Inclusive photon production in DIS off nuclei at small x

Raju Venugopalan

Brookhaven National Laboratory

POETIC VIII, Regensburg, March 19-22, 2018

Work in collaboration with Kaushik Roy,
based on arXiv:1802.09550, and work in preparation

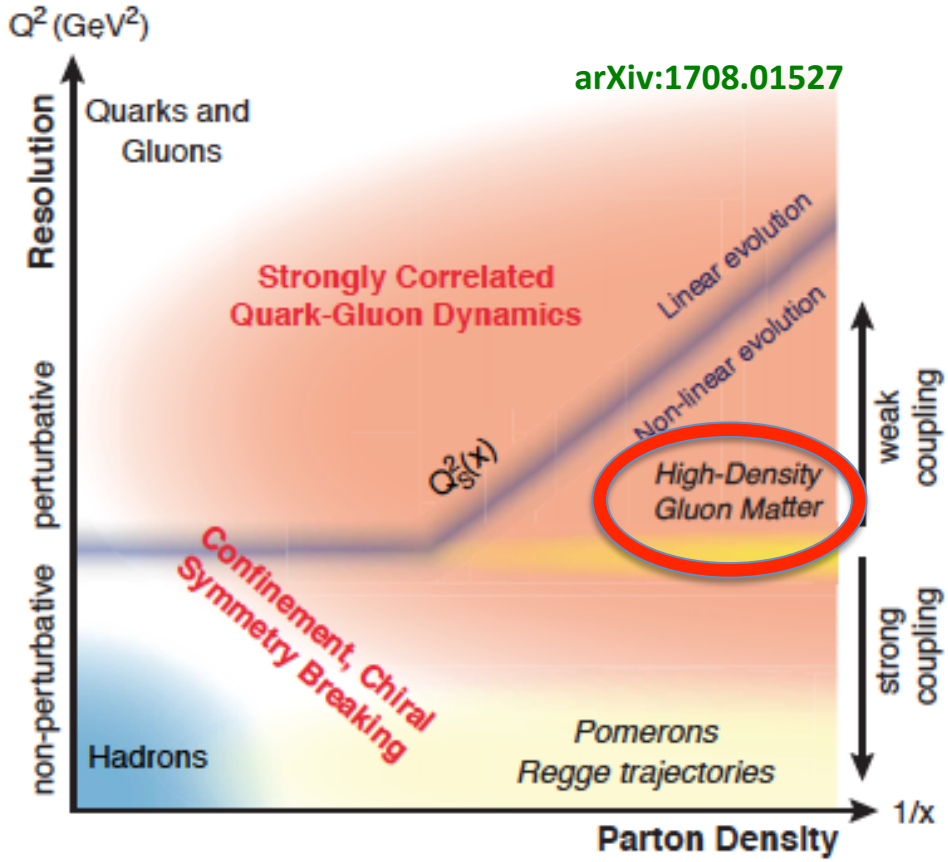
Acknowledgements: Previous work with Benic,Fukushima,Garcia-Montero on inclusive photon production in p+A (see Benic's talk)

Ian Balitsky, Yacine Mehtar-Tani,
Al Mueller, Anna Stasto and Andrey Tarasov,
for useful discussions on NLO computation

Talk Outline

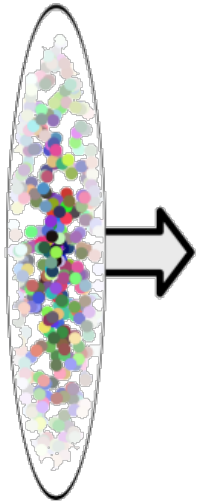
- The CGC EFT and RG evolution
- Inclusive photon production at LO, LLx and all-twist contributions
- Interesting limits (k_T & collinear factorization, di-jets)
- Fermion and Gluon shockwave propagators in the "wrong" light cone gauge – connection to Lipatov's Reggeon Field Theory
- The structure of higher order computations: the NLO inclusive photon impact factor

