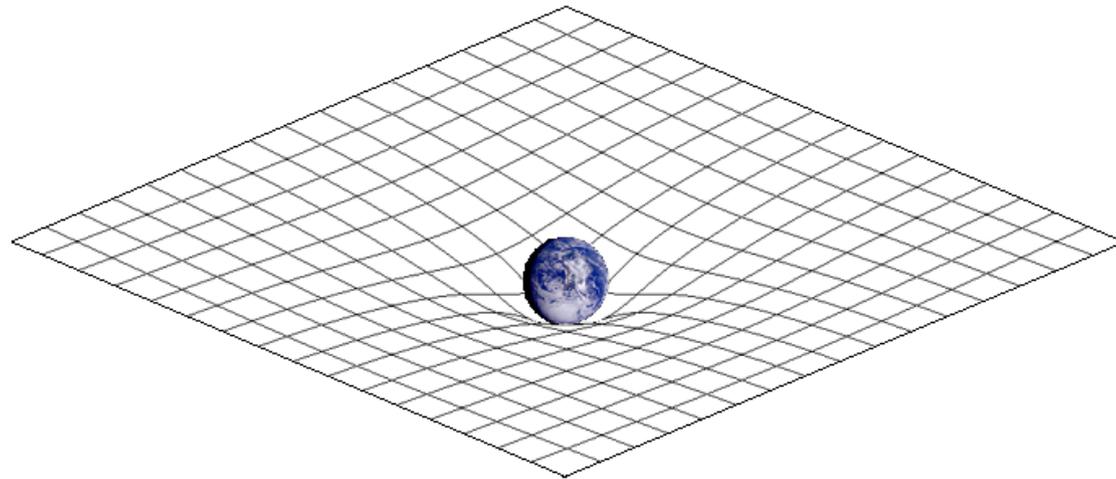


Gravitational and relativistic deflection of X-ray superradiance

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Shanghai Normal University, 11.07.2018



