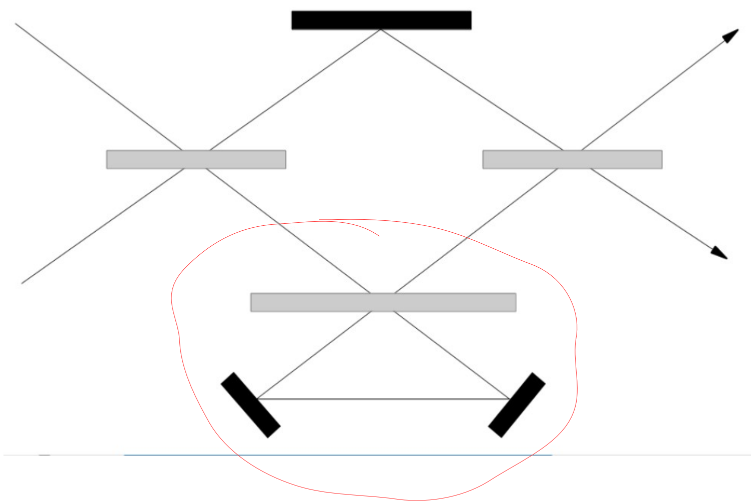
$$
\begin{array}{ccccc}
\psi_0^{(0)} & \searrow & & \nearrow & \psi_0^{(1)} \\
 & & U & & \\
\psi_1^{(0)} & \nearrow & & \searrow & \psi_1^{(1)}
\end{array}
\qquad
\begin{pmatrix} \psi_0^{(1)} \\ \psi_1^{(1)} \end{pmatrix} = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} \begin{pmatrix} \psi_0^{(0)} \\ \psi_1^{(0)} \end{pmatrix}
$$

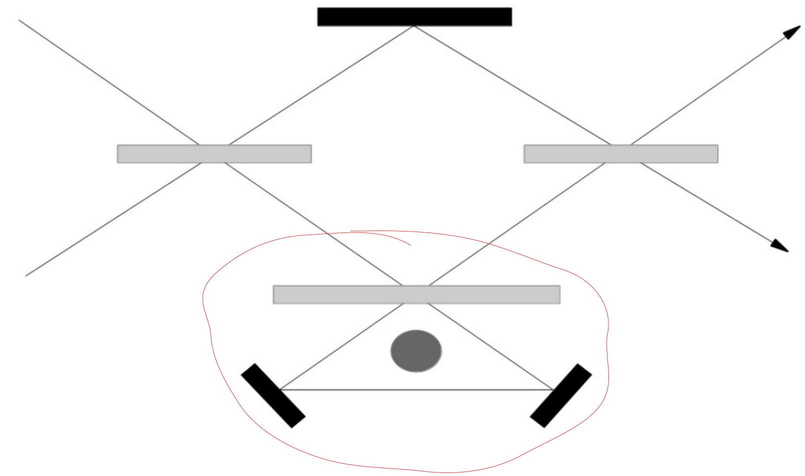
Looped unitary transformation is un-looped-space unitary.
Its explicit form

$$
\psi_0 = \sum_{j=1}^{\infty} \psi_0^{(j)} = \left(U_{00} + U_{01} V \frac{1}{1 - U_{11} V} U_{10} \right) \psi_0^{(0)} = \underset{\circlearrowleft}{U} \psi_0^{(0)}
$$

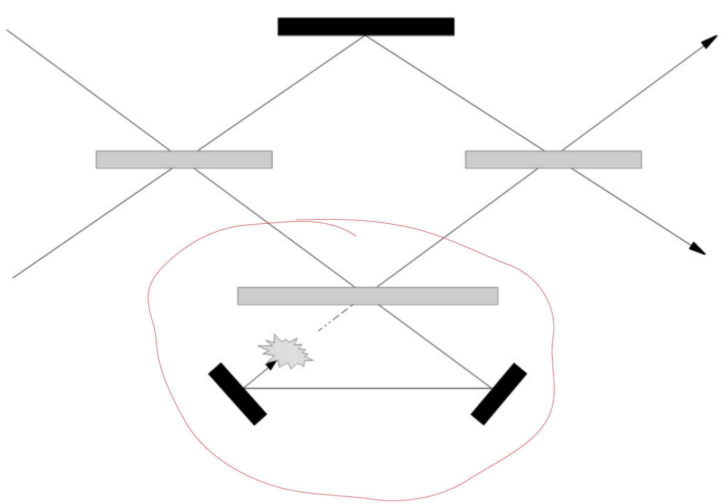
$$
\underset{\circlearrowleft}{U}^{*} \underset{\circlearrowleft}{U} = \underset{\circlearrowleft}{U} \underset{\circlearrowleft}{U}^{*} = \underset{\circlearrowleft}{1}
$$