The CGC EFT



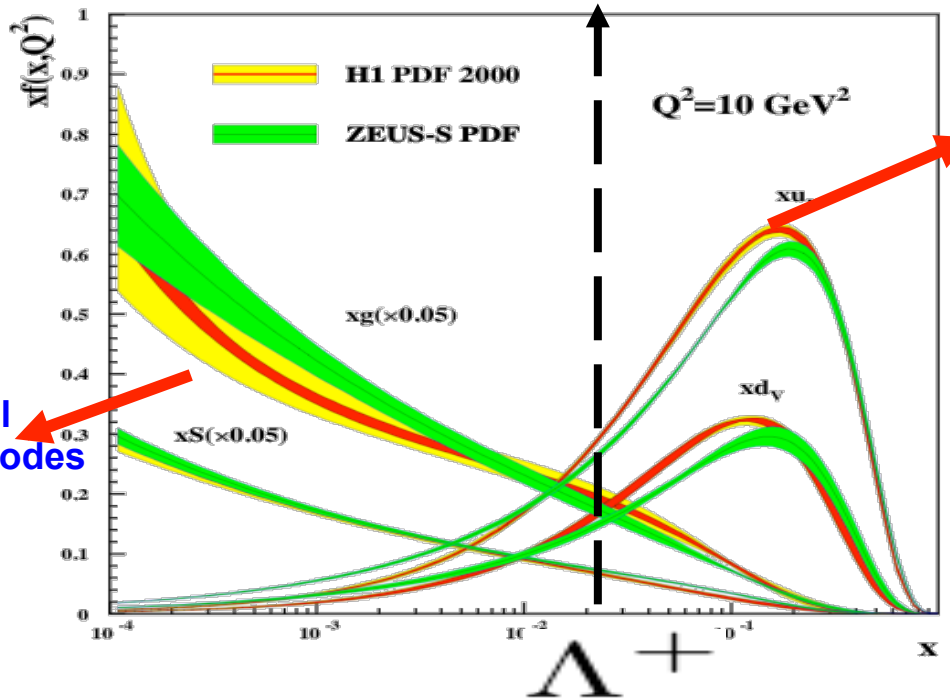
EFT in the Regge limit of $Q^2 >> \Lambda_{QCD}^2$ to study matter at high parton densities

The CGC EFT



Gelis,Iancu,Jalilian-Marian,RV,
arXiv:1002.0333

$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\mathbf{gg\ldots gg}\rangle$$



Large x modes- static sources for wee modes

Dynamical Small x modes

Born-Oppenheimer light cone separation natural for EFT

Susskind, Bardacki-Halperin

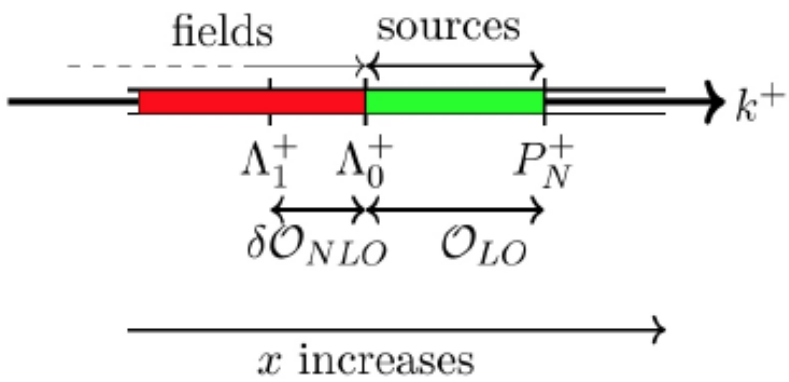
RG eqns describe evolution of wavefunction with energy

RG evolution with x in CGC EFT

Classical EFT: Expectation value of operator in hadron wavefunction at small x

$$\langle \mathcal{O} \rangle_{Y_A} = \int D\rho_A \, W_{Y_A}[\rho_A] \mathcal{O}[\rho_A]$$

Gauge invariant Weight Functional "density matrix" representing distribution of "classical" color sources



McLerran, RV
Jeon, RV

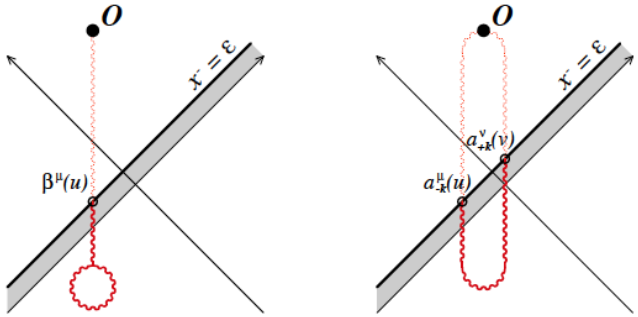
Successively integrate out quantum fluctuations ensuring $\alpha_S \, \mathrm{Ln}(\Lambda_0^+/\Lambda_1^+) << 1$

Ayala,Jalilian-Marian,McLerran, RV
Jalilian-Marian,Kovner,McLerran,Weigert

RG evolution with x in CGC EFT

The leading Log x NLO contribution can be expressed as

$$\mathcal{O}_{\rm NLO} = \alpha_S \log\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right) \mathcal{H}_{\rm JIMWLK} \otimes \mathcal{O}_{\rm LO}$$



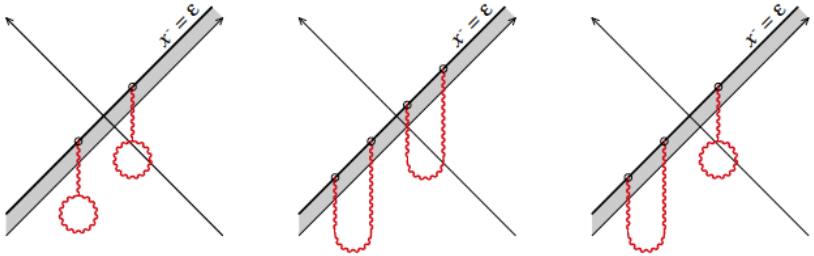
After an integration by parts, we obtain

