Non-paraxial effects in quantum scattering of wave packets.

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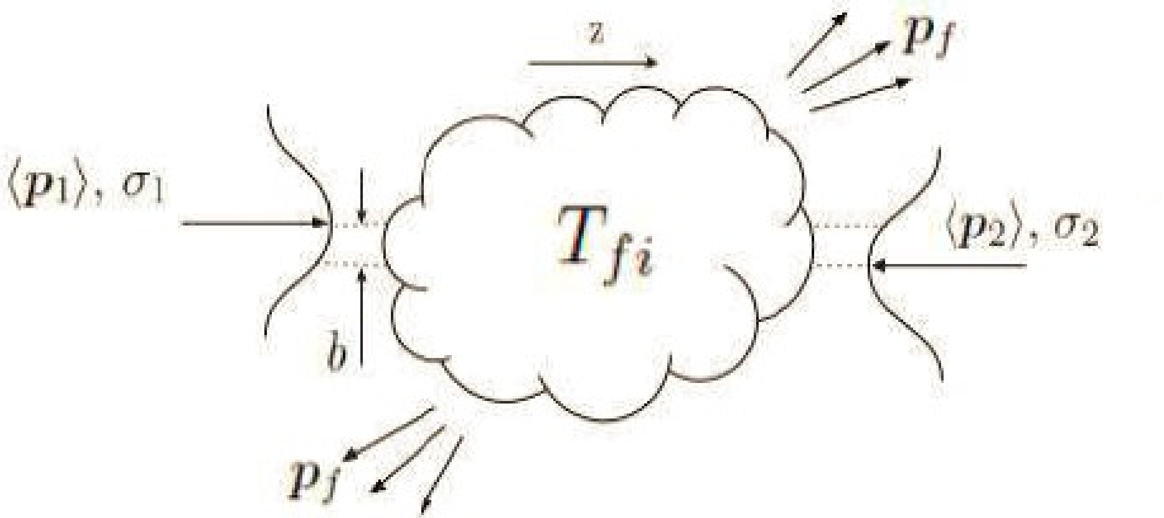
Consider a generic scattering problem 2 → N:



The paraxial approximation works when:

$\sigma \ll m$ ← the packets are narrow in p-space

$\sigma_\perp \gg \lambda_c = \hbar/mc$ ← and wide in x-space

Naively, the non-paraxial corrections are attenuated as

$$\sim \sigma^2/m^2 \sim \lambda_c^2/\sigma_\perp^2 \ll 1$$

For the LHC beam, it is less than 10^{-22}

For modern electron accelerators, it is less than 10^{-14} (and some 2-3 orders larger for ILC and CLIC)

For electron microscopes, it is less than 10^{-6} (!) Verbeeck, et al. 2011

Fortunately, there are also dynamical effects!

The plane-wave approximation in scattering

does *not* work if:

1. The impact parameters are large: an MD-effect (Novisibirsk)

Tikhonov 1982; Kotkin, Serbo, Schiller 1992

2. The initial particles are unstable *Ginzburg 1996; Melnikov, Serbo, 1997*

3. One describes neutrino oscillations *Akhmedov, Smirnov 2009; Akhmedov, Kopp, 2010*

4. The in-states are not Gaussian (!) *Jentschura, Serbo, 2011; Ivanov, 2011*

5. The quantum coherence is lost (!) *Sarkadi 2016; D.K., Serbo 2017*

To be addressed in this talk

Outline

(1) Non-Gaussian quantum states:

I. Vortex photons, electrons and neutrons with orbital angular momentum,

II. Airy photons and electrons,

III. Schrödinger cats,

IV. Their generalizations

(2) Non-paraxial wave packets and the Wigner functions

(3) Non-paraxial effects in scattering:

I. Finite momentum uncertainties, impact-parameters, “approximate” conservation laws, etc.

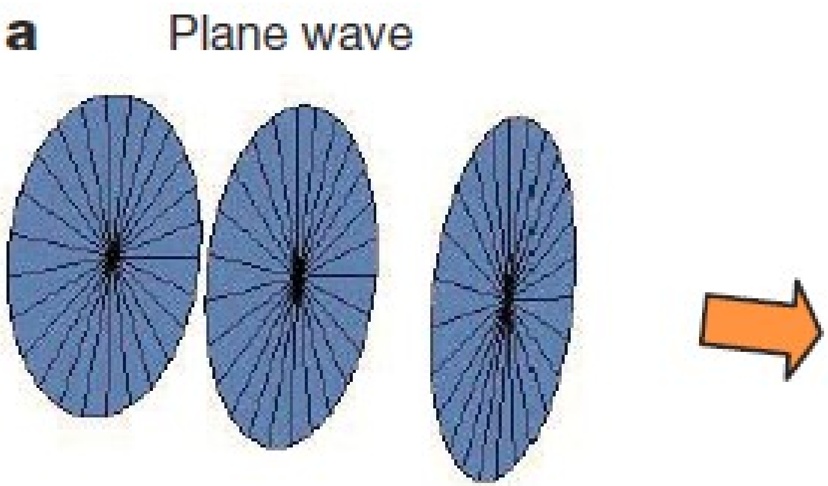
II. Enhancement of the non-paraxial corrections for highly twisted particles

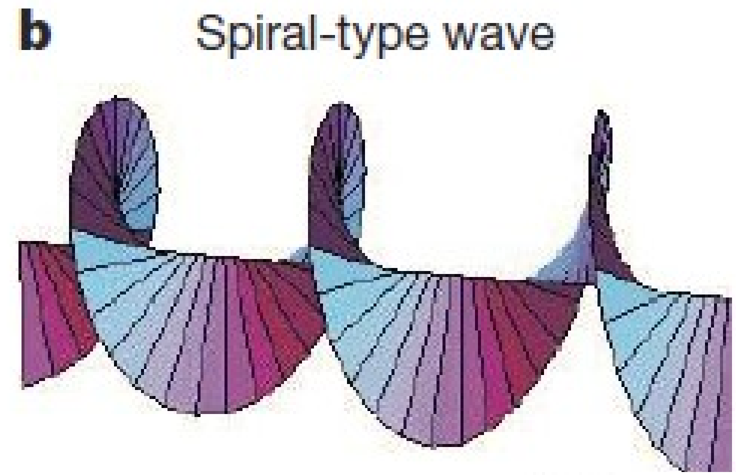
III. Cross sections grow dependent upon a phase of a scattering amplitude (say, hadronic or Coulomb one)

IV. Quantum decoherence and the Wigner functions' negativity may affect the cross section

Vortex particles with orbital angular momentum (OAM)

Twisted photons: Allen, et al. 1992



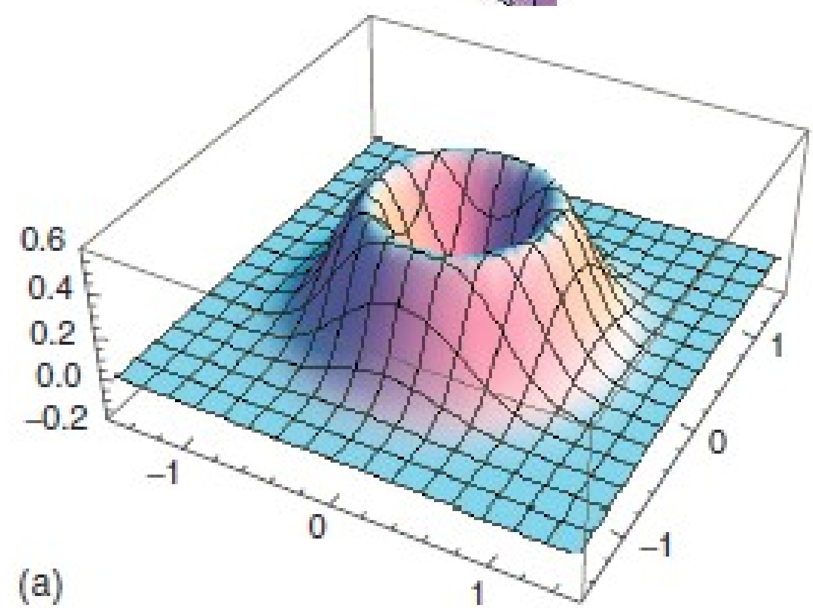


M. Uchida and A.Tonomura, Nature **464**, 737 (2010)

A Bessel-state of a free scalar particle:

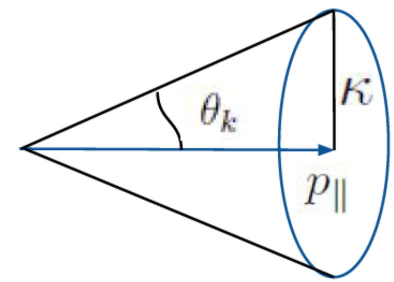
$$\psi(r) = N \; J_\ell(\kappa\rho) e^{-i\varepsilon t + i p_\parallel z + i\ell\phi_r}$$

Probability density
for a well-normalized packet



They form a complete and orthogonal set:

$$\langle p'_{\parallel},\kappa',\ell'|p_{\parallel},\kappa,\ell\rangle = (2\pi)^2 2\varepsilon(p)\,\delta(p_{\parallel}-p'_{\parallel})\frac{\delta(\kappa-\kappa')}{\kappa}\,\delta_{\ell\ell'}$$



$$\hat{\psi}(x) = \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2 2\varepsilon}(\langle x|p_{\parallel},\kappa,\ell\rangle\,\hat{a}_{\{p_{\parallel},\kappa,\ell\}} + \text{H.c.})$$

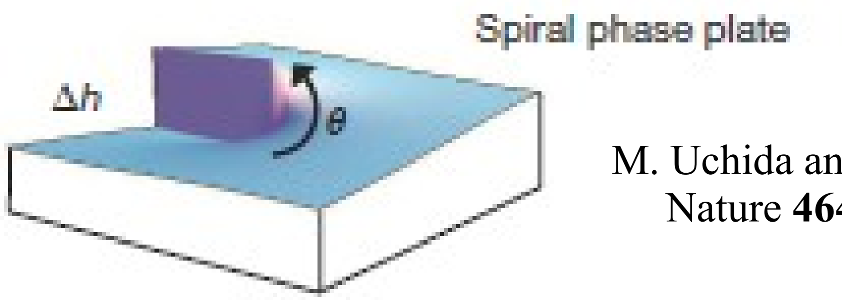
$$= \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2\sqrt{2\varepsilon}}\,(J_{\ell}(\kappa\rho)e^{-i\varepsilon t+ip_{\parallel}z+i\ell\phi_r}\hat{a}_{\{p_{\parallel},\kappa,\ell\}} + \text{H.c.})$$

$$[\hat{a}_{\{p_{\parallel},\kappa,\ell\}},\hat{a}^{\dagger}_{\{p'_{\parallel},\kappa',\ell'\}}] = (2\pi)^2\delta(p_{\parallel}-p'_{\parallel})\frac{\delta(\kappa-\kappa')}{\kappa}\,\delta_{\ell\ell'}$$

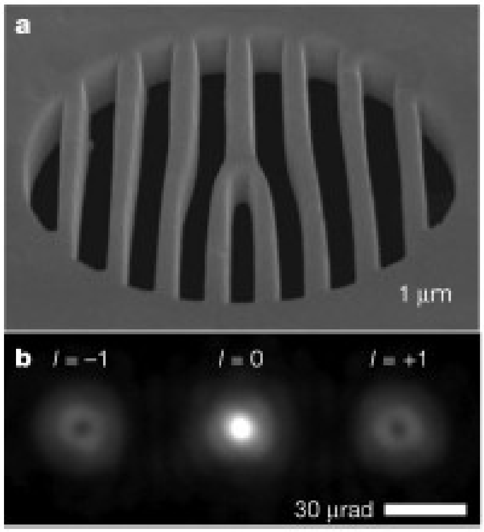
$$[\hat{\psi}(x),\hat{\psi}^{\dagger}(x')] = \sum_{\ell}\int \frac{dp_{\parallel}\kappa d\kappa}{(2\pi)^2 2\varepsilon}J_{\ell}(\kappa\rho)J_{\ell}(\kappa\rho')\,(e^{-i\varepsilon(t-t')+ip_{\parallel}(z-z')+i\ell(\phi_r-\phi'_r)} - \text{c.c.}).$$

D.K., PRA **91** (2015) 013847

Vortex electrons with E = 300 keV were generated in 2010:



M. Uchida and A. Tonomura, Nature **464**, 737 (2010)



*J. Verbeeck, et al., Nature **467**, 301 (2010)*

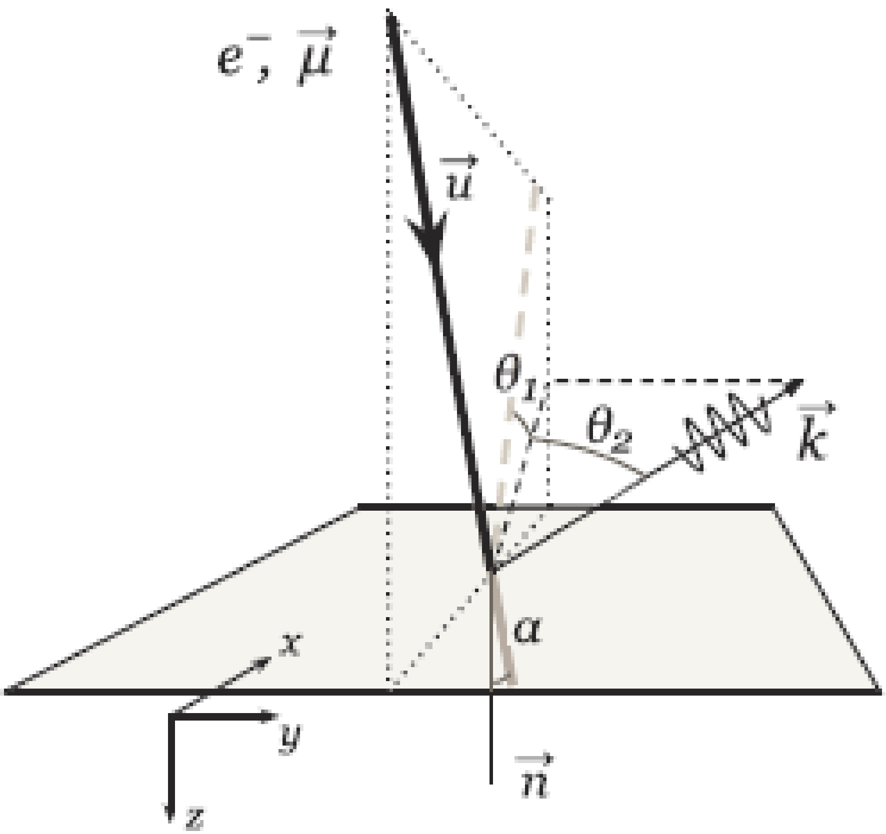
- They can be focused to a spot of 0.1 нм
  *J. Verbeeck, et al., Appl. Phys. Lett. **99**, 203109 (2011)*
- Their OAM can be as high as 1000!
  *E. Mafakheri, et al. Appl. Phys. Let. **110**, 093113 (2017)*
- Magnetic moment of the latter electrons is 3 orders of magnitude larger than the Bohr magneton!
  *K.Yu. Bliokh, et al., PRL **107**, 174802 (2011)*