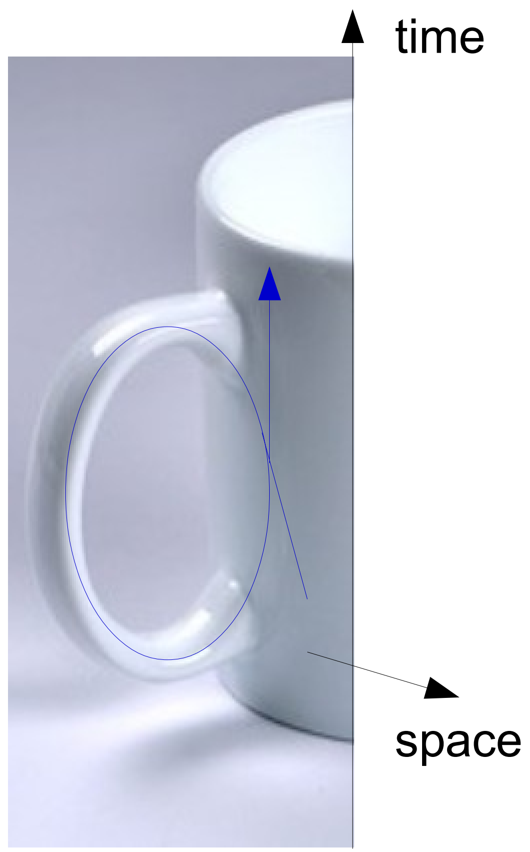
Operators in the un-looped subspace



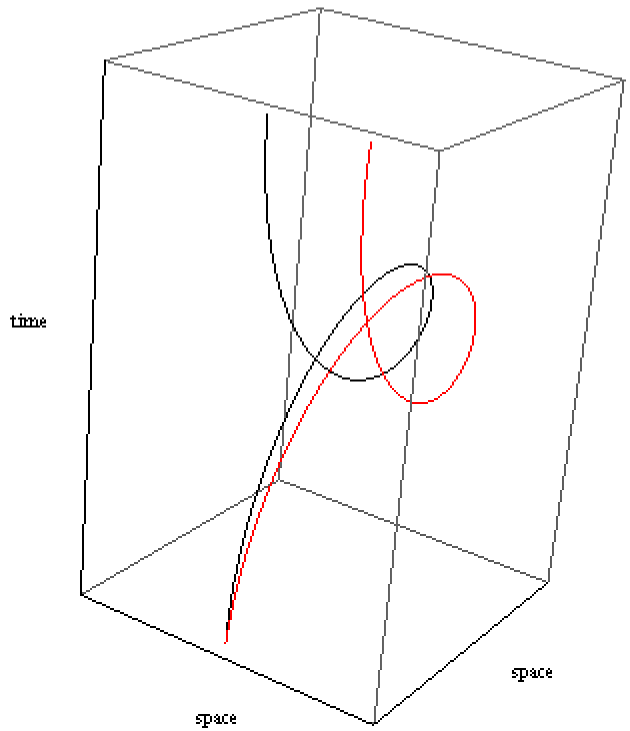


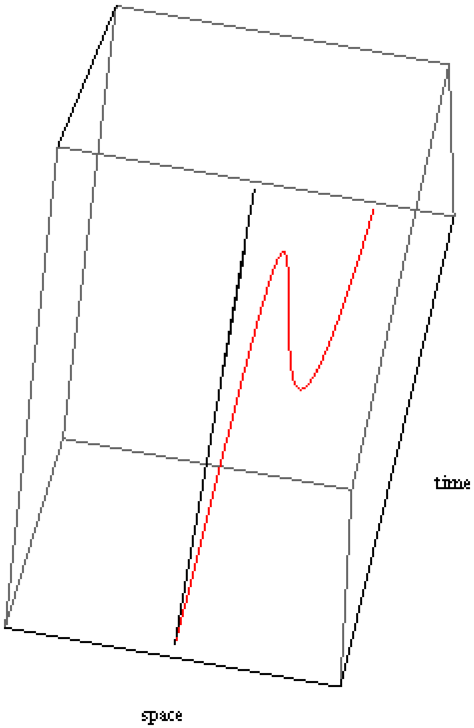
Grandfather paradox disappears
(Elitzur-Vaidman type effect)





