Vorticity, hydrodynamic helicity and polarization in baryon-rich matter

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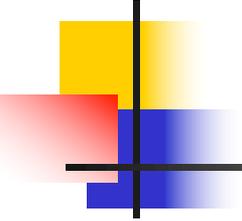
Based on the papers:

Phys. Rev. C82 (2010) with O. Rogachevsky

Phys. Rev. C88 (2013) with M. Baznat and K. Gudima

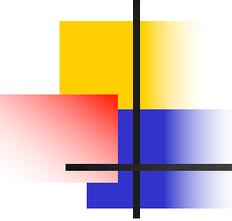*Phys. Rev C93 (2016) with M. Baznat and K. Gudima*

Phys.Rev. C95 (2017)

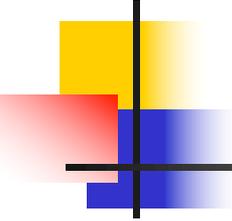*Phys. Rev. C97 (2018) with M. Baznat and K. Gudima*

Main Topics

- Rotation in heavy-ion collisions
- Λ Polarization in heavy-ion collisions
- Anomalous mechanism: 4-velocity as gauge field
- Comparison to thermal vorticity approach
- Chemical potential and Energy dependence: mass effects
- Rotation in heavy-ion collisions: Vortex sheets, helicity separation (helicity mirror structure and longitudinal vortisity quadrupole structure)
- Baryons vs antibaryons
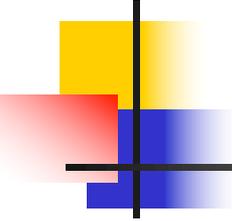- Vorticity and chaotic thermalization?
- Conclusions

Spin polarization

- Average normalized spin angular momentum: varies between 0 and 1
- P-odd quantity like any angular momentum
- Sensitive to P-odd effects
- Natural source in HIC – angular velocity and orbital angular momentum

Spin Physics in Heavy-Ion Collisions

- Spin-dependent observables might be manifested in HIC

- No beam polarization but plenty of effects in final state

- Especially interesting is the polarization of hyperons

- Self-analyzing: revealed in weak P-violating decay

- Related to P-odd effects in QCD medium:
  Vorticity and Hydrodynamic helicity

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (angular velocity ~ c/Compton wavelength)
- ~25 orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd: may lead to various P-odd effects

Rotation and polarization in HIC and related quantities

- Non-central collisions – orbital angular momentum is normal to the reaction plane (source of the induced global polarization?)

  $L = \Sigma \, r \times p$
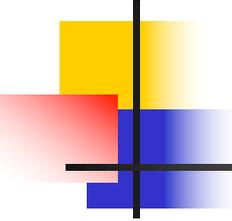
- Differential pseudovector – vorticity

  $\omega = \mathrm{curl}\ v$

- Pseudoscalar – helicity

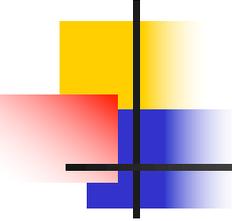 $H \sim \langle (v \, \mathrm{curl}\ v) \rangle$

- Maximal helicity – Beltrami chaotic flows
  $v \parallel \mathrm{curl}\ v$

Λ-polarisation

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – no direction normal to the scattering plane
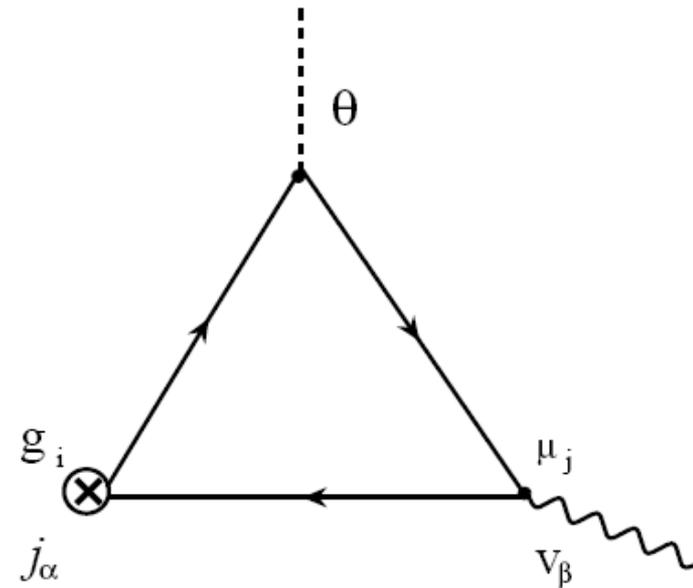- But is the randomization complete (smoothly from hadrons to ions)?!

Global Λ polarization

- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- **How to transform rotation to spin?**

- Gauge field -> 4-velocity (V.Zakharov) $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$

- CME (Kharzeev, McLerran,Warringa; Fukushima; Vilenkin;…) -> CVE

- Kharzeev, Zhitnitsky (2007) – EM current

- Straightforward generalization: any (e.g. baryonic) current – neutron asymmeries@NICA - Rogachevsky, Sorin, Teryaev (2010)

- Vorticity: $e_j \vec{H} \to \mu_j \vec{\nabla} \times \vec{V}$

- Induced currents $J_e^\gamma = \dfrac{N_c}{4\pi^2 N_f} \varepsilon^{\gamma\beta\alpha\rho} \partial_\alpha V_\rho \partial_\beta \left(\theta \sum_j e_j \mu_j \right)$
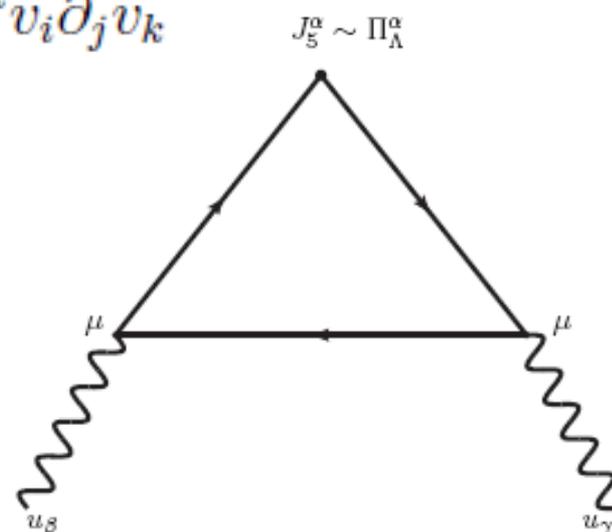
Axial Vortical Effect (AVE) – anomalous mechanism of polarization

- 4-Velocity is also a "GAUGE" FIELD $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, Teryaev, 2010)

AVE induced axial current (charge):

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Analogous to anomalous gluon contribution to nucleon spin

  (Efremov, Teryaev 88)

- 4-velocity instead of gluon field!

