NLO corrections for DIS structure functions in the dipole factorization

Guillaume Beuf

University of Jyväskylä

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Outline

- Introduction: dipole factorization for DIS at low x_{Bj}

- One-loop correction to the $\gamma^*_{T,L} \to q\bar{q}$ light-front wave-functions:

  Direct calculation

  G.B., PRD94 (2016)

- DIS at NLO in the dipole factorization (with massless quarks):

  Cancellation of the UV divergences between the $q\bar{q}$ and $q\bar{q}g$ terms

  Fixed order NLO results for $\sigma^{\gamma}_{T,L}(x_{Bj}, Q^2)$ (or $F_{T,L}$)

  G.B., PRD96 (2017)
  See also: Hänninen, Lappi, Paatelainen, arXiv:1711.08207

- A few words about low x_{Bj} LL resummation

Introduction

At low x_{Bj}, many DIS observables can be expressed within dipole factorization, including gluon saturation $\rightarrow$ rich phenomenology.

In particular: Dipole amplitude obtained from fits of HERA data for DIS structure functions in the dipole factorization at LO+LL with rcBK
Albacete *et al.*, PRD80 (2009); EPJC71 (2011)
Kuokkanen *et al.*, NPA875 (2012);
Lappi, Mäntysaari, PRD88 (2013)

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In the last 10 years, many theoretical (including numerical) progresses towards NLO/NLL accuracy for gluon saturation/CGC.

Obviously, DIS structure functions at NLO in the dipole factorization are required to push the fits beyond LO+LL accuracy.
$\rightarrow$ Precision gluon saturation physics both at HERA and at future ep/eA colliders

DIS at NLO: previous results

2 independent calculations had been performed earlier for NLO corrections to photon impact factor and/or DIS cross-section:

1. Balitsky, Chirilli, PRD83 (2011); PRD87 (2013)
   Using covariant perturbation theory. Results provided as
   - Current correlator in position space
   - Impact factor for $k_\perp$ factorization $\rightarrow$ Good for BFKL phenomenology

2. G.B., PRD85 (2012)
   Using light-front perturbation theory. Results provided as
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However, in both papers: only the $q\bar{q}g$ contribution explicitly calculated, and NLO corrections to the $q\bar{q}$ contribution obtained indirectly. Methods used for that:

In Balitsky, Chirilli, PRD83 (2011):
Matching with earlier vacuum results

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Unitary argument

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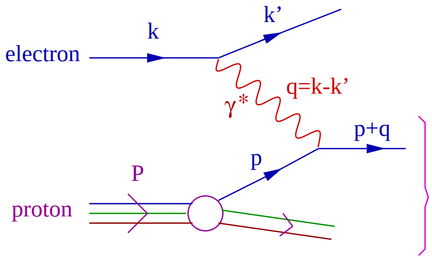
Unitary argument $\to$ wrong: missed photon finite WF renormalization

$\Rightarrow$ NLO $q\bar{q}$ terms needs to be calculated separately in LFPT

Kinematics for Deep Inelastic Scattering (DIS)



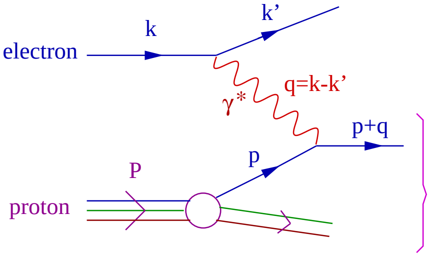
$$\frac{d\sigma^{ep\to e+X}}{dx_{Bj}\,d^2Q} = \frac{\alpha_{em}}{\pi x_{Bj} Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma}(x_{Bj},Q^2)+(1-y)\,\sigma_L^{\gamma}(x_{Bj},Q^2)\right]$$

Photon virtuality: $Q^2 \equiv -q^2 > 0$

Bjorken x variable: $x_{Bj} \equiv \frac{Q^2}{2P\cdot q} \in [0,1]$

Inelasticity: $y \equiv \frac{2P\cdot q}{(P+k)^2} = \frac{2P\cdot q}{s} \in [0,1]$ $\qquad x_{Bj}\, y\, s = Q^2$

Kinematics for Deep Inelastic Scattering (DIS)



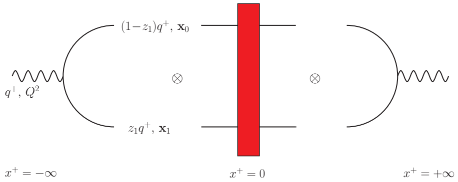
$$\frac{d\sigma^{ep\to e+X}}{dx_{Bj}\,d^2Q} = \frac{\alpha_{em}}{\pi x_{Bj} Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma}(x_{Bj},Q^2)+(1-y)\,\sigma_L^{\gamma}(x_{Bj},Q^2)\right]$$

