Operator Product Expansion in Wilson lines with sub-eikonal spin corrections

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March 21, 2018

Outline

- Motivation
- Brief review Operator Product Expansion at high-energy
- Operator Product Expansion at high-energy with sub-eikonal corrections
  - Quark propagator with sub-eikonal corrections
- Conclusions

Motivation

- Unpolarized DIS at low-x: dynamics is driven by gluon structure functions
  - gluon structure function grows as $(1/x)^{\lambda}$ with $\lambda > 1$.
- Polarized DIS at low-x: polarized gluon structure function grows as $(1/x)^{\lambda}$ with λ close to 0.
  - This implies that polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x spin TMDs and g_1 structure function are relevant
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.

Motivation

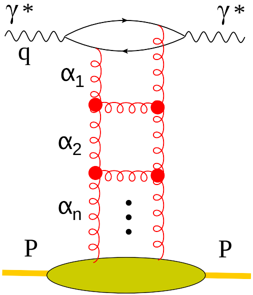
- Unpolarized DIS at low-x: dynamics is driven by gluon structure functions
  - gluon structure function grows as $(1/x)^{\lambda}$ with $\lambda > 1$.
- Polarized DIS at low-x: polarized gluon structure function grows as $(1/x)^{\lambda}$ with λ close to 0.
  - This implies that polarized quark and gluon structure functions are equally relevant.
- At Electron Ion Collider low-x spin TMDs and g_1 structure function are relevant
- Understand how the proton's spin arises from the intrinsic and orbital angular momenta of the constituent quarks and gluons.
- Compare with results obtained in the Leading Log approximation by Bartels-Ermolaev-Ryskin-(1995-1996) and recent work in Saturation formalism obtained by Kovchegov-Pytoniak-Sievert (20016-2017)

Unpolarized vs. Polarized DIS at low-x

- DGLAP: resums $\left(\alpha_s \ln \frac{Q^2}{\mu}\right)^n$ BFKL: resums $\left(\alpha_s \ln \frac{1}{x}\right)^n$
- overlap region resums $\left(\alpha_s \ln \frac{1}{x} \ln \frac{Q^2}{\mu}\right)^n$
- Scattering amplitude with fermion in t-channel in Regge limit we have $\left(\alpha_s \ln^2 \frac{1}{x}\right)^n$ contributions
  - such contribution not included in DGLAP asymptotic $x \to 0$
- Double Log of energy of quark distribution
  - unpolarized case: are not relevant since are suppressed by gluon distribution
  - polarized case: are relevant Bartels-Ermolaev-Ryskin-(1995-1996)

Semi-classical approach: Background field method

p_1^μ , p_2^μ light-cone vectors $\Rightarrow k^\mu = \alpha p_1^\mu + \beta p_2^\mu + k_\perp^\mu$
$\alpha_1 \gg \alpha_2 ... \gg \alpha_n$



Fields are ordered in their rapidities $\Rightarrow$

- large α gluons are treated as quantum fields
- low α fields are treated as classical fields

Boost of the fields

$$
\begin{aligned}
A_\bullet(x_\bullet, x_*, x_\perp) &\rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_*(x_\bullet, x_*, x_\perp) &\rightarrow \lambda^{-1}A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\
A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)
\end{aligned}
$$

λ is the boost parameter.

$$
\langle x|\frac{i}{\not{P} + i\epsilon}|y\rangle \rightarrow \langle x|\frac{i}{\not{p} + \alpha\frac{2}{s}\not{p}_2 A_\bullet + i\epsilon}|y\rangle
$$

$[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

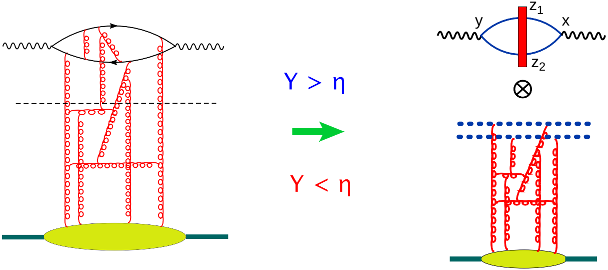
Eikonal interactions give a Wilson lines

$$U_z = [\infty p_1 + z_\perp, -\infty p_1 + z_\perp]$$

$$[x, y] = Pe^{ig \int_0^1 \, du (x-y)^\mu A_\mu (ux + (1-u)y)} \qquad p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