If the looped trajectory self-crosses then the crossing point is reflecting





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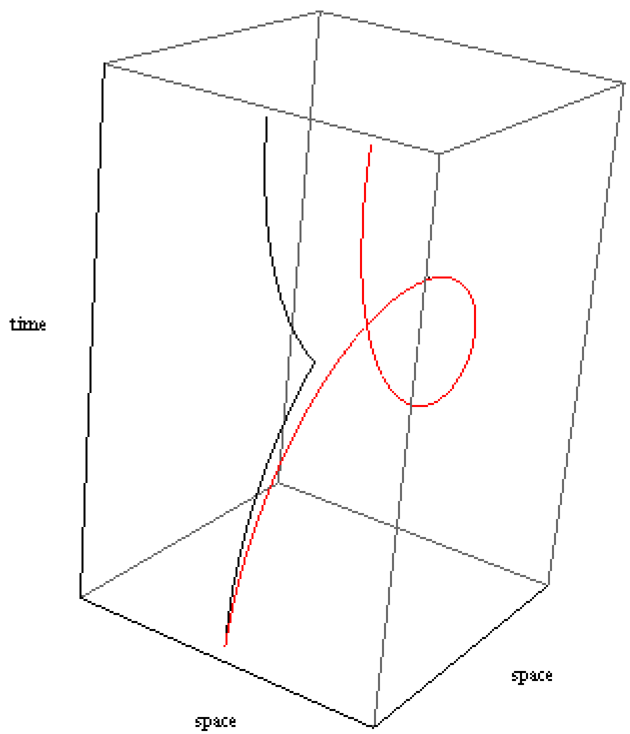
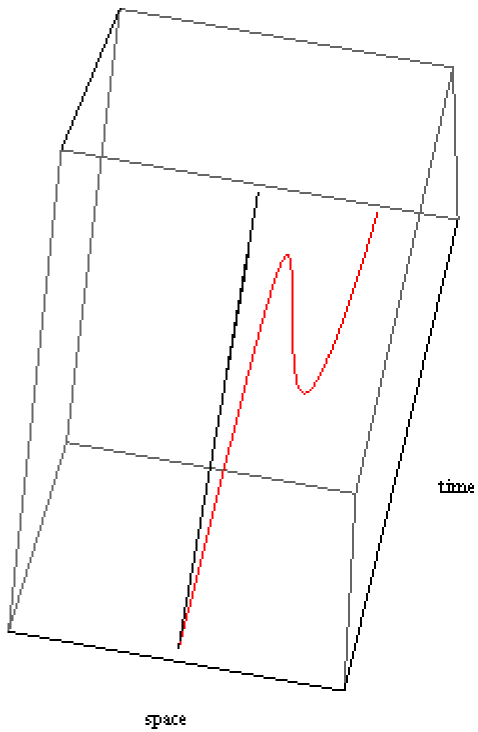
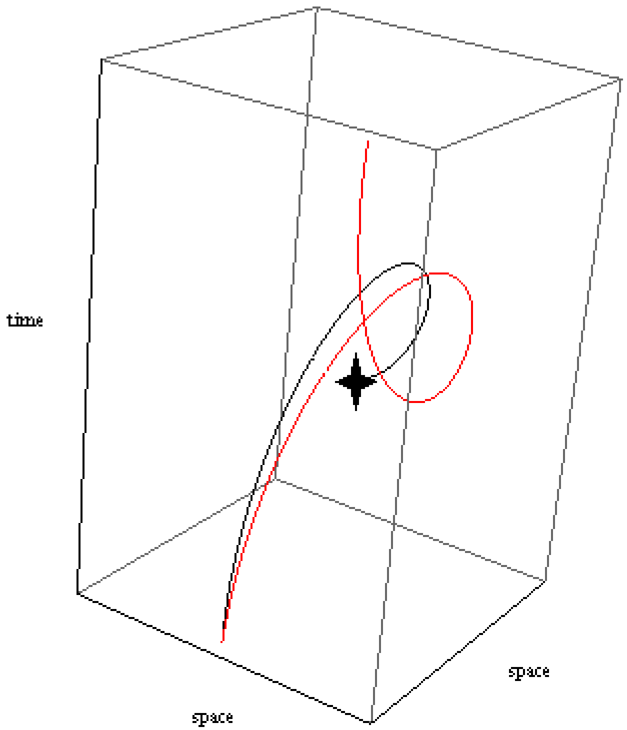