Other equivalent parametrization: structure functions F_i

$$\sigma^{\gamma}_{T,L}(x_{Bj},Q^2) = \frac{(2\pi)^2\alpha_{em}}{Q^2}\,F_{T,L}(x_{Bj},Q^2)$$

$$F_2 = F_T + F_L \;\text{ and }\; 2x_{Bj}\,F_1 = F_T$$

Dipole factorization for DIS at LO



$$\sigma^{\gamma}_{T,L}(x_{Bj}, Q^2) = \frac{4N_c\,\alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\, \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \times \mathcal{I}^{q\bar{q},LO}_{T,L}(x_{01}, z_1, Q^2)\, \mathrm{Re}\Big[1 - \langle \mathcal{S}_{01}\rangle_Y\Big]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

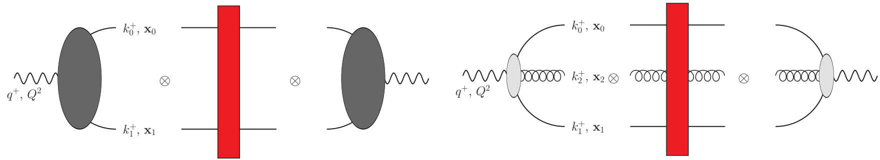
Dipole operator: $$\mathcal{S}_{01} = \frac{1}{N_c}\mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^{\dagger}(\mathbf{x}_1)\right)$$

with "rapidity" $Y \sim \log(1/x_{Bj})$ for $x_{Bj} \to 0$.
$\to$ Dependence of $\langle \mathcal{S}_{01}\rangle_Y$ on Y comes from high-energy (low-x_{Bj}) LL resummation.

Dipole factorization for eikonal DIS

Total cross section for (virtual) photon scattering on a gluon shockwave background, in light-front perturbation theory:

$$\sigma_\lambda^\gamma = 2N_c \widetilde{\sum_{q_0\bar{q}_1 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ - q^+)}{2q^+} \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}_{01}\right]$$
$$+ 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+}$$
$$\times \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1 g_2}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}_{012}^{(3)}\right] + O(\alpha_{em}\,\alpha_s^2)$$



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Dipole operator: $$\mathcal{S}_{01} \equiv \frac{1}{N_c} \mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1)\right)$$

"Tripole" operator: $$\mathcal{S}_{012}^{(3)} \equiv \frac{1}{N_c\, C_F} \mathrm{Tr}\left(t^b U_F(\mathbf{x}_0)\, t^a U_F^\dagger(\mathbf{x}_1)\right) U_A(\mathbf{x}_2)_{ba}$$

Dipole factorization for eikonal DIS

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&+ 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+} \\
&\qquad \times \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1 g_2}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}_{012}^{(3)}\right] + O(\alpha_{em}\,\alpha_s^2)
\end{aligned}$$

$\widetilde{\psi}_{\gamma_\lambda \to f}$: color-stripped light-front wavefunctions of the incoming photon for the Fock-state decomposition in mixed-space $(k^+, \mathbf{x})$

Calculation of the $\gamma_{T,L} \to q\bar{q}$ LF wave-functions at NLO

- Calculation done in Light-front perturbation theory for QCD+QED
- Cut-off k^+_{min} introduced to regulate the small k^+ divergences

  $\Rightarrow$ associated with low-x leading logs to be resummed with BK/JIMWLK evolution at the end
- UV divergences from various tensor transverse integrals, but no UV renormalization at this order (for massless quarks).

  $\Rightarrow$ UV divergences (and finite regularization artifacts) have to cancel at cross-section level

  $\Rightarrow$ Use (Conventional) Dimensional Regularization, and pay attention to rational terms in $(D-4)/(D-4)$

Diagrams for the $\gamma_T \to q\bar{q}$ LF wave-function only

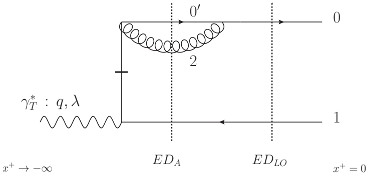


Diagram A'

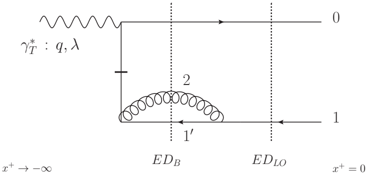


Diagram B'

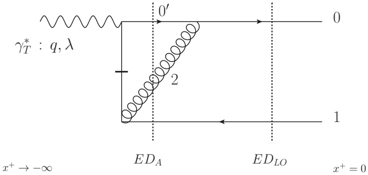


Diagram 1'

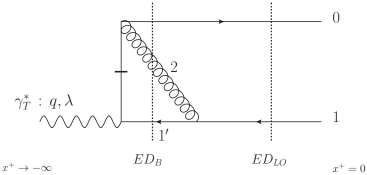
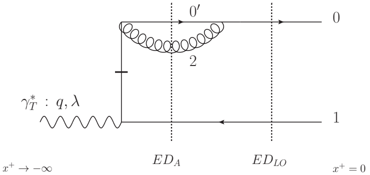


Diagram 2'

Diagrams for the $\gamma_T \to q\bar{q}$ LF wave-function only



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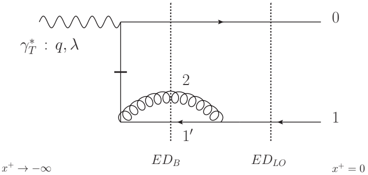
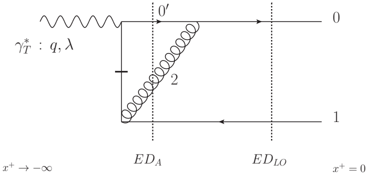


Diagram B'



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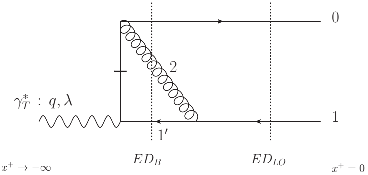


Diagram 2'

All four vanish due to Lorentz symmetry, for massless quarks.

