Protons, Ions and Photons as signatures of monopoles

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Introduction

We have used the measure of mangetic moments and two photon decays as possible signals of monopoles.

I posed myself the question: are there any other ways to find monopoles?

Although I will always discuss Dirac monopoles many of the results apply also to topological solitons since we will be discussing detection and not production.

It is clear that the biggest asset of a monopole is it huge magnetic charge g

$$eg = N/2 \quad , N= 1,2,\ldots$$

What is the interaction of charged particles with monopoles?
What is the interaction of charged particles with a magnetic dipole?
What is the most natural way for monopole-antimonopole pairs (monopolium) to disintegrate?

Later I assume that LHC can create monopole-antimonopole pairs and they might escape the interaction region, hopefully crossing MoEDALS's nuclear track detectors, and bind into the beam pipe or MoEDAL's trap detectors or disintegrate into photons.

Scattering of a charged spin 1/2 particle by a spinless magnetic monopole

In a beautiful paper (Physical Review D15 (1976) 2287) using fiber bundels and thus avoiding the Dirac string Yoichi Kazama, Chen Ning Yang and Alfred S. Goldhaber calculated among other things the cross section of a beam of spinors of charged Ze on a magnetic monopole (here N=1) central potential:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2k^2}\left(|T_{|q|}|^2 + 2q^2(\sin(\theta/2))^{4|q|-2} \right),$$

where q= Zeg = Z/2, k is the momentum of the beam and $T_{|q|}$

$$T_q(\Theta) = -qe^{-i\pi q}\frac{1}{\sin^2(\frac{1}{2}\Theta)}\left\{ \frac{\cos\frac{1}{2}\Theta}{1+\sin\frac{1}{2}\Theta}\left[1+\left(1+\frac{i\pi q}{2}\right)\sin\frac{1}{2}\Theta\right] + \frac{1}{2}\left(1+i\pi q-\frac{\pi^2 q^2}{4}\right)\frac{1-(\sin\frac{1}{2}\Theta)^{2q}}{\cos\frac{1}{2}\Theta}\sin^2(\tfrac{1}{2}\Theta)\right\} + U_q(\Theta),$$

and

$$U_q(\Theta) = 2\cos\tfrac{1}{2}\Theta(\sin\tfrac{1}{2}\Theta)^{2q}\sum_{n=0}^{\infty}(-1)^n R P_n^{2q,1}(\cos\Theta)$$

is an absolutely convergent series with terms behaving for large n like $n^{-2.5}$ were P are Jacobi polynomials. These equations consider the monopole as infinitely massive.

This expression looks impossible to understand but a plot might help. Lets draw a the ratio of this expression with Rutherford's cross section with some factors to avoid the difference in size

$$R(q,\theta) = \left(\frac{Z'e}{gv}\right)^2 \left(\frac{d\sigma}{d\Omega} \Big/ \frac{d\sigma}{d\Omega_R}\right)$$

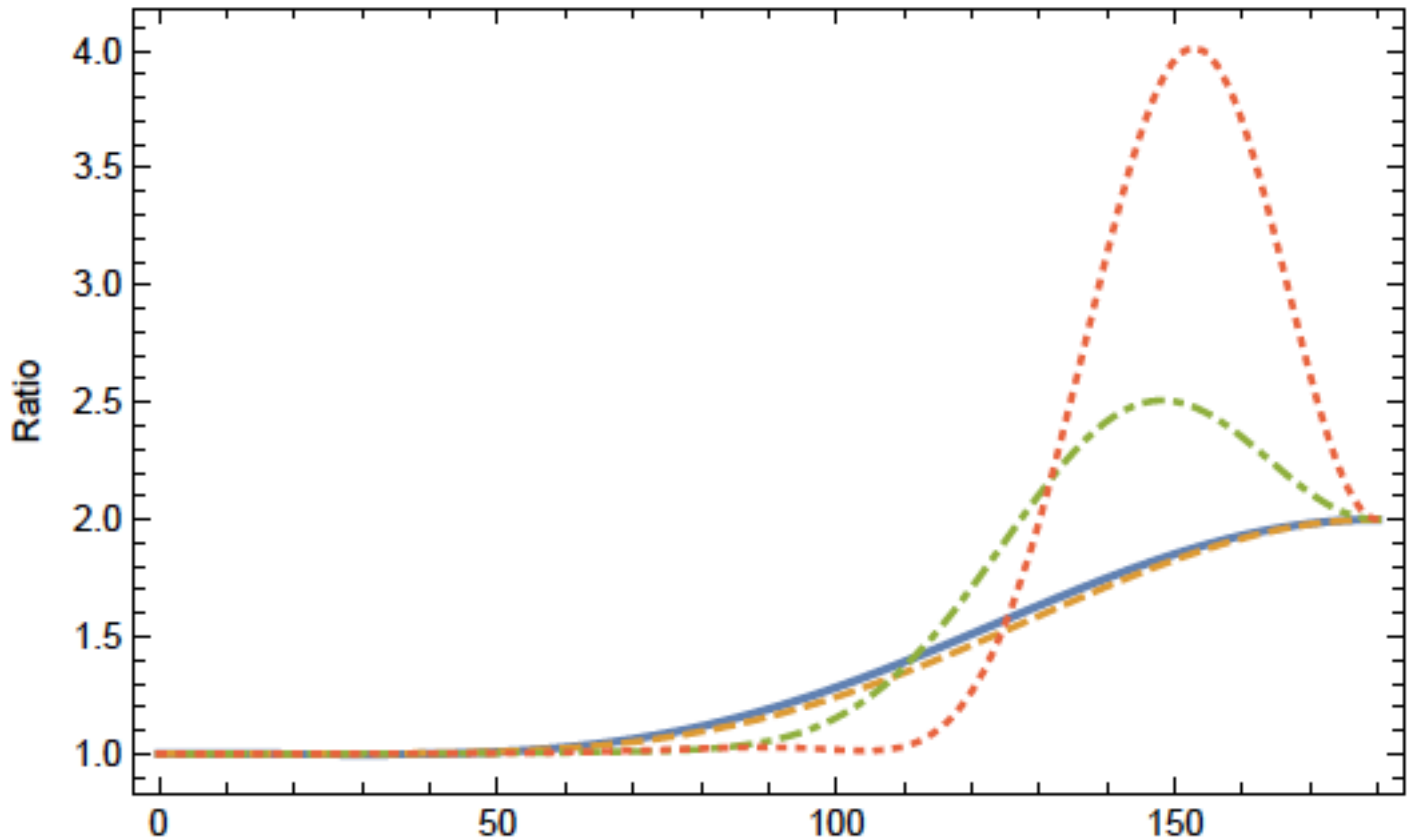
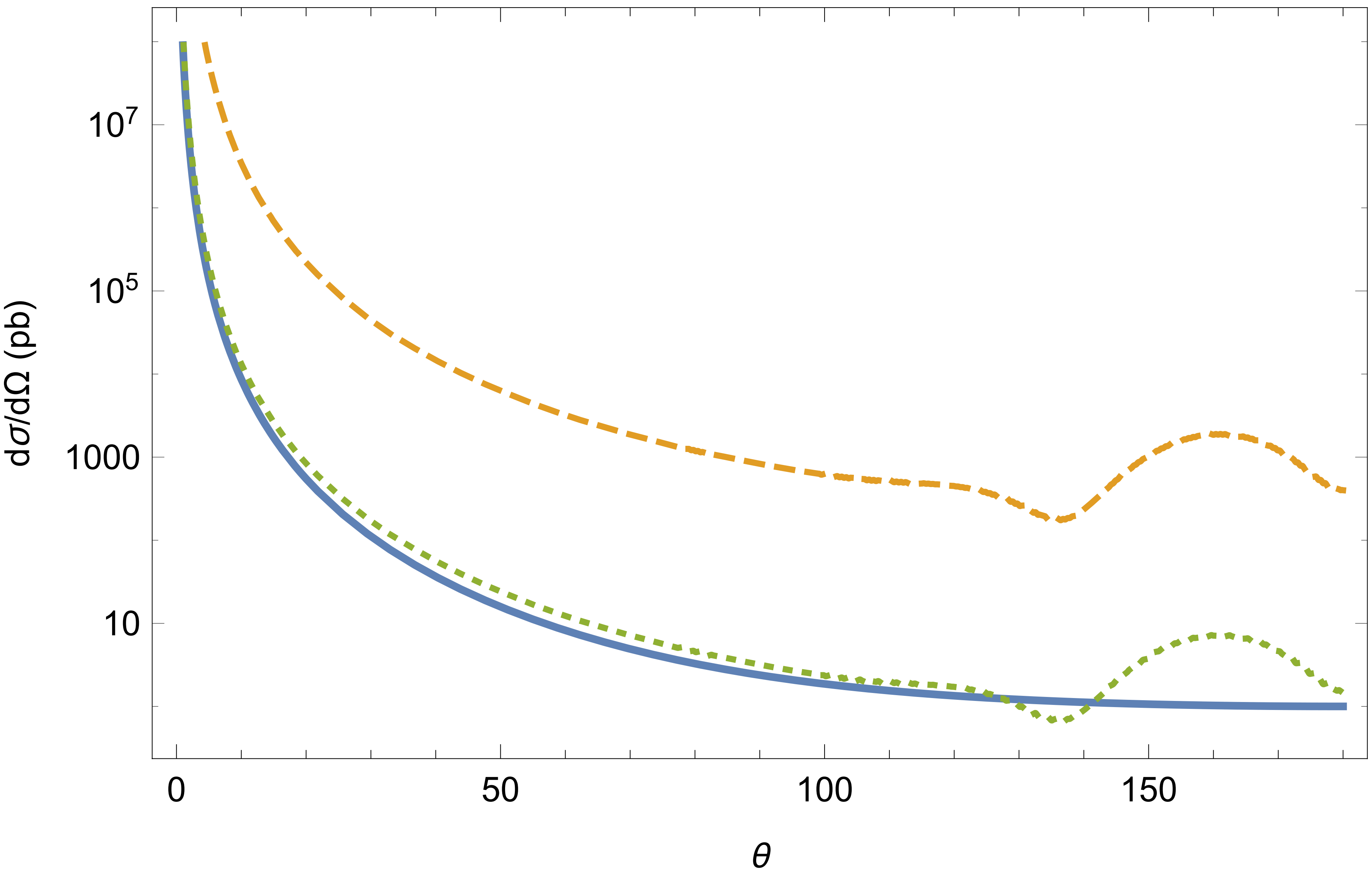


FIG. 1: We plot the function $R(q,\theta)$ defined in Eq. (3) for $q = 1/2$ (solid), q=2 (dashed), q=4 (dotdashed) and $q = 6$ (dotted).