From axial charge to polarization (and from quarks to confined hadrons) – analog of Cooper-Frye

- Analogy of matrix elements and classical averages (account for other charges!)

$$< p_n|j^0(0)|p_n >= 2p_n^0 Q_n \; ; \qquad < Q >\equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x\, j_{class}^0(x)}{N}$$

$$< p_n, \Pi_n|j_{5,i}^0(0)|p_n, \Pi_n >= 2a_{i,n}m_n\Pi_n^0$$

$$Q_{5,i,n} \rightarrow \frac{m_n a_{i,n}\Pi_n^0}{p_n^0} \; ; \qquad < \frac{a_i m \Pi^0}{p_0} >\equiv \frac{\sum_{n=1}^N \frac{m_n a_{i,n}\Pi_n^0}{p_n^0}}{N} = \frac{\int d^3x\, j_{5,class}^0(x)}{N} .$$

Anomaly for polarization

- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- T-dependent term - related to gravitational anomaly
- Lattice simulation: suppressed due to collective effects

Perturbative Quark mass effects

$$\langle p|\bar{Q}\gamma_\mu\gamma_5 Q|p\rangle = i\frac{N_c\alpha_s}{2\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\left\{1 - \int_0^1 dx\frac{2m_Q^2(1-x)}{m_Q^2 - p^2 x(1-x)}\right\}$$

$$= -i\frac{N_c\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_Q^2} + O(\frac{1}{m_Q^4}),$$

FIG. 2. Dependence of the anomaly coefficient on $r = 2m^2/k^2$

Comparison of methods

- Wigner function (Becattini et al)– induced axial current (triangle diagram– V.I. Zakharov) – Prokhorov ,Teryaev, Phys.Rev. D97 (2018) no.7, 076013

$$\alpha_\mu = \frac{1}{T} u^\nu \partial_\nu u_\mu = \frac{a_\mu}{T}, \quad w_\mu = \frac{1}{2T} \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha u^\beta = \frac{\omega_\mu}{T}$$

$$\langle : j_\mu^5 : \rangle = \Big(\frac{1}{6} \big[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \big] + \frac{\mu^2}{2\pi^2} \Big) \omega_\mu + \frac{1}{12\pi^2} (\omega \cdot a)\, a_\mu$$

$$\langle : j_\mu^5 : \rangle = 2\pi \, \mathrm{Im} \Big[\Big(\frac{1}{6}(T^2 + \varphi^2) + \frac{\mu^2}{2\pi^2} \Big) \varphi_\mu \Big] \qquad \varphi_\mu = \frac{a_\mu}{2\pi} + \frac{i\omega_\mu}{2\pi}$$

- ## Anomalous term reproduced
- ## New terms of higher order in vorticity

Similarity and difference of methods

- Anomalous method – massless limit
- Spin carriers: quarks
- Sign and scale of polarization – depends on hadron structure
- TD method –massive particles
- Spin carriers: hyperons
- Universal sign and weak (mass) dependence of scale

Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \, \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Rapid decrease with energy!
- Large chemical potential: appropriate for NICA/FAIR energies

One might compare the prediction below with the right panel figures

One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

Current development – more accurate STAR 200 GeV data (Z.Ye, T. Niida @QM2018)

- ## Drop at about of order of magnitude

STAR, arXiv:1805.04400

$$P_H(\Lambda) \; [\%] = 0.277 \pm 0.040(\text{stat}) \pm^{0.039}_{0.049} (\text{sys})$$

$$P_H(\bar{\Lambda}) \; [\%] = 0.240 \pm 0.045(\text{stat}) \pm^{0.061}_{0.045} (\text{sys})$$

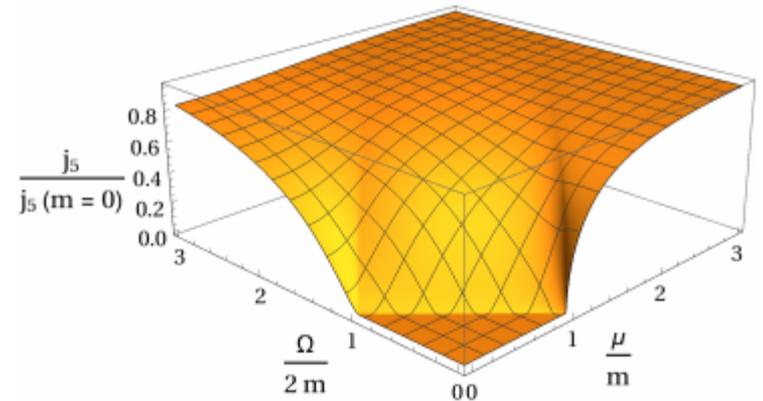- ## Square of chemical potential should decrease faster!

Possible explanation: role of non-perturbative mass effects (Prokhorov, Teryaev, Zakharov, arXiv:1805.12029)

- Wigner function approach (following F. Becattini et al.)

- Anomalous axial current reproduced in massless limit and momentum integration ("hidden anomaly")

- Angular velocity leads to "effective" chemical potential $\mu+\varpi/2$

Small chemical potential (comparable to mass)

- Result nullifies for small enough chemical potential and velocity

- Suggestion: for small enough potentials anomalous mechanism is transformed to "TD" one (TD/dynamics duality)

Simulation in QGSM (Kinetics -> HD)

Simulations in kinetic Quark-Gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes): Baznat, Gudima, Sorin,Teryaev (Phys. Rev. C 2013, 2016)

$50 \times 50 \times 100$ cells $\qquad dx = dy = 0.6 \text{ fm}, dz = 0.6/\gamma \text{ fm}$

- Velocity $\qquad \vec{v}(x, y, z, t) = \dfrac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$

- Vorticity – from discrete partial derivatives

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)

- Conservation of OAM with a good accuracy!