High-Energy Operator Product Expansion

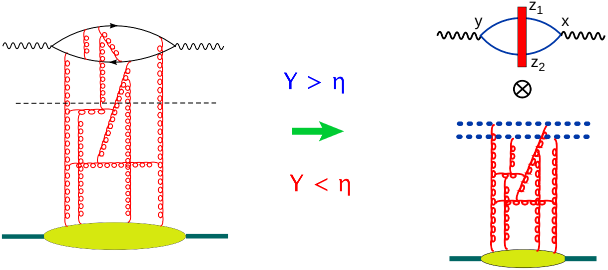
DIS amplitude is factorized in rapidity: η



$|B\rangle$ is the target state.

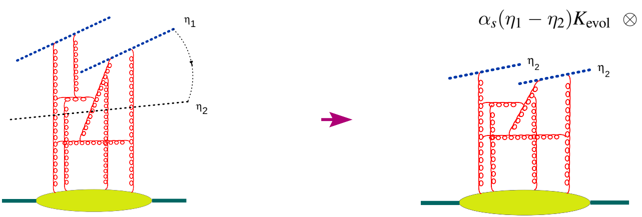
$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

High-Energy Operator Product Expansion



$$\langle B|T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}|B\rangle = \int \frac{d^2z_1 d^2z_2}{z_{12}^4}\, I^{\rm LO}_{\mu\nu}(x,y;z_1,z_2)\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle + \dots$$

- If we use a model to evaluate $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ we can calculate the DIS cross-section.
- If we want to include energy dependence to the DIS cross section, we need to find the evolution of $\langle B|\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}|B\rangle$ with respect to the rapidity parameter η.



- Separate fields in quantum and classical according to low and large rapidity.
  Formally we may write:

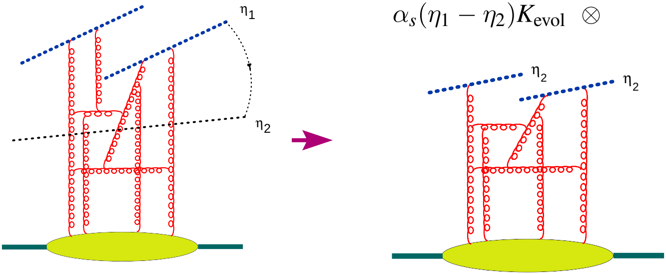
$$\langle B|\mathcal{O}^{\eta_1}|B\rangle \to \langle \mathcal{O}^{\eta_1}\rangle_A \to \langle \mathcal{O}'^{\eta_2}\otimes \mathcal{O}'^{\eta_1}\rangle_A$$

- Integrate over the quantum fields and get one-loop rapidity evolution of the operator $\mathcal{O}$

$$\langle \mathcal{O}^{\eta_1}\rangle_A = \alpha_s(\eta_1-\eta_2)K_{\rm evol}\otimes\langle \mathcal{O}'^{\eta_2}\rangle_A$$

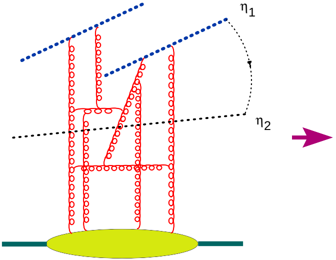
- Where in principle $\mathcal{O}$ and $\mathcal{O}'$ may be different operators.