We next plot the scattering cross sections for proton and ${}^{41}_{20}Ca$ (at 7TeV (dashed) and at 2.76 TeV per nucleon (dotted)):



Proton Monopole scattering

The wishful (smoking gun) signal for proton monopole scattering is recalling Rutherford’s words: “I was shooting with a gun to a gold foil and the bullets were bouncing back”, protons bouncing back! transverse is enough!!

But the monopole in the beam pipe is 3cm away from the beam, from the impact parameter formula

$$b = \frac{qE}{k^2} \cot(\theta/2)$$

we find that θ is practically 0 and therefore there is no smoking gun!!! It seems we cannot use LHC for that purpose.

Instead of using LHC or a magnetometer to detect monopoles (beam pipe or metal rods) we could use a small accelerator and scan a beam of charged particles through it. Note that the scattering cross section is inversely proportional to k^2 thus the scanning accelerator does not have to be powerful.

$$\frac{d\sigma}{d\Omega} = \frac{1}{2k^2}\left(|\, T_{|q|}\,|^2 + 2q^2(\sin(\theta/2)^{4|q|-2} \right),$$

How do we avoid background?

Rutherford backscattering spectrometry

We describe Rutherford backscattering as an elastic collision between a high kinetic energy particle from the incident beam (m_b) and a stationary particle located in the sample (m_t).

Considering the kinematics of of non-relativistic collision, $m_b >> k$ the energy E_1 of the scattered projectile is reduced from the initial energy E_0:

$$E_1 = kE_0$$

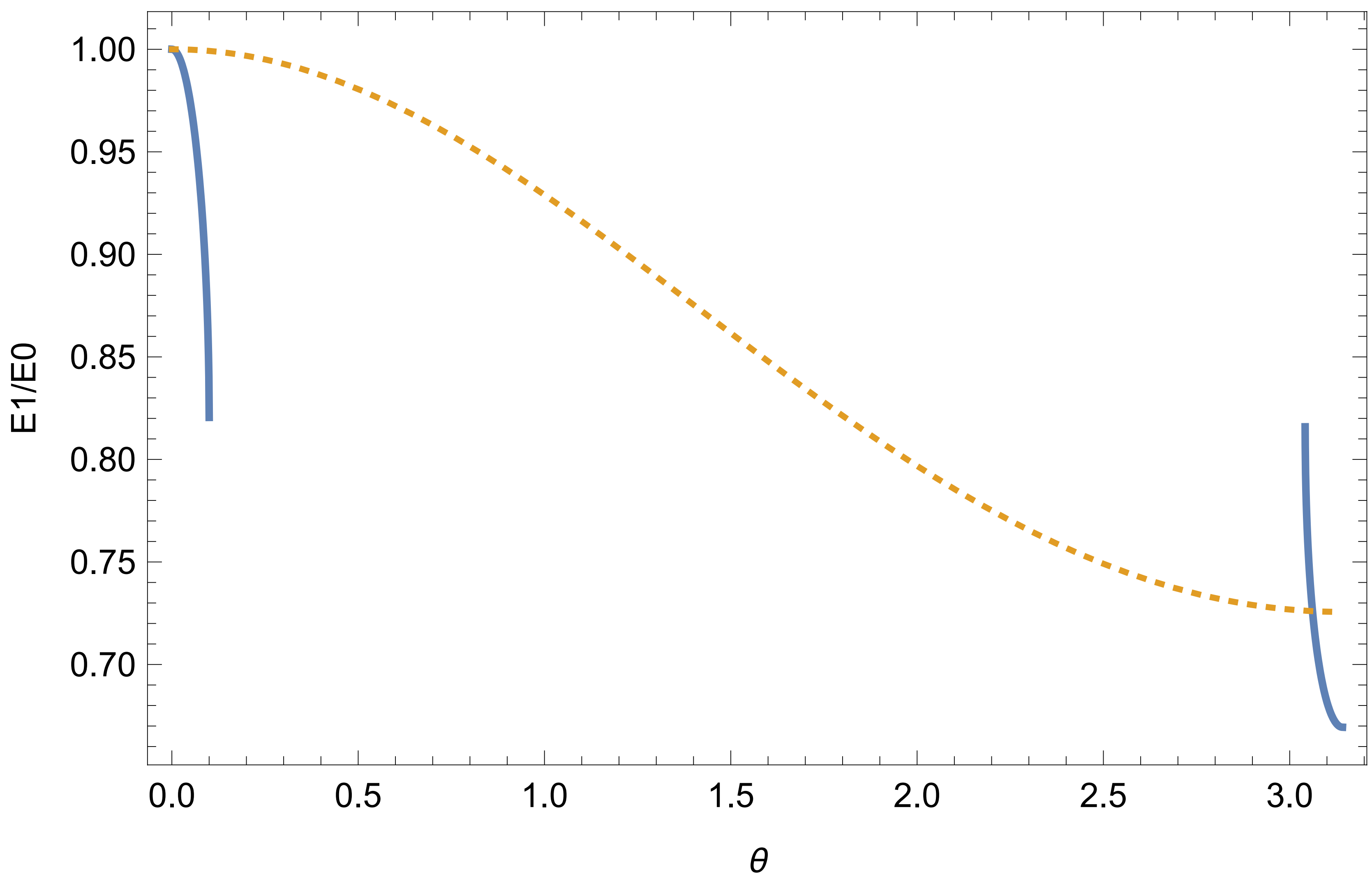
where k is known as the *kinematical factor*, and

$$k = \left(\frac{m_b \cos \theta_L \pm \sqrt{m_t^2 - m_b^2 (\sin \theta_L)^2}}{m_b + m_t} \right)^2$$

where b is the projectile, particle t is the target, and θ_L is the scattering angle of the projectile in the laboratory frame of reference. The plus sign is taken when the mass of the projectile is less than that of the target, otherwise the minus sign is taken. This equation can be easily generalized to relativistic kinematics.

The beam particles will scatter off the monopole but also off the beam pipe metal. BUT if $m_b > m_t$ there is no backscattering. Assuming that the monopole mass is much bigger than the beam particle mass, if beam ions backscatter this is a monopole signal, assuming no impurities with mass bigger than the beam particles.

Scanning of the beam pipe or MoEDAL's trap detectors by a low energy ion beam is another method to detecting monopoles (low k and low background)



Kinematical limits on low energy scattering of Calcium ions on Berylium targets (solid) and 500 GeV Monopole targets (solid)

Van der Meer Scanning or elastically scattered protons

Looking again at LHC. By using the special run conditions of the van der Meer scanning or elastically scattered protons, the LHC proton beam can be vertically displaced up to the maximal value where the beam protons tend to scrape the beam vaccuum chamber walls:

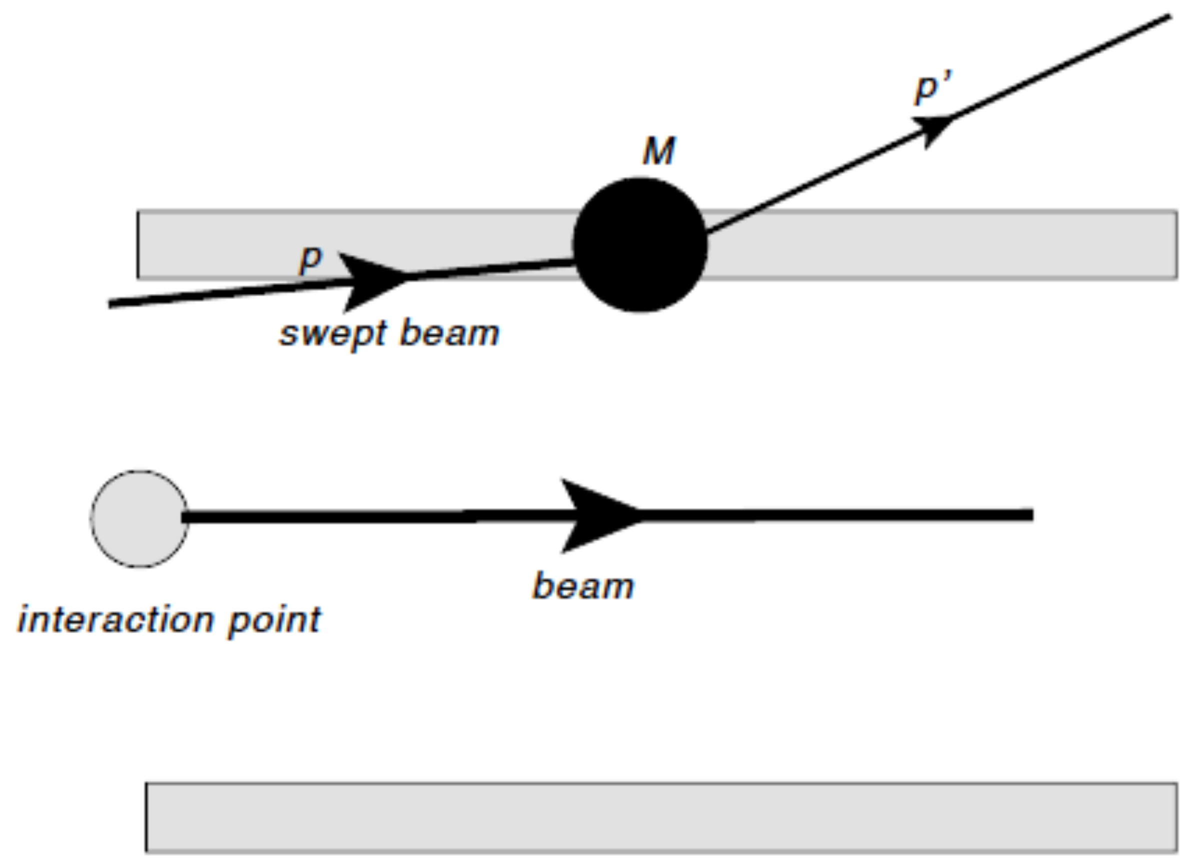


FIG. 4: Scattering of the swept beam off the bound monopole (M). The reach the monopole almost parallel to the initial beam.

In this case, the beam of protons might reach the monopole almost paralell to the beam pipe and with small impact parameter. We therefore reduce the scenario to the scattering of a proton by a central potential of the monopole field just described.

The problem we face is a knowledge of the luminosity. Just for the sake o this presentation we take low luminosity 10^32 cm^-2 s^-1 for 200 days. (recall that lower energy dramatically increases the scattering cross section)

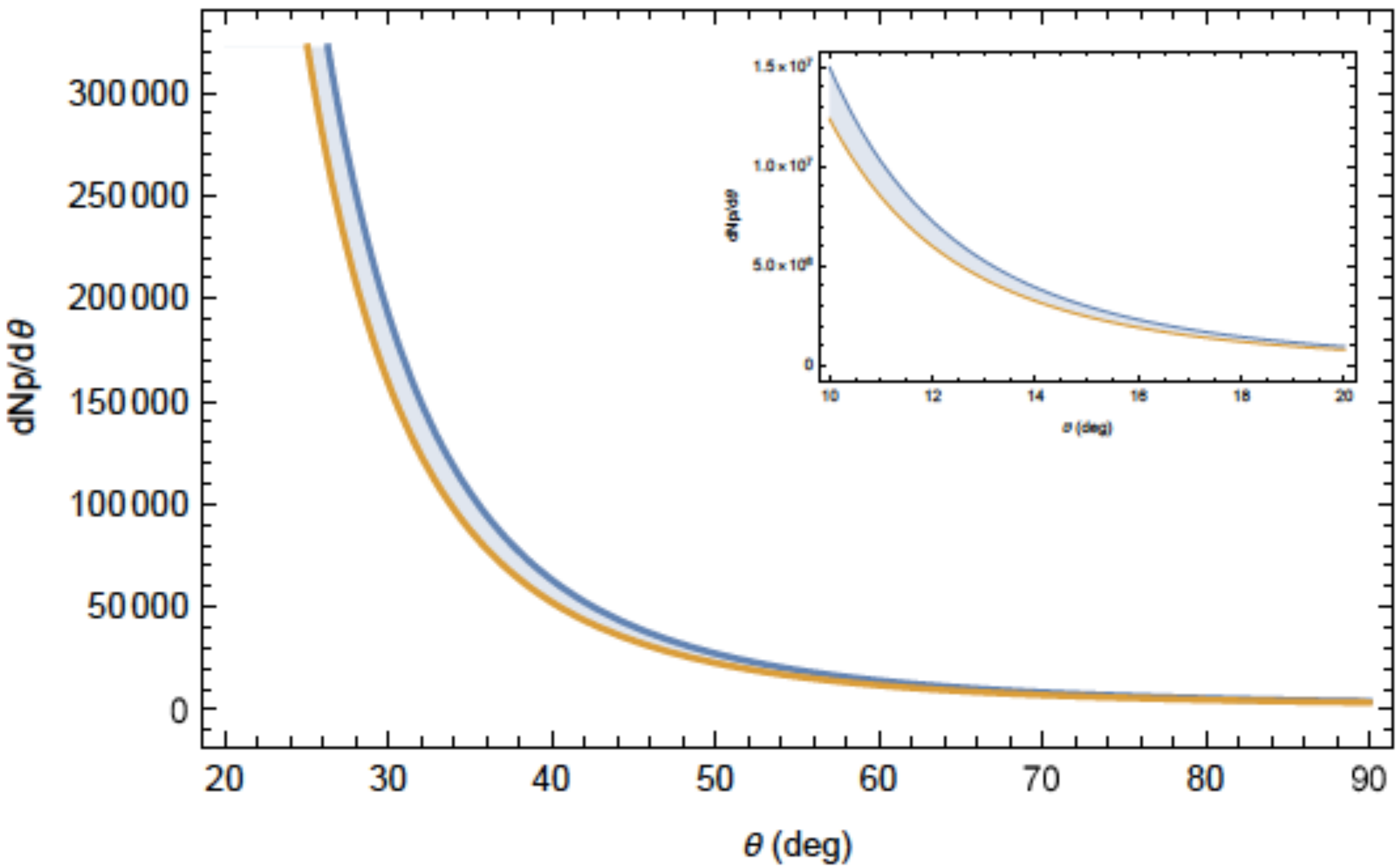
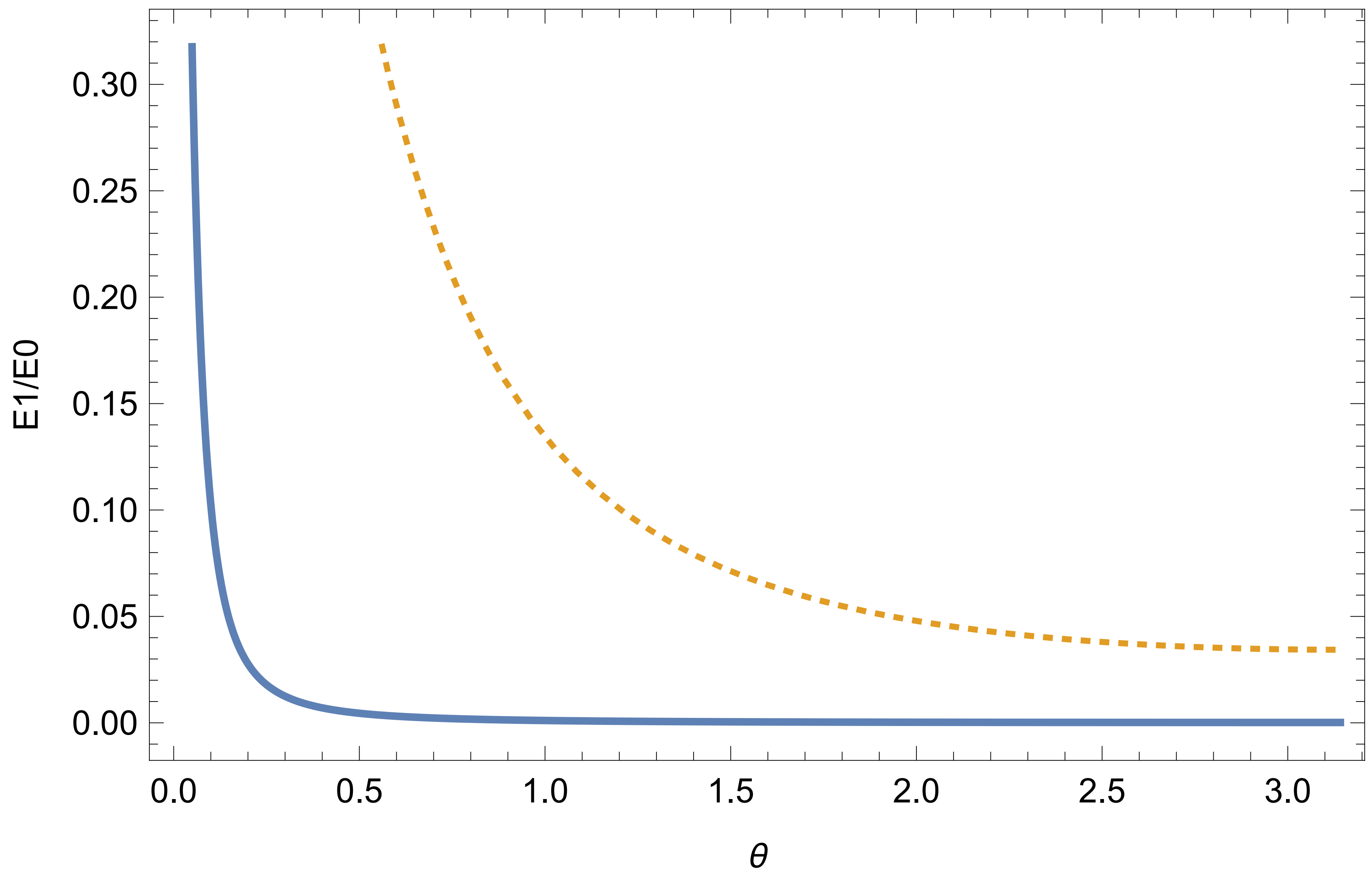


FIG. 5: Angular distribution of protons scattered by a monopole bound in the beam pipe. The inset shows small angle scattering. The lower curve corresponds to a monopole mass ten times the mass of the proton, $M = 10m_p$ and the upper curve the infinite monopole mass limit.



