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# Implicit assumptions in the proof of the Bell's inequality

David Oaknin

Rafael Advanced Defense Systems, Haifa, Israel

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# An explicitly local statistical model of hidden variables for Bell’s polarization states

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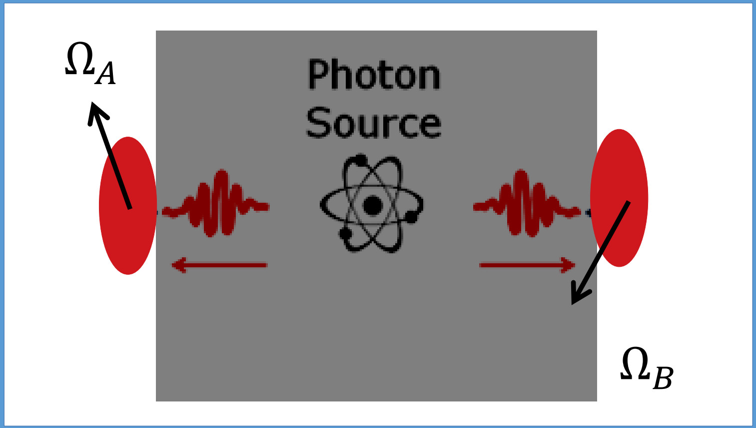
Rafael Advanced Defense Systems, Haifa, Israel

# Bell's experiment:




$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle^{(A)}\,|\downarrow\rangle^{(B)} \;+\; \mathrm{e}^{i\Phi}\,|\downarrow\rangle^{(A)}\,|\uparrow\rangle^{(B)}\right),$$

$\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ : eigenstates of Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes.

The particles' polarization is tested at two widely separated detectors along two arbitrary directions within the locally defined XY-planes.

Quantum Mechanics:

$$E(\Delta - \Phi) = -\cos(\Delta - \Phi)$$

$E(\Delta - \Phi)$ : Statistical correlation between binary outcomes of the two detectors.

$\Delta$ : Relative angle between the orientations of the two detectors.

The Bell's inequality states in an experimentally testable way that this prediction cannot be reproduced by any model of hidden variables that shares certain intuitive features.

In particular, the CHSH version of the inequality states that for any such model of hidden variables:

$$|E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \Delta) - E(\Delta_2 - \Delta)| \leq 2$$

for every set of values $(\Delta_{1,}\Delta_{2,}\Delta)$ .

On the other hand, according to the predictions of Quantum Mechanics this magnitude reaches a maximum value of $2\sqrt{2}$

Very carefully designed experimental tests have confirmed the predictions of Quantum Mechanics and, thus, ruled out all such models of hidden variables.

# The CHSH inequality's proof :

$S$ : space of possible configurations

$\rho(\lambda)$ : (density of) probability for each $\lambda \in S$ to happen in a single realization of the experiment.

$s_{\Omega_A}^{(A)}(\lambda)$, $s_{\Omega_B}^{(B)}(\lambda)$ : binary values - either $+1$ or $-1$, which describe the outcomes in each one of the two detectors if the polarization of their corresponding particles would be tested along directions $\Omega_A$ and $\Omega_B$, respectively.

Hence, for any $\lambda \in S$

$\Omega_A, \Omega'_A$ : two arbitrary directions for the polarization test of particle A

$\Omega_B, \Omega'_B$ : two arbitrary directions for the polarization test of particle B

$$s^{(A)}_{\Omega_A}(\lambda) * \left( s^{(B)}_{\Omega_B}(\lambda) + s^{(B)}_{\Omega'_B}(\lambda) \right) + s^{(A)}_{\Omega'_A}(\lambda) * \left( s^{(B)}_{\Omega_B}(\lambda) - s^{(B)}_{\Omega'_B}(\lambda) \right) = \pm 2$$

The CHSH inequality is then obtained by integration over the space of all possible hidden configurations.

The proof requires three physically well-defined angles, say

$$\Omega_B, \Omega'_B, \Omega'_A,$$

The fourth direction, $\Omega_A$ , serves as a reference direction.

This fourth direction can serve also as a reference to describe the possible hidden configurations $\lambda \in S$ of each pair of entangled particles: whatever $\lambda$ is, it must be defined with respect to a reference frame (as any physical property).