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle_{\rm LLx} = \int D\rho_A W_{\Lambda_1^+}[\rho_A] \mathcal{O}_{\rm LO}$$

Jalilian-Marian,Kovner,Leonidov,Weigert
Iancu,Leonidov,McLerran

The same structure as LO – iff W satisfies the JIMWLK equation

$$\frac{\partial}{\partial \log(\Lambda_0^+/\Lambda_1)} W_{\Lambda_1^+}[\rho_A] = -\mathcal{H}_{\rm JIMWLK} \otimes W_{\Lambda_1^+}[\rho_A]$$



Iterates figure above to all orders

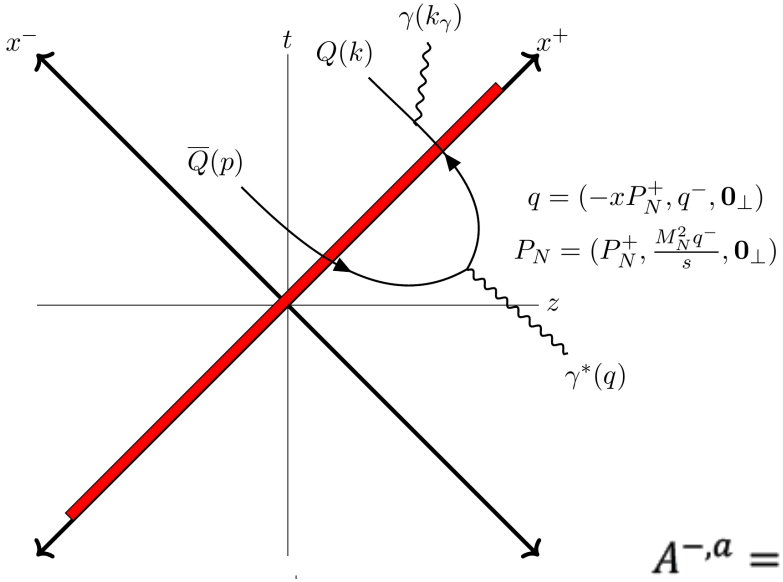
RG evolution with x in CGC EFT

This JIMWLK equation for $W[\rho_A]$ generates a hierarchy of equations for n-point lightlike Wilson line correlators – the Balitsky-JIMWLK hierarchy

For two-point "dipole" correlators (for large N_c and large A), one obtains the simplest nonlinear equation of high energy QCD demonstrating gluon saturation – the Balitsky-Kovchegov equation
In a low density limit, one recovers the BFKL equation

We will focus on DIS where the state-of-the art is fully inclusive DIS at NLO and discuss the next nontrivial computation
(and excellent probe of high parton densities)
inclusive photon production at LO & NLO

DIS inclusive photon production at LO

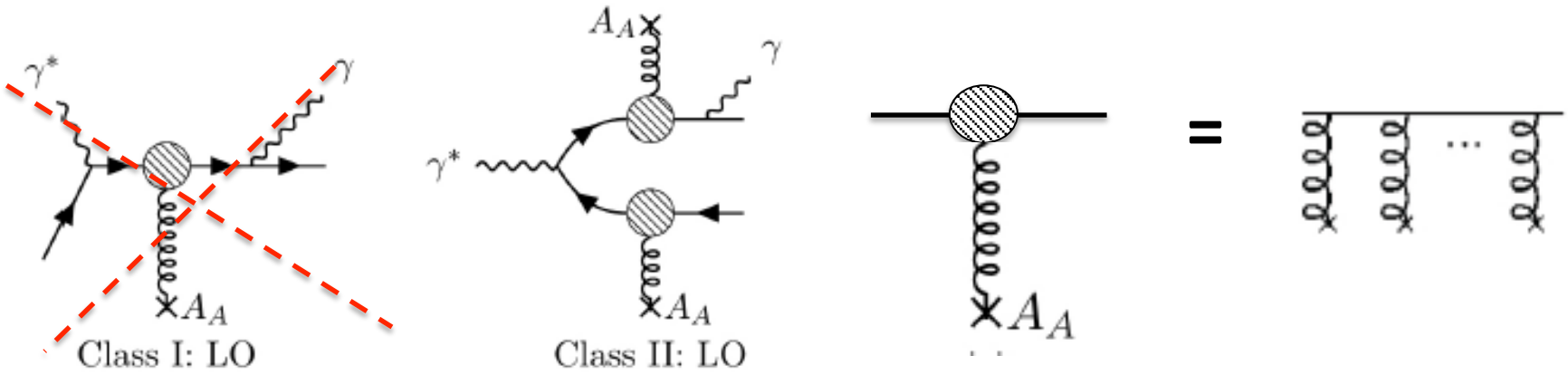


Right moving nucleus with momentum P_N^+ is Lorentz contracted in x^- direction

Glue fields satisfy Yang-Mills eqns.