Non-linear evolution equation

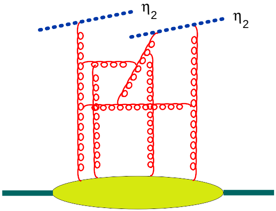


- Linear case $\quad \mathcal{O}^{\eta_1} = \alpha_s \Delta\eta\, K_{\rm evol} \ \otimes\ \mathcal{O}^{\eta_2}$

Non-linear evolution equation

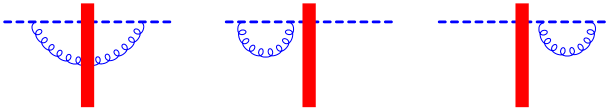


$\alpha_s(\eta_1 - \eta_2) K_{\rm evol} \; \otimes$



- Linear case $\quad \mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\rm evol} \; \otimes \; \mathcal{O}^{\eta_2}$
- Non-linear case $\quad \mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\rm evol} \; \otimes \; \{\mathcal{O}^{\eta_2}\mathcal{O}^{\eta_2}\}$

Non-linear evolution equation

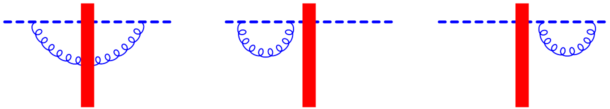


$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta\int\frac{d^2z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

$\Delta\eta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

Non-linear evolution equation



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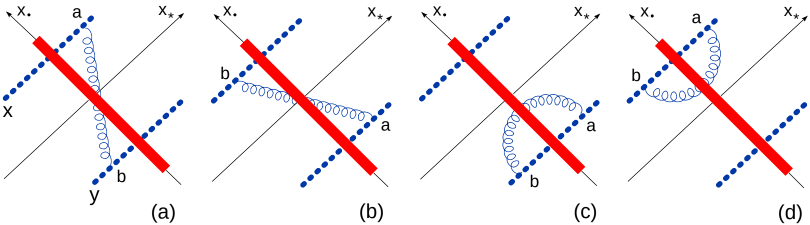
$$\{U_x^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij}, \quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij}, \quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

- Obtain a set of rules that allow one to get the LO evolution of any trace or product of traces of Wilson lines
- Hierarchy of evolution equation: B-JIMWLK equation.

Leading order evolution equation

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \ldots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad$ $x_* = \sqrt{\frac{s}{2}}x^+$

Non-linear evolution equation: Balitsky-Kovchegov equation

$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) = \frac{\alpha_s N_c}{2\pi^2}\int\frac{d^2z\,(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

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- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.

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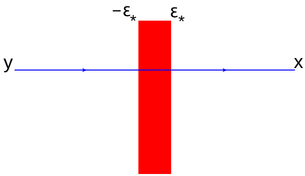
- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): describes proliferation of gluons.
- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK eqn
  - background field method: describes recombination process.
- Note: if $x_\perp \rightarrow z_\perp$ or $y_\perp \rightarrow z_\perp$ divergences cancel out.

Shock-wave with finite width



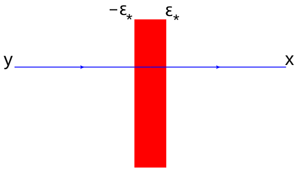
$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$

$$A_\bullet(x_\bullet, x_*, x_\perp) \rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_*(x_\bullet, x_*, x_\perp) \rightarrow \lambda^{-1}A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$
$$A_\perp(x_\bullet, x_*, x_\perp) \rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp)$$

λ is the boost parameter

- $p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$
- small α gluons are classical fields   large α gluons are quantum fields.
- Longitudinal sized classical fields: $\epsilon_* = \frac{\alpha s}{l_\perp^2}$ with $l_\perp$ trans. mom. of classical fileds
- Distance traveled by quantum fields: $z_* = \frac{\alpha s}{k_\perp^2}$ with $k_\perp$ trans. mom. of classical fileds
- We are in the case $l_\perp \sim k_\perp$

Shock-wave with finite width



$$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$$

$$\begin{aligned} A_\bullet(x_\bullet, x_*, x_\perp) &\rightarrow \lambda A_\bullet(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\ A_*(x_\bullet, x_*, x_\perp) &\rightarrow \lambda^{-1}A_*(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \\ A_\perp(x_\bullet, x_*, x_\perp) &\rightarrow A_\perp(\lambda^{-1}x_\bullet, \lambda x_*, x_\perp) \end{aligned}$$