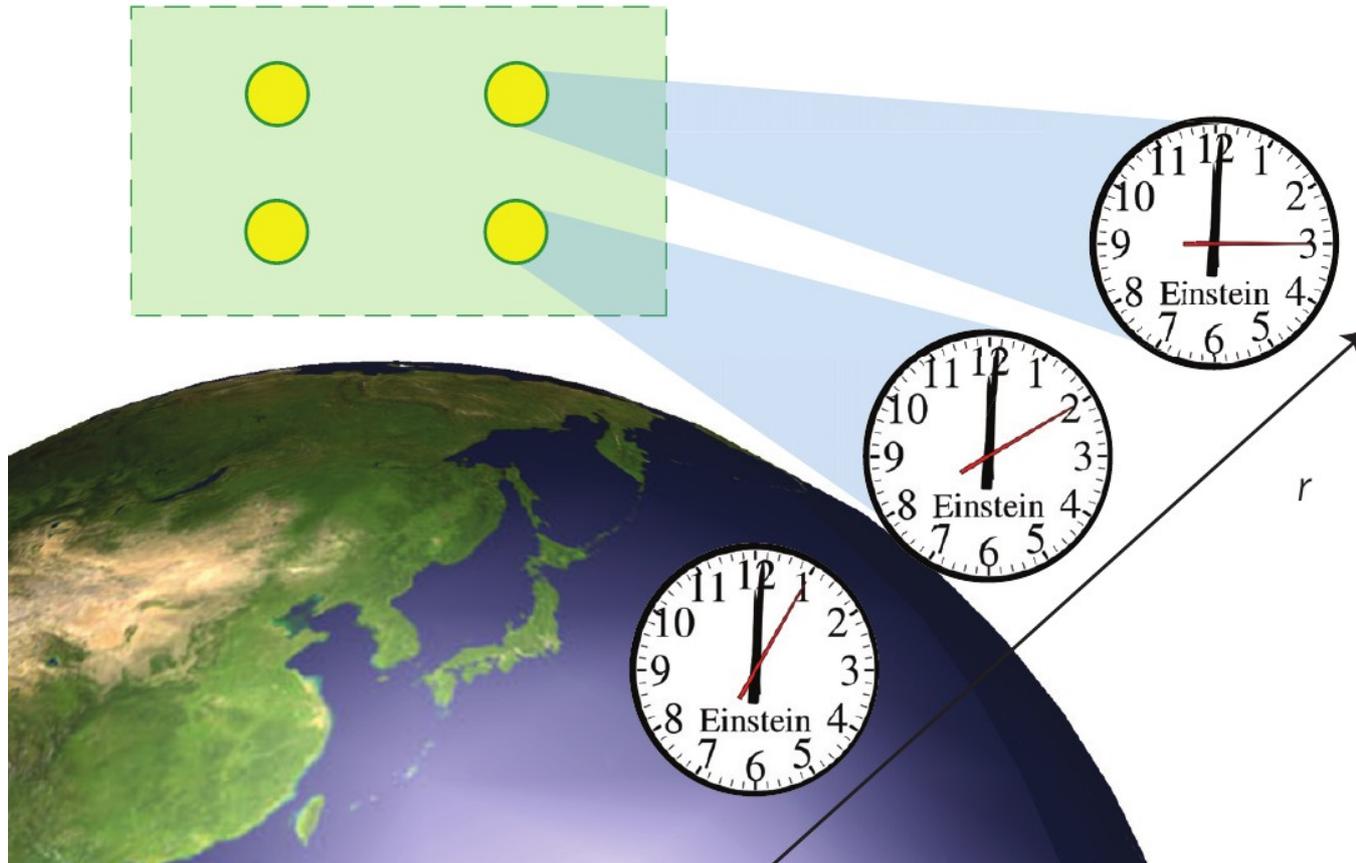
上海师范大学

SHANGHAI NORMAL UNIVERSITY

Mathematics & Science College

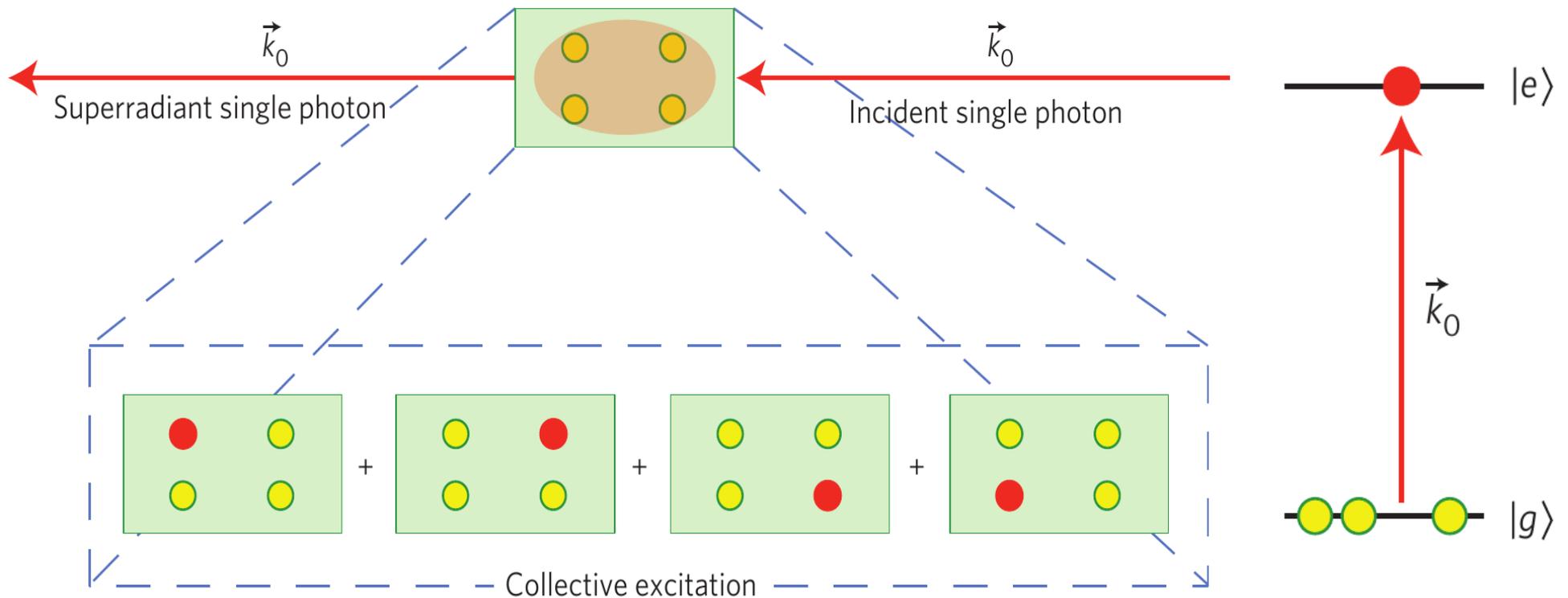
Time runs not uniformly in curved space-times

$$d\tau^2 = \left(1 - \frac{2GM_E}{rc^2}\right) c^2 dt^2 - \left(1 - \frac{2GM_E}{rc^2}\right)^{-1} dr^2 - r^2 d\Omega^2$$