Diagrams for γ_T and γ_L LFWFs: 3 steps graphs

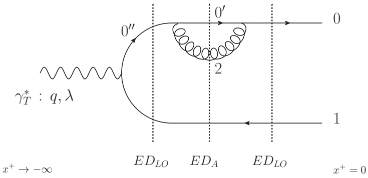
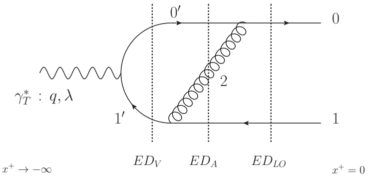


Diagram A



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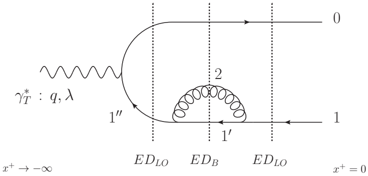

Diagram B

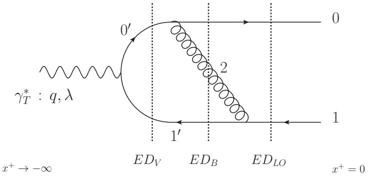

Diagram 2

Diagrams for γ_T and γ_L LFWFs: 2 steps graph

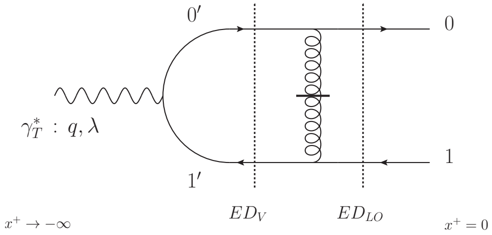

Diagram 3

- In the γ_T case: vanishes due to Lorentz symmetry (massless quark case)
- In the γ_L case: non-zero, and cancels the unphysical power-like small k^+ divergence of the other vertex correction graphs.

Results for NLO $\gamma_{T,L} \to q\bar{q}$ LFWFs in momentum space

$$\psi_{\gamma^*_{T,L}\to q_0\bar{q}_1} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right)\, \mathcal{V}^{T,L}\right]\, \psi^{\text{tree}}_{\gamma^*_{T,L}\to q_0\bar{q}_1} + \mathcal{O}(e\,\alpha_s^2)$$

$$\begin{aligned}\mathcal{V}^L =\; & 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right]\left[\Gamma\left(2-\frac{D}{2}\right)\left(\frac{\overline{Q}^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2} - 2\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right)\right]\\ & +\frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O\left(D-4\right)\end{aligned}$$

$$\mathcal{V}^T = \mathcal{V}^L + 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right]\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\mathbf{P}^2}\right)\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right) + O\left(D-4\right)$$

Notations: $\overline{Q}^2 \equiv \frac{k_0^+ k_1^+}{(q^+)^2}\, Q^2$,

and relative transverse momentum: $\mathbf{P} \equiv \mathbf{k}_0 - \frac{k_0^+}{q^+}\,\mathbf{q} = -\mathbf{k}_1 + \frac{k_1^+}{q^+}\,\mathbf{q}$

Remark: results consistent with the ones of Boussarie, Grabovsky, Szymanowski and Wallon, JHEP11(2016)149

Results for NLO $\gamma_{T,L} \to q\bar{q}$ LFWFs in mixed space

$$\widetilde{\psi}_{\gamma^*_{T,L}\to q_0\bar{q}_1} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right)\, \widetilde{\mathcal{V}}^{T,L}\right] \widetilde{\psi}^{\text{tree}}_{\gamma^*_{T,L}\to q_0\bar{q}_1} + \mathcal{O}(e\,\alpha_s^2)$$

$$\begin{aligned}\widetilde{\mathcal{V}}^{T} &= \widetilde{\mathcal{V}}^{L} + O\,(D-4)\\ &= 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left[\frac{1}{(2-\frac{D}{2})} - \Psi(1) + \log\left(\pi\,\mu^2\,\mathbf{x}_{01}{}^2\right)\right]\\ &\quad + \frac{1}{2}\left[\log\left(\frac{k^+_0}{k^+_1}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O\,(D-4)\end{aligned}$$

- In mixed space: NLO corrections $\Rightarrow$ rescaling of the LO $\gamma_{T,L} \to q\bar{q}$ LFWFs by a factor independent of the photon polarization and virtuality !
- Leftover logarithmic UV and low k^+ divergences to be dealt with at cross-section level.