λ is the boost parameter

sub-eikonal terms go like $\frac{1}{\lambda}$

$$\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle \rightarrow \langle x|\not{P}\frac{i}{p^2+2\alpha A_\bullet + ig\frac{2}{s}\not{p}_2\gamma^i F_{\bullet i} + \frac{1}{2}F_{ij}\sigma^{ij} + \ldots + i\epsilon}|y\rangle$$

- Note: $[\hat{\alpha}, \hat{A}^{cl}_\mu] = 0$ with $\alpha = \sqrt{\frac{2}{s}}p^+$ and $\not{p}_2 \propto \gamma^+$

$$e^{i\frac{\hat{p}_\perp^2}{\alpha s}z_*}\hat{A}_\bullet(z_*)\,e^{-i\frac{\hat{p}_\perp^2}{\alpha s}z_*} \simeq A_\bullet(z_*) - \frac{z_*}{\alpha s}\{p^i, F_{\bullet i}(z_*)\} - \frac{z_*^2}{2\alpha^2 s^2}\{p^j, \{p^i, D_j F_{\bullet i}(z_*)\}\} + \ldots$$

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad \sqrt{\frac{2}{s}}p^+$

$$\langle x|\frac{i}{\not P + i\epsilon}|y\rangle$$

$$= \left[\int_0^{+\infty}\frac{đ\alpha}{2\alpha}\theta(x_* - y_*) - \int_{-\infty}^0 \frac{đ\alpha}{2\alpha}\theta(y_* - x_*)\right] e^{-i\alpha(x_\bullet - y_\bullet)}\frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\Big\{\not p \not p_2[x_*, y_*]\not p$$

$$+\frac{ig}{2\alpha}\int_{y_*}^{x_*} d\frac{2}{s}\omega_*\Big[\not p\,\not p_2\,[x_*, \omega_*]\frac{1}{2}F^{\perp}_{\mu\nu}(x_*)\sigma^{\mu\nu}[\omega_*, y_*]\not p + \not p\not p_2\{p^i, [x_*, \omega_*]\frac{2}{s}\omega_* F_{i\bullet}(\omega_*)[\omega_*, y_*]\}\not p$$

$$+g\int_{\omega_*}^{x_*} d\frac{2}{s}\omega'_*\,\not p\not p_2\,\frac{2}{s}(\omega_* - \omega'_*)[x_*, \omega'_*]F^i{}_\bullet(\omega'_*)[\omega'_*, \omega_*]F_{i\bullet}[\omega_*, y_*]\,\not p + \not p\not p_2(\ldots)\Big]\Big\}e^{i\frac{\hat{p}^2}{\alpha s}y_*}|y_\perp\rangle + O(\frac{1}{\lambda^2})$$

- $\not p\not p_2(...)$ stands for “other similar terms” not included for brevity

Quark propagator with sub-eikonal corrections

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^- \qquad \sqrt{\frac{2}{s}}p^+$

$$\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle$$

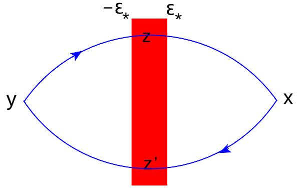
$$= \left[\int_0^{+\infty}\frac{\not{d}\alpha}{2\alpha}\theta(x_*-y_*) - \int_{-\infty}^0\frac{\not{d}\alpha}{2\alpha}\theta(y_*-x_*)\right] e^{-i\alpha(x_\bullet-y_\bullet)}\frac{1}{\alpha s}\langle x_\perp|\, e^{-i\frac{\hat{p}_\perp^2}{\alpha s}x_*}\Big\{\not{p}\not{p}_2[x_*,y_*]\not{p}$$