K. Schwarzschild, Pr. Akad. d. Wiss. 3.2.1916, 189, (1916)

Probing time evolution with superradiant excitation

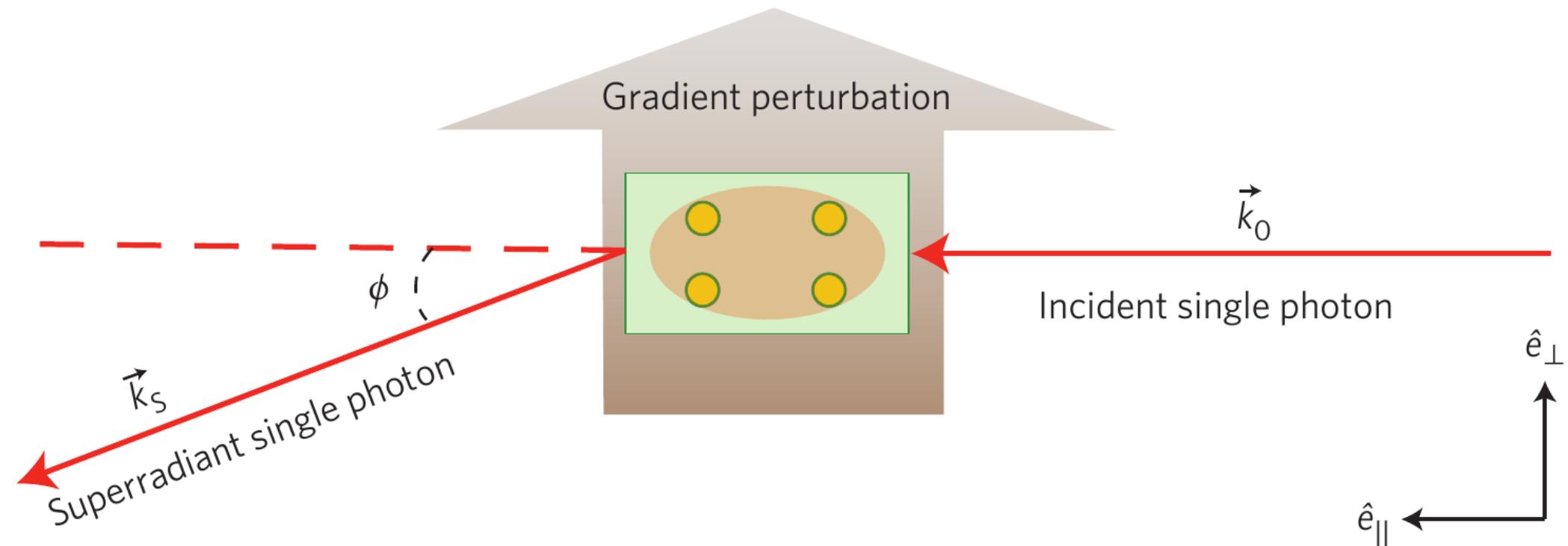


$$|E\rangle = \frac{1}{\sqrt{N}} \sum_l^N e^{i[\vec{k}_0 \cdot \vec{r}_l - \omega_l(\vec{R} + \vec{r}_l)t]} |g\rangle |e_l\rangle$$