Relativistic kinematics for 7 TeV proton beams on Berylium targets (solid) and 500 GeV Monopole target (dotted)

The result is a Rutherford type scattering which can be detected by the set up used for CEP (Central Exclusive Production) when grazing the beam through a beam pipe with large luminosity. This beam van der Meer sweeping or elastically scattered protons done before replacement could be a very efficcient way of detecting monopoles.

What about MoEDAL?

As we have seen the scattering cross section depends on q=Z/2

$$\frac{d\sigma}{d\Omega} = \frac{1}{2k^2}\left(|T_{|q|}|^2 + 2q^2(\sin(\theta/2)^{4|q|-2} \right),$$

Thus it can be applied for ions. Due to the slow convergence of the series we have done the calculation for Ca, but certainly the isotopes about Pb would be experimentally more convenient.

For 100 days with a luminosity of 10^27 cm^-2 s^-1

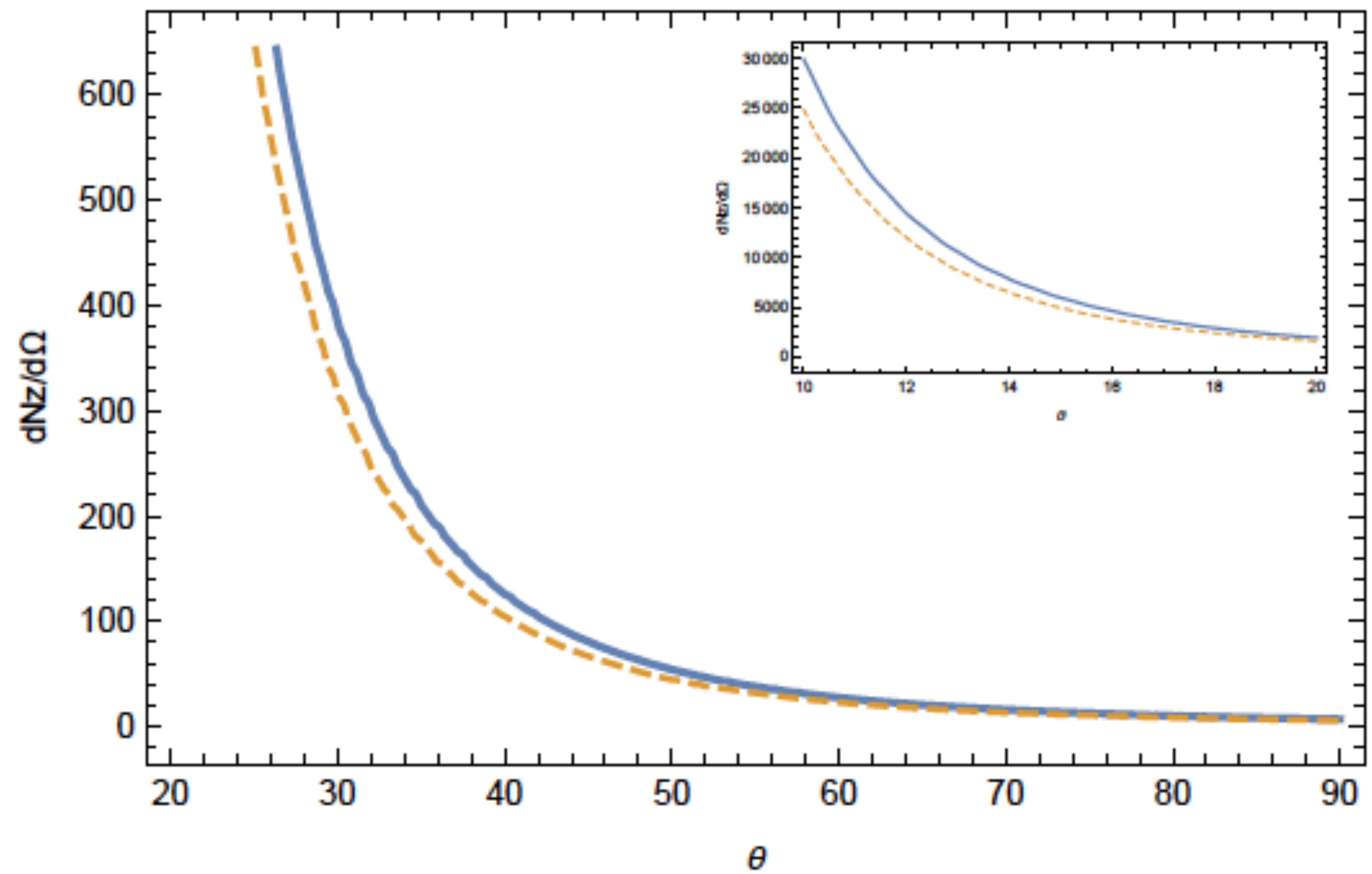
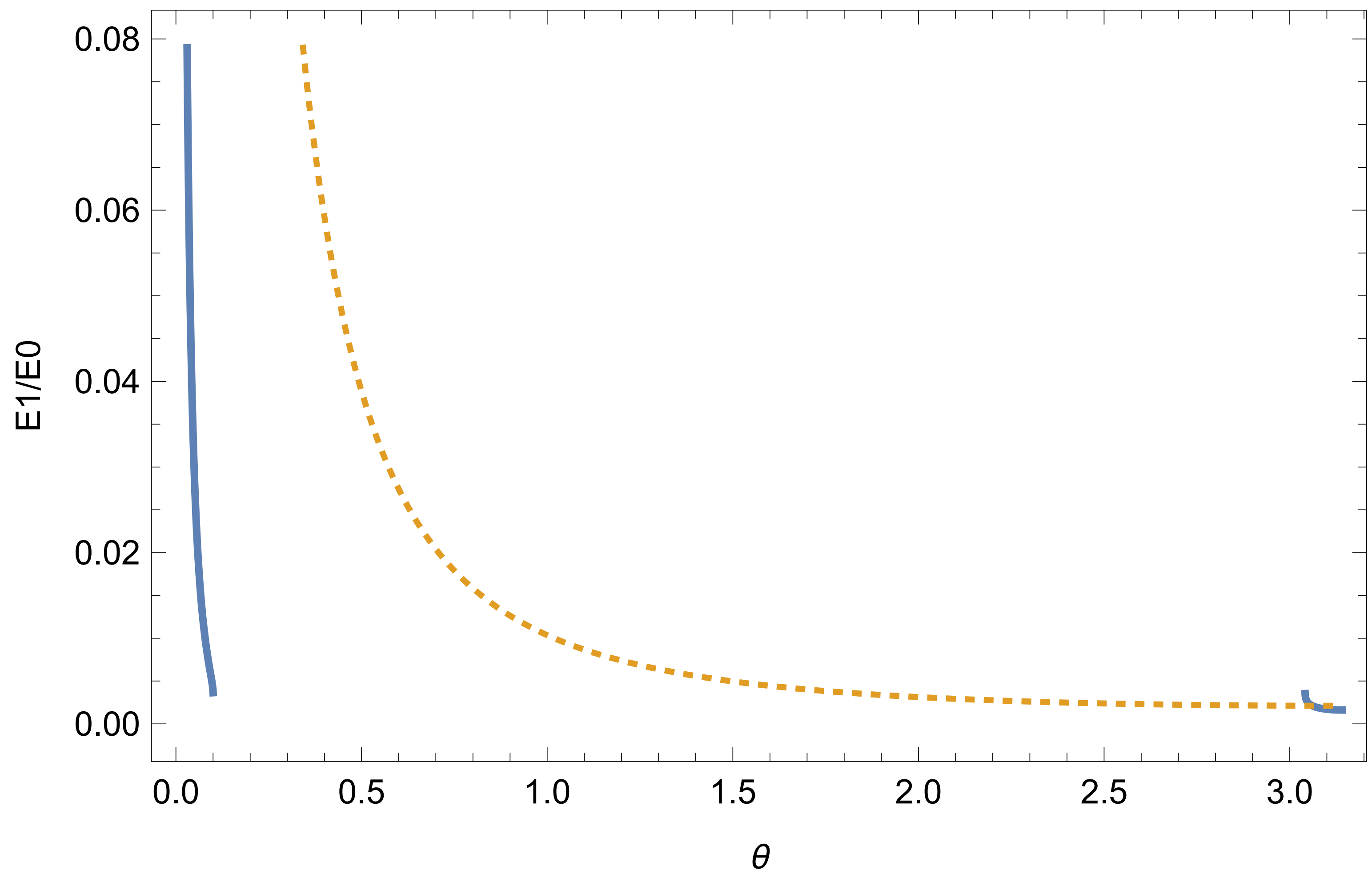


FIG. 3: Angular distribution of Z=20 spin 1/2 ions scattered by a monopole bound in the beam pipe. The inset shows small angle scattering. The lower curve corresponds to a monopole mass ten times the mass of the ion, $M = 10m_Z$, and the upper curve to the infinite monopole mass limit.

