Deep Inelastic Scattering in the Dipole Picture at Next-to-Leading Order

Henri Hänninen

March 22, 2018

B. Ducloué, H. Hänninen, T. Lappi, and Y. Zhu, *Deep inelastic scattering in the dipole picture at next-to-leading order*, Phys. Rev. D 96, 094017 (2017).

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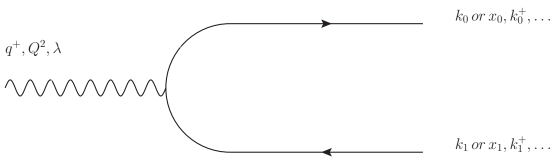
Leading Order DIS in the dipole picture



Leading order virtual photon fluctuation.

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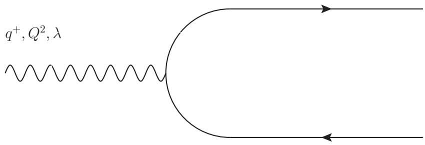


Leading order virtual photon fluctuation.

In leading order dipole picture $\gamma^* p$ cross section is of the form

$$\sigma_{L,T}^{\mathrm{LO}}(x_{Bj}, Q^2) = 4 N_{\mathrm{c}} \alpha_{em} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0, \mathbf{x}_1} \mathcal{K}_{L,T}^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, x_{Bj}),$$

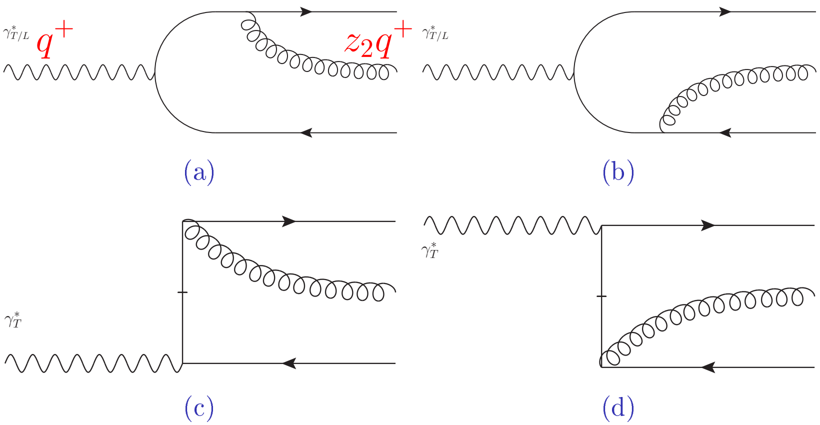
where

$$\mathcal{K}_L^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = 4Q^2 z_1^2 (1-z_1)^2 K_0^2(QX_2) \left(1 - S_{01}(X)\right),$$

$$\mathcal{K}_T^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = Q^2 z_1 (1-z_1) \left(z_1^2 + (1-z_1)^2\right) K_1^2(QX_2) \left(1 - S_{01}(X)\right)$$

with $\int_{\mathbf{x}_0} := \int \frac{\mathrm{d}^2 \mathbf{x}_0}{2\pi}$, $X_2^2 := z_1(1-z_1)\mathbf{x}_{01}^2$ and X is the momentum fraction at which the Wilson line correlator S_{01} is evaluated.

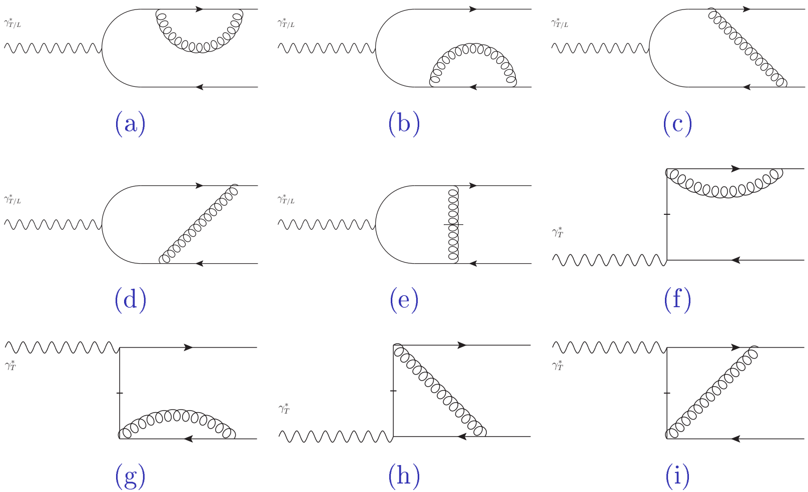
Next-to-Leading Order: Tree-level diagrams



Virtual photon fluctuation diagrams relevant to the scattering at next-to-leading order. Cause a logarithmic divergence for $\sigma^{\mathrm{NLO}}_{L,T}$ as $z_2 \to 0$.