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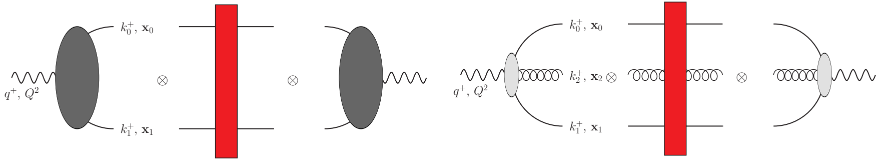
$$\widetilde{\psi}_{\gamma^*_{T,L} \to q_0 \bar{q}_1} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right) \widetilde{\mathcal{V}}^{T,L}\right] \widetilde{\psi}^{\text{tree}}_{\gamma^*_{T,L} \to q_0 \bar{q}_1} + \mathcal{O}(e\, \alpha_s^2)$$

$$\begin{aligned}
\widetilde{\mathcal{V}}^T &= \widetilde{\mathcal{V}}^L + O\,(D-4) \\
&= 2\left[\log\left(\frac{k^+_{\text{min}}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left[\frac{1}{\left(2-\frac{D}{2}\right)} - \Psi(1) + \log\left(\pi\,\mu^2\,\mathbf{x}_{01}{}^2\right)\right] \\
&\quad + \frac{1}{2}\left[\log\left(\frac{k^+_0}{k^+_1}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2} + \frac{1}{2} + O\,(D-4)
\end{aligned}$$

- In mixed space: NLO corrections $\Rightarrow$ rescaling of the LO $\gamma_{T,L} \to q\bar{q}$ LFWFs by a factor independent of the photon polarization and virtuality !
- $(D-4)/(D-4)$ rational term $1/2$: from γ^μ algebra in D dimensions in CDR $\Rightarrow$ UV regularization scheme dependent! Not present in FDH, see Hänninen, Lappi, Paatelainen (2017)

From LFWFs to DIS cross-section



$\widetilde{\psi}^{\gamma^*_{T,L}}_{\gamma^*_{T,L}\to\bar{q}}$ now known at NLO accuracy in Dim Reg.

$\Rightarrow$ Need to be combined with the $q\bar{q}g$ contribution in the dipole factorization formula at NLO

$\Rightarrow$ $\widetilde{\psi}_{\gamma^*_{T,L}q\bar{q}g}$ is required also in Dim Reg, in order to cancel UV divergences as well as scheme dependent artifacts.

Tree-level diagrams for $\gamma_L \to q\bar{q}g$ LFWFs

2 diagrams contribute to $\gamma_L \to q\bar{q}g$:

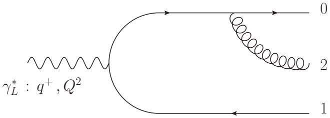


Diagram (a)

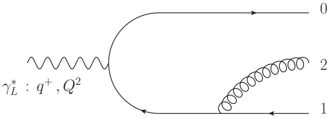


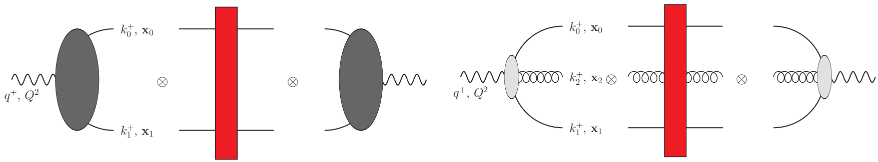
Diagram (b)

$\to$ Standard calculation in momentum space using LFPT rules, but to be done in dimensional regularization

Then: Fourier transform to mixed space

$\gamma_T \to q\bar{q}g$ case: 2 additional diagrams contribute, with the intermediate fermion line now instantaneous

From LFWFs to DIS cross-section



$$\sigma^{\gamma}_{T,L} = \sigma^{\gamma}_{T,L}\Big|_{q\bar{q}} + \sigma^{\gamma}_{T,L}\Big|_{q\bar{q}g} + O(\alpha_{em}\,\alpha_s^2)$$

with

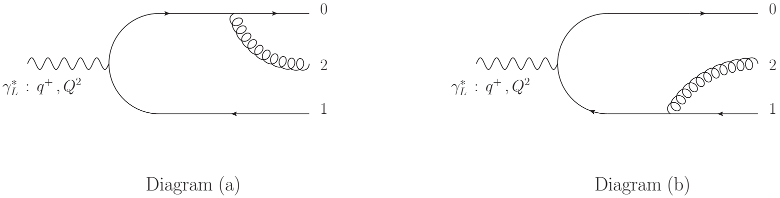
$$\sigma^{\gamma}_{\lambda}\Big|_{q\bar{q}g} = 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+} \times \left|\widetilde{\psi}_{\gamma_\lambda \to q_0\bar{q}_1 g_2}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}^{(3)}_{012}\right] + O(\alpha_{em}\,\alpha_s^2)$$

All the necessary building blocks for $\sigma^{\gamma}_{T,L}$ at NLO now known.

