Second moment of the pion distribution amplitude from Lattice QCD

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and the RQCD collaboration

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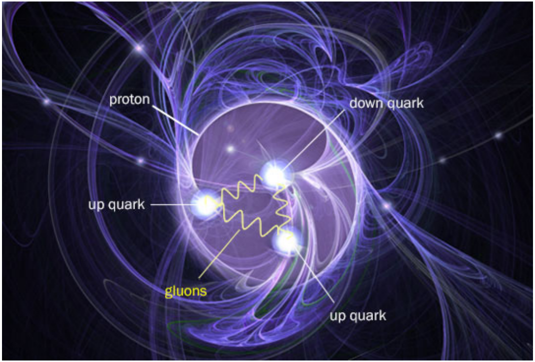
Hadrons' internal structure

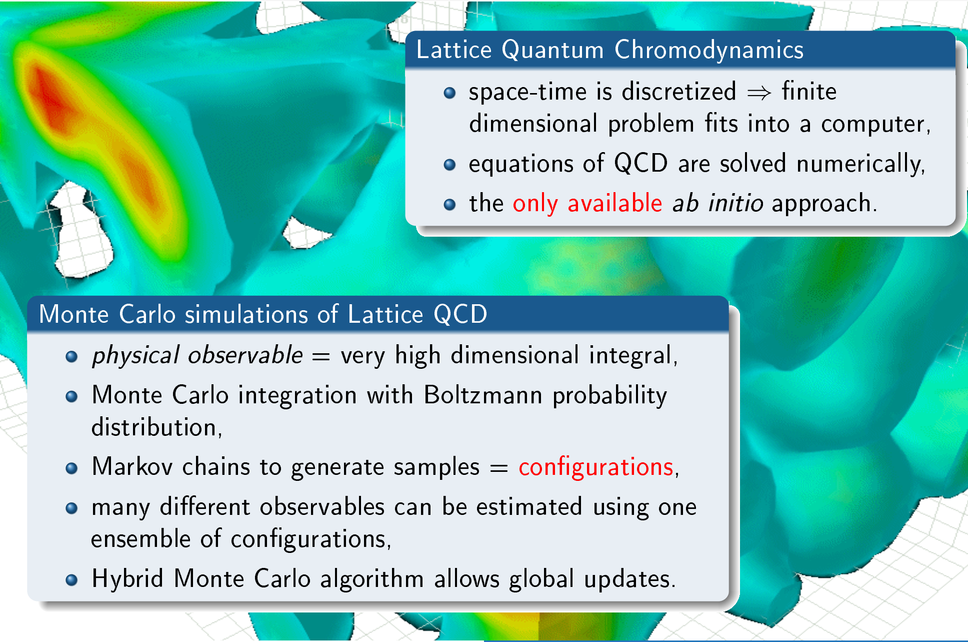
Standard Model of elementary particles: electrons, muons, quarks, gluons, photons, $W^{\pm}$, Z, Higgs, ...



Credit: *Brookhaven National Lab website*

Experiment: HERA, LHC, Electron-Ion Collider study hadron structure functions and try to discover the origin of mass,
Theory: Quantum Chromodynamics (QCD) is the theory describing the interations of quarks and gluons.

Quantum Chromodynamics



Lattice Quantum Chromodynamics

- space-time is discretized ⇒ finite dimensional problem fits into a computer,
- equations of QCD are solved numerically,
- the only available *ab initio* approach.

Monte Carlo simulations of Lattice QCD

- *physical observable* = very high dimensional integral,
- Monte Carlo integration with Boltzmann probability distribution,
- Markov chains to generate samples = configurations,
- many different observables can be estimated using one ensemble of configurations,
- Hybrid Monte Carlo algorithm allows global updates.

Pion distribution amplitude

Definition

Pion DA is the quantum amplitude that the pion moving with momentum P is built of a pair of quark and antiquark moving with momentum xP and $(1-x)P$ respectively.

Relevance

Pion photoproduction: two off-shell photons provide the hard scale necessary for the factorization into the perturbative and non-perturbative parts. Transition form factor measured most recently experimetaly by BaBar '09 and Belle '12.

Implementation

2nd moment of the pion DA, $\langle \xi^2 \rangle$, can be obtained numerically from two-point correlation functions.