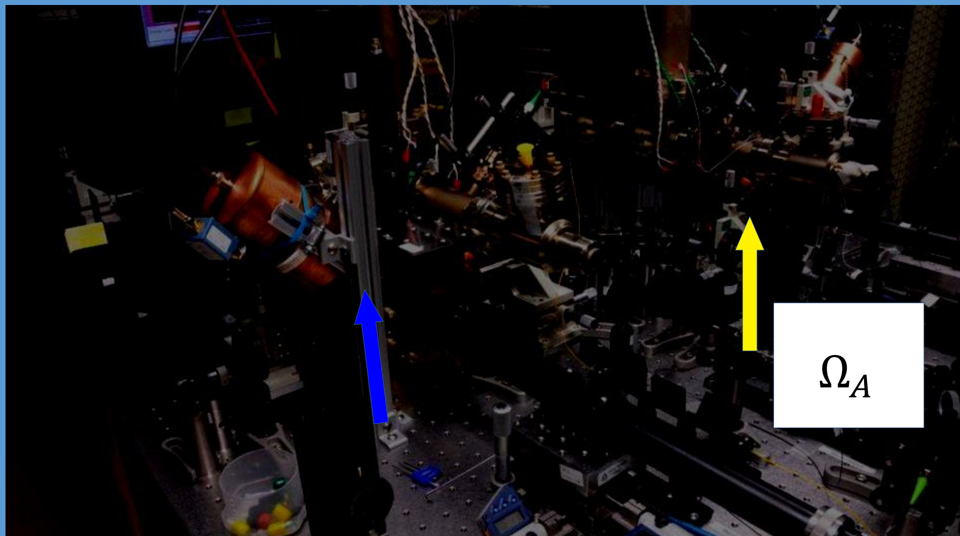
The proof requires three physically well-defined angles, say

$$\Omega_B, \Omega'_B, \Omega'_A,$$

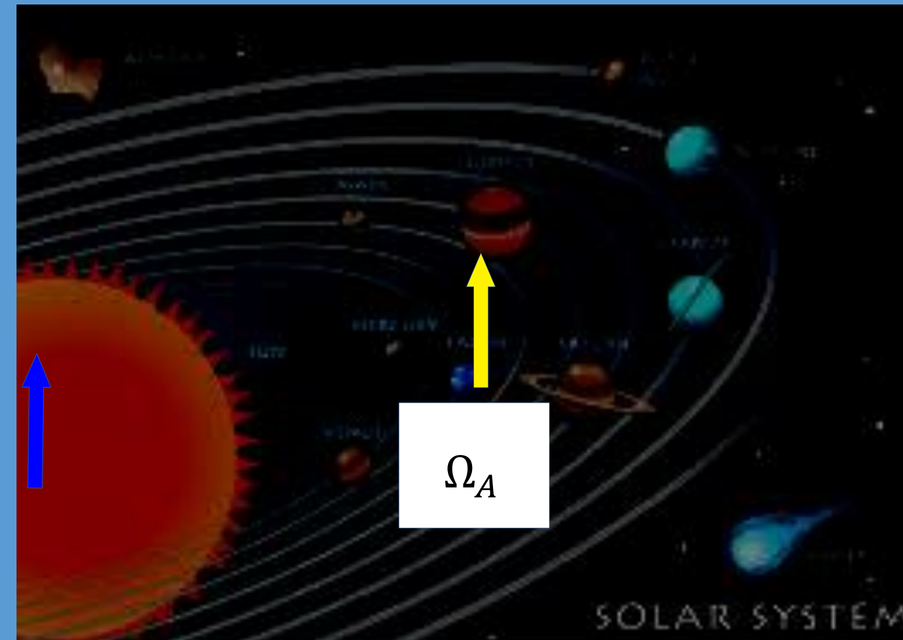
The fourth direction, $\Omega_A$ , serves as a reference direction.

The orientation of this reference direction is an spurious (unphysical/irrelevant) gauge degree of freedom.