M. Scully et al. , Phys. Rev. Lett. 96, 010501 (2006)

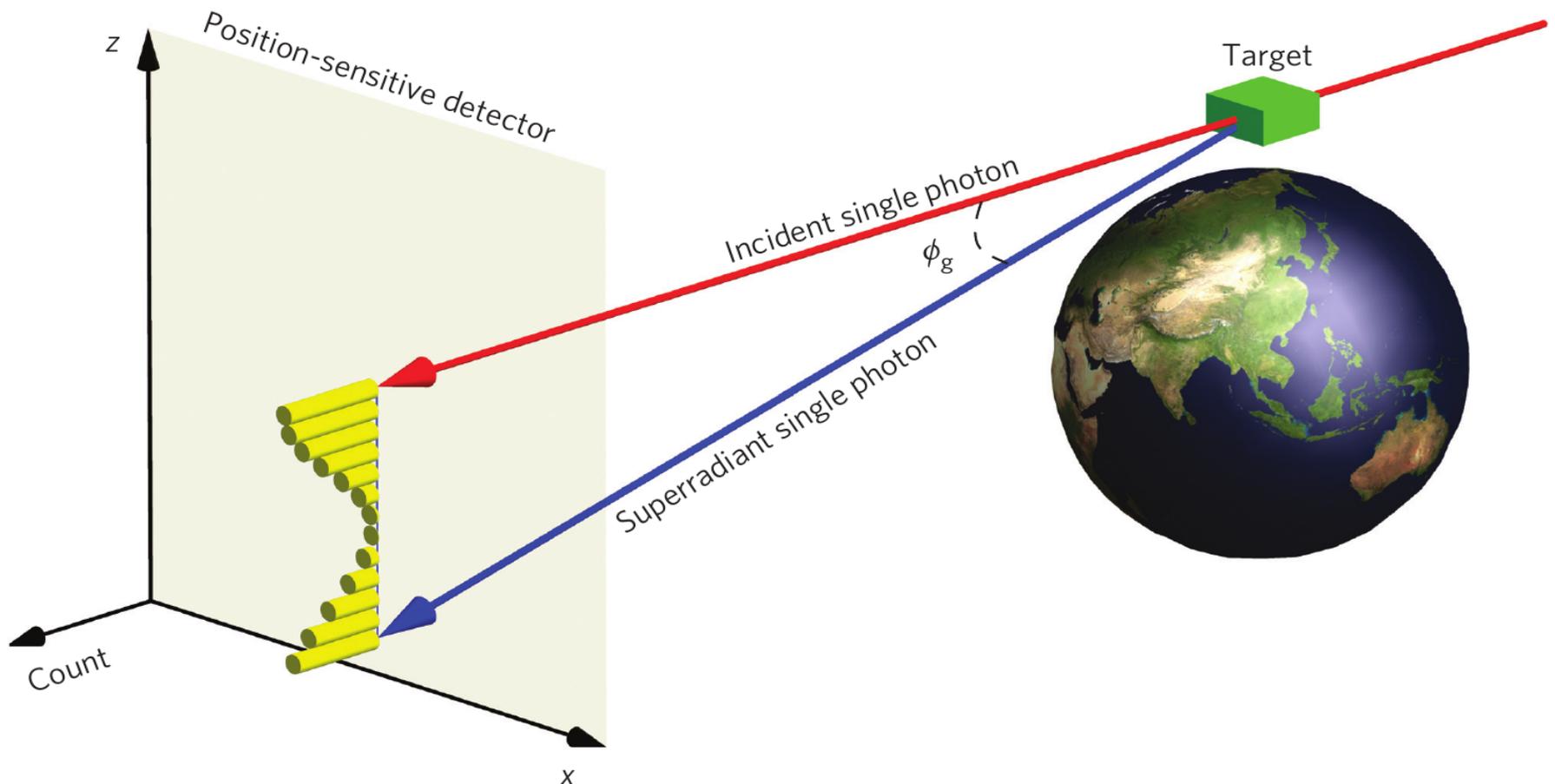
Time gradient induces tilt of X-ray excitation phase fronts

$$\omega_l(\vec{R} + r_l) \approx \omega_l + \vec{\nabla}\omega(\vec{R})\hat{e}_\perp \cdot \vec{r}_l \quad \Rightarrow \quad \vec{k}_s(t) = \vec{k}_0 - \vec{\nabla}\omega t \hat{e}_\perp$$