For Pb “at the same momentum as Ca” one expects approximately a factor 1600 more, i.e. a few thousand ions per day in the transverse direction. If it is at the “same momentum per nucleon” than the Z^2 factor is killed by a A^2 factor. It is important to remember that low momentum increases the discovery potential.



Kinematics for Calcium beams at 2.5 TeV for Berylium targets (solid) and 500 GeV Monopole targets (dotted).

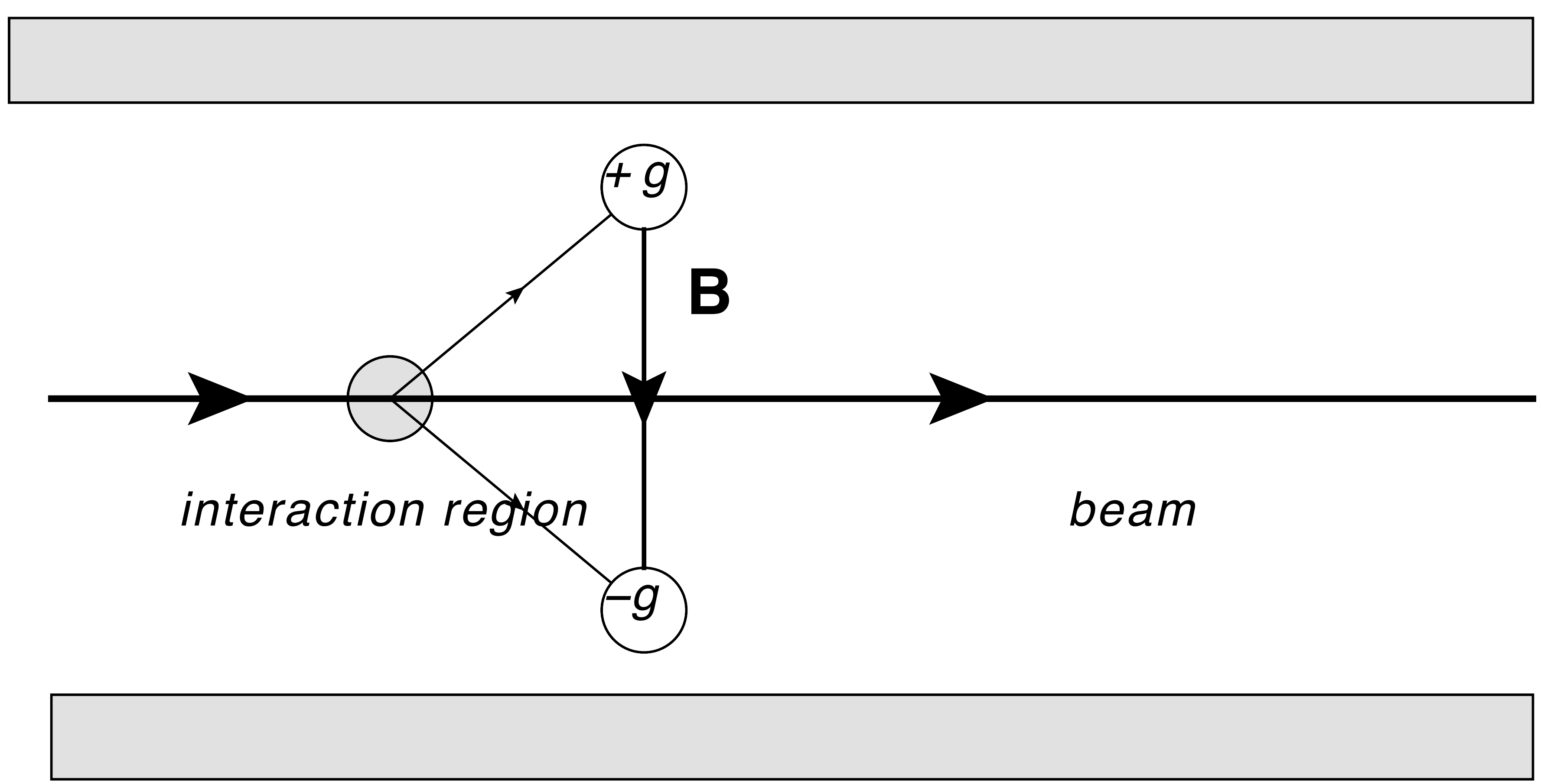
Conclusions

- We are not discussing here about the probability of creating monopoles but we are assuming they exist and can bound to metals.
- If monopoles exist their scattering with charge particles is large and their coupling with protons produces sizeable effects!
- The smoking gun of monopole detection would be protons coming out in the transverse direction or even if the backward direction, a repetition of Rutherford’s discovery result.
- The high mass of the monopole determines appropriate kinematical cuts for monopole detection. In particular in the case of ion scattering the cuts are absolute.
- The scattering cross section is inversely proportional to the momentum squared, therefore scanning of material by low energy accelerators would increase the detection capabilities.
- Van der Meer scanning or elastic scattered protons together with the CEP set up can lead to detection capabilities at LHC.
- MoEDAL may be suited for heavy ion scattering of monopoles, in particular for Pb a few thousand ions per day.
- Certainly this calculation is a simplified model of reality and would require a profound geometrical study but the message to be transmitted is that if a monopole exists scattering numbers with charged particles tend to be large :
- “The mistery is in the production cross section not the detection cross section”.

Magnetic dipole scattering

At LHC one will hopefully produce pairs of monopole-antimonopole. If they do not annihilate, they create a magnetic dipole field. If they make a bound state, monopolium, in an excited state, they also create a magnetic dipole moment. Details for future work.

Let us assume that we are near threshold, i.e. the pair moves away from each other very slowly, until the most probably they bind in the beam pipe. Again we are not discussing production cross section, we are assuming that the pair is produced.



What happens to the beam of LHC? In the dipole case the Dirac string disappears, thus it is a non singular system. I need no fibre bundles. I apply the Bohr approximation corrected for relativistic effects which leads to

$$\frac{d\sigma}{d\omega}(\theta)|_{nr} = \frac{4m_b^2 q^2}{k^4}\left(\frac{\sin(kdsin\theta)}{1-\cos\theta}\right)^2$$

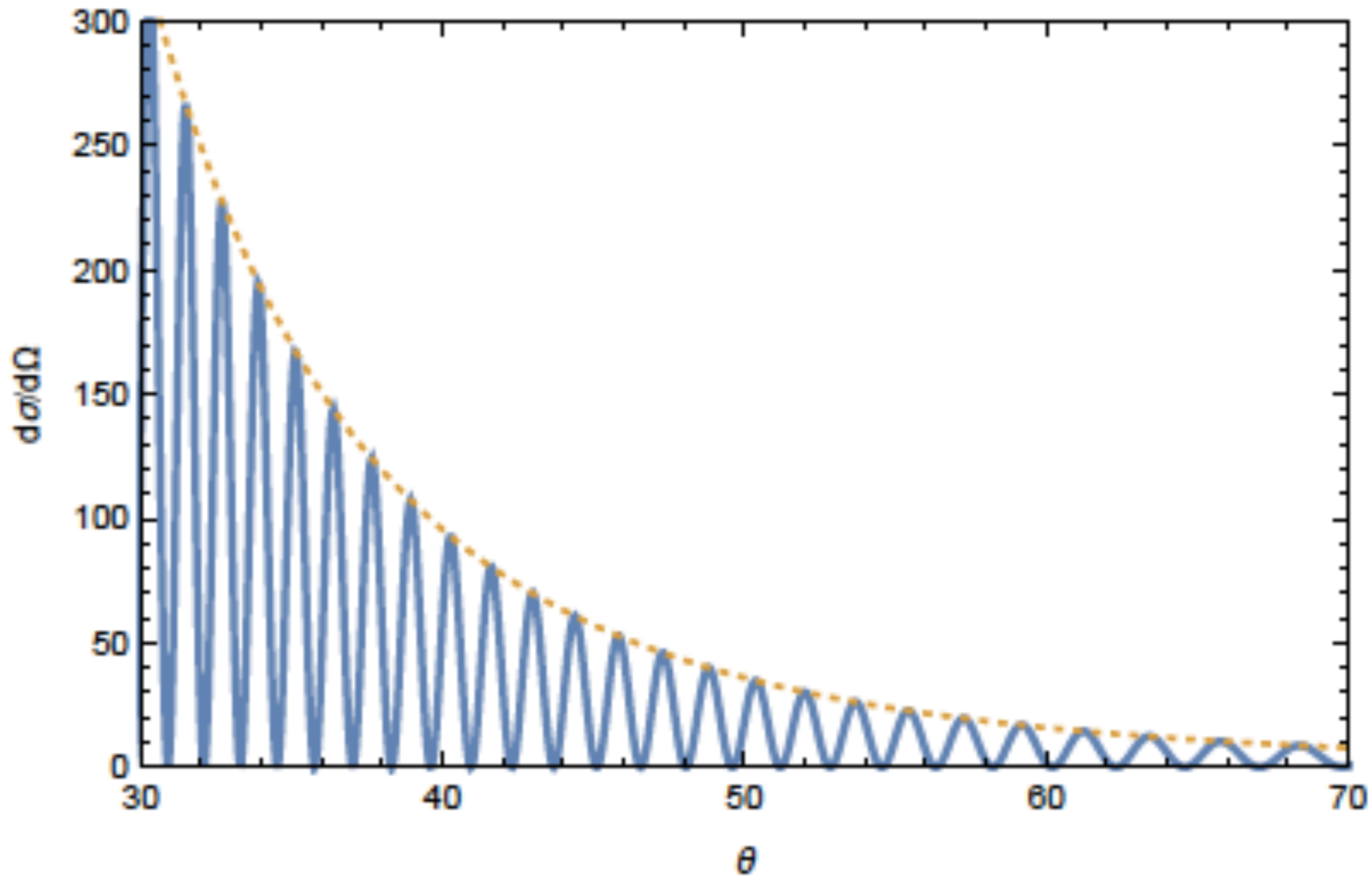
$$\frac{d\sigma}{d\omega}(\theta) = \frac{E^2 - k^2\sin(\theta/2)}{m_b^2}\frac{d\sigma}{d\omega}(\theta)|_{nr}\,.$$