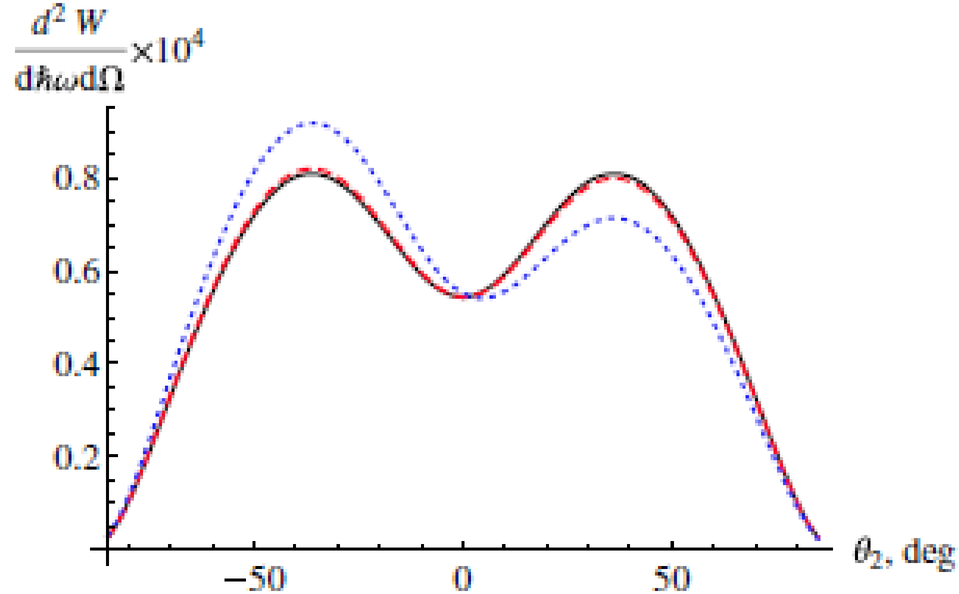
Vortex particles with orbital angular momentum (OAM)

The huge magnetic moment → “Orbital light”:

Transition radiation:

Angular asymmetry of ~ 0.1 – 1%





I.P. Ivanov, D.K., PRL **110** (2013) 264801

FIG. 2 (color online). Distribution of the forward TR over θ_2 for $\ell = 0$ (black solid line), $\ell = 1000$ (red dashed line), and $\ell = 10000$ (blue dotted line). Parameters are $\alpha = 70°$, $\theta_1 = -40°$, $\hbar\omega = 5$ eV.

Berry, Balazs 1979

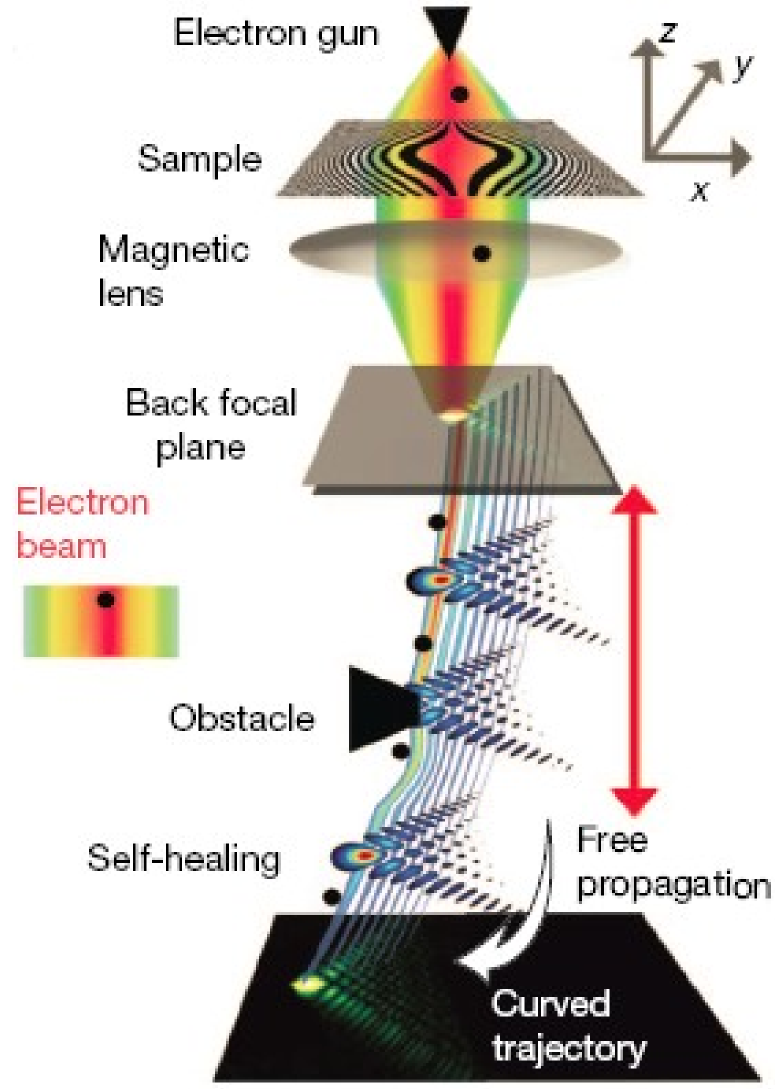
For an ideal Airy beam:

1. There is no spreading

2. A curved path in free space

3. Self-healing after scattering

Experimental realization
for 200 keV electrons →



The figure from N. Voloch-Bloch, et al., Nature **494** (2013) 331

Lorentz-invariant non-Gaussian packets beyond the paraxial regime:

$$\psi(p) = \frac{2^{3/2}\pi}{\sigma} \frac{e^{-m^2/\sigma^2}}{\sqrt{K_1(2m^2/\sigma^2)}} \exp\left\{\frac{(p-\bar{p})^2}{2\sigma^2}\right\}$$

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2\varepsilon} |\psi(p)|^2 = 1,$$

$$p^2 = \bar{p}^2 = m^2$$

Naumov, Naumov 2010

In the paraxial regime, this turns into a 3D Gaussian packet:

$$\frac{(p-\bar{p})^2}{2\sigma^2} = -\frac{1}{2\sigma^2} (\delta_{ij} - \bar{u}_i \bar{u}_j) (\boldsymbol{p} - \bar{\boldsymbol{p}})_i (\boldsymbol{p} - \bar{\boldsymbol{p}})_j + \mathcal{O}((\boldsymbol{p} - \bar{\boldsymbol{p}})^3)$$

Mean energy:

$$\langle \varepsilon \rangle = \int d^3x \, T^{00} = \bar{\varepsilon} \frac{K_2(2m^2/\sigma^2)}{K_1(2m^2/\sigma^2)} = \bar{\varepsilon} \left(1 + \frac{3}{4}\frac{\sigma^2}{m^2} + \mathcal{O}(\sigma^4/m^4)\right)$$

Non-paraxial correction!