Earth gravity deflects X-ray superradiance

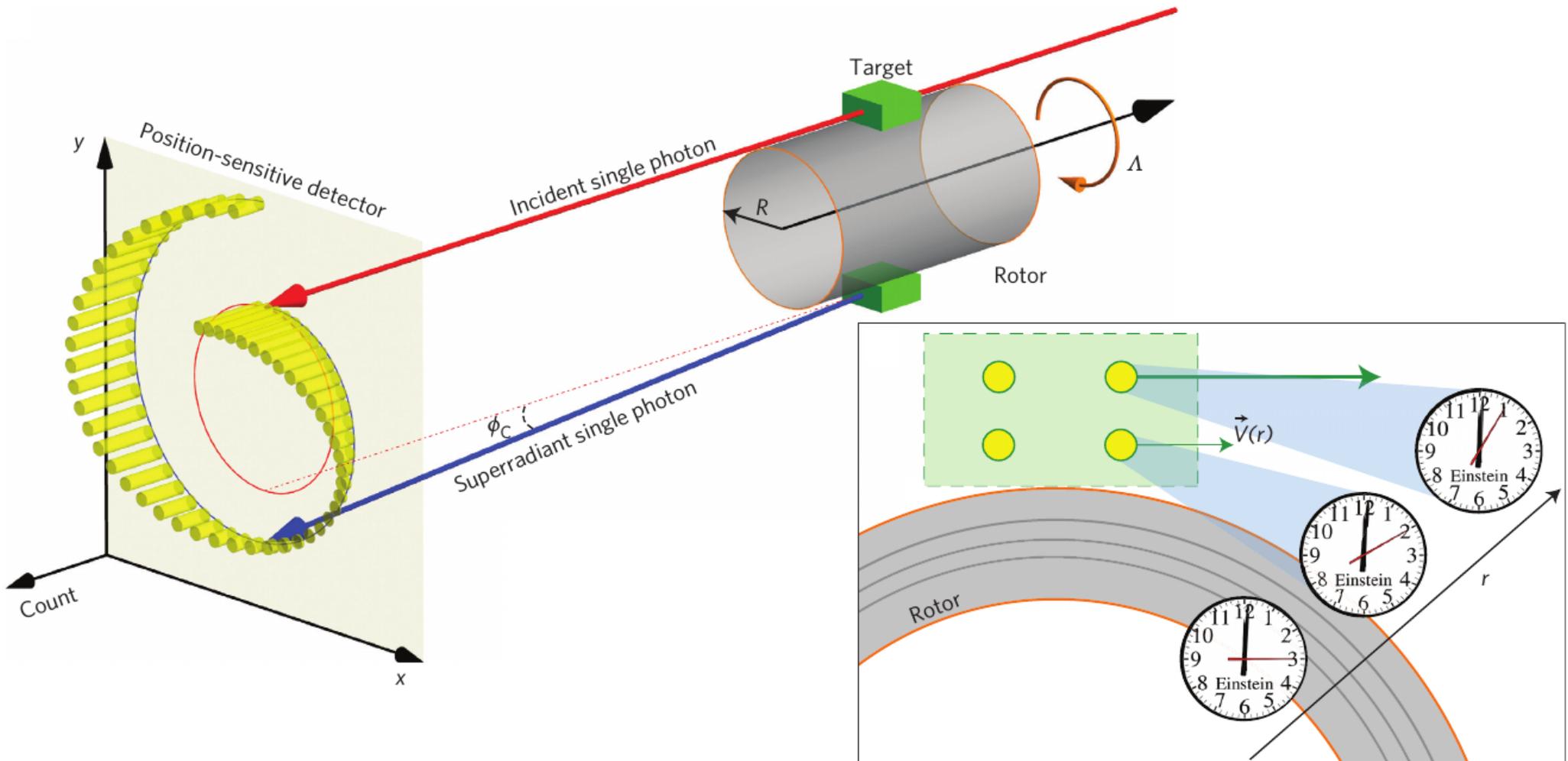
$$\phi_g \approx \tan^{-1} \left[\frac{|\vec{\nabla}\omega|t}{|\vec{k}_0|} \right] = \tan^{-1} \left[\frac{GM_E t}{cr_E^2 \sqrt{1 - \frac{2GM_E}{c^2 r_E}}} \right] \approx 1.9 \cdot 10^{-6} \text{ } ^\circ \text{ s}^{-1} t$$



Centripetal motion deflects X-ray superradiance

$$\phi_C \approx \tan^{-1} \left[\frac{R\Lambda^2 t}{\sqrt{c^2 - \Lambda^2 R^2}} \right] \approx 185^\circ \text{ s}^{-1} t$$