Calculated by G. Beuf, Phys. Rev. D **96**, 074033 (2017).

Next-to-Leading Order: One-gluon-loop diagrams



Loop diagrams relevant at next-to-leading order.

Calculated by G. Beuf, Phys. Rev. D **94**, 054016 (2016).

Full NLO DIS cross section in the dipole picture

Next-to-Leading Order $\gamma^* p$ cross section can be partitioned as

$$\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{IC}} + \sigma_{L,T}^{qg} + \sigma_{L,T}^{\text{dip}},$$

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where the NLO contributions are[1]:

$$\sigma_{L,T}^{qg} = 8 N_\mathrm{c} \alpha_{em} \frac{\alpha_s C_\mathrm{F}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int^{1-z_1} \frac{\mathrm{d}z_2}{z_2} \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}_{L,T}^{\mathrm{NLO}} \left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right),$$

$$\sigma_{L,T}^{\mathrm{dip}} = 4 N_\mathrm{c} \alpha_{em} \frac{\alpha_s C_\mathrm{F}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \times \int_{\mathbf{x}_0,\mathbf{x}_1} \mathcal{K}_{L,T}^{\mathrm{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X^{\mathrm{dip}}) \left[\frac{1}{2} \ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right].$$

z_2 = gluon momentum fraction.

[1] G. Beuf, Phys. Rev. D **96**, 074033 (2017).

Subtraction of the soft gluon divergence

- Find a natural lower limit for gluon fractional momentum z_2.

[2] E. Iancu et al. , JHEP **12** (2016) 041; B. Ducloué et al., Phys. Rev. D **95** 114007 (2017).

Subtraction of the soft gluon divergence

- Find a natural lower limit for gluon fractional momentum z_2.
- BK evolution of the target can be considered using two distinct variables: probe longitudinal momentum and target momentum fraction $X = \Delta k^- / P^-$.

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- For single inclusive pA dominant $k_T \sim pA$ hard scale.[2]

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- For single inclusive pA dominant $k_T \sim pA$ hard scale.[2]

We assume the same for DIS, i.e. $k_T \sim Q$, and set $X(z_2) \equiv x_{Bj}/z_2$, kinematical limit to $X(z_2) < x_0$, i.e. $z_2 > x_{Bj}/x_0$.

[2] E. Iancu et al. , JHEP **12** (2016) 041; B. Ducloué et al., Phys. Rev. D **95** 114007 (2017).

First subtraction scheme: the 'unsubtracted' form

With the above assumptions for the subtraction scheme we get

$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{IC}} + \sigma_{L,T}^{qg,\mathrm{unsub.}} + \sigma_{L,T}^{\mathrm{dip}} \,,$$

where $\sigma_{L,T}^{\mathrm{IC}}$ is the LO result with non-evolved $S_{01}(X = x_0)$ and

$$\sigma_{L,T}^{qg,\mathrm{unsub.}} = 8 N_{\mathrm{c}} \alpha_{em} \frac{\alpha_s C_{\mathrm{F}}}{\pi} \sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{x_{Bj}/x_0}^{1-z_1} \frac{\mathrm{d}z_2}{z_2} \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathcal{K}_{L,T}^{\mathrm{NLO}} \left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right),$$

$$X(z_2) = x_{Bj}/z_2,$$

$$z_2 = \text{gluon momentum fraction.}$$

Second subtraction scheme: the 'subtracted' form

$$\sigma_{L,T}^{\mathrm{NLO}} = \sigma_{L,T}^{\mathrm{IC}} + \sigma_{L,T}^{qg,\mathrm{unsub.}} + \sigma_{L,T}^{\mathrm{dip}}$$

can be rewritten in an equivalent form:

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$$\sigma_{L,T}^{qg,\mathrm{sub.}} = 8N_{\mathrm{c}}\alpha_{em}\frac{\alpha_s C_{\mathrm{F}}}{\pi}\sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{x_{Bj}/x_0}^1 \frac{\mathrm{d}z_2}{z_2} \times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \Big[\theta(1-z_1-z_2)\mathcal{K}_{L,T}^{\mathrm{NLO}}\left(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right) - \mathcal{K}_{L,T}^{\mathrm{NLO}}\left(z_1, 0, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)\right)\Big].$$

Numerical results

Few details about the published numerical results:

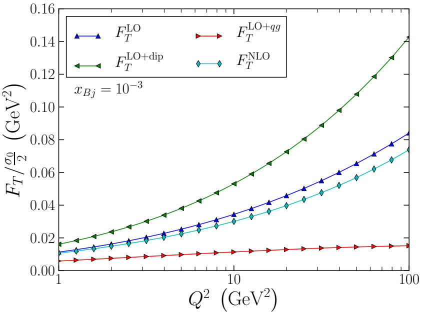
- Impact parameter dependence neglected $\to$ plotting $F_{L,T}/(\sigma_0/2)$.
- Leading order BK, with McLerran-Venugopalan initial condition.