**Lab frame: laboratory's table**

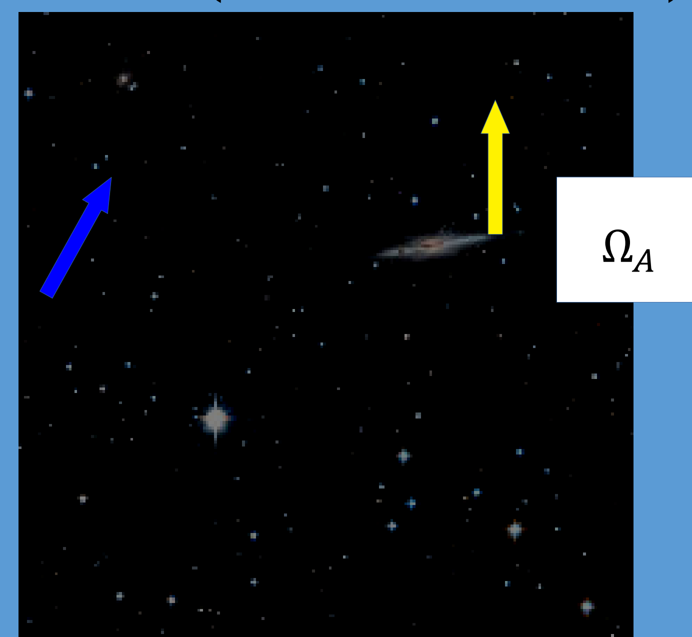



**Lab frame: Sun's axis**




**Lab frame: the Galaxy's center**


**Lab frame: the center of the local Supercluster (the Great Attractor)**



THE PROOF OF THE CHSH INEQUALITY SEEMS STRAIGHTFORWARD AND INDISPUTABLE.

NONETHELESS, IT INVOLVES A SUBTLE, THOUGH CRUCIAL, ASSUMPTION THAT:

a) IT IS NOT FULFILLED BY THE ACTUAL EXPERIMENTAL SET-UP

b) IT IS NOT REQUIRED BY FUNDAMENTAL PHYSICAL PRINCIPLES

How can we properly define the relative orientation between $\Omega_A$ and $\Omega'_A$, if we are defining the orientation of detector A as our reference ?

It would be possible if the four binary values

$$s^{(A)}_{\Omega_A}(\lambda), s^{(A)}_{\Omega'_A}(\lambda), s^{(B)}_{\Omega_B}(\lambda), s^{(B)}_{\Omega'_B}(\lambda)$$

could be obtained for single pairs of entangled particles.

In the actual experimental set-up the polarization of each particle in a single pair can be tested only along one direction !