$R = 5 \text{ mm}$
 $\Lambda = 70 \text{ kHz}$



Deflection angles in experiment

crystal	E_e (keV)	coherence time τ_{coh}	$\phi_g(\tau_{coh})$ (degrees)	$\phi_c(\tau_{coh})$ (degrees)
^{45}Sc	12.4	459 ms	8.6×10^{-7}	90
^{57}Fe	14.41	141 ns	2.6×10^{-13}	2.6×10^{-5}
^{67}Zn	93.31	13.09 μs	2.5×10^{-11}	2.4×10^{-3}
^{73}Ge	13.28	4.21 μs	7.9×10^{-12}	7.8×10^{-4}
^{109}Ag	88.03	57.13 s	1.1×10^{-4}	90
^{181}Ta	6.24	8.73 μs	1.6×10^{-11}	1.6×10^{-3}
^{182}Ta	16.27	408 ms	7.7×10^{-7}	90
$^{229}\text{Th}:\text{CaF}_2$	0.0078	1 ms	1.9×10^{-9}	0.18

- $^{229}\text{Th}:\text{CaF}_2$ routinely used in experiment (see nuclear lighthouse effect)
- ^{109}Ag deflects $1.1 \times 10^{-4}^\circ \Rightarrow$ minimal target size: $37 \mu\text{m}!$

Our model

Wavefunction:

$$|\psi\rangle = |G\rangle \otimes \sum_{\vec{u}} B_{\vec{u}}(t) |1\rangle_{\vec{u}} + \frac{1}{\sqrt{N}} \sum_{\ell} D_{\ell}(t, \vec{r}_{\ell}) e^{i\vec{k}_0 \cdot \vec{r}_{\ell}} |g\rangle |e_{\ell}\rangle \otimes |0\rangle$$

Hamiltonian:

$$\begin{aligned} \hat{H} = & \hbar \sum_{\vec{k}} \nu_{\vec{k}} \hat{a}_{\vec{k}}^{\dagger} \hat{a}_{\vec{k}} + \hbar \sum_j [\omega + \Delta(r_j, z_j, t)] |e_j\rangle \langle e_j| \\ & + \hbar \sum_{j, \vec{k}} g_{\vec{k}} \left[e^{i\vec{k} \cdot \vec{r}_j} \hat{\sigma}_+^j \hat{a}_{\vec{k}} + e^{-i\vec{k} \cdot \vec{r}_j} \hat{\sigma}_-^j \hat{a}_{\vec{k}}^{\dagger} \right] \end{aligned}$$

Deflecting phase:

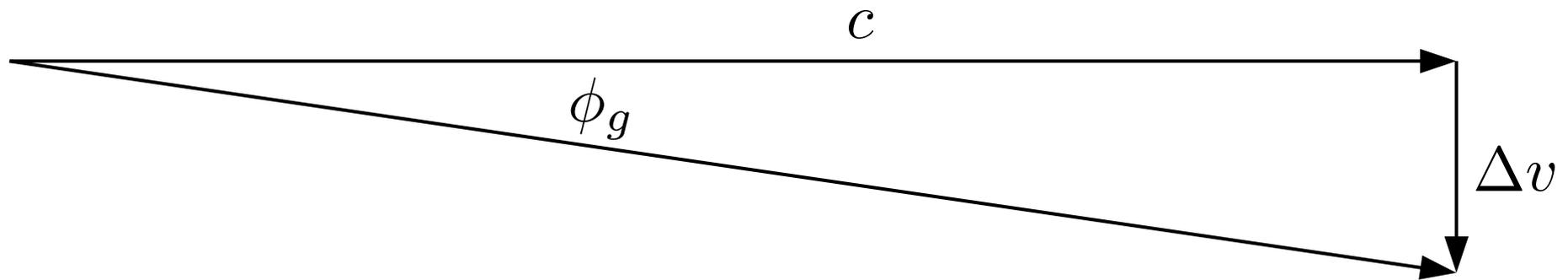
$$\Delta(\vec{r}_j, t) = \alpha + [Q\hat{e}_{\perp} + \beta(t)\hat{e}_{\parallel}] \cdot \vec{r}_j$$

Schrödinger equation:

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} \hat{H} |\psi\rangle$$

Comparison with dropping objects

Deflection of X-ray superradiance: $\phi_g \approx \frac{GM_E}{cr^2} \Delta t$



$$a = \frac{\Delta v}{\Delta t} \approx \frac{\phi_g c}{\Delta t} = \frac{GM_E}{r^2}$$

→ acceleration as for
massive particles in Newtonian gravity

Liao, Ahrens, Nature Photonics 9, 169 (2015)

Conclusions & Outlook

- quantum phase 'drops' like massive objects
- analog for centrifugal acceleration in special relativity
- possible experiment with ^{229}Th nucleus
- possible probe of on subatomic scales?
- comparison with freely propagating light ?

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Thank you!

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