whose validity is determined by

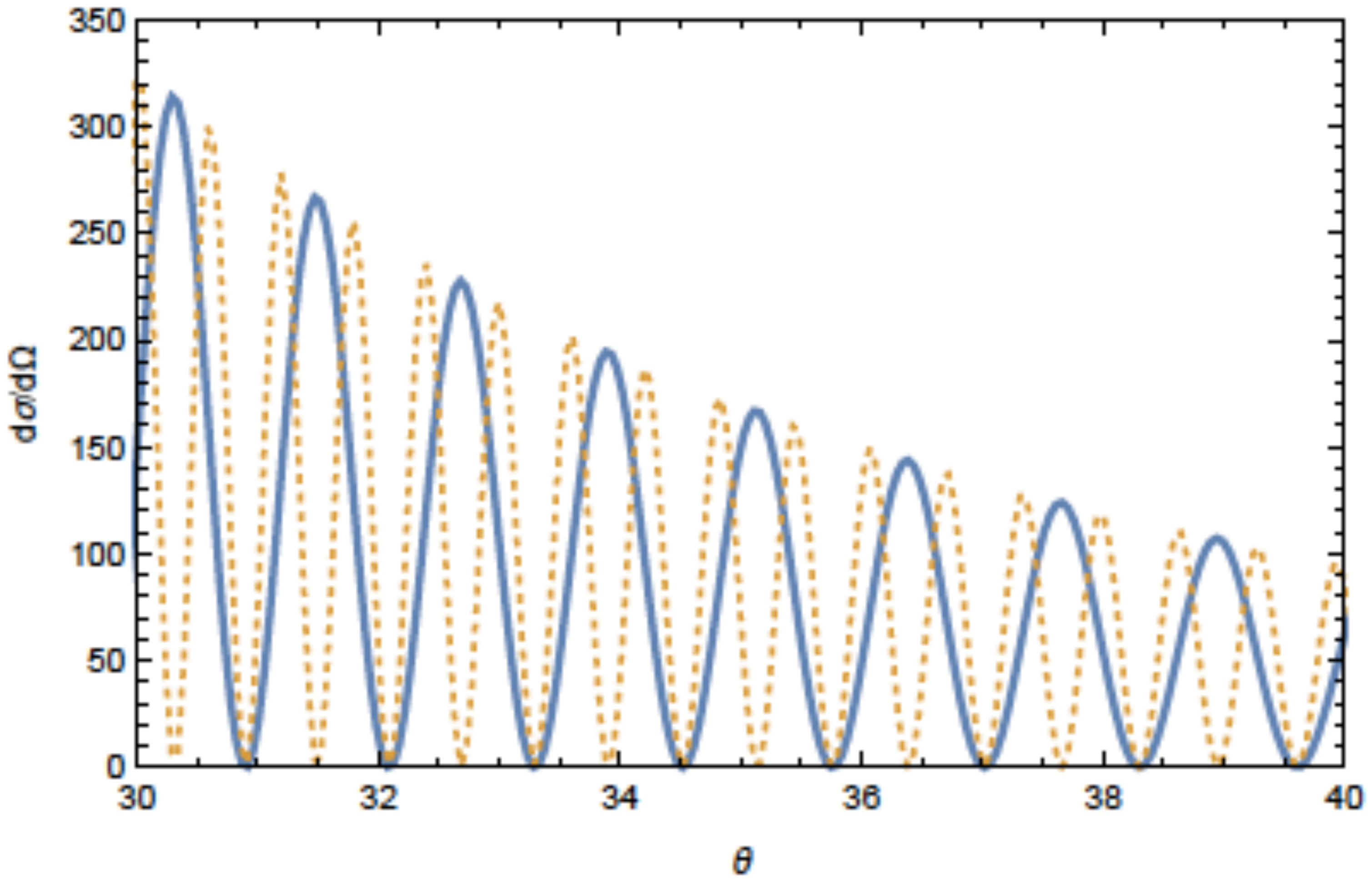
$$|\frac{m_b q\pi}{k}| < < 1.$$

Since for LHC k >> m_b q this condition is satisfied. For example for a Pb ion beam at 2.76 TeV per nucleon the factor is 0.0466 and for a proton beam at 7 TeV $ 0.0002$.

Again equations are not very illuminating. Let us plot the result for pole distance = 0.01 fm

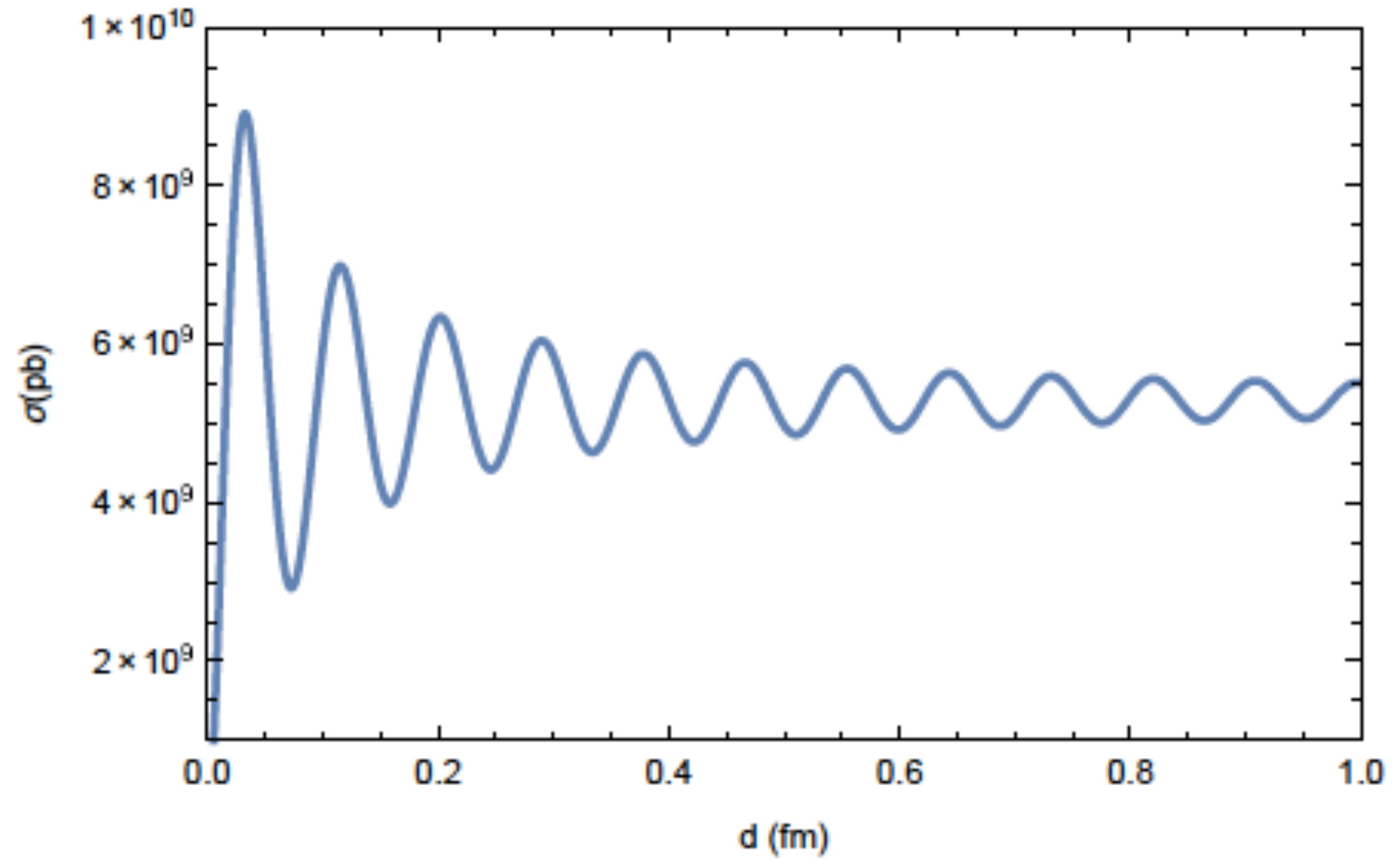


What happens if we double the distance?



In our scenario the distance will be much larger and therefore the experimental outcome will be like the envelope of the maxima which does not depend on the pole distance. Again we see a Rutherford type scenario, i.e. particles coming out in the transverse and backward directions.

How does the total cross section change with distance?