A relativistic generalization for a vortex boson is

$$\psi_\ell(p) = \frac{2^{3/2}\pi}{\sigma^{|\ell|+1}\sqrt{|\ell|!}}\, p_\perp^{|\ell|}\, \frac{e^{-m^2/\sigma^2}}{\sqrt{K_{|\ell|+1}(2m^2/\sigma^2)}}\, \exp\left\{\frac{(p-\bar{p})^2}{2\sigma^2} + i\ell\phi_p\right\}$$

They are orthogonal: $$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2\varepsilon}\,[\psi_{\ell'}(p)]^*\,\psi_\ell(p) = \delta_{\ell,\ell'}$$

An exact solution to the Klein-Gordon equation:

$$\psi_\ell(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\varepsilon}\,\psi_\ell(p)e^{-ipx} = \frac{(i\rho)^{|\ell|}}{\sqrt{2|\ell|!}\,\pi}\,\frac{\sigma^{|\ell|+1}}{\varsigma^{|\ell|+1}}\,\frac{K_{|\ell|+1}(\varsigma m^2/\sigma^2)}{\sqrt{K_{|\ell|+1}(2m^2/\sigma^2)}}\, e^{i\ell\phi_r}$$

$$\varsigma = \frac{1}{m}\sqrt{(\bar{p}_\mu + ix_\mu\sigma^2)^2} = \mathrm{inv},\ \mathrm{Re}\,\varsigma > 0$$

And analogously for a fermion...

D.K., ArXiv: 1803.09150; 1803.10166

A mean momentum of such a vortex packet is

$$\langle p_\ell^\mu \rangle = \{\langle \varepsilon_\ell \rangle, \langle \boldsymbol{p}_\ell \rangle\} = \{\bar{\varepsilon}, \bar{\boldsymbol{p}}\} \frac{K_{|\ell|+2}(2m^2/\sigma^2)}{K_{|\ell|+1}(2m^2/\sigma^2)} \simeq \{\bar{\varepsilon}, \bar{\boldsymbol{p}}\} \left(1 + \left(\frac{3}{4} + \frac{|\ell|}{2}\right) \frac{\sigma^2}{m^2}\right)$$

The packet's invariant mass:

$$m_\ell^2 = \langle p_\ell \rangle^2 \simeq m^2 \left(1 + \left(\frac{3}{2} + |\ell|\right) \frac{\sigma^2}{m^2}\right)$$

$$\frac{\delta m_\ell}{m_{\text{inv}}} \simeq \frac{\delta m_\ell}{m} \lesssim 10^{-3}$$

For the electrons with

$|\ell| \sim 10^3 \quad \sigma_\perp \gtrsim 0.1 \text{ nm}$

For the vortex electron's magnetic moment:

$$\boldsymbol{\mu}_f = \frac{1}{2} \int d^3r\, \mathbf{r} \times \bar{\psi}_f(x) \gamma \psi_f(x) \simeq \frac{1}{2\bar{\varepsilon}} (\zeta + \hat{\boldsymbol{z}}\, \ell) \Big(1 + \mathcal{O}(|\ell| \sigma^2/m^2)\Big)$$

Enhancement due to the OAM!

D.K., ArXiv: 1803.09150; 1803.10166

Scattering of wave packets and the Wigner functions

$$S_{fi} = \langle pw|\hat{S}|i\rangle = \int \prod_{i=1}^{N} \frac{d^3 p_i}{(2\pi)^3}\, \psi_i(p_i) S_{fi}^{(pw)}$$

Is there a small parameter?

The plane-wave limit: $\sigma_i \to 0, \quad p_i \to p_i' \to \langle p_i \rangle$ therefore $\frac{p_i + p_i'}{2} \to \langle p_i \rangle,\ p_i - p_i' \to 0$

In the new variables $\frac{p_i + p_i'}{2} \to p_i,\ p_i - p_i' \to k_i$ we get $|k_i| \ll |p_i|$ when $\sigma \ll m$

A density matrix in these new variables is called a Wigner function

Wigner 1932

The scattering probability can be expressed via the Wigner functions:

$$dW = |S_{fi}|^2 \prod_{f=3}^{N_f+2} V \frac{d^3 p_f}{(2\pi)^3} = \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3}\, d\sigma(\boldsymbol{k}, \boldsymbol{p}_{1,2})\, \mathcal{L}^{(2)}(\boldsymbol{k}, \boldsymbol{p}_{1,2}),$$

Kotkin, Serbo, Schiller, Int. J. Mod. Phys. A7 (1992) 4707

$$d\sigma(\boldsymbol{k}, \boldsymbol{p}_{1,2}) = (2\pi)^4\, \delta\Big(\varepsilon_1(\boldsymbol{p}_1 + \boldsymbol{k}/2) + \varepsilon_2(\boldsymbol{p}_2 - \boldsymbol{k}/2) - \varepsilon_f\Big)\, \delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_f)$$
$$\times T_{fi}^{(pw)}(\boldsymbol{p}_1 + \boldsymbol{k}/2, \boldsymbol{p}_2 - \boldsymbol{k}/2) T_{fi}^{*\,(pw)}(\boldsymbol{p}_1 - \boldsymbol{k}/2, \boldsymbol{p}_2 + \boldsymbol{k}/2) \frac{1}{\upsilon(\boldsymbol{p}_1, \boldsymbol{p}_2)} \prod_{f=3}^{N_f+2} \frac{d^3 p_f}{(2\pi)^3},$$

Matches the customary cross section when **k** = 0!

$$\mathcal{L}^{(2)}(\boldsymbol{k}, \boldsymbol{p}_{1,2}) = \upsilon(\boldsymbol{p}_1, \boldsymbol{p}_2) \int dt\, d^3 r\, d^3 R\, e^{i\boldsymbol{k}\boldsymbol{R}}\, \underline{n_1}(\boldsymbol{r}, \boldsymbol{p}_1, t) \underline{n_2}(\boldsymbol{r} + \boldsymbol{R}, \boldsymbol{p}_2, t),$$

$$\upsilon(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}}{\varepsilon_1(\boldsymbol{p}_1)\varepsilon_2(\boldsymbol{p}_2)} = \sqrt{(\boldsymbol{u}_1 - \boldsymbol{u}_2)^2 - [\boldsymbol{u}_1 \times \boldsymbol{u}_2]^2},$$

the Wigner functions

What do we lose in the paraxial regime?

For a non-relativistic Airy beam: $\psi(\boldsymbol{p}) = \pi^{3/4}\left(\frac{2}{\sigma}\right)^{3/2} \exp\left\{ -i\boldsymbol{r}_0\boldsymbol{p} - \frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{2\sigma^2} + \frac{i}{3}\left(\xi_x^3 p_x^3 + \xi_y^3 p_y^3\right)\right\}$

The exact Wigner function is
(not everywhere positive)

$$n(\boldsymbol{r},\boldsymbol{p},t;\boldsymbol{\xi}) = 2^{13/3}\,\frac{\pi}{\sigma^2\xi_x\xi_y}\exp\left\{ -\sigma^2(z-\langle z\rangle)^2 - \frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{\sigma^2} + \right.$$
$$\left. + \frac{1}{\sigma^2\xi_x^3}\left(x-\langle x\rangle + \xi_x^3p_x^2 + \frac{1}{6\sigma^4\xi_x^3}\right) + \frac{1}{\sigma^2\xi_y^3}\left(y-\langle y\rangle + \xi_y^3p_y^2 + \frac{1}{6\sigma^4\xi_y^3}\right)\right\},$$
$$\times \mathrm{Ai}\left[\frac{2^{2/3}}{\xi_x}\left(x-\langle x\rangle + \xi_x^3p_x^2 + \frac{1}{4\sigma^4\xi_x^3}\right)\right]\mathrm{Ai}\left[\frac{2^{2/3}}{\xi_y}\left(y-\langle y\rangle + \xi_y^3p_y^2 + \frac{1}{4\sigma^4\xi_y^3}\right)\right]$$