Pion distribution amplitude

Definition, Braun *et al.*, '15

$$\langle 0|\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n)|\pi(p)\rangle =$$

$$= if_\pi(p\cdot n)\int_0^1 dx e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_\pi(x,\mu^2)$$

Neglecting isospin breaking effects $\phi_\pi(x)$ is symmetric under the interchange of momentum fraction $x \to (1-x)$

$$\phi_\pi(x,\mu^2) = \phi_\pi(1-x,\mu^2)$$

Moments of the momentum fraction difference

$$\xi = x - (1-x)$$

are interesting

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x,\mu^2)$$

$$\phi_\pi(x,\mu^2) = 6u(1-u)\left[1 + \sum_n a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1)\right]$$

Local operators, Braun *et al.*, '15

The nonlocal operator can be Taylor expanded and expressed in terms of local operators with derivatives

$$\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \ldots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{i+1}}$$

where

$$\mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{k+l}} = \bar{d}(0)\overleftarrow{D}_{(\mu_1} \ldots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \ldots \overrightarrow{D}_{\mu_{k+l}} \gamma_{\rho)}\gamma_5 u(0)$$

Consequently,

$$i^{k+l}\langle 0|\mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{k+l}}|\pi(p)\rangle = if_\pi p_{(\rho} p_{\mu_1} \ldots p_{\mu_{k+l})} \langle x^l (1-x)^k \rangle$$

Pion distribution amplitude on the lattice

Lattice operators for the 2nd moment, Braun *et al.*, '15

Two operators local are relevant

$$\mathcal{O}^{-}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

and

$$\mathcal{O}^{+}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

We estimate the following correlation functions

$$C_{\rho}(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}_{\rho}(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

$$C^{\pm}_{\rho\mu\nu}(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}^{\pm}_{\rho\mu\nu}(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

Lattice operators for the 2nd moment, Braun *et al.*, '15

From the correlation functions we construct ratios

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$

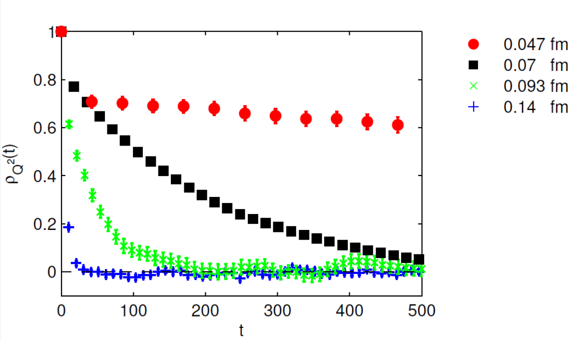
which exhibit plateaux and which we fit to extract the value $R^{\pm}_{\rho\mu\nu,\sigma}$. Finally,

$$\langle\xi^2\rangle^{\overline{\mathrm{MS}}} = \zeta_{11}R^{-} + \zeta_{12}R^{+},$$
$$a_2^{\overline{\mathrm{MS}}} = \frac{7}{12}\left[5\zeta_{11}R^{-} + (5\zeta_{12} - \zeta_{22})R^{+}\right]$$

where ζ_{ij} are renormalization constants estimated nonperturbatively.

Landscape of ensembles

Periodic boundary conditions

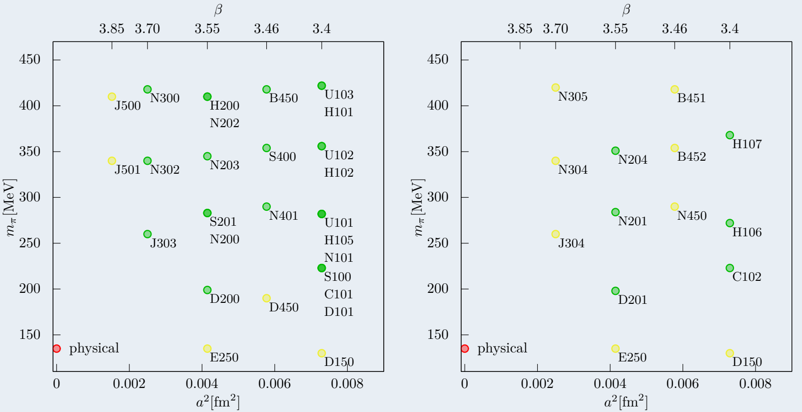


Credit: *ALPHA Collaboration, Nucl.Phys. B845 (2011) 93-119*

A very severe critical slowing down of the topological charge in pure Yang-Mills theory has been observed when using the HMC algorithm, implying that the simulations scale as a^{-10}.