Both fixed ($\alpha_s = 0.2$) and parent dipole running coupling were used:

$$\alpha_{\rm s}(\mathbf{x}_{01}^2) = \frac{12\pi}{(11N_{\rm c} - 2n_{\rm f})\ln\left(\frac{4e^{-2\gamma_e}}{\mathbf{x}_{01}^2\Lambda_{\rm QCD}^2}\right)},$$

with $\Lambda_{\rm QCD} = 0.241$ GeV.

Results: 'unsubtracted' scheme, fixed coupling



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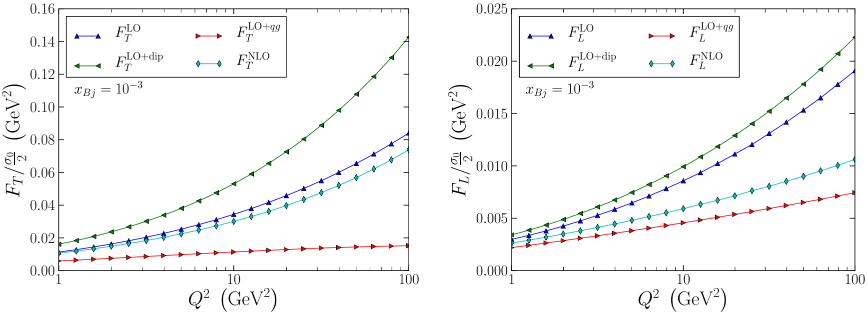

LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_s = 0.2$.

- Full NLO corrections are moderate.
- In this scheme there are large cancellations between the NLO contributions.

A possible approximative scheme: 'x_{Bj}-subtracted'

As an approximation set $X(z_2) \equiv x_{Bj}$ and $x_{Bj}/x_0 \to 0$.
Motivation: S_{01} is evaluated only at x_{Bj}.

[3] G. A. Chirilli, B.-W. Xiao and F. Yuan, Phys. Rev. Lett. **108** (2012) 122301

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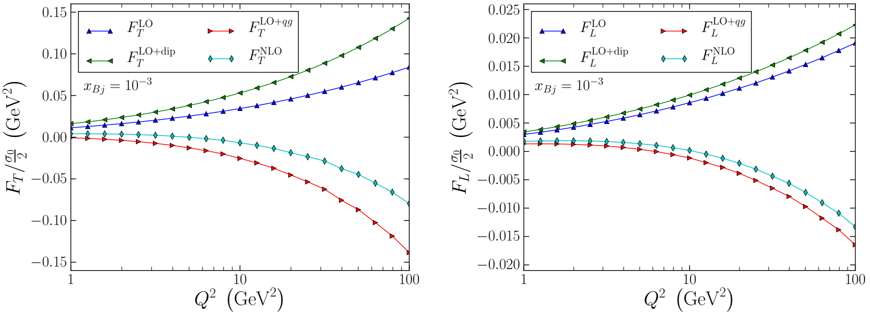
Motivation: S_{01} is evaluated only at x_{Bj}.

$$\begin{aligned}
\sigma_{L,T}^{\mathrm{NLO},x_{Bj}-\mathrm{sub.}} &= \sigma_{L,T}^{\mathrm{LO}} + \sigma_{L,T}^{qg,\mathrm{sub.}*} + \sigma_{L,T}^{\mathrm{dip}}, \\
\sigma_{L,T}^{qg,\mathrm{sub.}*} &= 8N_{\mathrm{c}}\alpha_{em}\frac{\alpha_s C_{\mathrm{F}}}{\pi}\sum_f e_f^2 \int_0^1 \mathrm{d}z_1 \int_{0}^1 \frac{\mathrm{d}z_2}{z_2} \\
&\times \int_{\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \Big[\theta(1-z_1-z_2)\mathcal{K}_{L,T}^{\mathrm{NLO}}(z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,x_{Bj}) \\
&\qquad - \mathcal{K}_{L,T}^{\mathrm{NLO}}(z_1,0,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,x_{Bj})\Big].
\end{aligned}$$

Analogous to a subtraction scheme used for single inclusive particle production in pA.[3]

[3] G. A. Chirilli, B.-W. Xiao and F. Yuan, Phys. Rev. Lett. **108** (2012) 122301