$$+\frac{ig}{2\alpha}\int_{y_*}^{x_*}d\frac{2}{s}\omega_*\Big[\not{p}\not{p}_2\,[x_*,\omega_*]\frac{1}{2}F^\perp_{\mu\nu}(x_*)\sigma^{\mu\nu}[\omega_*,y_*]\not{p}+\not{p}\not{p}_2\{p^i,[x_*,\omega_*]\frac{2}{s}\omega_*F_{i\bullet}(\omega_*)[\omega_*,y_*]\}\not{p}$$

$$+g\int_{\omega_*}^{x_*}d\frac{2}{s}\omega'_*\,\not{p}\not{p}_2\,\frac{2}{s}(\omega_*-\omega'_*)[x_*,\omega'_*]F^i{}_\bullet(\omega'_*)[\omega'_*,\omega_*]F_{i\bullet}[\omega_*,y_*]\,\not{p}+\not{p}\not{p}_2(\dots)\Big]\Big\}e^{i\frac{\hat{p}^2}{\alpha s}y_*}|y_\perp\rangle + O(\frac{1}{\lambda^2})$$

- $\not{p}\not{p}_2(...)$ stands for "other similar terms" not included for brevity
- Leading-eikonal term
- Sub-eikonal terms

Quark propagator with sub-eikonal corrections



Let $|B\rangle$ be proton or nuclear target

$$\langle B|J^\mu(x)J^\nu(y)|B\rangle \quad \to \quad \langle J^\mu(x)J^\nu(y)\rangle_A = \mathrm{Tr}\{\gamma^\mu\langle x|\frac{i}{\not{P}+i\epsilon}|y\rangle\,\gamma^\nu\langle y|\frac{i}{\not{P}+i\epsilon}|y\rangle\}$$

OPE in Wilson lines with sub-eikonal correction

$$\langle P|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)|P\rangle = \mathrm{tr}\Big\{\langle\langle x|\frac{i}{\not P + i\epsilon}|y\rangle\rangle\gamma^\nu\langle\langle y|\frac{i}{\not P + i\epsilon}|x\rangle\rangle\gamma^\mu\Big\}$$

$$= \int d^2z d^2z' \, I^{\mu\nu}(x,y;z,z')\langle P|\mathrm{tr}\{U_z U^\dagger_{z'}\}|P\rangle$$

$$+ \int d^2z d^2z' \, I^{\mu\nu,\, ij}(x,y,;z,z') \int_{y_*}^{x_*} d\omega_* \Big[\langle P|\mathrm{tr}\{U_z[-\infty,\omega_*]_{z'}\, gF_{ij}[\omega_*,+\infty]_{z'}\}|P\rangle + \mathrm{c.c.}\Big]$$

$$+ \ldots$$

$$I^{\mu\nu}(x,y;z,z') \propto \mathrm{tr}\{(\not x - \not z)\not p_2\,(\not y - \not z)\gamma^\nu(\not y - \not z')\not p_2\,(\not y - \not z')\gamma^\mu\}$$

$$I^{\mu\nu\,,ij}(x,y;z,z') \propto \mathrm{tr}\{(\not x - \not z)\not p_2\,(\not y - \not z)\gamma^\nu(\not y - \not z')\not p_2\,\sigma^{ij}(\not x - \not z')\gamma^\mu\}$$

$\sigma^{ij} = \frac{1}{2}[\gamma^i,\gamma^j]$ with $i,j = 1,2$

- Besides the dipole, we have now new operators

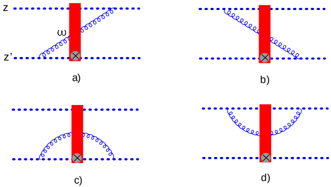
Evolution of sub-eikonal operator

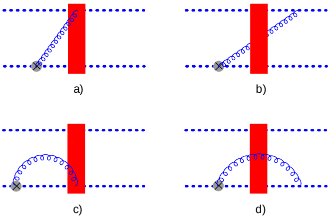
Consider, for example, the following sub-eikonal operator

$$\int_{y_*}^{x_*} d\omega_*\, \mathrm{tr}\{U_z[-\infty,\omega_*]_{z'}\, gF_{ij}(\omega_*, z'_\perp)[\omega_*, +\infty]_{z'}\}$$