UV divergences of the $q\bar{q}g$ contribution

There are 2 UV divergences in $\sigma_\lambda^\gamma\Big|_{q\bar{q}g}$:

- At $\mathbf{x}_2 \to \mathbf{x}_0$ for $|(a)|^2$ contribution
- At $\mathbf{x}_2 \to \mathbf{x}_1$ for $|(b)|^2$ contribution



In both of these limits, the gluon loop shrinks to a tiny loop across the shockwave, and : $\mathcal{S}_{012}^{(3)} \to \mathcal{S}_{01}$ (color coherence)

$\Rightarrow$ Makes the cancellation of UV divergences between $q\bar{q}$ and $q\bar{q}g$ contributions possible

Subtraction of UV divergences for the $q\bar{q}g$ contribution

In practice, construct UV subtraction terms to make the cancellation of UV divergences explicit (analog to standard treatment of soft and collinear divergences cancelling between real and virtual corrections)

$$\sigma^{\gamma}_{T,L} = \left[\sigma^{\gamma}_{T,L}\Big|_{q\bar{q}} + \sigma^{\gamma}_{T,L}\big|_{UV,|(a)|^2} + \sigma^{\gamma}_{T,L}\big|_{UV,|(b)|^2}\right] + \left[\sigma^{\gamma}_{T,L}\Big|_{q\bar{q}g} - \sigma^{\gamma}_{T,L}\big|_{UV,|(a)|^2} - \sigma^{\gamma}_{T,L}\big|_{UV,|(b)|^2}\right] + O(\alpha_{em}\,\alpha_s^2)$$

where, for example for γ_L :

$$\left[\sigma^{\gamma}_{L}\Big|_{q\bar{q}g} - \sigma^{\gamma}_{L}\big|_{UV,|(a)|^2} - \sigma^{\gamma}_{L}\big|_{UV,|(b)|^2}\right] = 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+} \times \mathrm{Re}\left\{ \left|\widetilde{\psi}_{\gamma_L\to q_0\bar{q}_1 g_2}\right|^2 \left[1 - \mathcal{S}^{(3)}_{012}\right] - \left[\mathcal{C}^L_{UV,|(a)|^2} + \mathcal{C}^L_{UV,|(b)|^2}\right]\left[1 - \mathcal{S}_{01}\right]\right\}$$

Subtraction of UV divergences for the $q\bar{q}g$ contribution

$$\left[\sigma_L^\gamma\Big|_{q\bar{q}g} - \sigma_L^\gamma\big|_{UV,|(a)|^2} - \sigma_L^\gamma\big|_{UV,|(b)|^2}\right] = 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2}} \frac{2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+)}{2q^+}$$

$$\times \mathrm{Re}\left\{ \left|\widetilde{\psi}_{\gamma_L\to q_0\bar{q}_1 g_2}\right|^2 \left[1-\mathcal{S}_{012}^{(3)}\right] - \left[\mathcal{C}^L_{UV,|(a)|^2} + \mathcal{C}^L_{UV,|(b)|^2}\right]\left[1-\mathcal{S}_{01}\right]\right\}$$

Requirements:

- $\left|\widetilde{\psi}_{\gamma_L\to q_0\bar{q}_1 g_2}\right|^2 \sim \mathcal{C}^L_{UV,|(a)|^2}$ for $\mathbf{x}_2 \to \mathbf{x}_0$
- $\left|\widetilde{\psi}_{\gamma_L\to q_0\bar{q}_1 g_2}\right|^2 \sim \mathcal{C}^L_{UV,|(b)|^2}$ for $\mathbf{x}_2 \to \mathbf{x}_1$, in order to subtract the 2 UV divergences
- $\mathcal{C}^L_{UV,|(a)|^2}$ and $\mathcal{C}^L_{UV,|(b)|^2}$ should not lead to other new divergences (UV, coll., ...)
- $\int \mathrm{d}^{D-2}\mathbf{x}_2\, \mathcal{C}^L_{UV,|(a)|^2}$ and $\int \mathrm{d}^{D-2}\mathbf{x}_2\, \mathcal{C}^L_{UV,|(b)|^2}$ should be calculable analytically for generic D

Combining the UV terms with the $q\bar{q}$ contribution

Calculating the UV subtraction terms for generic D, one finds:

$$\sigma_L^\gamma\Big|_{dipole} \equiv \left[\sigma_L^\gamma\Big|_{q\bar{q}} + \sigma_L^\gamma|_{UV,|(a)|^2} + \sigma_L^\gamma|_{UV,|(b)|^2}\right]$$

$$= 2N_c \widetilde{\sum_{q_0\bar{q}_1 \text{ F. states}}} \frac{2\pi\delta(k_0^+ + k_1^+ - q^+)}{2q^+} \mathrm{Re}\left[1 - \mathcal{S}_{01}\right] \left|\widetilde{\psi}^{\text{tree}}_{\gamma_L \to q_0\bar{q}_1}\right|^2$$

$$\times \left[1 + \frac{\alpha_s C_F}{\pi} \left(\widetilde{\mathcal{V}} + \widetilde{\mathcal{V}}_{UV,|(a)|^2} + \widetilde{\mathcal{V}}_{UV,|(b)|^2}\right)\right] + O(\alpha_{em}\,\alpha_s^2)$$

Transverse photon case: same treatment of UV divergences $\Rightarrow$ same $\widetilde{\mathcal{V}}_{UV,|(a)|^2}$ and $\widetilde{\mathcal{V}}_{UV,|(b)|^2}$ (and $\widetilde{\mathcal{V}}$)

