Unequal Rapidity Correlators in the Dilute Limit of JIMWLK

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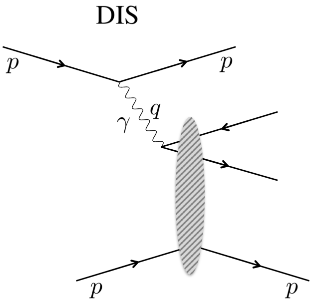
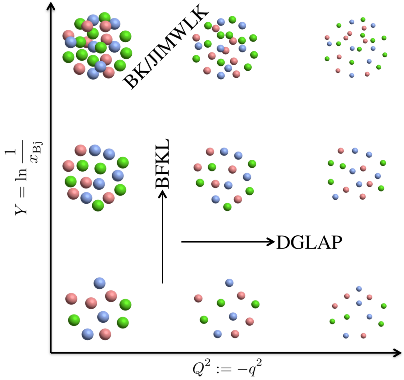
Outline

- Context: saturation physics, particle production
- JIMWLK evolution: $\rightarrow$ usual **Fokker-Planck** picture
  $\rightarrow$ lesser studied **Langevin** picture[1]
- Dilute limit $\rightarrow$ BFKL equation
- Outlook: calculating two-particle correlations

[1] Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469], Kovner & Lublinsky [JHEP 0611 (2006) 083], Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

Colour Glass Condensate

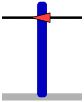




Particle Production

- Cross sections for particle production $\rightarrow$ Wilson lines:

$$U_{\boldsymbol{x}}^{\dagger} := P \exp\left\{ig \int dx^{+} \alpha_{\boldsymbol{x}}^{a}(x^{+}) t^{a}\right\} =$$



- Correlators $\left\langle \ldots \right\rangle_{Y}$, e.g. dipole $\left\langle \frac{\mathrm{tr}\left(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^{\dagger}\right)}{N_c} \right\rangle$

Fokker-Planck JIMWLK

- Evolution in the target field as a function of rapidity
- Nonlinear, renormalisation group equation
- Functional differential equation ≈ Balitsky hierarchy

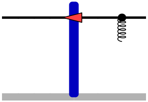
$$\frac{d}{dY}\left\langle \ldots \right\rangle_Y = -H_{\mathrm{JIMWLK}}\left\langle \ldots \right\rangle_Y$$

$$H_{\mathrm{JIMWLK}} := -\frac{\alpha_s}{2\pi^2}\int d^2 z\, \tilde{\mathcal{K}}_{\boldsymbol{xzy}}\left(L^a_{\boldsymbol{x}} - U^{\dagger ab}_{\boldsymbol{z}} R^b_{\boldsymbol{x}}\right)\left(L^a_{\boldsymbol{y}} - U^{\dagger ac}_{\boldsymbol{z}} R^c_{\boldsymbol{y}}\right)$$

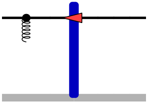
$$\tilde{\mathcal{K}}_{\boldsymbol{xzy}} := \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

JIMWLK in Diagrams

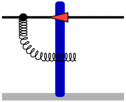
$$L^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} := ig\delta_{\boldsymbol{u}\boldsymbol{x}} t^a U^\dagger_{\boldsymbol{x}} \propto$$



$$R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} := ig\delta_{\boldsymbol{u}\boldsymbol{x}} U^\dagger_{\boldsymbol{x}} t^a \propto$$



$$U^{\dagger ab}_{\boldsymbol{z}} R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} \propto$$



Langevin JIMWLK

- Fokker-Planck dynamics $\leftrightarrow$ Langevin dynamics (better suited to numerics)
- Discretise rapidity difference: $Y - Y_A = \epsilon N$ with $\mathbb{Z} \ni N \to \infty, \epsilon \to 0$
- JIMWLK Langevin equation for a Wilson line

$$U^\dagger_{\boldsymbol{x},n+1} = e^{i\epsilon g \alpha^L_{\boldsymbol{x},n}} U^\dagger_{\boldsymbol{x},n} e^{-i\epsilon g \alpha^R_{\boldsymbol{x},n}}$$

- Colour rotations

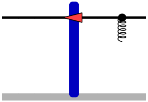
$$\alpha^L_{\boldsymbol{x},n} := \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n} t^a, \qquad \alpha^R_{\boldsymbol{x},n} := \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b$$

where $\mathcal{K}^i_{\boldsymbol{xy}} := \dfrac{(\boldsymbol{x}-\boldsymbol{y})^i}{(\boldsymbol{x}-\boldsymbol{y})^2}$ (Weizsäcker-Williams)

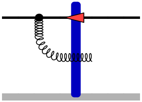
- Local Gaussian white noise $\langle \nu^{i,a}_{\boldsymbol{x},m} \nu^{jb}_{\boldsymbol{y},n} \rangle = \dfrac{1}{\epsilon} \delta^{ij} \delta^{ab} \delta_{mn} \delta_{\boldsymbol{xy}}$ where $\nu^{i,a}_{\boldsymbol{x},m} \in \mathbb{R}$