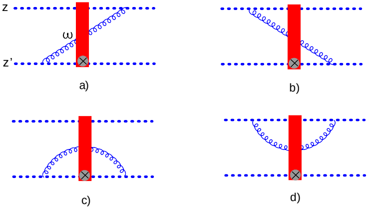
Background field method: split fields in quantum and classical and integrate out the quantum fields

Sample of diagrams:





sample of BK-type diagrams



- $\int_0^{+\infty} \frac{d\alpha}{\alpha}$ rapidity divergence
- if $\omega_\perp \to z_\perp$ divergence cancel out.
- if $\omega_\perp \to z'_\perp$ divergence does not cancel out.
  - we have $\left(\alpha_s \ln^2 \frac{1}{x}\right)$ type of contribution

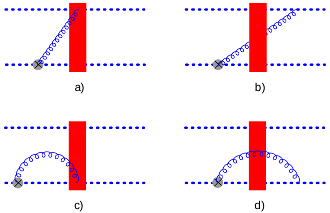
Summing real and virtual diagrams we get

$$\int_{-\infty}^{+\infty} d\omega_* \langle \mathrm{tr}\{[\infty, -\infty]_z [-\infty, \omega_*]_{z'} gF_{ij}(\omega_*)[\omega_*, \infty]_{z'}\}\rangle_{\mathrm{BK-type}}$$

$$= \frac{\alpha_s}{2\pi^2} \int_{-\infty}^{+\infty} d\omega_* \int_0^{+\infty} \frac{d\alpha}{\alpha} \int d^2\omega \, \frac{(z-z')_\perp^2}{(z-\omega)_\perp^2 (z'-\omega)_\perp^2}$$

$$\times \Big[\mathrm{tr}\{U_z U_\omega^\dagger\}\mathrm{tr}\{U_\omega[-\infty, \omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\} - N_c \mathrm{tr}\{U_z[-\infty, \omega_*]_{z'} \, gF_{ij}(\omega_*, z'_\perp)[\omega_*, \infty]_{z'}\}\Big]$$

- Diagram with gluon propagator without sub-eikonal corrections has no rapidity divergence



- $\Rightarrow$ Need sub-eikonal corrections also in gluon propagator.

Gluon propagator in the light-cone gauge with sub-eikonal corrections is

$$\langle \mathrm{T}A_\mu(x)A_\nu(y)\rangle = \Big[-\int_0^{+\infty}\frac{\bar{d}\alpha}{2\alpha}\theta(x_*-y_*) + \int_{-\infty}^{0}\frac{\bar{d}}{2\alpha}\theta(y_*-x_*)\Big]$$

$$\times\langle x_\perp|\, e^{-i\frac{p_\perp^2}{\alpha s}x_*}\,\mathcal{O}(x_*,y_*)\,e^{i\frac{p_\perp^2}{\alpha s}y_*}\,|y_\perp\rangle + i\langle x|\frac{p_{2\mu}p_{2\nu}}{p_*^2}|y\rangle$$

$$\mathcal{O}(x_*,y_*) = [x_*,y_*] - \frac{2ig}{\alpha s^2}\int_{y_*}^{x_*} dz_*\,\big(z_*\{p^j,[x_*,z_*]F_{\bullet j}[z_*,y_*]\} + \dots\big)$$

see Balitsky – Tarasov 2016

Conclusions

- Quark propagator with sub-eikonal corrections is good for
  - spin-dependent TMDs: SIDIS, Weizsäcker-Williams TMD at low-x
  - spin g_1 structure function at low-x
- New operators appears if we consider spin
- OPE at high-energy extended to include sub-eikonal spin corrections
  - Calculate the corresponding Impact Factor at LO
- Sub-eikonal corrections to BK-equation are now also possible
  - Although these are suppressed in the unpolarized case
- Future...include NLO corrections with spin