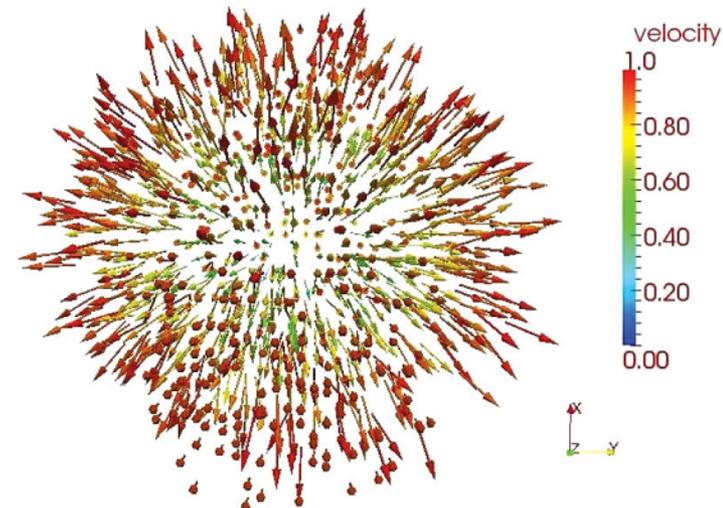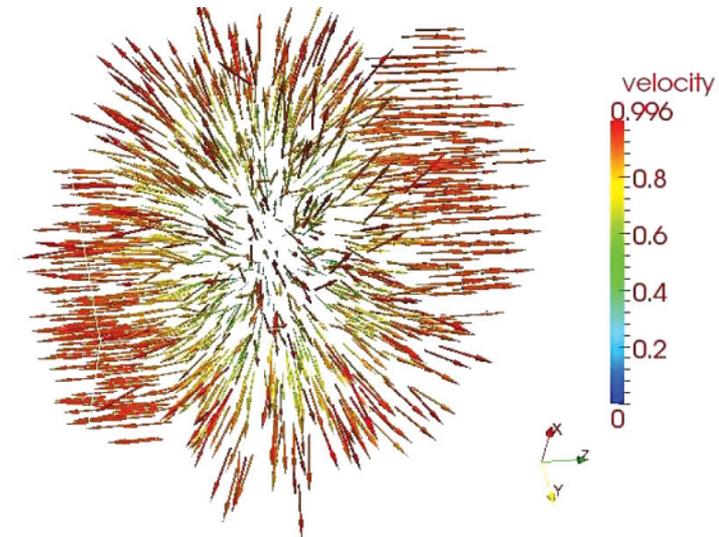# Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)

Distribution of velocity ("Little Bang")

- 3D/2D projection

- z-beams direction
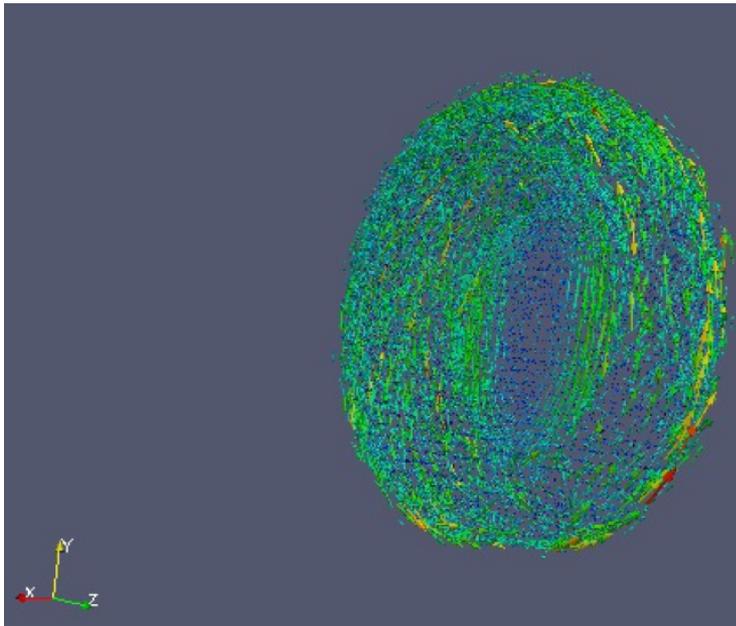
- x-impact paramater
- Little Hubble law

$$< v/c >= v_0/c + H\rho.$$
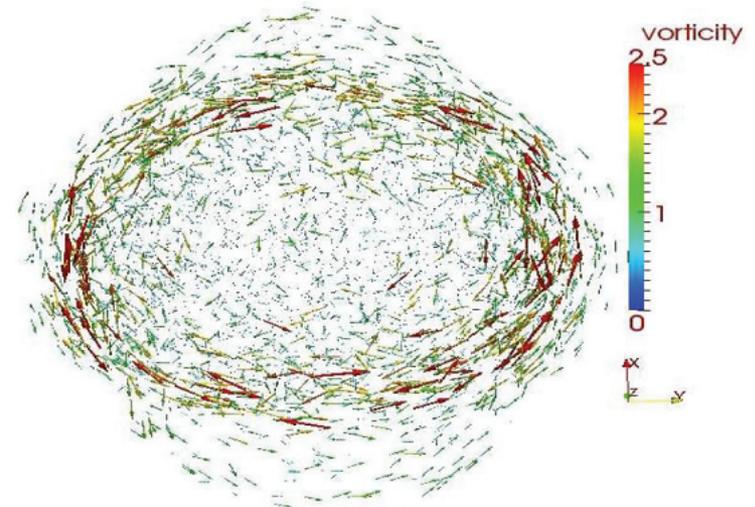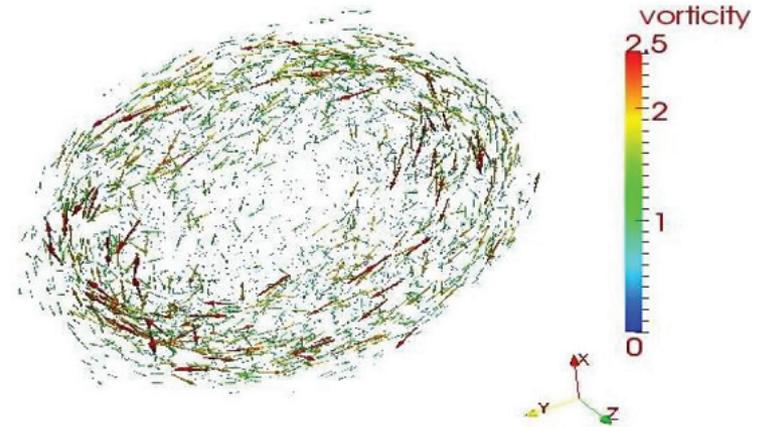
$$H = 0.024 \div 0.028 \, (\text{fm}/\text{c})^{-1}$$