Colour Rotations

$$\alpha^L_{\boldsymbol{x},0} U^\dagger_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},0} t^a \propto$$

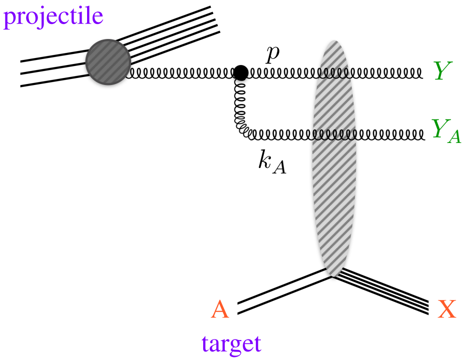


$$\alpha^R_{\boldsymbol{x},0} U^\dagger_{\boldsymbol{x},0} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},0} U^{\dagger ab}_{\boldsymbol{z},0} t^b \propto$$



Example

- Inclusive two-gluon production at unequal rapidities: $g + A \to g + g + X$



Cross Section

- Inclusive two-gluon production at unequal rapidities[1]:

$$\frac{d\sigma_{2g}}{dY\,d^2\boldsymbol{p}\,dY_A d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \times \left\langle H(\boldsymbol{k}_A)\left\langle \frac{\mathrm{tr}\left(\bar{U}_{\bar{\boldsymbol{x}},A}U^{\dagger}_{\boldsymbol{x},A}\right)}{(N_c^2-1)}\right\rangle_{Y-Y_A}\Bigg|_{\bar{U}_A=U_A}\right\rangle_{Y_A}$$

$$H(\boldsymbol{k}_A) := \frac{1}{4\pi^3}\int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{i\boldsymbol{k}_A\cdot(\bar{\boldsymbol{y}}-\boldsymbol{y})}\int_{\boldsymbol{u}\boldsymbol{v}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}}\mathcal{K}^i_{\bar{\boldsymbol{y}}\boldsymbol{v}}\left(L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{y}}R^b_{\boldsymbol{u}}\right)\left(\bar{L}^a_{\boldsymbol{v}} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}}}\bar{R}^c_{\boldsymbol{v}}\right)$$

$$\langle\hat{\mathcal{O}}\rangle_{Y_A} := \int [DU][D\bar{U}]W_{Y_A}[U,\bar{U}]\mathcal{O}$$

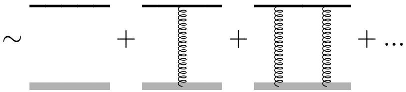
$$\langle\hat{\mathcal{O}}\rangle_{Y-Y_A} := \int [DU\,D\bar{U}]W_{Y-Y_A}\left[U,\bar{U}\middle|U_A,\bar{U}_A\right]\hat{\mathcal{O}}$$

[1] Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

Our Goal

- Study unequal rapidity correlators in JIMWLK
- For analytical understanding $\rightarrow$ start with BFKL
- Dilute limit of Langevin picture should give BFKL equation
- Expand Wilson lines

$$U^{\dagger}_{\boldsymbol{x},n} =: e^{i\lambda_{\boldsymbol{x},n}} = \mathbb{1} + i\lambda_{\boldsymbol{x},n} - \frac{1}{2}\lambda^2_{\boldsymbol{x},n} + \mathcal{O}(\lambda^3)$$



(unitarity $\Rightarrow \lambda_{\boldsymbol{x},n} = \lambda^T_{\boldsymbol{x},n}$ where $\lambda_{\boldsymbol{x},n} \in \mathrm{su}(N_c)$)

BFKL Equation

- Expand known BK equation in λ:

$$\frac{d}{dY}\left\langle \frac{\mathrm{tr}\left(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right)}{N_c}\right\rangle = \frac{\alpha_s}{\pi^2}\frac{N_c}{2}\int_{\boldsymbol{z}} \tilde{\mathcal{K}}_{\boldsymbol{xyz}}\left\langle \frac{\mathrm{tr}\left(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{z}}\right)}{N_c}\frac{\mathrm{tr}\left(U_{\boldsymbol{z}}U^\dagger_{\boldsymbol{y}}\right)}{N_c} - \frac{\mathrm{tr}\left(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right)}{N_c}\right\rangle$$

  becomes

$$\frac{1}{\epsilon}\left\langle(\lambda^a_{\boldsymbol{x},n+1}-\lambda^a_{\boldsymbol{y},n+1})^2-(\lambda^a_{\boldsymbol{x},n+1}-\lambda^a_{\boldsymbol{y},n+1})^2\right\rangle$$
$$= -\frac{\alpha_s}{\pi^2}N_c\int_{\boldsymbol{z}}\tilde{\mathcal{K}}_{\boldsymbol{xyz}}\left\langle \lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{z},n}+\lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{y},n}-\lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{y},n}-\lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{z},n}\right\rangle$$