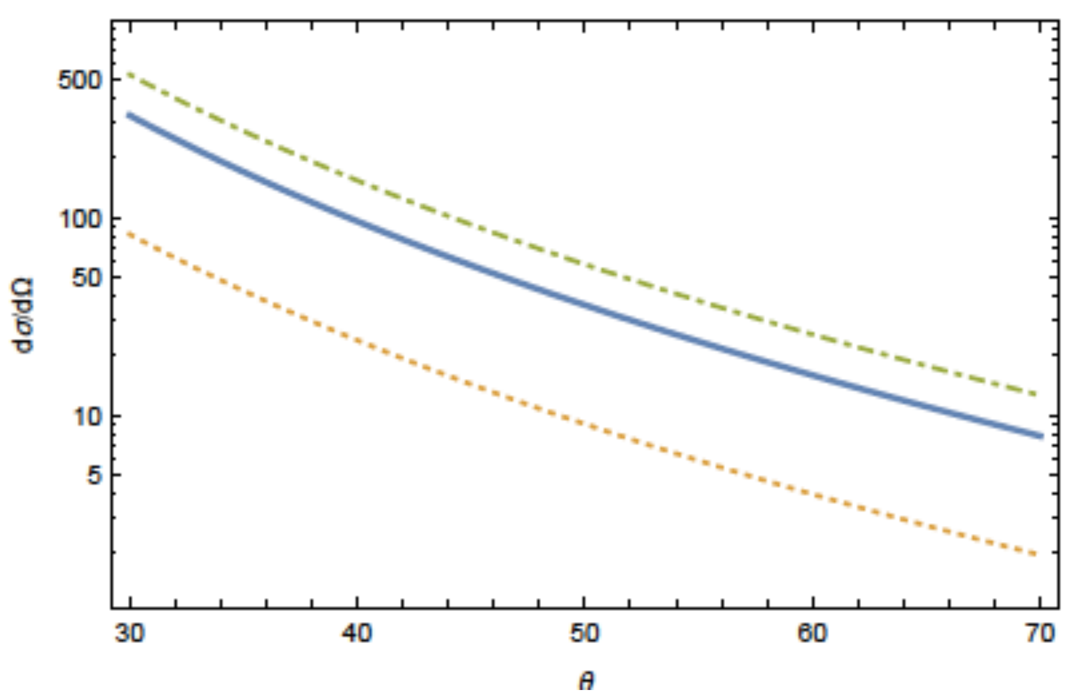


I have not jet studied how far can I go in d with this approximation.

Which effect is bigger the dipole moment effect or the monopole effect. In order to take into account the minima I define an average cross section

$$\overline{\frac{d\sigma}{d\theta}} = \frac{\sigma}{\sigma_{max}} \frac{d\sigma_{max}}{d\theta}.$$

We show the result for ions



The advantage in this case is luminosity since the dipole scatters the actual beam.

Conclusions

- magnetic dipole scattering can also produces a Rutherford type scenario.
- as long a $k^2 > m_b$ lower momenta imply greater cross sections.
- ion scattering is favored over proton scattering (good for MoEDAL)
- this scenario has no problem with luminosity, but the problem arises with time since the pair will be moving away from each other.
- If bound we still have to study the validiy of the approximation as a function of pole distance since d~3cm looks more prone to pole scattering like in the first presentation than to a 6cm magnetic dipole scattering.
- In any case at low energy better detection capabilities

Multi-photon annihilation of monopolium

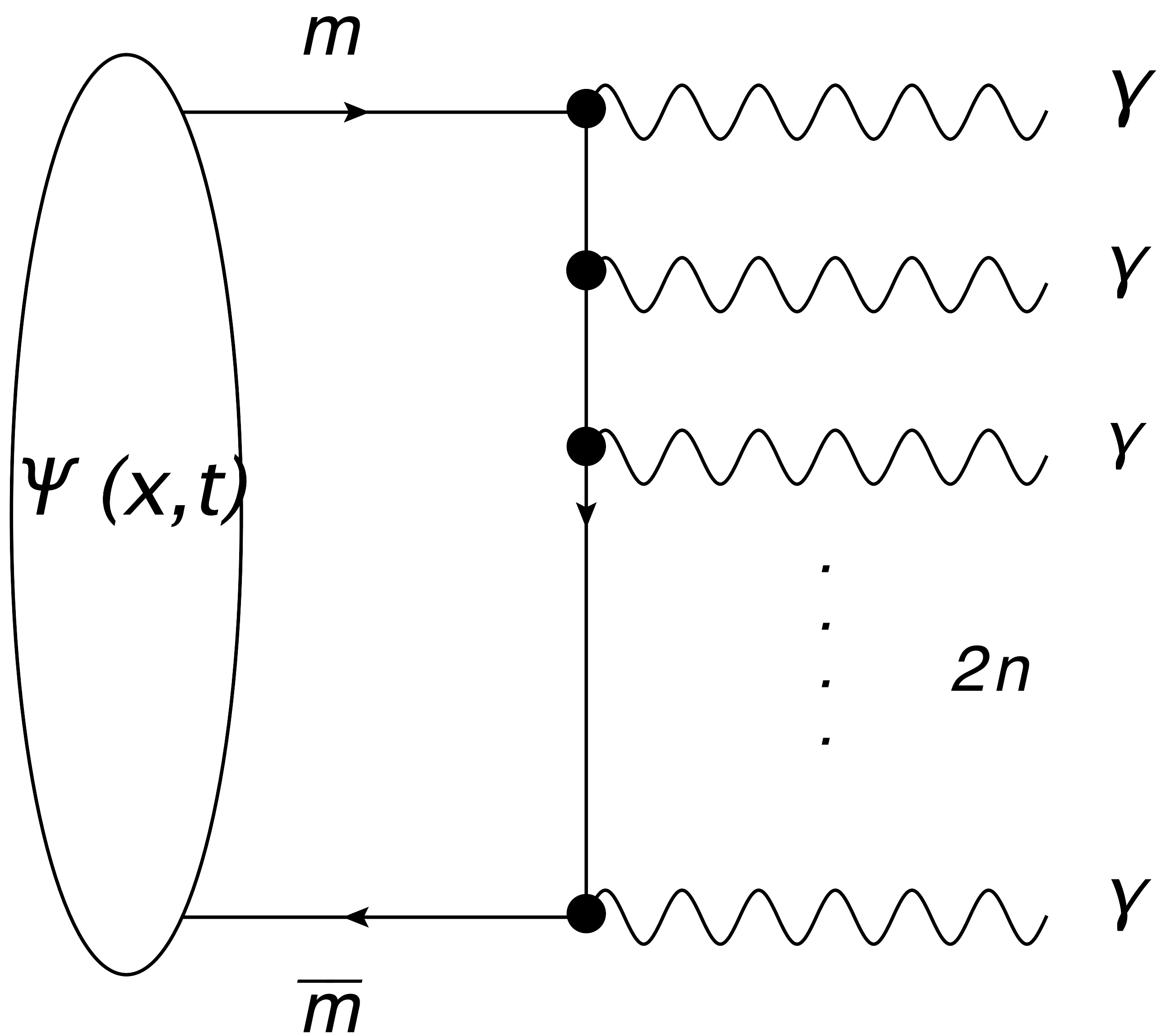
Four to two photon decay of positronium

$$\frac{\Gamma_4}{\Gamma_2} = 0.277 \left(\frac{\alpha}{\pi}\right)^2$$

Let us assume that the monopole-photon coupling is analogous to the electron-photon coupling except for an effective vertex characterized by the dressed monopole magnetic charge g (Zwanziger1970):

$$\frac{\Gamma_4}{\Gamma_2} \sim F_{42} \left(\frac{\alpha_g}{\pi}\right)^2 \quad \cdots \quad \frac{\Gamma_{2n}}{\Gamma_2} \sim F_{2n2} \left(\frac{\alpha_g}{\pi}\right)^{2n-2},$$

The F's represent the contribution of all the Feynman amplitudes to the process shown as subindex after extracting the contribution of the magnetic charge, which is explicitly shown.



In the Figure we show one of the 2 n! contributions to the amplitude for a 2n photon decay to leading order, and we note that this type of contributions in the above ratios are determined only by vertices and propagators.

Let us discuss first an educated estimation for large n. In the rest frame of the bound system the annihilation into many photons leads to an average momentum for each photon much smaller than the mass of monopolium and therefore much smaller than the mass of the monopole. In order to make an estimation of the above ratios we consider that in the propagators the monopole mass dominates over the momentum and therefore the calculation of the width, in units of monopole mass, depends exclusively on three factors: the number of diagrams (2 n)!, the photons' symmetry factor 1/(2 n)! and the phase space of the outgoing massless particles, namely

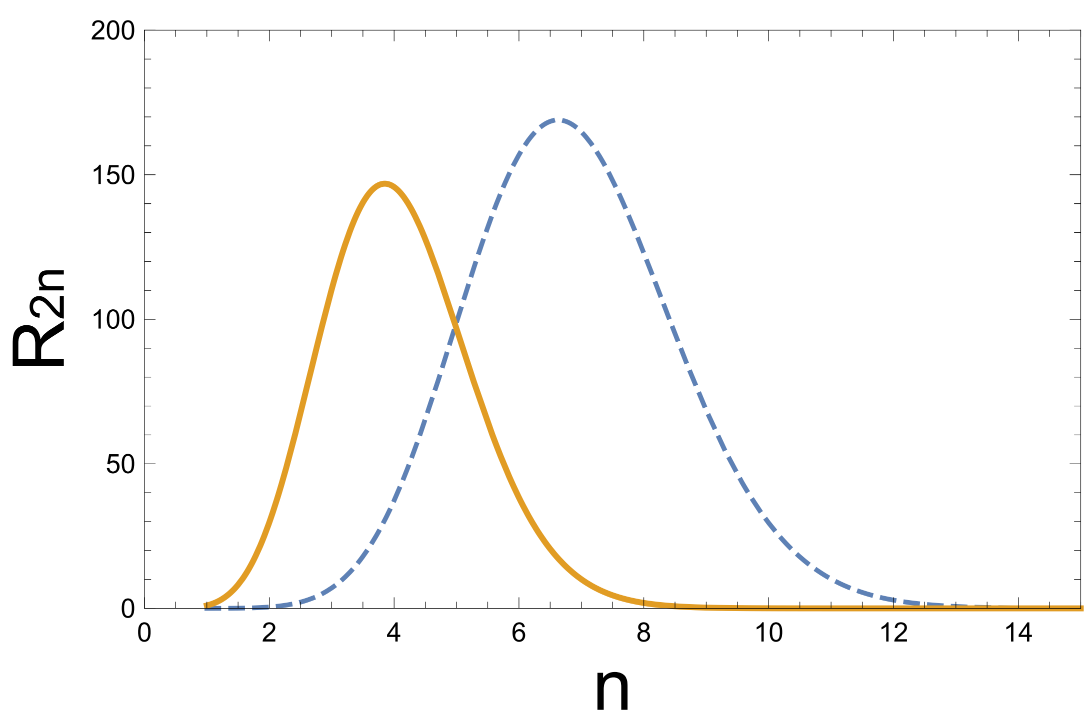
$$(phsp)^{2n} = \frac{1}{2}\frac{1}{(4\pi)^{4n-3}}\frac{M^{4n-4}}{\Gamma(2n)\Gamma(2n-1)}$$