Landscape of ensembles

Coordinated Lattice Simulations collaboration



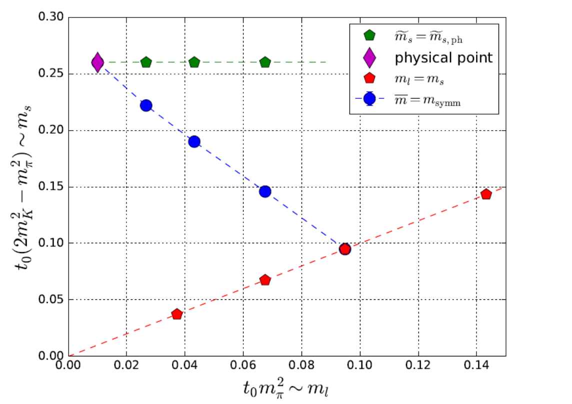
Credit: *J. Simeth, Univ. Regensburg*

CLS

CLS: CERN, DESY, Univ. Regensburg, Univ. Mainz, Univ. Madrid, Univ. Munster, Univ. Odensee, Jagiellonian Univ., Univ. Milano, Univ. Dublin

Landscape of ensembles

Coordinated Lattice Simulations collaboration

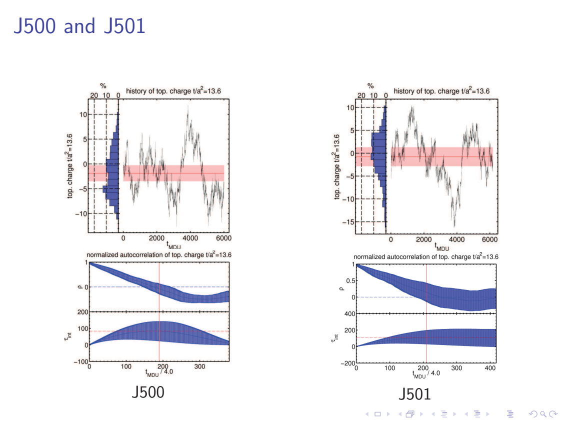


Credit: *W. Söldner, Univ. Regensburg*

Two trajectories lead to the physical point, a third trajectory as generated for non-perturbative renormalization.

Challenges: autocorrelations

Autocorrelation of the topological charge

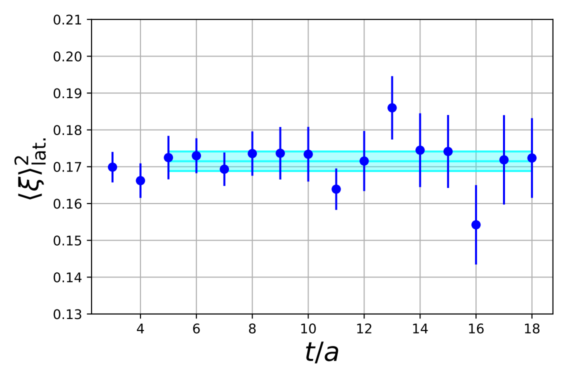


Credit: *J. Simeth, Univ. Regensburg*

In spite of using open boundary conditions we still experience growing autocorrelations times: ~ 200 configurations at $a = 0.039$ fm.

Plateau fit example

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$



We use momentum smearing (Bali *et al.* '16) to reduce signal-to-noise problem (Braun *et al.* '17).

2^{nd} moment of the pion distribution amplitude

Combined fit

We perform a combined fit to all data points: all lattice spacings and all pion/kaon masses along the three trajectories with the ChPT inspired fit ansatz

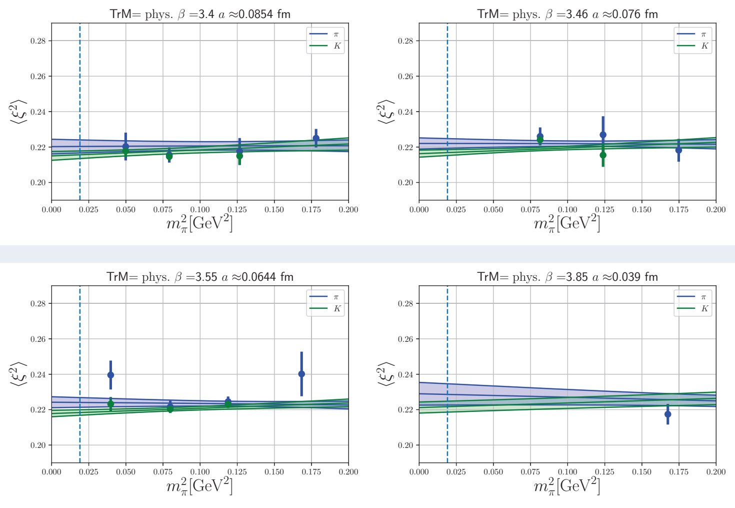
$$\overline{M}^2 = \frac{2m_K^2 + m_\pi^2}{3}, \qquad \delta M^2 = m_K^2 - m_\pi^2$$