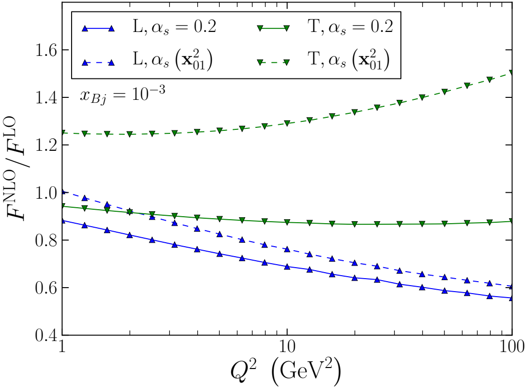
Results: approximative scheme, fixed coupling



LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_s = 0.2$ and using the x_{Bj}-subtraction procedure.

- The approximations break the scheme and lead to negative NLO structure functions.
- Similar to the negativity issue seen with single inclusive pA.

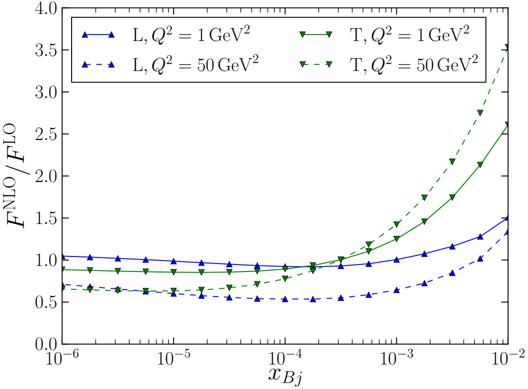
Results: 'unsubtracted' scheme, fixed v. running coupling



NLO/LO ratio for F_L and F_T as a function of Q^2 at $x_{Bj} = 10^{-3}$ with fixed (solid) and running (dashed) coupling.

- NLO corrections sensitive to subtraction scheme details e.g. running coupling due to the large cancellations.

Results: 'unsubtracted' scheme, running coupling



NLO/LO ratio for F_L and F_T as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (solid) and $Q^2 = 50$ GeV^2 (dashed) with running coupling.

- By scheme construction $\sigma^{qg} \xrightarrow[x_{Bj}\to x_0]{} 0 \implies$ large transient effect at large x_{Bj} especially for F_T.

Conclusions

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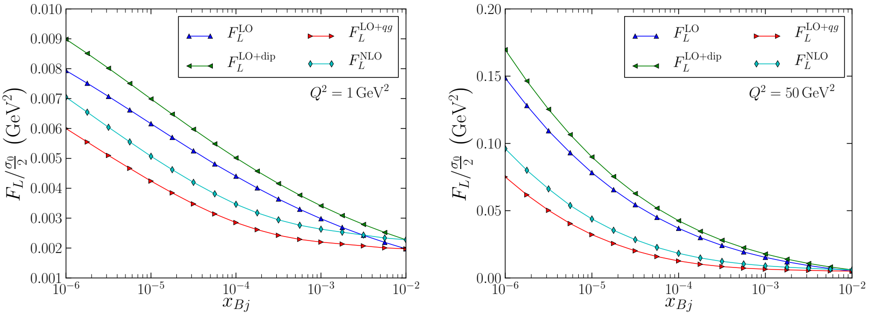
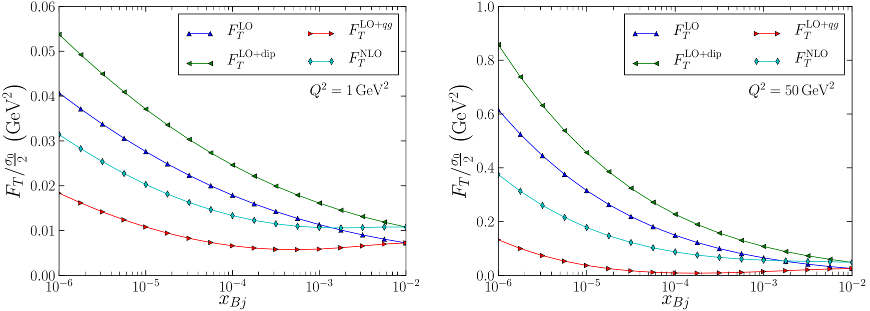
- Fit to HERA data.
- Extension of impact factor results for massive quarks.

Thank you!

Backup slides



LO and NLO contributions to F_L as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (left) and $Q^2 = 50$ GeV^2 (right) with $\alpha_s = 0.2$.



LO and NLO contributions to F_T as a function of x_{Bj} at $Q^2 = 1$ GeV^2 (left) and $Q^2 = 50$ GeV^2 (right) with $\alpha_{\rm s} = 0.2$.