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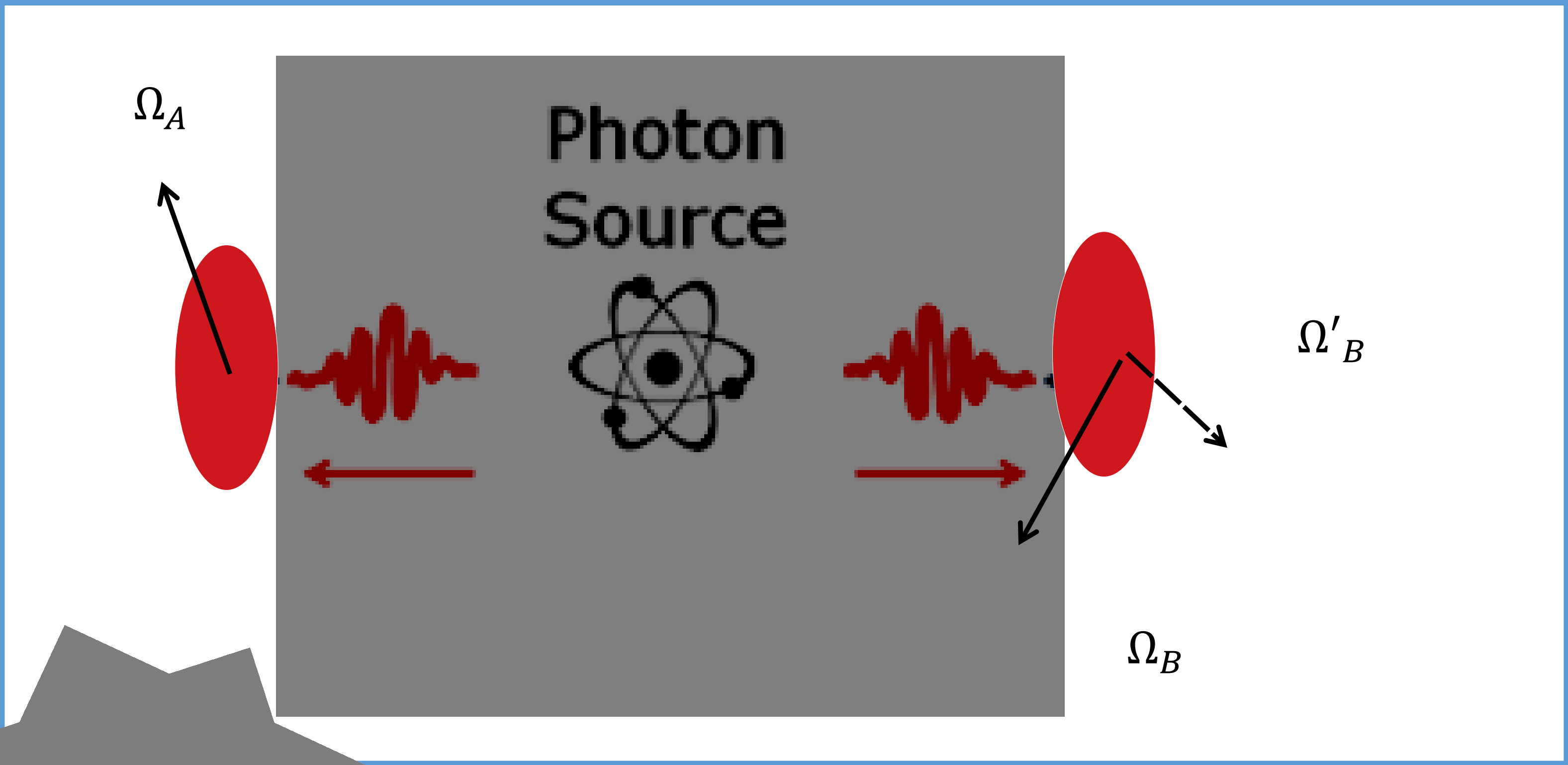
It still would be possible if there would exists a preferred absolute frame of reference to which we can define the experimental set-up

Principle of relativity: we are fully entitled to choose the orientation of detector A as a reference direction to describe the hidden configuration of each single pair of entangled particles !

**In the actual experimental set-up only the relative angle between the orientation of the two detectors is a physically well-defined observable, while their absolute orientation is an unphysical/irrelevant gauge degree of freedom.**




Lab frame !

Pencil

It makes no sense to attempt to distinguish a situation in which the pencil/lab is kept fixed while the reference detector is rotated (crucial for the proof of CHSH inequality !), from a situation in which the reference direction is kept fixed while the pencil/lab is rotated (which is irrelevant !)

Gauge Symmetry: In order to build a model of hidden variables we may only need to define the binary values (gauge-fixing condition):

$$s_{\Omega_A}^{(A)}(\lambda), s_{\Omega_B}^{(B)}(\lambda), s_{\Omega'_B}^{(B)}(\lambda), s_{\Omega_B-\Delta}^{(B)}(\lambda), s_{\Omega'_B-\Delta}^{(B)}(\lambda)$$

This is quite similar to Gupta-Bleuler formalism for QED: a proper norm is defined only for physical states.

However, it is straightforward to check that the proof of the CHSH inequality does not necessarily hold for such models.

$$s_{\Omega_A}^{(A)}(\lambda) * \left( s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda) \right)$$

$$+ s_{\Omega_A}^{(A)}(\lambda) * \left( s_{\Omega_B-\Delta}^{(B)}(\lambda) - s_{\Omega'_B-\Delta}^{(B)}(\lambda) \right)$$

Are this arguments only relevant to models of hidden variables ? Do they apply also to Quantum Mechanics ?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle^{(A)}\,|\downarrow\rangle^{(B)} \;+\; \mathrm{e}^{i\Phi}\,|\downarrow\rangle^{(A)}\,|\uparrow\rangle^{(B)}\right),$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ are bases of eigenstates of the Pauli operators $\sigma_Z^{(A,B)}$ along Z-axes locally defined at the sites of each one of the particles.