velocity
0.996
0.8
0.6
0.4
0.2
0

velocity
1.0
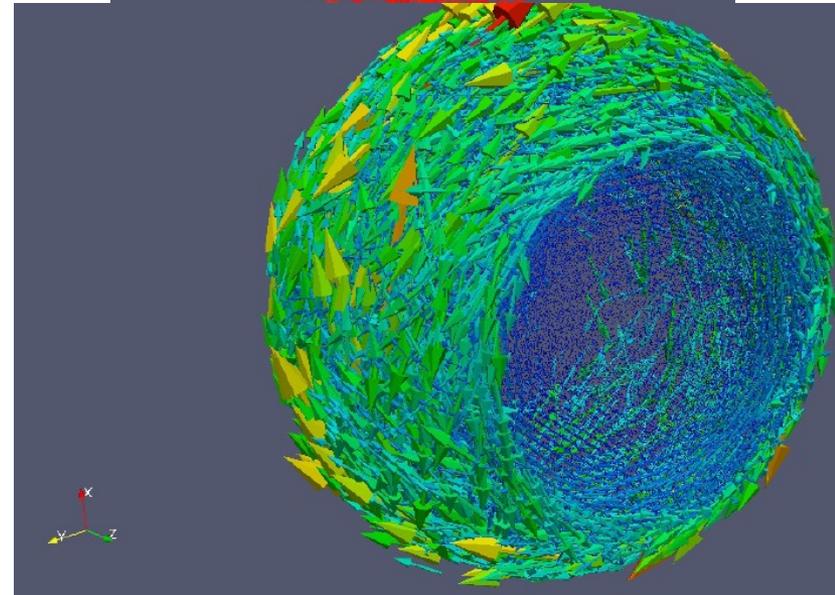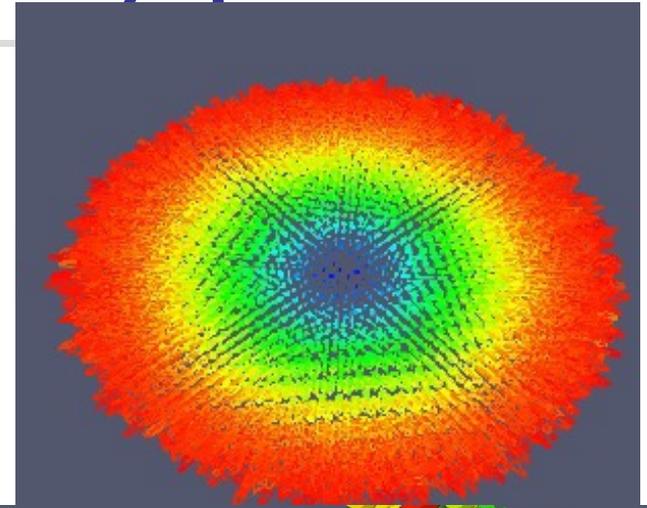0.80
0.60
0.40
0.20
0.00

Distribution of vorticity ("Little galaxies")

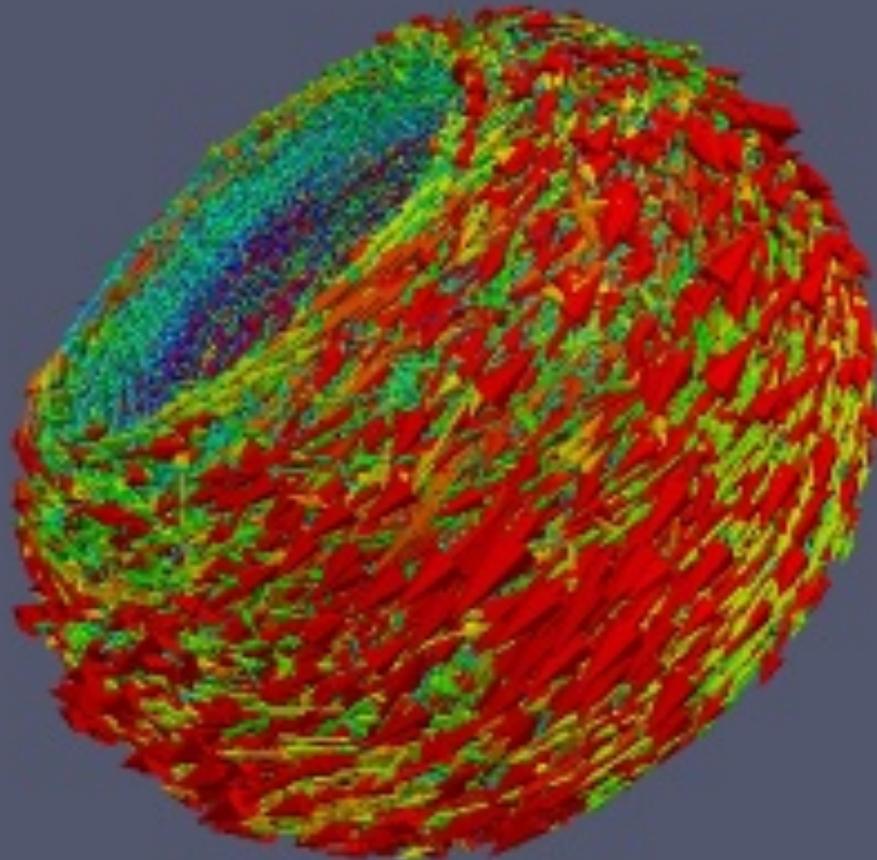- Layer (on core - corona borderline) patterns
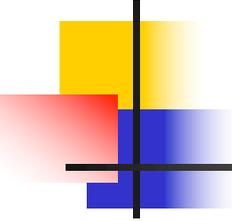
Velocity and vorticity patterns
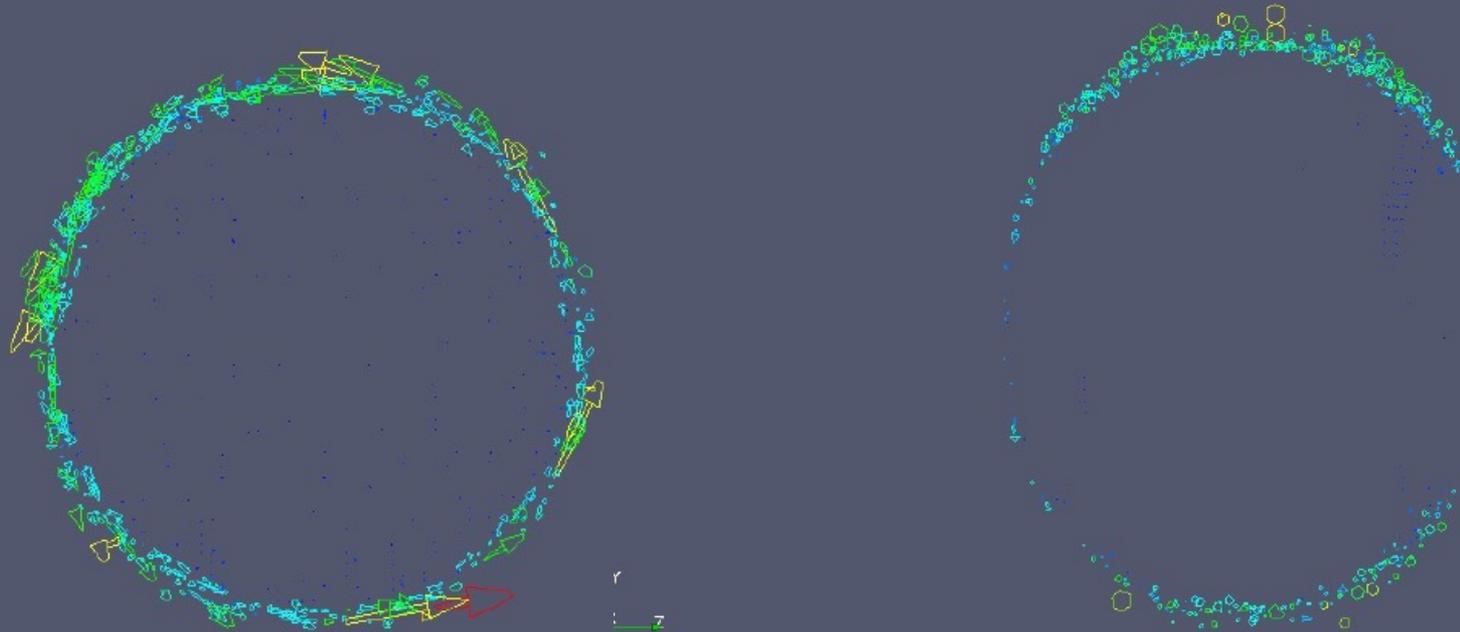
- Velocity