$$[D_\mu, F^{\mu\nu}](x) = g\delta^{\nu+}\delta(x^-)\rho_A(x_\perp)$$

$A^{-,a} = 0$, $F_{ij}^a = 0$ with $A^{+,a}, A^{i,a}$ static (independent of x^+)



Suppressed at small x

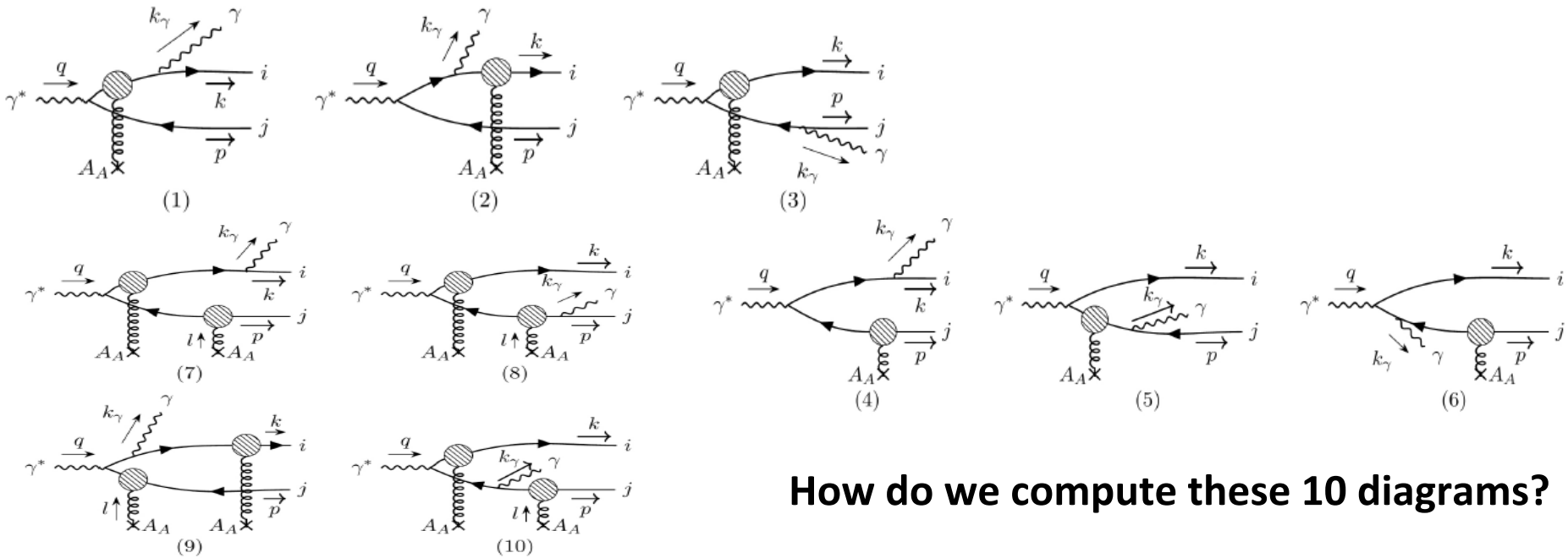
DIS inclusive photon production at LO

$$e(\tilde{l}) + A(P) \longrightarrow e(\tilde{l}') + Q(k) + \bar{Q}(p) + \gamma(k_\gamma) + X$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi y^2}{64\pi^3 Q^2}\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_k}\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3 2E_p}\frac{\mathrm{d}^3\mathbf{k}_\gamma}{(2\pi)^3 2E_{k_\gamma}}\frac{1}{2q^-}\left(\frac{1}{2}\sum_{\mathrm{spins},\lambda}\left\langle|\tilde{\mathcal{M}}|^2\right\rangle_{Y_A}\right)(2\pi)\delta(P^- - q^-)$$

$$\frac{1}{2}\sum_{\mathrm{spins},\lambda}\langle|\tilde{\mathcal{M}}|^2\rangle_{Y_A} = L_{\mu\nu}X^{\mu\nu}$$

$L_{\mu\nu}$ is well known-the lepton tensor
$X^{\mu\nu}$ - the hadron tensor for inclusive photon production is what we compute



How do we compute these 10 diagrams?

Shockwave fermion propagator

McLerran,RV; hep-ph/9402335, hep-ph/9809427

Solve Dirac equation in LC gauge background field ($A^-=0$, $A^+=0$, $F_{ij}=0$) on either side of source – match solutions at $x^-=0$

LC propagator can be expressed as

$$S_{\rm LC}(x,y) = G(x^-,x_\perp) S_{\rm Lorenz}\, G^\dagger(y^-,y_\perp)$$

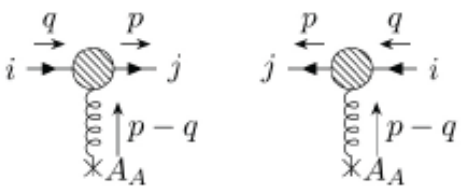
where $G(x^-,x_\perp) = \theta(-x^-) + \theta(x^-)\tilde{U}(x_\perp)$

$$\tilde{U}(x_\perp) = \mathcal{P}_- \exp\left[-ig\int_{-\infty}^{\infty} dz^- \frac{\rho_A^a(z^-,x_\perp)t^a}{\nabla_\perp^2}\right]$$