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

Expanding around $D = 4$:

$$\widetilde{\mathcal{V}}_{UV,|(a)|^2} + \widetilde{\mathcal{V}}_{UV,|(b)|^2} = -2\left[\frac{1}{\left(2-\frac{D}{2}\right)} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right)\right] \times \left[\log\left(\frac{k^+_{\min}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right] - \frac{1}{2} + O(D-4)$$

Combining the UV terms with the $q\bar{q}$ contribution to σ_L^γ

Expanding around $D = 4$:

$$\widetilde{\mathcal{V}}_{UV,|(a)|^2} + \widetilde{\mathcal{V}}_{UV,|(b)|^2} = -2\left[\frac{1}{\left(2-\frac{D}{2}\right)} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right)\right] \times \left[\log\left(\frac{k^+_{\min}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right] - \frac{1}{2} + O(D-4)$$

But in the $q\bar{q}$ contribution to $\sigma^\gamma_{T,L}$:

$$\widetilde{\mathcal{V}} = 2\left[\frac{1}{\left(2-\frac{D}{2}\right)} - \Psi(1) + \log\left(\pi\, \mathbf{x}_{01}{}^2\, \mu^2\right)\right]\left[\log\left(\frac{k^+_{\min}}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right] + \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2} + \frac{1}{2} + O(D-4)$$

$\Rightarrow$ Cancelation of:

- the UV divergence
- the $k^+_{\min}$ dependence
- the $\pm 1/2$ rational term : strong hint of UV regularization scheme independence

Results for $\sigma^{\gamma}_{T,L}$ at NLO

Results after cancellation of UV divergences (but before LL resummation):

$$\begin{aligned} \sigma^{\gamma}_{T,L} &= \sigma^{\gamma}_{T,L}\big|_{\text{dipole}} + \sigma^{\gamma}_{T,L}\big|_{q\to g} + \sigma^{\gamma}_{T,L}\big|_{\bar{q}\to g} \\ &= \sigma^{\gamma}_{T,L}\big|_{\text{dipole}} + 2\,\sigma^{\gamma}_{T,L}\big|_{q\to g} \end{aligned}$$

where:

$$\sigma^{\gamma}_{T,L}\big|_{\text{dipole}} = 4N_c\,\alpha_{em}\sum_f e_f^2 \int_0^1 dz \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\,\mathrm{Re}\left[1-\mathcal{S}_{01}\right]$$
$$\times\mathcal{I}_{T,L}(|\mathbf{x}_{01}|,z,Q^2)\left\{1+\left(\frac{\alpha_s\,C_F}{\pi}\right)\left[\frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2-\frac{\pi^2}{6}+\frac{5}{2}\right]\right\}$$

$$\begin{aligned} \mathcal{I}_L(|\mathbf{x}_{01}|,z,Q^2) &= 4z^2(1-z)^2\,Q^2\left[\mathrm{K}_0\left(Q\sqrt{z(1-z)}|\mathbf{x}_{01}|\right)\right]^2 \\ \mathcal{I}_T(|\mathbf{x}_{01}|,z,Q^2) &= z(1-z)\left[z^2+(1-z)^2\right]\,Q^2\left[\mathrm{K}_1\left(Q\sqrt{z(1-z)}|\mathbf{x}_{01}|\right)\right]^2 \end{aligned}$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

$$\left[\sigma^\gamma_{T,L}\big|_{q\bar{q}g} - \sigma^\gamma_{T,L}\big|_{UV,|(a)|^2} - \sigma^\gamma_{T,L}\big|_{UV,|(b)|^2}\right] = 2\,\sigma^\gamma_{T,L}\big|_{q\to g}$$

Changing variable to momentum fractions:

$$\sigma^\gamma_L\big|_{q\to g} = 4N_c\,\alpha_{em}\sum_f e_f^2\int_{\frac{k^+_{\min}}{q^+}}^1 dz\, 4Q^2z^2(1-z)^2\,\frac{\alpha_s C_F}{\pi}\int_{\frac{k^+_{\min}}{zq^+}}^1 d\xi\int\frac{d^2\mathbf{x}_0}{2\pi}\int\frac{d^2\mathbf{x}_1}{2\pi}\int\frac{d^2\mathbf{x}_2}{2\pi}$$
$$\times\left\{\frac{\left[1+(1-\xi)^2\right]}{\xi}\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}-\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]\left[\left(\mathrm{K}_0(QX_{012})\right)^2\mathrm{Re}\left(1-\mathcal{S}^{(3)}_{012}\right)-\left(\mathbf{x}_2\to\mathbf{x}_0\right)\right]\right.$$
$$\left.+\xi\,\frac{(\mathbf{x}_{20}\cdot\mathbf{x}_{21})}{x_{20}^2x_{21}^2}\left(\mathrm{K}_0(QX_{012})\right)^2\mathrm{Re}\left(1-\mathcal{S}^{(3)}_{012}\right)\right\}$$

with:

$$Q^2X_{012}^2\equiv\frac{Q^2}{(q^+)^2}\left[k_0^+k_1^+x_{01}^2+k_0^+k_2^+x_{02}^2+k_1^+k_2^+x_{12}^2\right]=\frac{q\bar{q}g\ \text{form. time}}{\gamma^*\ \text{life time}}$$