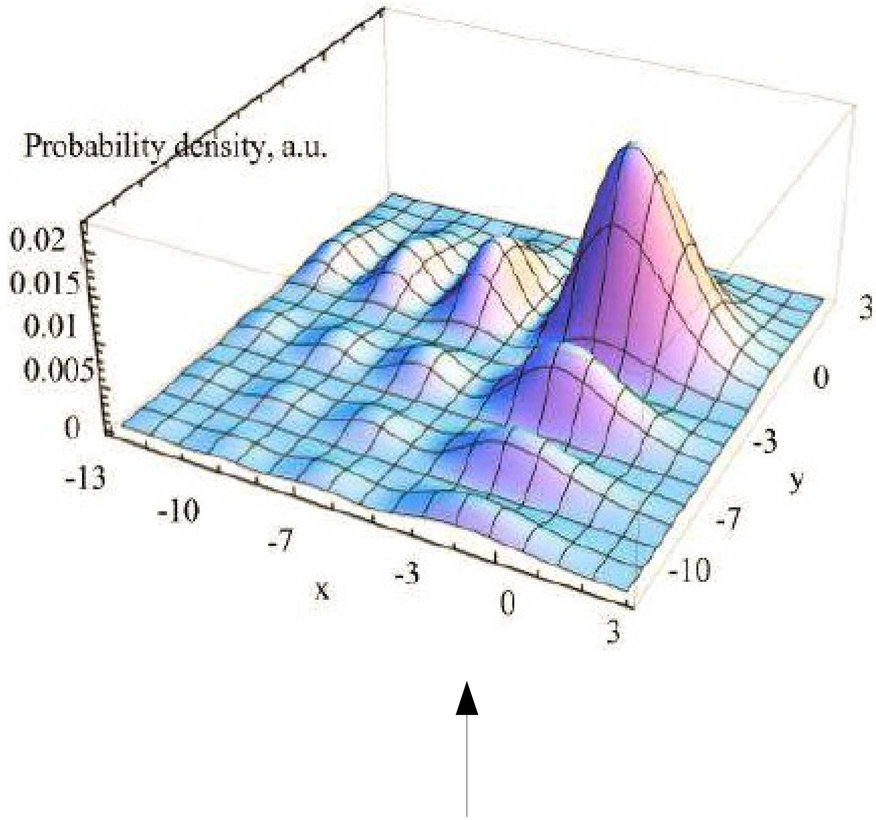
The approximate/paraxial one is
(everywhere positive)

$$n(\boldsymbol{r},\boldsymbol{p},t;\boldsymbol{\xi}) = 8\,\exp\left\{-\frac{(\boldsymbol{p}-\langle\boldsymbol{p}\rangle)^2}{\sigma^2} - \sigma^2\left(\boldsymbol{r}-\langle\boldsymbol{r}\rangle+\boldsymbol{\eta}\right)^2\right\}$$

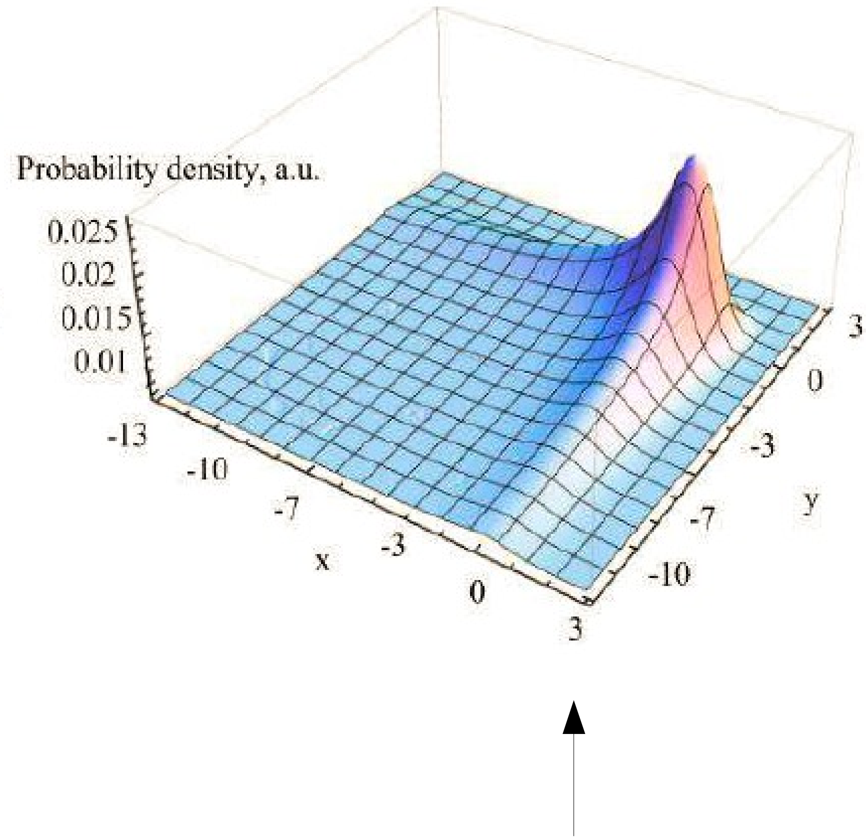
$$\boldsymbol{\eta} \equiv \boldsymbol{\eta}(\boldsymbol{p}_\perp) = \{\xi_x^3p_x^2, \xi_y^3p_y^2, 0\}$$

Possible quantum decoherence is lost!

Non-paraxial– vs. paraxial Wigner functions



The exact Airy



The paraxial Airy

$$m = 1, \sigma/\langle p\rangle_z = 1/5,\ \xi_x = \xi_y = 2/\sigma,\ r_0 = z = t = \langle p\rangle_\perp = 0$$

Non-paraxial effects in scattering

We represent the scattering amplitude as follows: $T_{fi} = |T_{fi}| \exp\{i\zeta_{fi}\}$

$$T_{fi}(p_1 + k/2, p_2 - k/2) T^*_{fi}(p_1 - k/2, p_2 + k/2) \approx$$

$$\approx \left(|T_{fi}|^2 + \frac{1}{4} k_i k_j C_{ij} + \mathcal{O}(k^4) \right) \exp\left\{ i k \underline{\partial_{\Delta p} \zeta_{fi}} + \mathcal{O}(k^3) \right\}$$

$$\partial_{\Delta p} = \frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2},$$

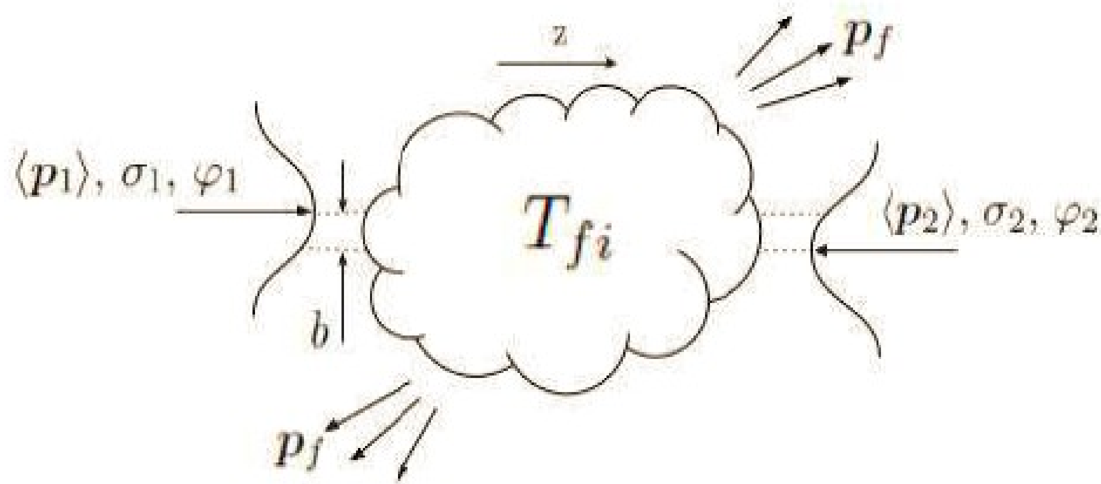
$$C_{ij}(\mathbf{p}_1, \mathbf{p}_2) = |T_{fi}| \partial_{\Delta p_i} \partial_{\Delta p_j} |T_{fi}| - (\partial_{\Delta p_i} |T_{fi}|)(\partial_{\Delta p_j} |T_{fi}|)$$

$$\tilde{b}_\varphi = b - \frac{\partial \varphi_1(p_1)}{\partial p_1} + \frac{\partial \varphi_2(p_2)}{\partial p_2} - \left(\frac{\partial}{\partial p_1} - \frac{\partial}{\partial p_2} \right) \zeta_{fi}.$$