- Vorticity pattern –
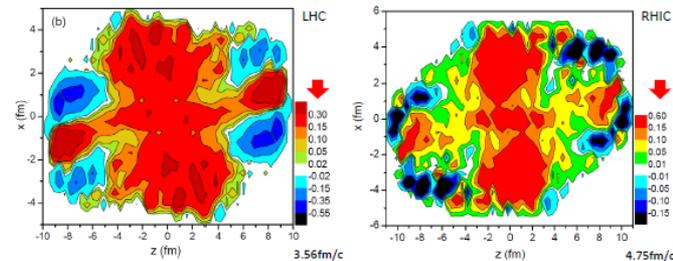 vortex sheets

Vortex sheet

Sections of vorticity patterns
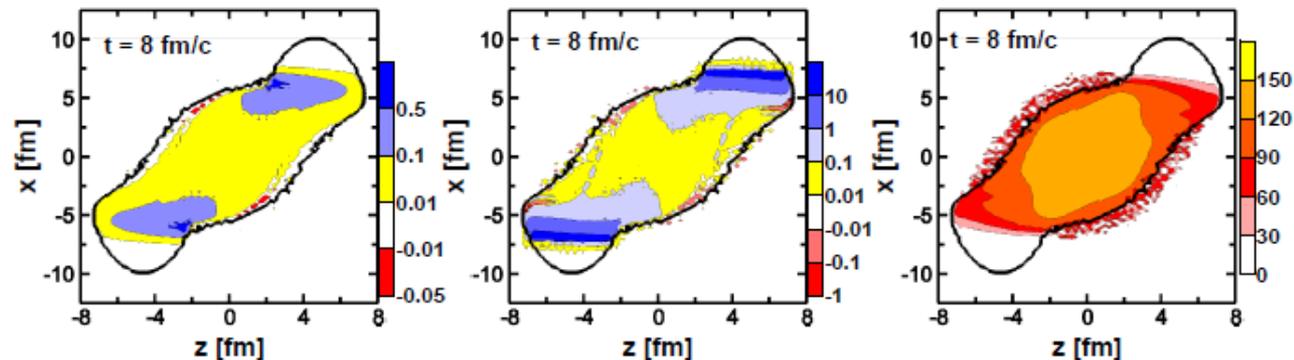
- Front and side views