The mom. space Lorenz gauge propagator has the simple form

$$S_{\rm Lorenz}(q,p) = (2\pi)^4\delta^{(4)}(q-p) + S_0(q)\mathcal{T}(q,p)S_0(p)$$

Effective vertex



$$\mathcal{T}(q,p) = 2\pi\delta(p^- - q^-)\,{\rm sign}(p^-)\gamma^- \int_{x_\perp} e^{i(q_\perp - p_\perp)\cdot x_\perp}\left[\tilde{U}^{{\rm sign}(p^-)} - 1\right]$$

DIS inclusive cross-section at LO

Roy, RV; arXiv:1802.09550

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^2\mathbf{k}_{\gamma\perp}\mathrm{d}\eta_{k_\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2}\frac{1}{2q^-}\int_0^{+\infty}\frac{\mathrm{d}k^-}{k^-}\int_0^{+\infty}\frac{\mathrm{d}p^-}{p^-}\int_{\mathbf{k}_\perp,\mathbf{p}_\perp} L^{\mu\nu}\tilde{X}_{\mu\nu}(2\pi)\delta(P^- - q^-)$$

$$L^{\mu\nu} = \frac{2e^2}{Q^4}\left[(\tilde{l}^\mu \tilde{l}'^\nu + \tilde{l}^\nu \tilde{l}'^\mu) - \frac{Q^2}{2}g^{\mu\nu}\right]$$

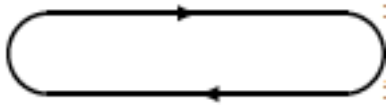
Trace over Dirac matrices

$$\tilde{X}_{\mu\nu} = \int_{\mathbf{x}_\perp,\mathbf{y}_\perp,\mathbf{x}'_\perp,\mathbf{y}'_\perp,\mathbf{l}_\perp,\mathbf{l}'_\perp} e^{-i(\mathbf{P}_\perp-\mathbf{l}_\perp).\mathbf{x}_\perp - i\mathbf{l}_\perp.\mathbf{y}_\perp + i(\mathbf{P}_\perp-\mathbf{l}'_\perp).\mathbf{x}'_\perp + i\mathbf{l}'_\perp.\mathbf{y}'_\perp}\ \tau_{\mu\nu}{}^{q\bar{q},q\bar{q}}(\mathbf{l}_\perp,\mathbf{l}'_\perp|\mathbf{P}_\perp)\ \boxed{\Xi(\mathbf{x}_\perp,\mathbf{y}_\perp;\mathbf{x}'_\perp,\mathbf{y}'_\perp)}$$

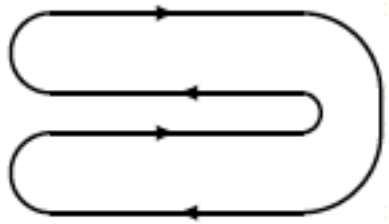
All the nonperturbative info about strongly correlated gluons is in

$$\Xi(\mathbf{x}_\perp,\mathbf{y}_\perp;\mathbf{x}'_\perp,\mathbf{y}'_\perp) = 1 - D(\mathbf{x}_\perp,\mathbf{y}_\perp) - D(\mathbf{y}'_\perp,\mathbf{x}'_\perp) + Q(\mathbf{x}_\perp,\mathbf{y}_\perp;\mathbf{y}'_\perp,\mathbf{x}'_\perp)$$

Dipoles: $D(x_\perp,y_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\left(\tilde{U}(x_\perp)\tilde{U}^\dagger(y_\perp)\right)\rangle_{Y_A}$

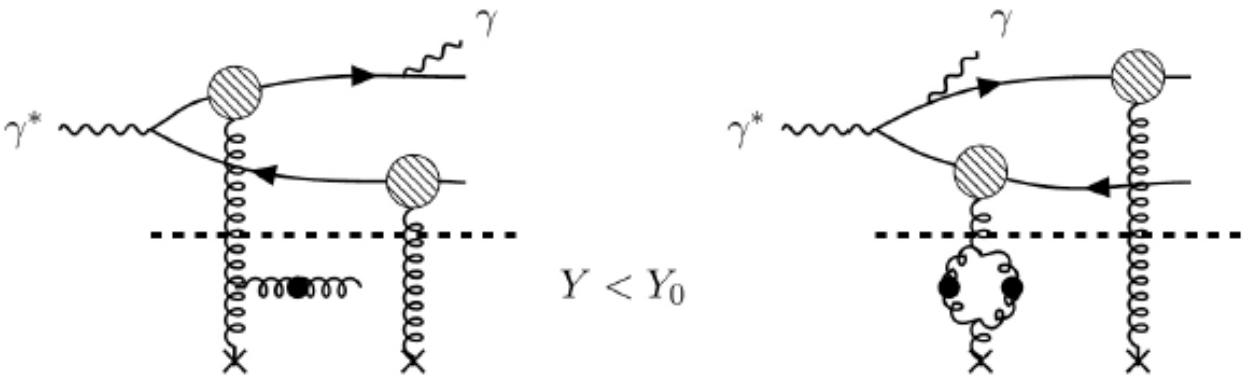


Quadrupoles: $Q(x_\perp,y_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\left(\tilde{U}(y'_\perp)\tilde{U}^\dagger(x'_\perp)\tilde{U}(x_\perp)\tilde{U}^\dagger(y_\perp)\right)\rangle_{Y_A}$