If you are able to enter a timelike wormhole, it means there is no way out! You will never cross your own worldline again



If the loop in principle allows you to cross your own worldline, you will reflect from the entrance to the wormhole.
This is essentially a version of the Elitzur-Vaidman effect.

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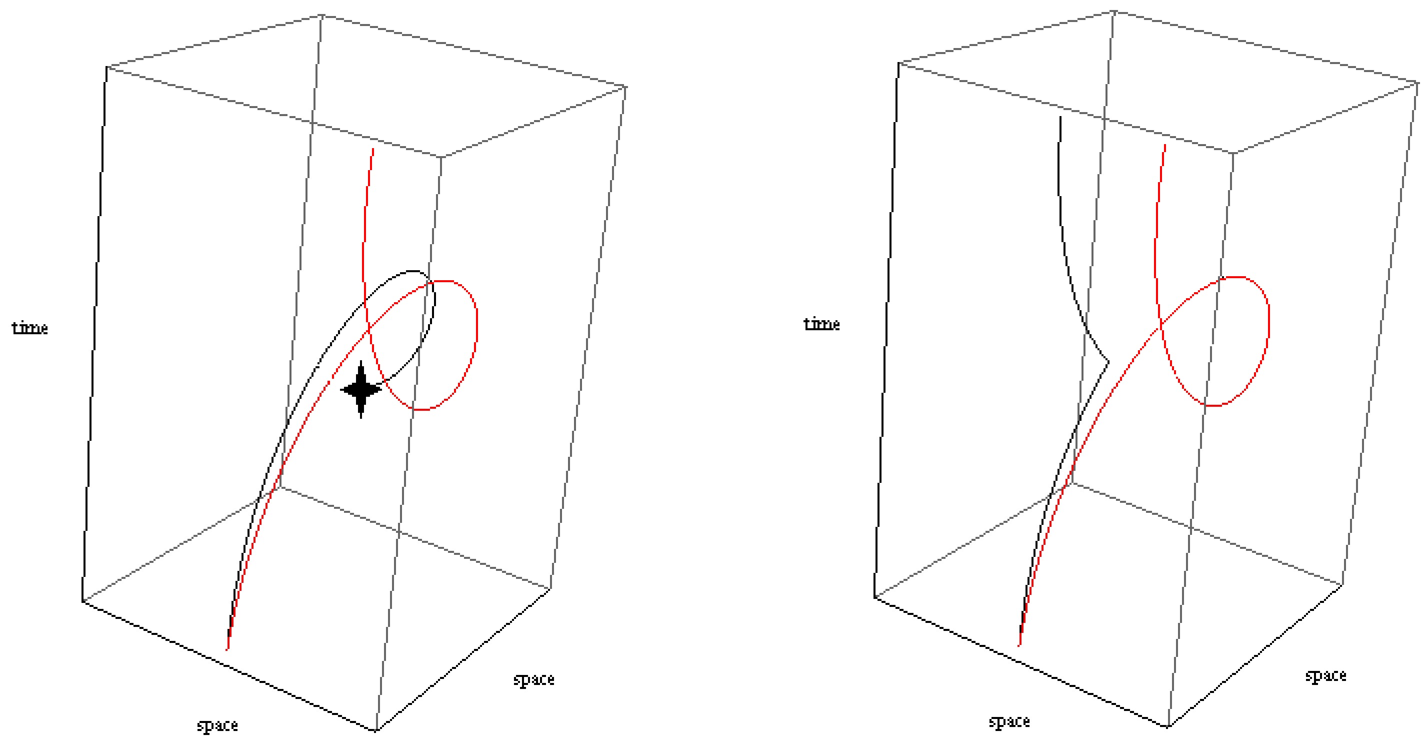

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