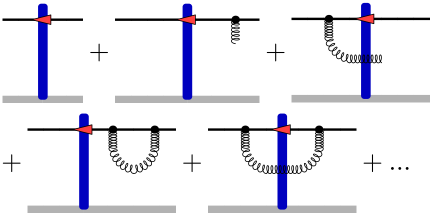
  $\rightarrow$ BFKL equation in terms of λ

- Find required quantities like $\lambda_{\boldsymbol{x},n+1}\lambda_{\boldsymbol{y},n+1}$ from Langevin JIMWLK equation

Expansion in ϵ

- Expand Langevin JIMWLK equation in ϵ:

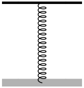
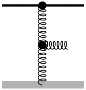
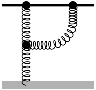
$$U^{\dagger}_{\boldsymbol{x},n+1} = U^{\dagger}_{\boldsymbol{x},n} + \int_{\boldsymbol{z}} \left(\frac{ig}{\sqrt{4\pi^3}} \epsilon \nu^{i,a}_{\boldsymbol{z},n} \mathcal{K}^i_{\boldsymbol{xz}} - \frac{g^2}{4\pi^3} \epsilon \mathcal{K}^i_{\boldsymbol{xz}} \mathcal{K}^i_{\boldsymbol{xz}} t^a \right) \times \left(t^a U^{\dagger}_{\boldsymbol{x},n} - U^{\dagger}_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b \right) + \mathcal{O}(\epsilon^2)$$

$U^{\dagger}_{\boldsymbol{x},1} \sim$  $+ \ldots$

Dilute Limit in Langevin Picture

- Expand this equation in λ:

$$\lambda_{\boldsymbol{x},n+1} = \lambda_{\boldsymbol{x},n} + \int_{\boldsymbol{z}} \left(\frac{ig}{\sqrt{4\pi^3}} \epsilon \nu^{i,a}_{\boldsymbol{z},n} \mathcal{K}^i_{\boldsymbol{xz}} - \frac{g^2}{4\pi^3} \epsilon \mathcal{K}^i_{\boldsymbol{xz}} \mathcal{K}^i_{\boldsymbol{xz}} t^a \right) \times [t^a, \lambda_{\boldsymbol{x},n} - \lambda_{\boldsymbol{z},n}] + \mathcal{O}(\lambda^2) + \mathcal{O}(\epsilon^2)$$

$\lambda_{\boldsymbol{x},1} \sim$  $+$  $+$  $+ \ldots$

- Calculate $\lambda_{\boldsymbol{x},n+1} \lambda_{\boldsymbol{y},n+1}$ from this equation

BFKL Recovered

- This satisfies BFKL equation found above in terms of λ! ✓

$$\frac{1}{\epsilon}\left\langle(\lambda^a_{\boldsymbol{x},n+1}-\lambda^a_{\boldsymbol{y},n+1})^2-(\lambda^a_{\boldsymbol{x},n+1}-\lambda^a_{\boldsymbol{y},n+1})^2\right\rangle$$
$$=-\frac{\alpha_s}{\pi^2}N_c\int_{\boldsymbol{z}}\tilde{\mathcal{K}}_{\boldsymbol{xyz}}\left\langle\lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{z},n}+\lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{y},n}-\lambda^a_{\boldsymbol{x},n}\lambda^a_{\boldsymbol{y},n}-\lambda^a_{\boldsymbol{z},n}\lambda^a_{\boldsymbol{z},n}\right\rangle$$

where $\tilde{\mathcal{K}}_{\boldsymbol{xyz}}=\mathcal{K}^i_{\boldsymbol{xz}}\mathcal{K}^i_{\boldsymbol{xz}}+\mathcal{K}^i_{\boldsymbol{yz}}\mathcal{K}^i_{\boldsymbol{yz}}-2\mathcal{K}^i_{\boldsymbol{xz}}\mathcal{K}^i_{\boldsymbol{yz}}$

- Unequal rapidity correlation → need Langevin equations for $RU^\dagger$, RU, $LU^\dagger$, LU
- Cross section contains $\left(L^a_{\boldsymbol{u}}-U^{\dagger ab}_{\boldsymbol{y}}R^b_{\boldsymbol{u}}\right)\left(\bar{L}^a_{\boldsymbol{v}}-\bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}}}\bar{R}^c_{\boldsymbol{v}}\right)\frac{\mathrm{tr}\left(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}}\right)}{N_c}$

Summary & Outlook

- Saturation physics $\rightarrow$ JIMWLK
- Focker Planck vs. Langevin pictures
- First step: BFKL from Langevin JIMWLK
- Dilute limit

Outlook

- Unequal rapidity correlators $\rightarrow$ bilocal quantities LU etc. that appear in cross sections