These eigenstates are defined up to a phase and, therefore, the phase $\Phi$ in the above expression has not been properly defined yet.

How can we properly define it ?

Choose an arbitrary experimental setting and use it as a definition of parallel directions $\Delta = 0$ between the orientations of the two detectors.

Use the experimental correlations between their outcomes to properly define the phase $\Phi$ of the entangled state in the chosen reference setting.

With respect to this reference setting we can now properly define a relative rotation $\Delta \neq 0$ in the orientation of the two detectors.

**We can take advantage of this gauge symmetry to build a explicitly local statistical model that reproduces the predictions of Quantum Mechanics for Bell's states and fulfills the constraints of 'free-will'.**

THE MODEL:

We consider an infinite set of possible hidden configurations distributed over the unit circle.

The orientation of detector A sets a reference direction along this circle and its associated set of coordinates $\lambda_A \in [-\pi, \pi]$ . The probability density of each hidden configuration to happen is given by:

$$g(\lambda_A) = -\left(\frac{1}{4}\right)|\sin(\lambda_A)|$$

Similarly, the orientation of detector B sets its own reference direction along this circle with its own associated set of coordinates $\lambda_B \in [-\pi, \pi]$.

Both sets of coordinates are related by a transformation law

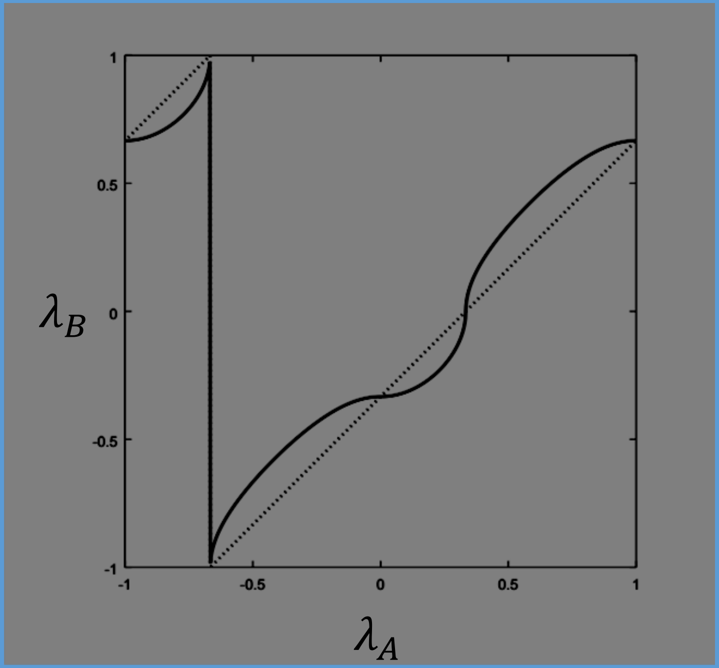
$$\lambda_B = -L(\lambda_A; \Delta - \Phi)$$

Moreover, symmetry considerations demand that the probability density of each hidden configuration to happen must given in the new set of coordinates by:

$$g(\lambda_B) = -\left(\frac{1}{4}\right)|\sin(\lambda_B)|$$

Since the probability of each possible hidden configuration must be independent of the set of coordinates ("free-will") we must have:

$$g(\lambda_B)\, d\lambda_B = g(\lambda_A)\, d\lambda_A$$




That is,

$$|d[\cos(\lambda_B)]| = |d[\cos(\lambda_A)]|$$

This demand fixes the transformation law

$$\lambda_B = -L(\lambda_A; \Delta - \Phi)$$

as a function of the parameter $\Delta - \Phi$

This transformation law is additive in the following sense:

Let $L_0$ be a setting for which $\lambda_B = -L(\lambda_A; 0)$

If we use it as a definition of parallel directions between the two detectors, the entangled state corresponds to $\Delta = 0$ , $\Phi = 0$ .