UV-subtracted $q\bar{q}g$ contribution to σ_L^γ

$$\left[\sigma^{\gamma}_{T,L}|_{q\bar{q}g} - \sigma^{\gamma}_{T,L}|_{UV,|(a)|^2} - \sigma^{\gamma}_{T,L}|_{UV,|(b)|^2}\right] \quad = \quad 2\,\sigma^{\gamma}_{T,L}|_{q\to g}$$

Changing variable to momentum fractions:

$$\sigma^{\gamma}_{L}|_{q\to g} \quad = \quad 4N_c\,\alpha_{em}\sum_f e_f^2 \int_{\frac{k^+_{\min}}{q^+}}^{1} dz\, 4Q^2z^2(1-z)^2\,\frac{\alpha_s C_F}{\pi}\int_{\frac{k^+_{\min}}{zq^+}}^{1} d\xi \int\frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int\frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}$$

$$\times\Bigg\{\frac{\left[1+(1-\xi)^2\right]}{\xi}\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}-\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]\left[\Big(\mathrm{K}_0(QX_{012})\Big)^2\mathrm{Re}\Big(1-\mathcal{S}^{(3)}_{012}\Big)-\Big(\mathbf{x}_2\to\mathbf{x}_0\Big)\right]$$

$$+\xi\,\frac{(\mathbf{x}_{20}\cdot\mathbf{x}_{21})}{x_{20}^2x_{21}^2}\Big(\mathrm{K}_0(QX_{012})\Big)^2\mathrm{Re}\Big(1-\mathcal{S}^{(3)}_{012}\Big)\Bigg\}$$

with:

$$Q^2X_{012}^2 = Q^2\left[(1-\xi)z(1-z)x_{01}^2+\xi(1-\xi)z^2x_{20}^2+\xi z(1-z)x_{21}^2\right]$$

UV-subtracted $q\bar{q}g$ contribution to σ_T^γ

$$
\begin{aligned}
\sigma_T^\gamma\Big|_{q\to g} \;=\;& 4N_c\,\alpha_{em}\sum_f e_f^2\int_{\frac{k^+_{\min}}{q^+}}^1 dz\, z(1-z)\,\frac{\alpha_s C_F}{\pi}\int_{\frac{k^+_{\min}}{zq^+}}^1 d\xi\int\frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int\frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\\
&\times\Bigg\{\Big[z^2+(1-z)^2\Big]\,\frac{\Big[1+(1-\xi)^2\Big]}{\xi}\,\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}-\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]\\
&\qquad\times\left[Q^2\Big(\mathrm{K}_1\left(QX_{012}\right)\Big)^2\mathrm{Re}\Big(1-\mathcal{S}^{(3)}_{012}\Big)-\Big(\mathbf{x}_2\to\mathbf{x}_0\Big)\right]\\
&\quad+\xi\Bigg[\Big[z^2+(1-z)^2\Big]\frac{(\mathbf{x}_{20}\cdot\mathbf{x}_{21})}{x_{20}^2x_{21}^2}+2z(1-z)(1-\xi)\frac{(\mathbf{x}_{20}\cdot\mathbf{x}_{21})}{x_{20}^2X_{012}^2}\\
&\qquad-\frac{z(1-\xi)(1-z+z\xi)}{X_{012}^2}\Bigg]Q^2\Big(\mathrm{K}_1\left(QX_{012}\right)\Big)^2\mathrm{Re}\Big(1-\mathcal{S}^{(3)}_{012}\Big)\Bigg\}
\end{aligned}
$$

Choice of evolution variable for low x_{Bj} resummation

In general, various scheme are possible for low x_{Bj} LL resummation, and even several choice of evolution variable to formulate them:

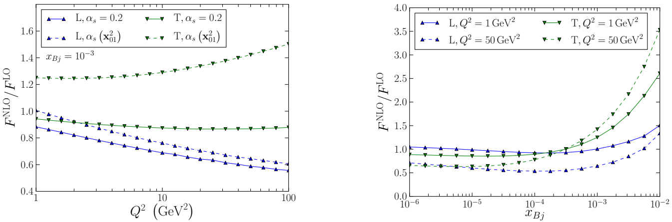
- k^- as evolution variable:
  - Best choice physically for DIS: optimize the matching of BK and DGLAP evolutions for the target
  - But not suitable in our formalism: kinematics parameterized by k^+ and $\mathbf{k}$ or $\mathbf{x}$. Very hard to keep track of the k^-s through the loop or Fourier integrals.
- k^+ as evolution variable:
  - Most practical choice in our formalism
  - But a priori incorrect DLL limit for target evolution

Best solution: use k^+ as evolution variable for convenience, but use the BK equation with kinematical improvement rather than the naive one, in order to recover a smooth matching of BK and DGLAP
G.B., PRD89 (2014)
see also: Iancu *et al.*, PLB744 (2015)