Building blocks of high energy QCD

LLx RG evolution of the cross-section



The leading LLx one loop real and virtual contributions below the quark loop ($Y<Y_A$) in rapidity can be absorbed in the JIMWLK evolution of $W_{YA}[\rho_A]$

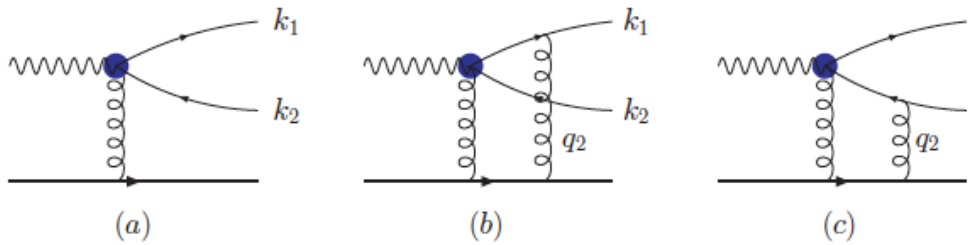
− thereby also giving the LLx evolution equations for Dipole and Quadrupole Wilson line correlators

Some interesting limits

When $k_\gamma \rightarrow 0$, the amplitude satisfies the Low-Burnett-Kroll theorem:

$$\mathcal{M}_\mu(\mathbf{q},\mathbf{k},\mathbf{p},\mathbf{k}_\gamma) \to -(eq_f)\epsilon^*_\alpha(\mathbf{k}_\gamma,\lambda)\left(\frac{p^\alpha}{p.k_\gamma} - \frac{k^\alpha}{k.k_\gamma}\right)\mathcal{M}^{NR}_\mu(\mathbf{q},\mathbf{k},\mathbf{p})$$

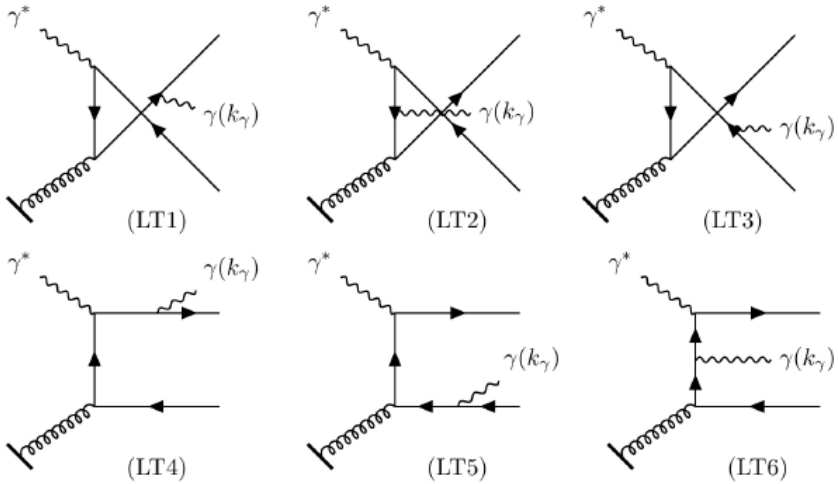
Polarization vector × Vectorial structure depending only on momenta of emitted particles × Non-radiative DIS amplitude



Recover the result of Dominguez, Marquet, Xiao and Yuan for di-jet production -as they show, it is sensitive to the gluon Weizsäcker-Williams distribution for large pair momenta

Some interesting limits

At high P_T, recover leading twist k_T and collinear factorization expressions



$$\tilde{X}^{\rm LT}_{\mu\nu} = \frac{2\alpha_S}{N_C}\frac{\phi_{Y_A}(P_\perp)}{P_\perp^2}\Theta_{\mu\nu}(P_\perp)$$

Here ϕ is the unintegrated gluon distribution and Θ represents the Dirac trace

$$\tilde{X}^{\rm coll.}_{\mu\nu} = \frac{2\alpha_S\pi^2}{N_C}(2\pi)^2\delta^{(2)}(p_\perp + k_\perp + k_{\gamma\perp})\, x_A G_A(x_A, Q^2) \lim_{P_\perp\to 0}\frac{\Theta_{\mu\nu}(P_\perp)}{P_\perp^2}$$

Inclusive photon production is directly sensitive to the nuclear gluon distribution - result at small x agrees exactly with Aurenche et al (Z. Phys. C24, 309 (1984))