Vortex sheets

- ## Naturally appears in kinetic models
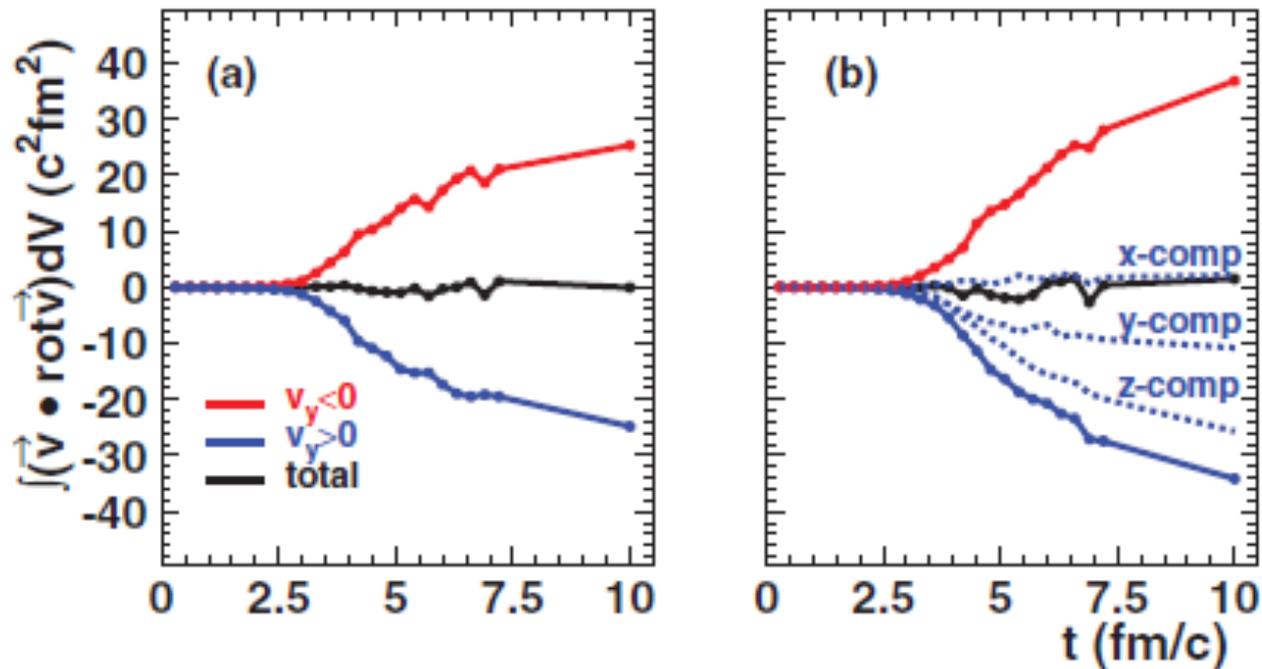- ## Absent in viscous HD (L. Csernai et al)

- Appears in 3 fluid dynamics model

(Yu. Ivanov, A. Soldatov, Phys.Rev. C97 (2018) no.4, 044915**)**

Helicity separation in QGSM
PRC88 (2013) 061901

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (Teryaev, Usubov, PRC92 (2015) 014906

Transverse and longitudinal vorticity

- Transverse vorticity – the same sign on the two sides of reaction plane

- Change of velocity sign ($v_y \sim sign(y)$) leads to helicity separation $h_y \sim sign(y)$

- Longitudinal vorticity – must have quadrupole structure to provide mirror structure of helicity

$$h = h_x + h_y + h_z \sim sign(y); v_z \sim sign(x); \omega_z \sim sign(x)sign(y)$$

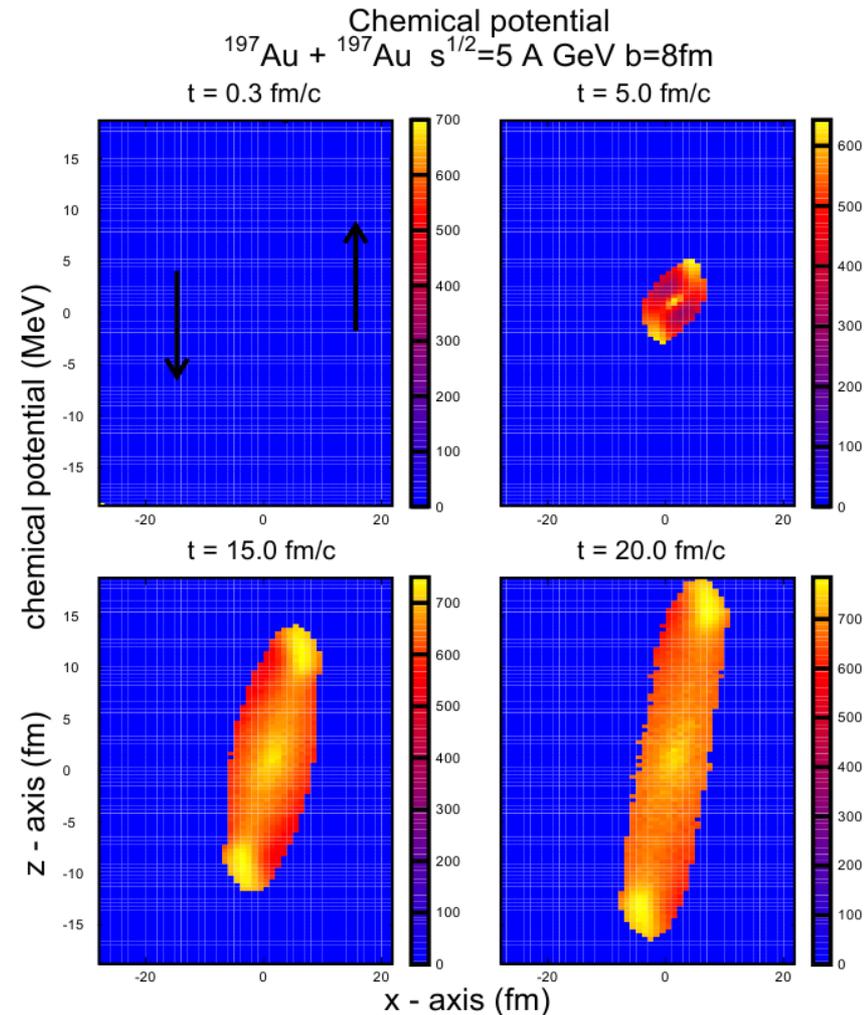$$h_z = \omega_z v_z = (sign(x))^2 sign(y) = sign(y)$$

Quadrupole structures – averaged in cylinders along z

Chemical potential : Kinetics -> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: y=0

Chemical potential
^{197}Au + ^{197}Au $s^{1/2}$=5 A GeV b=8fm

Strange chemical potential (polarization of Lambda is carried by strange quark!)

- Non-uniform in space and time

Strange chemical potential
^{197}Au + ^{197}Au $s^{1/2}$=5 A GeV b=8fm

Temperature

Temperature
$^{197}Au + ^{197}Au$ $s^{1/2}=5$ A GeV b=8fm

Axial charge and polarization: role of kinematics

- Analogy of matrix elements and classical averages (account for other charges!)