The amplitude's phase

Impact-parameter

Phases of the in-states



Non-paraxial effects in scattering

The first correction to the plane-wave cross section:

$$d\sigma = dN/L \approx d\sigma^{(pw)} + \underline{d\sigma^{(1)}}$$

provided the packets do not spread much during the collision: $t_{\text{col}} \ll t_{\text{diff}} \sim \dfrac{\sigma_b}{u_\perp} \sim \sigma_b^2 \varepsilon$

$$\frac{d\sigma^{(1)}}{d\sigma^{(pw)}} = \text{“geometric” terms} + \text{dynamic terms}$$

$\sim \dfrac{\sigma_1^2}{m_1^2}$ and $\sim \dfrac{\sigma_2^2}{m_2^2}$ (→ “geometric” terms)

Also depend on the phases and on *an overlap of the in-states* (→ dynamic terms)

D.K., JHEP **03** (2017) 049

Non-paraxial effects in scattering

Interference of the incoming packets is governed by

$$\left(\frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_2^2}\right)^{-1} = \frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

← Due to a finite overlap of two non-orthogonal packets!

A corresponding term in the cross section is:

$$\frac{d\sigma^{(1)}}{d\sigma^{(pw)}} \propto \frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\left[\frac{\Delta \boldsymbol{u}}{|\Delta \boldsymbol{u}|} \times \left[\frac{\Delta \boldsymbol{u}}{|\Delta \boldsymbol{u}|} \times \langle \boldsymbol{b}_\varphi \rangle\right]\right] \cdot \left(\frac{\partial}{\partial \boldsymbol{p}_2} - \frac{\partial}{\partial \boldsymbol{p}_1}\right) \zeta_{fi}\Big|_{\boldsymbol{p}_{1,2}=\langle \boldsymbol{p}\rangle_{1,2}}$$

$$\Delta \boldsymbol{u} = \boldsymbol{u}_1 - \boldsymbol{u}_2 \qquad \boldsymbol{b}_\varphi = \boldsymbol{b} - \frac{\partial \varphi_1(\boldsymbol{p}_1)}{\partial \boldsymbol{p}_1} + \frac{\partial \varphi_2(\boldsymbol{p}_2)}{\partial \boldsymbol{p}_2}$$

This results in an azimuthal asymmetry:

$$\mathcal{A}[\boldsymbol{b}_\varphi] = \frac{dW[\boldsymbol{b}_\varphi] - dW[-\boldsymbol{b}_\varphi]}{dW[\boldsymbol{b}_\varphi] + dW[-\boldsymbol{b}_\varphi]} = \frac{d\sigma[\boldsymbol{b}_\varphi] - d\sigma[-\boldsymbol{b}_\varphi]}{d\sigma[\boldsymbol{b}_\varphi] + d\sigma[-\boldsymbol{b}_\varphi]} = \frac{d\sigma^{(1)}[\boldsymbol{b}_\varphi] - d\sigma^{(1)}[-\boldsymbol{b}_\varphi]}{2d\sigma^{(pw)}} + \mathcal{O}(\sigma^4)$$

D.K., JHEP **03** (2017) 049

Non-paraxial effects in scattering

$$\mathcal{A} = \frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\left[\frac{\Delta u}{|\Delta u|}\times\left[\frac{\Delta u}{|\Delta u|}\times\langle b_\varphi\rangle\right]\right]\cdot\left(\frac{\partial}{\partial p_2}-\frac{\partial}{\partial p_1}\right)\zeta_{fi}\Big|_{p_{1,2}=\langle p\rangle_{1,2}}$$

There are two scenarios:

1. Off-center collision of the Gaussian beams

2. Central collision of non-Gaussian beams (vortex particles, Airy beams, etc.)

For a 1 + 2 → 3 + 4 process in the collider frame:

$$\mathcal{A} = 4\frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\langle b_\varphi\rangle p_3\,\frac{\partial\zeta_{fi}(s,t)}{\partial t}$$

← Shows how the phase changes with the transferred momentum!

It is odd with respect to $\phi_3 \to \phi_3 \pm \pi$

An up-down asymmetry!

D.K., JHEP **03** (2017) 049

Non-paraxial effects in scattering

We take identical relativistic beams and small scattering angles: *ee* → *X, pp* → *X, etc.*

$$\mathcal{A} \approx -2\,\frac{\lambda_c}{\sigma_b}\,\cos\phi_{sc}\,\frac{1}{\gamma}\frac{\partial\,\zeta_{fi}}{\partial\theta_{sc}}$$

Just a linear "geometric" suppression!

In QED (West, Yennie, 1968): $\frac{1}{\gamma}\frac{\partial\,\zeta_{fi}}{\partial\theta_{sc}} \sim \frac{\alpha_{em}}{\gamma\theta_{sc}}$ ⇒ $\mathcal{A} = \mathcal{O}\left(\frac{\lambda_c}{\sigma_b}\frac{\alpha_{em}}{\gamma\theta_{sc}}\right)$ ← Lorentz invariant!

For Moeller scattering of 300 keV electrons focused to 0.1 nm
and $\theta_{sc} \sim 10^{-2} - 10^{-1}$ we have:

$$|A| \sim 10^{-4} - 10^{-3} \quad (!)$$

Similar estimates were also obtained
by Ivanov, et al. 2016

Non-paraxial effects in scattering

A parameter which is usually employed: $\rho = \mathrm{Re}T_{fi}/\mathrm{Im}T_{fi} = 1/\tan\zeta_{fi}$

Once the Coulomb phase is known, one can retrieve also the hadronic phase!

Eur. Phys. J. C (2016) 76:661
DOI 10.1140/epjc/s10052-016-4399-8

THE EUROPEAN PHYSICAL JOURNAL C

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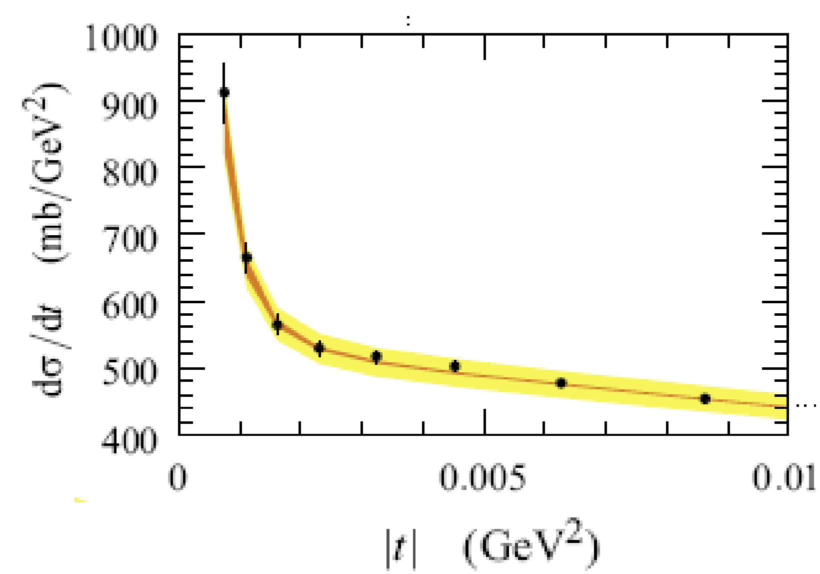
Regular Article - Experimental Physics

Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb–nuclear interference region: determination of the ρ-parameter and the total cross-section

TOTEM Collaboration

$|t|$, from 6×10^{-4} to $0.2\,\mathrm{GeV}^2$

The region of a Coulomb-hadronic interference →



Non-paraxial effects in scattering

For the same models as were used by TOTEM, one can estimate the asymmetry induced by the hadronic phase:

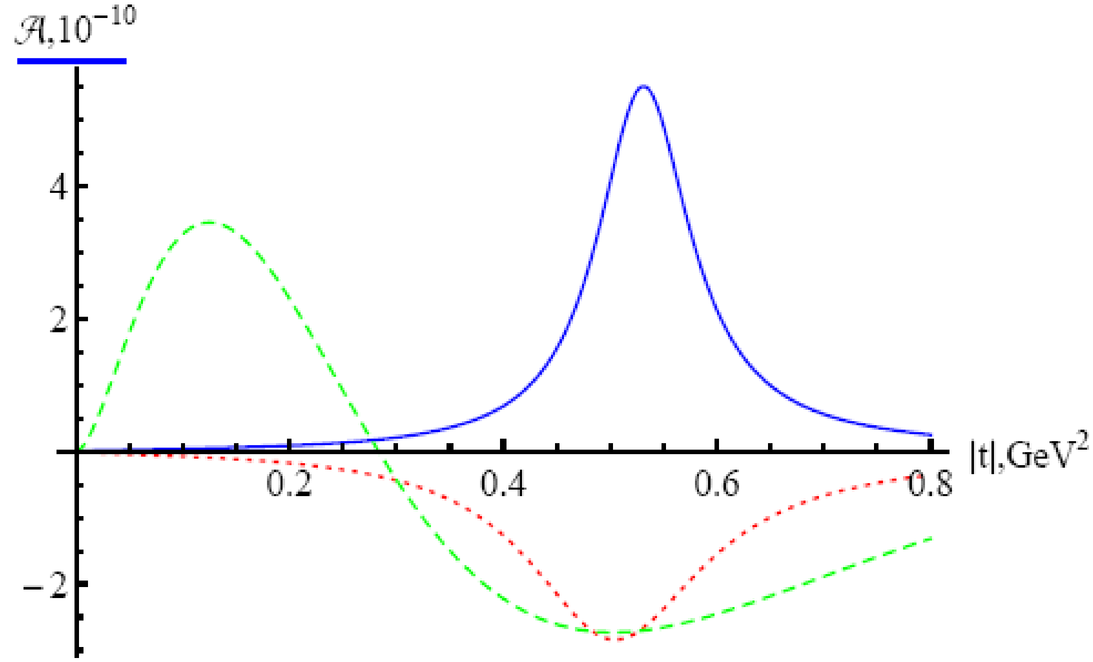
$$\frac{\partial \zeta_{fi}}{\partial t} = -\frac{\tau}{\tau^2 + (t+|t_0|)^2}$$ – the so-called standard parametrization, red dotted line

$$\frac{\partial \zeta_{fi}}{\partial t} = -\frac{\rho t_d}{(\rho t_d)^2 + (t-t_d)^2}$$ – the one by Bailly et al. (EHS-RCBC Collaboration), Z. Phys. C **37**, 7 (1987) blue solid line

$$\frac{\partial \zeta_{fi}}{\partial t} = \zeta_1(\kappa + \nu t)\left(\frac{-t}{1\,\mathrm{GeV}^2}\right)^{\kappa-1} e^{\nu t}$$ – the so-called peripheral parametrization V. Kundrát and M. Lokajíček, Z. Phys. C **63**, 619 (1994) the green dashed line

For pp-collisions the beams are too wide...

$$\frac{\lambda_c}{\sigma_b} \sim 10^{-11}.$$



Non-paraxial effects in scattering

In the paraxial regime, the quantum decoherence does not reveal itself,
and the Wigner functions stay everywhere positive (the WKB approximation).

In order to probe negative values of a Wigner function in scattering:

Beam-beam collision $\rightarrow$ beam + atomic target

An analogous small parameter

$$a/\sigma_b \qquad a \approx 0.053 \text{ nm is a Bohr radius}$$

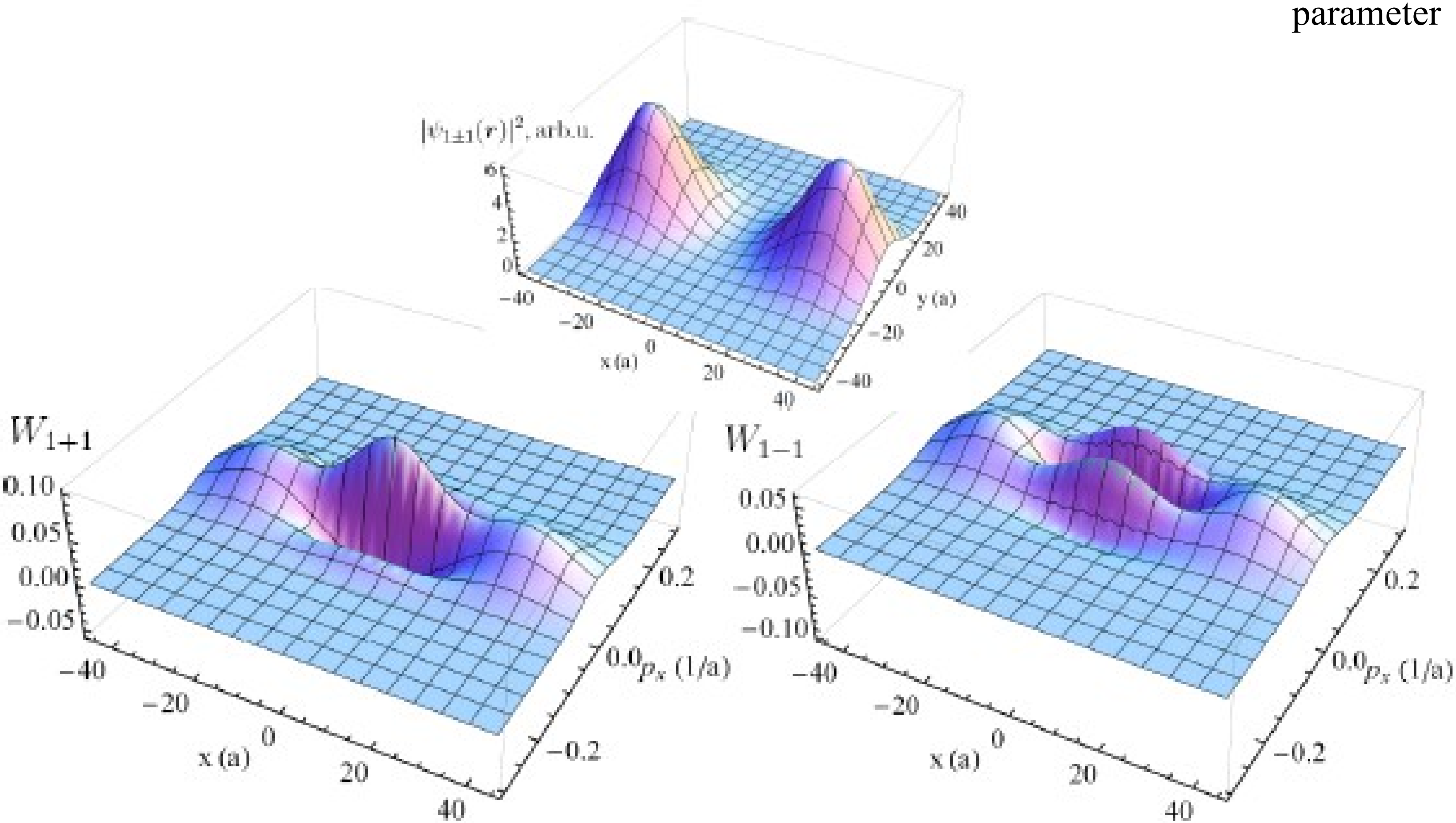
is 137 times larger than λ_c/σ_b !