Structure of higher order computations: Shockwave propagators

Convenient to work in the "wrong" light cone gauge $A^-=0$ for this problem (Gauge links in pdf definitions are unity in the right LC gauge $A^+=0$)

Just as for the Lorenz gauge quark propagator, the gluon propagator has an identical simple form where G_0 is the free propagator in $A^-=0$ gauge

$$G^{\mu\nu;ab}(p,p') = (2\pi)^4\delta^{(4)}(p-p')G_0^{\mu\nu;ab} + G_0^{\mu\rho;ac}\mathcal{T}_{\rho\sigma;cd}G_0^{\sigma\nu;db}(p')$$

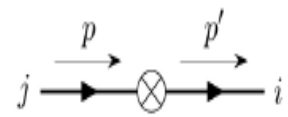
McLerran, RV; hep-ph/9402335
Ayala,Jalilian-Marian,McLerran, RV;,hep-ph/9508302
Balitsky, Belitsky, hep-ph/0110158

Note the slightly different form of the vertices–they will be useful In higher order computations

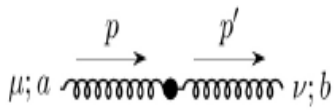
This structure of vertices is identical to the quark-quark-reggion and gluon-gluon-reggeon in Lipatov's Reggeon EFT

Bondarenko,Lipatov,Pozdynyakov, Prygarin,arXiv:1708.05183

Hentschinski, arXiv:1802.06755

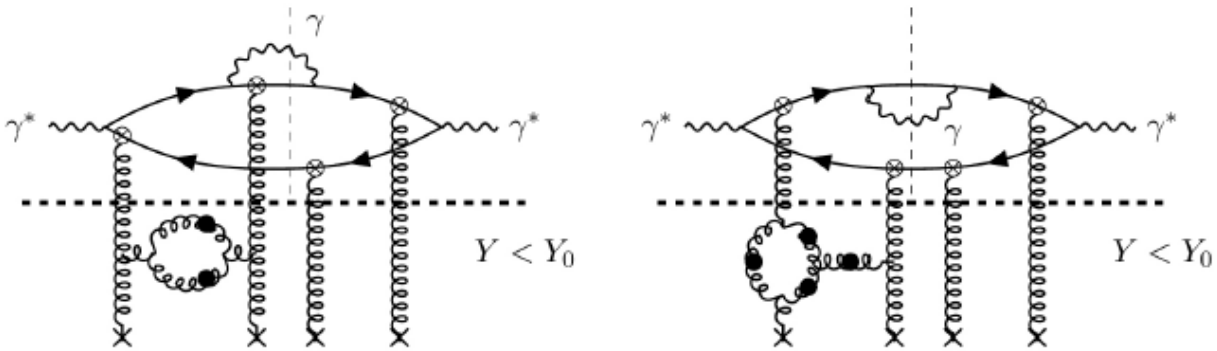


$$\mathcal{T}_{ij}(p,p') = (2\pi)\delta(p^- - p'^-)\gamma^- \mathrm{sign}(p^-)\int d^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp).\mathbf{z}_\perp}\tilde{U}^{\mathrm{sign}(p^-)}(\mathbf{z}_\perp)_{ij}$$



$$\mathcal{T}_{\mu\nu;ab}(p,p') = -2\pi\delta(p^- - p'^-)\times(2p^-)g_{\mu\nu}\ \mathrm{sign}(p^-)\int d^2\mathbf{z}_\perp e^{i(\mathbf{p}_\perp - \mathbf{p}'_\perp).\mathbf{z}_\perp}U_{ab}^{\mathrm{sign}(p^-)}(\mathbf{z}_\perp)$$

DIS inclusive photon production at NLO



NLO contributions of this sort − give rise to NLLx JIMWLK evolution
--significant progress in this direction

Balitsky, Chirilli, arXiv:1309.7644
Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521
Kovner,Lublinsky,Mulian, arXiv:1310.0378
Lublinsky, Mulian, arXiv:1610.03453

Our method of computation, entirely in momentum space,
is different from most of these approaches − so would be interesting to compare.
This is work in progress with Roy.