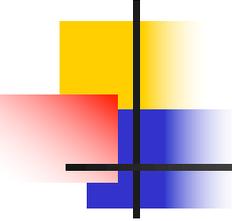$$< p_n|j^0(0)|p_n >= 2p_n^0 Q_n \qquad < Q >\equiv \frac{\sum_{n=1}^{N} Q_n}{N} = \frac{\int d^3x \, j_{class}^0(x)}{N}$$

- Lorentz boost: compensate the sign of helicity

$$< p_n, \Pi_n|j_{5,i}^0(0)|p_n, \Pi_n >= 2a_{i,n}m_n\Pi_n^0 \qquad\qquad Q_{5,i,n} \to \frac{m_n a_{i,n}\Pi_n^0}{p_n^0}$$

$$\Pi^{\Lambda,lab} = \left(\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}\right) = \frac{\Pi_0^\Lambda}{m_\Lambda}(p_y, 0, p_0, 0)$$

$$< \Pi_0^\Lambda > = \frac{m_\Lambda \Pi_0^{\Lambda,lab}}{p_y} =< \frac{m_\Lambda}{N_\Lambda \, p_y} > Q_5^s \equiv< \frac{m_\Lambda}{N_\Lambda \, p_y} > \frac{N_c}{2\pi^2}\int d^3x \, \mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k$$
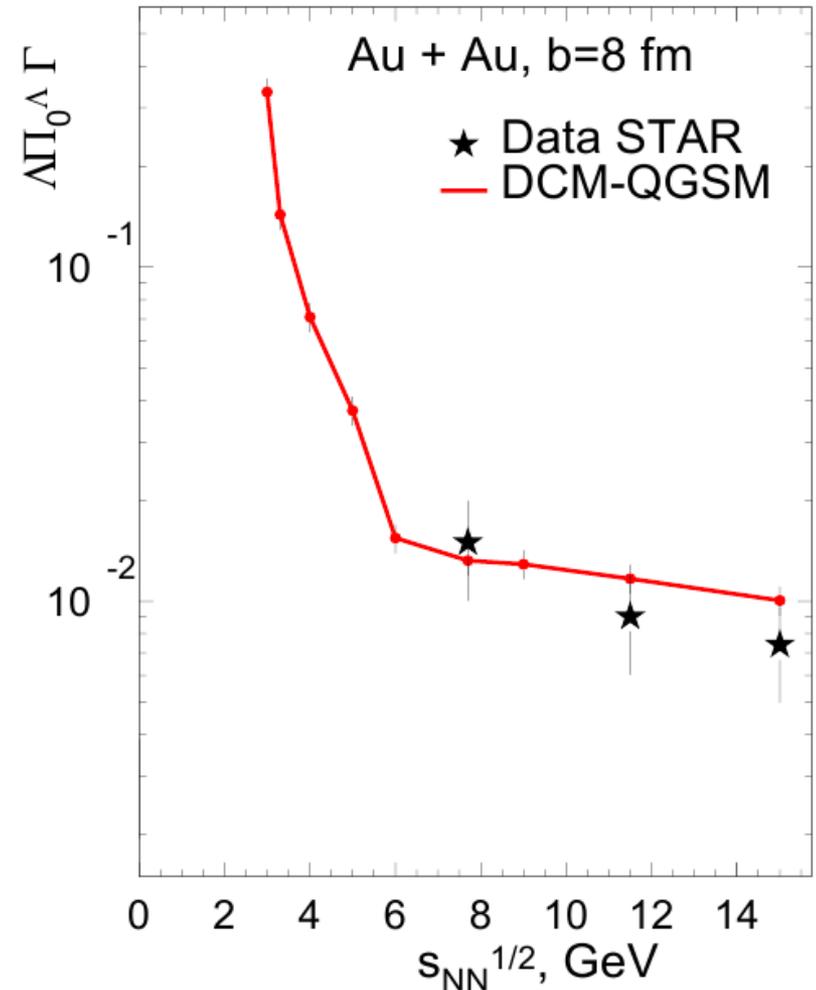
Axial charge and properties of polarization

- Polarizationis enhanced for particles with small transverse momenta – azimuthal dependence naturally emerges
- Antihyperons: same sign (C-even axial charge) and larger value (smaller N)
- More pronounced at lower energy BUT
- Baryon/antibaryon splitting due to magnetic field – increase with energy
- Recent STAR data support polarization for particles with angles close to reaction plane and closeness of baryons and antibaryons polarization at 200 GeV
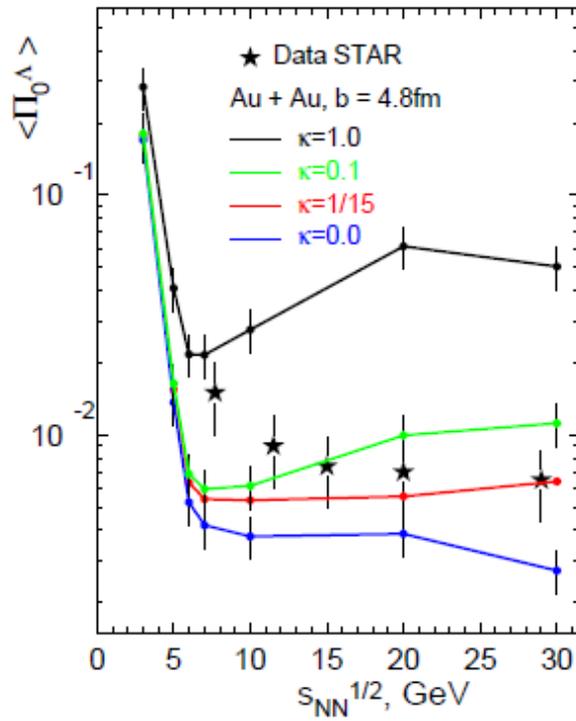
Energy dependence

- Growth at low energy

- Close to STAR data

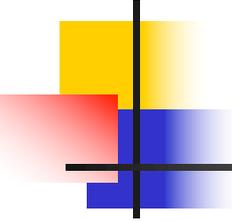- Structure – due to low number of Lambdas