For electron beams focused to 0.1 nm or less,
one can enter the non-paraxial regime!

Non-paraxial effects in scattering

A Schrödinger's cat state $|\boldsymbol{r}_0\rangle \pm |-\boldsymbol{r}_0\rangle$
has a not-everywhere positive Wigner function

$\boldsymbol{r}_0$ is an impact parameter



In the Born approximation, the number of scattering events is:

$$\frac{d\nu}{d\Omega} = N_e \int d^2b\, d^2p\; n(\boldsymbol{b})\, W(\boldsymbol{b},\boldsymbol{p}) (f(\boldsymbol{Q}-\boldsymbol{p}))^2$$

The target's transverse profile

The projectile's Wigner function

The Born amplitude

For a wide Gaussian target of hydrogen in the ground 1s state:

$$\frac{d\nu_{1\pm1}}{d\Omega} = \mathcal{N}_{1\pm1} \int\limits_0^\infty dx\, e^{-xg}\, \frac{x + x^2 + x^3/6}{1 + xa^2/(8\sigma_\perp^2)} \left(\cosh\left(\frac{\boldsymbol{b}_0 \cdot \boldsymbol{r}_0}{\Sigma^2}\right) e^{-r_0^2/(2\Sigma^2)} \pm \right.$$

$$\left. \pm \cos\left(2\boldsymbol{r}_0 \cdot \boldsymbol{p}_f \frac{xa^2/(8\sigma_\perp^2)}{1 + xa^2/(8\sigma_\perp^2)}\right) \exp\left\{ - \frac{r_0^2}{2\sigma_\perp^2 (1 + xa^2/(8\sigma_\perp^2))} \right\} \right)$$

Quantum interference does contribute to the cross section already in the Born approximation!

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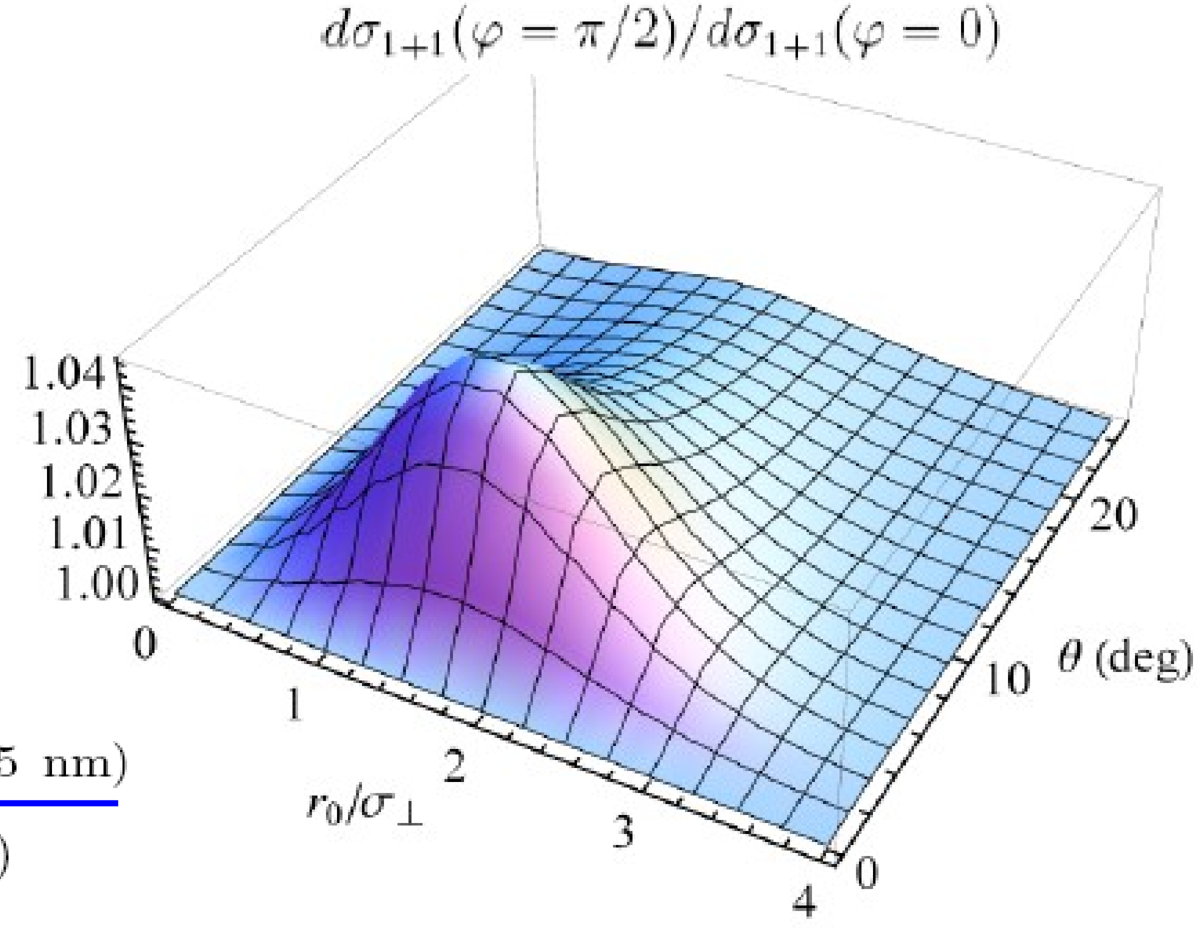
The quantum interference results in an angular asymmetry:



$\sigma_\perp = 2a \approx$ <u>0.1 nm (FWHM$\approx$ 0.25 nm)</u>

$p_i = p_f = 20/a \; (\varepsilon_{\text{kin}} = 5.6 \text{ keV})$

$\theta = 10^\circ$

Several per cent!

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Conclusion

- The non-paraxial effects in scattering can be attenuated as λ_c/σ_b,

  and not always as λ_c^2/σ_b^2.

- They can arise due to

  1. Quantum interference between the incoming packets
  2. Destructive interference due to negative values of the Wigner functions
  3. Large OAM of the vortex particles

- Say, for elastic scattering of the vortex electrons: $d\sigma^{(1)}/d\sigma_{\text{pw}} \sim |\ell|\lambda_c^2/\sigma_b^2 \gtrsim \alpha_{em}^2 = 1/137^2$,

  which can compete with the NNLO loop corrections in QED

- Generally, these effects can reach 0.1 – 10% for different types of the beams,

  either already available at electron microscopes (vortex, Airy, etc.)

  or producible in near future (cats, twisted cats, etc.)...

Why can these effects be interesting for particle physics?

1) They yield information on a quantum state of the particle:

Quantum tomography via scattering/annihilation?

2) For hadrons, such quantum numbers as OAM can couple to the internal degrees of freedom of partons:

News means for the proton spin puzzle?

3) They describe a number of new *specifically quantum phenomena*, such as

- Quantum decoherence in scattering (beyond the WKB regime),
- The spin-orbit interaction and its enhancement for highly twisted beams, etc.

Some of the predicted phenomena can already be studied experimentally!