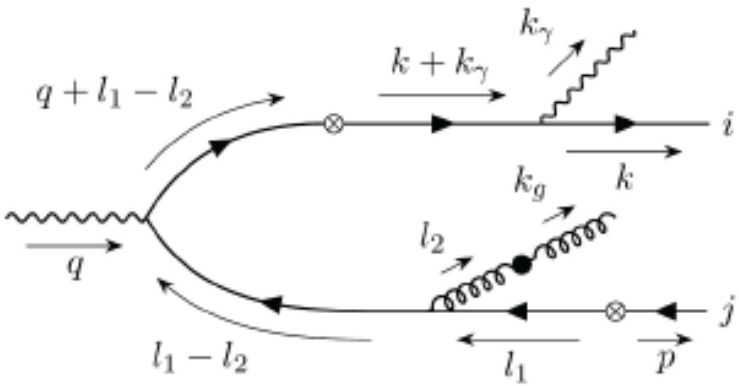
The NLO inclusive photon impact factor

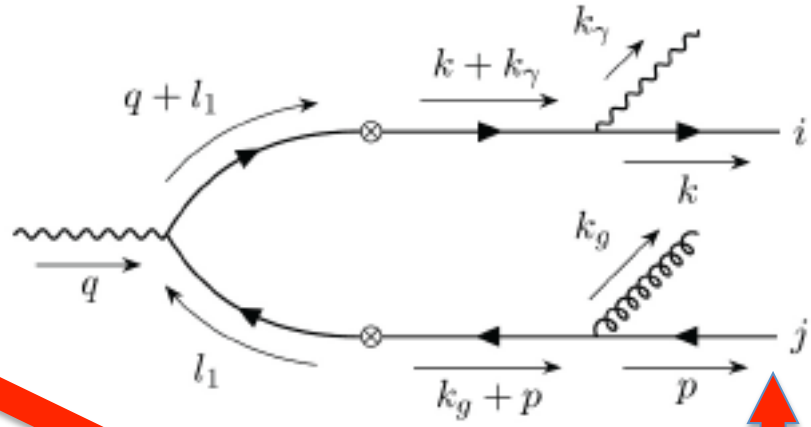
Several computations exist for fully inclusive DIS – subtleties in choice of scheme, etc.

Balitsky,Chirilli, arXiv:1009.4729
Beuf, arXiv:1606.00777, 1708.06557
Hanninen, Lappi, Paatelainen, 1711.08207
also, Boussarie, Grabovsky, Szymanowski, Wallon,1606.00419

Real contributions to the inclusive photon impact factor (10 of each kind) :

Roy, RV, in preparation



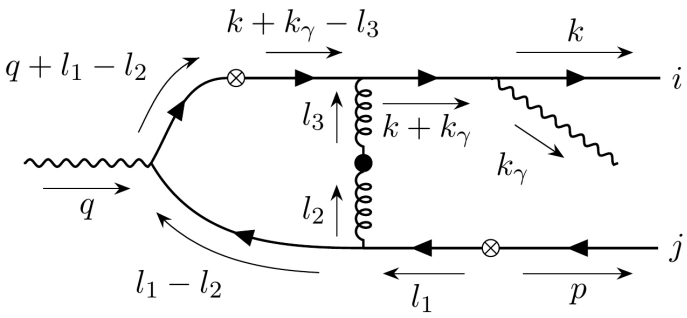


$$\mathcal{M}^{\mathrm{NLO;real}}_{\mu\alpha;b} = 2\pi(eq_f)^2 g\,\delta(q^- - P^-_{\mathrm{tot}})\,\mathrm{d}\Pi_{\mathrm{real}}\,\overline{u}(\boldsymbol{k})\Big\{T^{q\bar{q}g+\gamma}_{\mu\alpha}\Big[\Big(\tilde{U}(\boldsymbol{x}_{1\perp})t^a\tilde{U}^\dagger(\boldsymbol{x}_{2\perp})\Big)_{ij}U_{ab}(\boldsymbol{x}_{3\perp}) - (t_b)_{ij}\Big]$$
$$+ T^{q(\bar{q}+g)\gamma}_{\mu\alpha}\Big[\Big(\tilde{U}(\boldsymbol{x}_{1\perp})\tilde{U}^\dagger(\boldsymbol{x}_{2\perp})t_b\Big)_{ij} - (t_b)_{ij}\Big] + T^{(q+g)\bar{q}\gamma}_{\mu\alpha}\Big[\Big(t_b\tilde{U}(\boldsymbol{x}_{1\perp})\tilde{U}^\dagger(\boldsymbol{x}_{2\perp})\Big)_{ij} - (t_b)_{ij}\Big]v(\boldsymbol{p})$$

The NLO inclusive photon impact factor

Roy, RV, in preparation

Virtual contributions:



These computations are nearing completion − very efficient in mom.space ...
A few points can be made:

Collinear gluon emissions cancel between real and virtual contributions to the cross-section

Care must be taken to treat logs in rapidity − one convenient scheme is tp subtract these and absorb in RG evolution.

Our computations suggest that is is natural to have a scheme whereby the coupling runs with Q_S

Summary and Outlook

- **Inclusive photon production in inclusive DIS is a clean probe of the dynamics of strongly correlated gluons at high energies.**

- **A rich structure in terms of 2-point and 4-point Wilson line correlators is seen. Well-known results for inclusive di-jet production (and extraction of the Weizsäcker-Williams gluon distribution) are recovered in the soft photon limit. In the leading twist limit, the cross-section is directly proportional to the nuclear gluon distribution.**

- **The structure of dressed quark and gluon propagators in the "wrong" light cone gauge $A^{-}=0$ is very simple and facilitates higher order computations in momentum space (using otherwise standard techniques in pQCD)**

- **The computation of the NLO inclusive photon impact factor is nearing completion and will be presented shortly.**