Recipe for the BK/JIMWLK LL resummation

1. Assign k^+_{min} to the scale set by the target: $k^+_{\text{min}} = \frac{Q_0^2}{2x_0\, P^-} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}\, q^+$
2. In the UV-subtracted $q\bar{q}g$ contribution ($q \to g$), promote the dipole and tripole to $\langle \mathcal{S}_{01} \rangle_{Y_2^+}$ and $\langle \mathcal{S}^{(3)}_{012} \rangle_{Y_2^+}$, with $Y_2^+ = \log(k_2^+/k^+_{\text{min}}) = \log(\xi z q^+/k^+_{\text{min}})$
3. Choose a factorization scale $k_f^+ \lesssim k_0^+, k_1^+$, corresponding to a range for the high-energy evolution $Y_f^+ \equiv \log\left(\frac{k_f^+}{k^+_{\text{min}}}\right) = \log\left(\frac{x_0\, Q^2\, k_f^+}{x_{Bj}\, Q_0^2\, q^+}\right)$
4. In the *dipole* contribution in the observable, make the replacement
$$\langle \mathcal{S}_{01} \rangle_0 \quad = \quad \langle \mathcal{S}_{01} \rangle_{Y_f^+} - \int_0^{Y_f^+} dY^+ \left(\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+}\right)$$
with both terms calculated with the same evolution equation
5. Combine the second term with the $q \to g$ NLO correction in order to remove the low x_{Bj} LL it contains.

DIS at NLO: preliminary numerical study



Ducloué, Hänninen, Lappi, Zhu (2017)
Numerical results with a simplified LL factorization/resummation scheme:

- NLO results overall well behaved
- But: sign of NLO correction to F_T changes sign when switching to running coupling (parent dipole), due to large transient effects in x_{Bj}

$\Rightarrow$ Need to check in case of more realistic RC prescriptions and/or high-energy LL factorization scheme

See talk by H. Hänninen on thursday

Further refinement of the low x_{Bj} resummation

The gluon resummed at LL should have a $k^+_{\text{min}} < k^+_2 < k^+_f \lesssim k^+_0, k^+_1$

$\Rightarrow$ If $z = k^+_0/q^+$ is integrated down to 0 or up to 1, the bounds of that interval for k^+_2 might cross

$\Rightarrow$ Pathological behavior?

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$\Rightarrow$ If $z = k^+_0/q^+$ is integrated down to 0 or up to 1, the bounds of that interval for k^+_2 might cross
$\Rightarrow$ Pathological behavior?

Cure for that issue: Restrict the integration in z to the interval $z_{\min} < z < 1 - z_{\min}$ for all contributions to $\sigma^{\gamma}_{T,L}$, where
$z_{\min} \equiv \frac{k^+_{\min}}{q^+} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}$
and choose the factorization scale $k^+_f = \min\{z, 1-z\}\, q^+$.

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and choose the factorization scale $k^+_f = \min\{z, 1-z\}\, q^+$.

The appearance of $z_{\min}$ in the integration bounds for z is a power suppressed effect at high-energy/low x_{Bj}, beyond the accuracy of the formalism, but seems to make the LL resummation better defined.
$\rightarrow$ Idea to be further explored...

NLO DIS with massive quarks

Calculations presented so far: only with massless quarks

But: charm and bottom contributions sizable at HERA

$\Rightarrow$ Need to calculate as well NLO DIS with massive quarks in the dipole factorization

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Features/complications appearing in the massive case:

- New contributions induced by quark helicity flip vertices
  $\Rightarrow$ More algebra, not really a problem
- Loop or Fourier transform integrals not fully doable analytically anymore
  $\Rightarrow$ Final results for the extra contributions given as integrals over Feynman/Schwinger parameters.
- Quark mass renormalization has to be performed
  $\Rightarrow$ Conceptually tricky in Light-Front perturbation theory: energy denominators mass *vs.* vertex mass

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Calculations presented so far: only with massless quarks

But: charm and bottom contributions sizable at HERA

$\Rightarrow$ Need to calculate as well NLO DIS with massive quarks in the dipole factorization

Status of the NLO calculations in the massive quarks case:

- $\sigma_L^\gamma(x_{Bj}, Q^2)$ or F_L:
  - Finished, but need further checks.
  - Renormalization in on-shell scheme for the mass in the denominators.

- $\sigma_T^\gamma(x_{Bj}, Q^2)$ or F_T:
  - Partially done, calculation under way.

G. B., Lappi, Paatelainen, *in preparation*

Conclusion

1. Direct calculation of $\gamma_{T,L} \to q\bar{q}$ LFWFs at one-gluon-loop order, both in momentum and in mixed space
2. Full NLO corrections to F_L and F_T from the combination of the $q\bar{q}$ and $q\bar{q}g$ contributions, with consistent method to cancel UV divergences

Phenomenology outlook : All ingredients (soon) available for fits to HERA data at NLO+LL accuracy, and hopefully NLO+NLL accuracy, in the dipole factorization, including gluon saturation.

Theory outlook :

- Extension to the case of massive quarks: under way
- Application of the NLO $\gamma_{T,L} \to q\bar{q}(g)$ LFWFs to calculate other DIS observables at NLO?
- Comparison to other calculations of photon impact factor at NLO ?
  Bartels et al.(2001-2004); Balitsky, Chirilli (2011-2013)