$$\langle \xi^2 \rangle_\alpha = \left(1 + c_0 a + c_1 a \overline{M}^2 + c_2^\alpha a \delta M^2\right) \begin{cases} \langle \xi^2 \rangle_0 + \overline{A M}^2 - 2\delta A \delta M^2, & \alpha = \pi, \\ \langle \xi^2 \rangle_0 + \overline{A M}^2 + \delta A \delta M^2, & \alpha = K \end{cases}$$

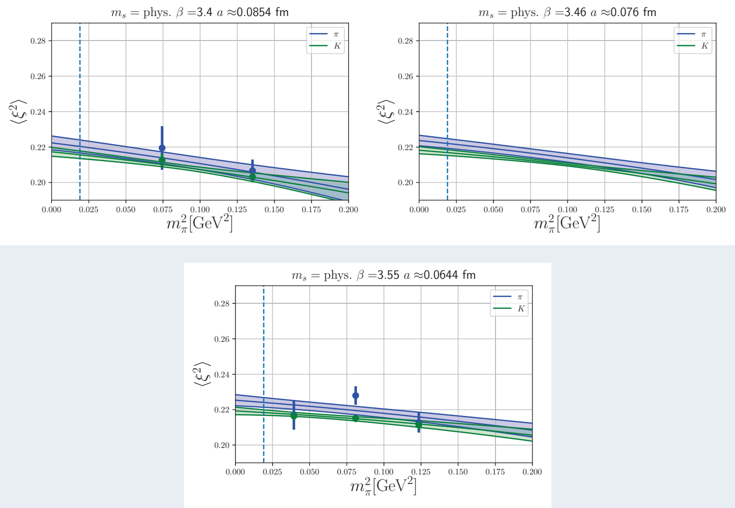
and $\overline{A}$ and δA are combinations of low energy constants.
$\Rightarrow$ 7 fit parameters

2^{nd} moment of the pion distribution amplitude

Continuum extrapolation: blue for pion, green for kaon

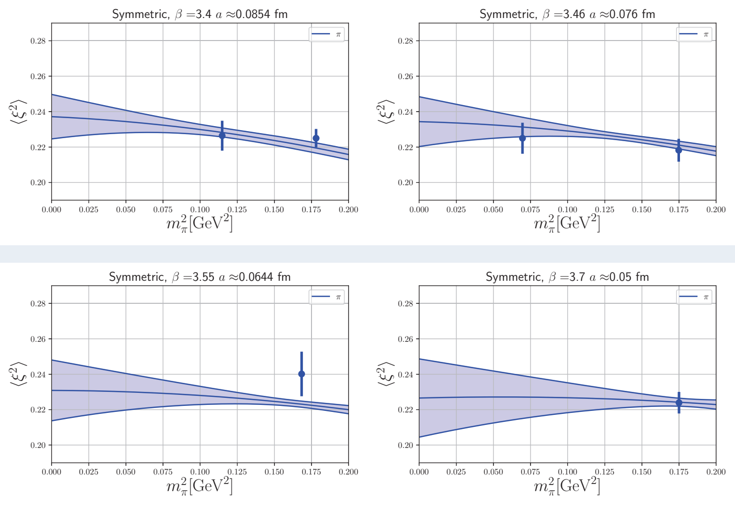


Continuum extrapolation: blue for pion, green for kaon



2^{nd} moment of the pion distribution amplitude

Continuum extrapolation: blue for pion, green for kaon



Conclusions

Pion distribution amplitude

Our preliminary result

$$\langle \xi^2 \rangle = 0.236 \pm 0.012$$

is the first ever continuum determination from First Principles. Our previous value at finite lattice spacing and for $N_f = 2$ was 0.236 ± 0.008.

Kaon and eta distribution amplitude

We also measured the kaon first and second moments and can infer from the combined fit the moment of the eta distribution amplitude.

Full x-dependence of the pion DA

In a separate project we are currently estimating non-perturbatively the full x dependence of the pion DA: Braun *et al.* '18

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