Note that this equation leads to $\Gamma_2/\Gamma_2 = 1$ and for n=2 and M=2m, one recovers the parapositronium case, $\Gamma_4/\Gamma_2 = \left(\frac{\alpha_g}{\pi}\right)^2$, with the interference factor missing.

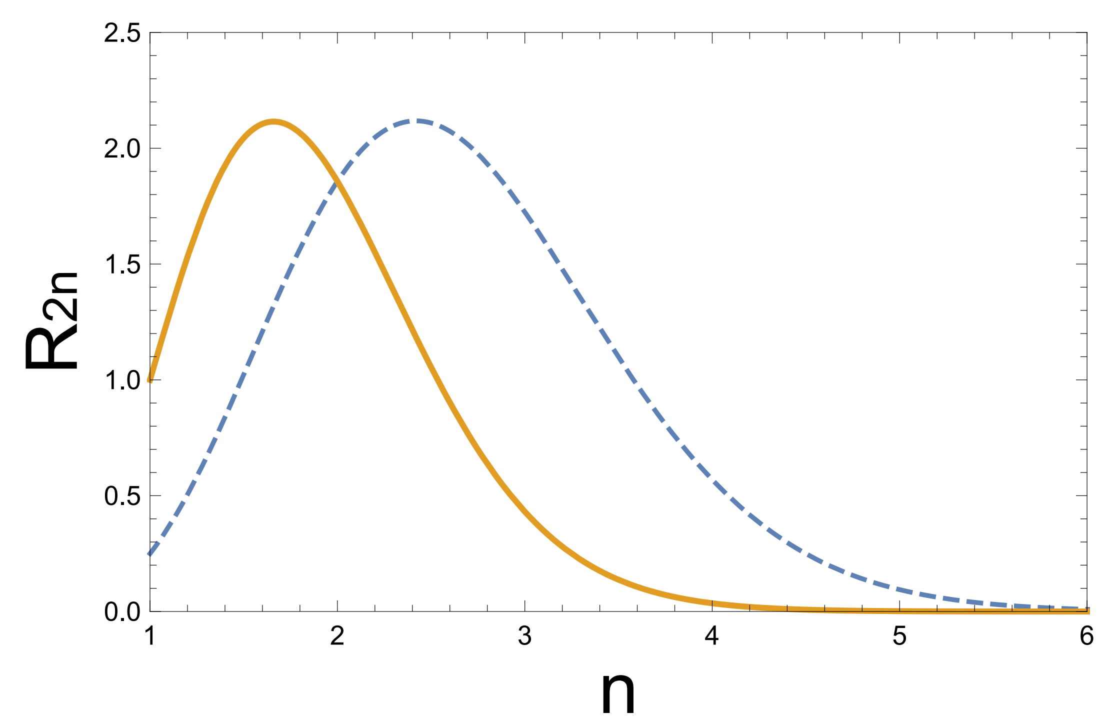
In order to incorporate this effect we make a second estimate. In the first estimate we have assumed p^2 to be very small compared with m^2 in the propagator an approximation valid for large n. Let us assume for the second estimate that on the contrary $p^2 \sim m^2$, an approximation which might be adequate for small n. This aproximation leads to

$$\frac{\Gamma_{2n}}{\Gamma_2} = \left(\frac{1}{2}\right)^{2n-2} \left(\frac{\alpha_g}{\pi}\right)^{2n-2} \left(\frac{M}{2m}\right)^{4n-4} \frac{2n!}{2!(2n-1)!(2n-2)!}$$

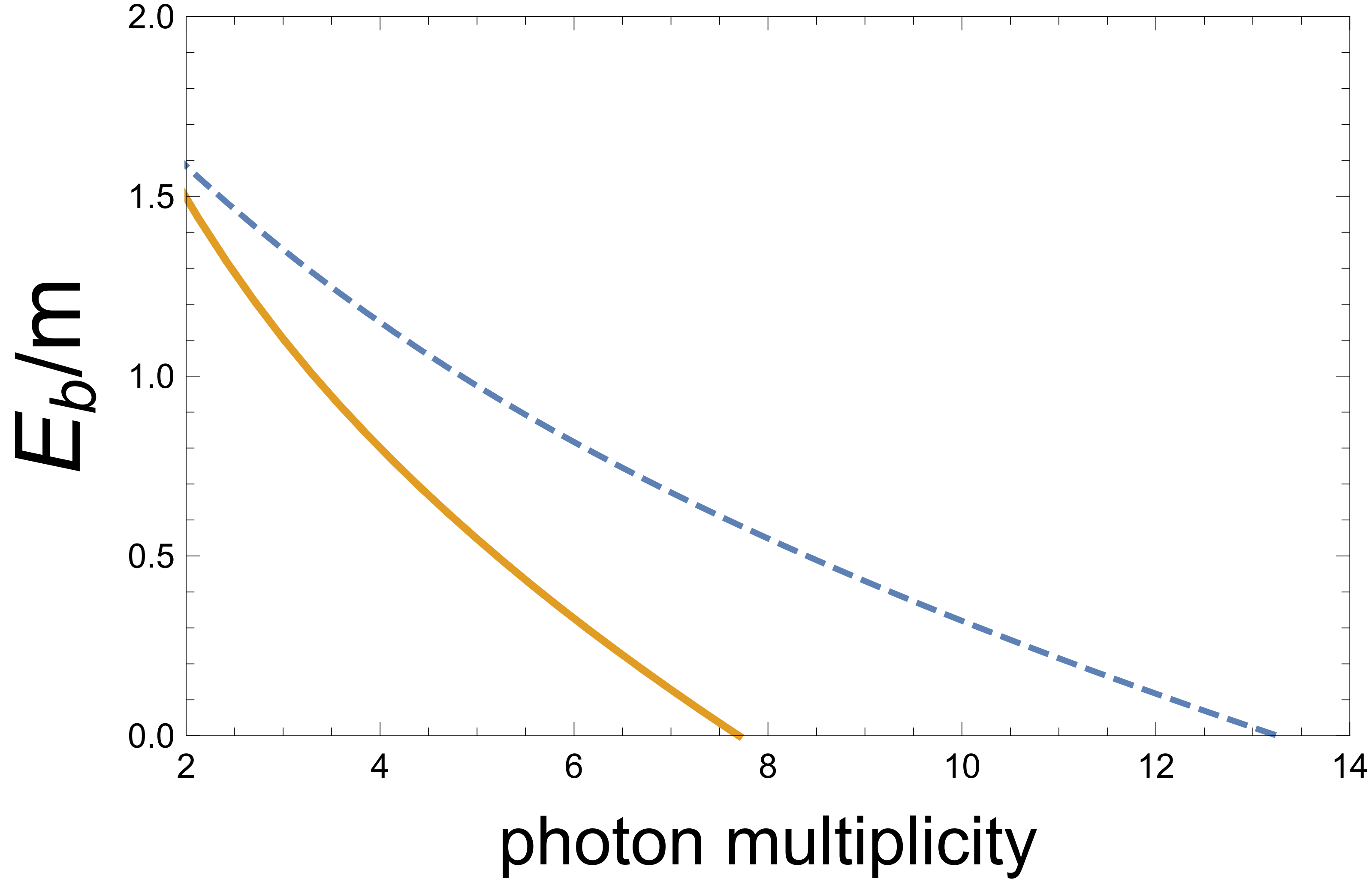
For n=2 this factor is 0.25 which is very close to true calculation to leading order 0.277. We show results with and without this factor to determine a region of confidence.



Monopoles



Monopolium



Photon multiplicity vs binding energy

Conclusions

-A “natural” scenario, due to the large coupling constant of the monopole, is that monopole-antimonopole or monopolium decay into photons might proceed via multi-photon decays.

-In view of the fact that the exact dynamics of monopoles and their properties are not available, large multiplicity of photon events might be the signal for the discovery of these elusive particles. Experiments should be ready to incorporate this feature into their analysis.

Final comment

The properties of the (anti)monopoles are rich both in their interaction among themselves, monopolium, and in their interaction with charged particles. In here we have assumed their existence and have studied how their properties can lead to the design of new experiments or different experimental scenarios. However, to go from the drawing board of the theoretician to the reality of an experiment many steps are needed which require the joint effort of theorists and experimentalists. I believe that the properties reviewed here are worth the effort.