Consider now a new setting $L_1$ that is obtained from the former by a relative rotation of the detectors by an angle $\Delta$ . The two new sets of coordinates are related by

$$\lambda'_B = -L(\lambda_A; \Delta)$$

If we use this new setting as a reference $\Delta_1 = 0$, the entangled state corresponds to $\Phi_1 = -\Delta$

Hence, if we now consider a third experimental setting $L_2$ that is obtained from $L_1$ by a relative rotation of the two detectors by an angle $\Delta_2$ the two final sets of coordinates are related by

$$\lambda''_B = -L(\lambda_A; \Delta_2 - \Phi_1) = -L(\lambda_A; \Delta_2 + \Delta_1)$$

That is, the last setting $L_2$ is related to the original one $L_0$ by a relative rotation of the detectors by an angle $\Delta_1 + \Delta_2$

Finally, we define the response function of detector A as:

$$s^{(A)}(\lambda_A) = \begin{cases} +1, if\ \lambda_A \in [0, +\pi) \\ -1, if\ \lambda_A \in [-\pi, 0) \end{cases}$$

Similarly, we define the response function of detector B as:

$$s^{(B)}(\lambda_B) = \begin{cases} +1, if\ \lambda_B \in [0, +\pi) \\ -1, if\ \lambda_B \in [-\pi, 0) \end{cases}$$

These definitions are explicitly local since they depend only on the orientation of the hidden configuration with respect to the corresponding detector !

Therefore,

$$s^{(A)}(\lambda_A) = +1 \ \wedge \ s^{(B)}\big(\lambda_B(\lambda_A)\big) = +1, if\ \lambda_A \in [0, \Delta - \Phi)$$
$$s^{(A)}(\lambda_A) = +1 \ \wedge \ s^{(B)}\big(\lambda_B(\lambda_A)\big) = -1, if\ \lambda_A \in [\Delta - \Phi, \pi)$$
$$s^{(A)}(\lambda_A) = -1 \ \wedge \ s^{(B)}\big(\lambda_B(\lambda_A)\big) = +1, if\ \lambda_A \in [\Delta - \Phi - \pi, 0)$$
$$s^{(A)}(\lambda_A) = -1 \ \wedge \ s^{(B)}\big(\lambda_B(\lambda_A)\big) = -1, if\ \lambda_A \in [-\pi, \Delta - \Phi - \pi)$$

Hence, the correlation between the outcomes of the two detectors is given by:

$$E(\Delta - \Phi) = \int_0^{\Delta-\Phi} g\,(\lambda_A)\, d\lambda_A + \int_{-\pi}^{\Delta-\Phi-\pi} g\,(\lambda_A)\, d\lambda_A - \int_{\Delta-\Phi}^{\pi} g\,(\lambda_A)\, d\lambda_A - \int_{\Delta-\Phi-\pi}^{0} g\,(\lambda_A)\, d\lambda_A = -\cos(\Delta - \Phi)$$

# SUMMARY & CONCLUSIONS:

**The proof of the Bell's inequality relies on an unjustified implicit assumption, which:**

**a) is not fulfilled by the actual set-up of the experiments that test it;**

**b) is not required by fundamental physical principles and, indeed, it is at odds with the principle of relativity.**

**Namely, the proof of the inequality implicitly assumes the existence of an absolute preferred frame of reference with respect to which the orientation of the devices that test the particles' polarization can be defined.**

**Hence, the inequality cannot actually distinguish between the predictions of quantum mechanics for Bell's states and those of models of hidden variables that do not comply with this unjustified assumption.**

# SUMMARY & CONCLUSIONS:

**It is possible to build a local model of hidden variables that reproduces the predictions of Quantum Mechanics for the Bell's polarization states of two entangled particles and fulfills the constraints of 'free-will', once the gauge degrees of freedom involved are properly identified.**

D.Oaknin, "Solving the EPR paradox: an explicit local statistical model for the singlet", arxiv:1411.5704

**Similar analysis has been done for the GHZ polarization state of three entangled particles and for a single spin-1 particle (qutrit).**

D.Oaknin, "Solving the Greenberger-Horne-Zeilinger paradox: an explicit local statistical model for the GHZ state", arxiv:1709.00167

D.Oaknin, "Bypassing the Kochen-Specker theorem: an explicit local statistical model for the qutrit", arxiv:1805.04935