The role of (gravitational anomaly related) T² term

- Different values of coefficient probed

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + k\frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$
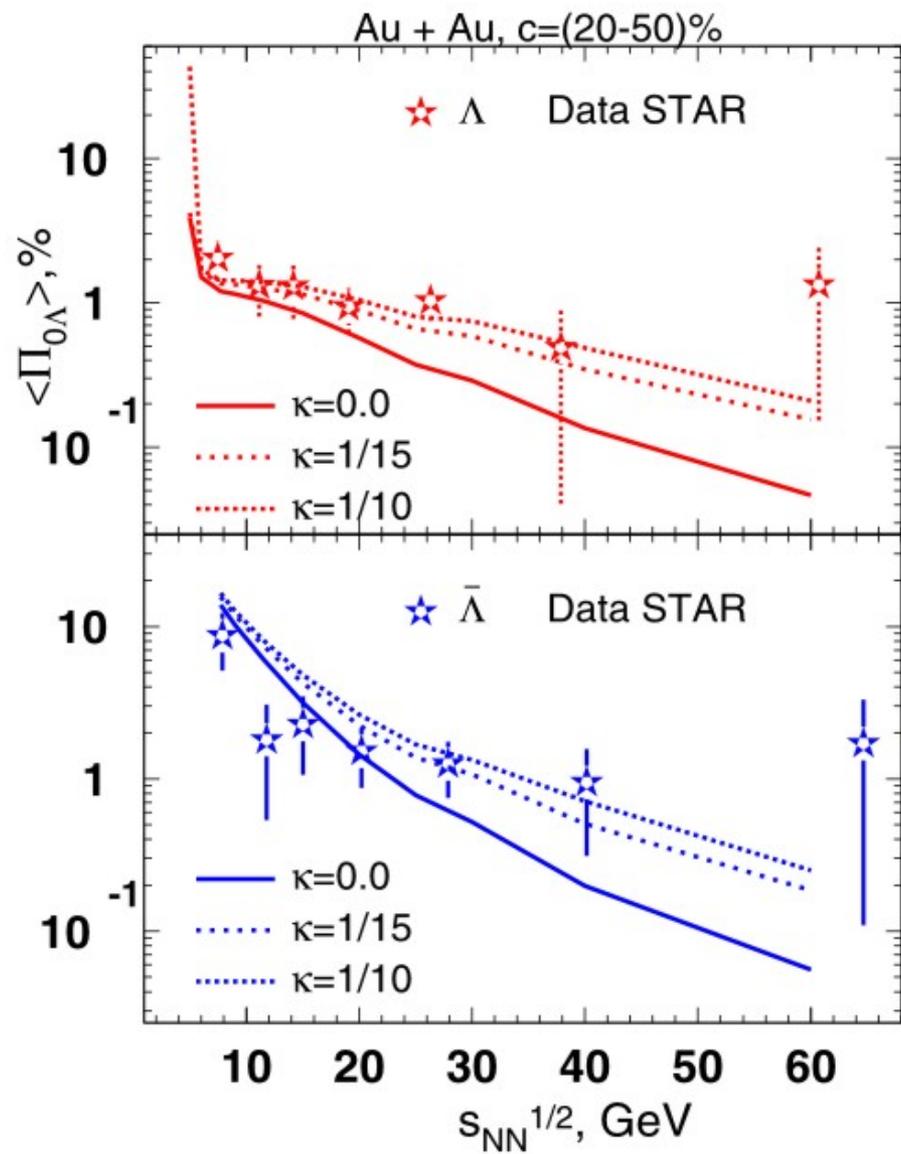
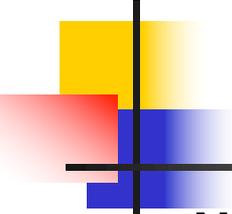- LQCD suppression by collective effects supported

Lambda vs Antilambda and role of vector mesons

- Difference at low energies too large – same axial charge carried by much smaller number of hadrons

- Strange axial charge may be also carried by K* mesons

- Λ - accompanied by (-,anti 0) K* mesons with two sea quarks – small corrections

- Anti Λ – accompanied by more numerous (+,0) K* mesons with single (sea) strange antiquark

Λ vs Anti Λ

Arnold-Beltrami flows

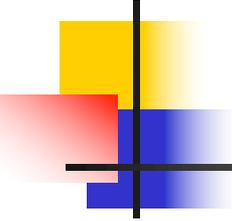- **Nonrelativistic incompressible fluids with vorticity parallel to velocity**

$$\omega_i \equiv \epsilon_{ijk}\,\partial_j\,v_k = m\,v_i$$

- **Compatible with Euler equation for steady flows**

$$v_j\,\partial_j\,v_i = -\frac{1}{\rho}\partial_i\,p$$

- **Bernoulli condition is valid in the whole volume of the fluid**

$$\partial_i\,\phi = 0 \qquad\qquad \phi = \frac{p}{\rho} + \frac{v^2}{2}$$
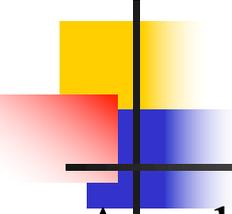
Chaotic streamlines

$$\frac{d}{dt}x^i(t) = u^i(x(t), t)$$

Arnold's theorem:

For flows taking place on compact three manifolds, the only velocity fields able to produce chaotic streamlines are those satisfying Beltrami equation.
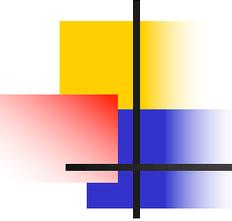
&

Topological conception of contact structures, each of which admits a representative contact vector field also satisfying Beltrami equation.
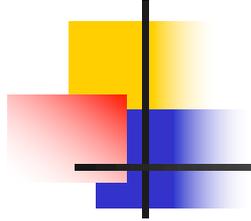
Chaotic thermalization?

- Arnold-Beltrami flows – Lagrangian turbulence

- Simple explanation: Bernoulli in the volume – the streamlines come close to each other

- Chaotic advection: laminar flows result in the chaotic motion of passive admixture

- Fast Dynamo problem, the spontaneous generation of a exponential growing magnetic field in a flow of conducting fluid with vorticity

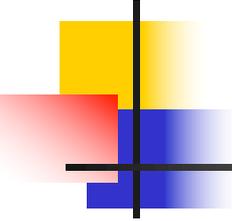- Possible role in the fast thermalization (complementary description)?!

Conclusions/Outlook

- Polarization – new probe of anomaly in quark-gluon matter (to be studied at NICA

- Generated by femto-vortex sheets

- Energy dependence predicted and confirmed

- Same sign and larger magnitude of antihyperon polarization: splitting decreases with energy (contradicts to explanation due to magnetic moment/field ; supported by the data)

- T-dependent term due to gravitational anomaly may be extracted from the data

- Anomalous vs TD: massless quarks vs massive hadrons – may be dual at smaller chemical potentials

THANK YOU FOR ATTENTION!

Spin effects in heavy ion collisions may be used as a complementary probe spin physics program

involving all the NICA detectors (MPD, SPD, BM@N),

Polarization at NICA/MPD (A. Kechechyan)

- QGSM Simulations and recovery accounting for MPD acceptance effects

AuAu (LAQGSM)