

Deep Inelastic Scattering in the Dipole Picture at Next-to-Leading Order

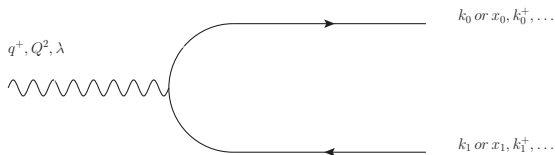
Henri Hänninen

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B. Ducloué, H. Hänninen, T. Lappi, and Y. Zhu, *Deep inelastic scattering in the dipole picture at next-to-leading order*, Phys. Rev. D 96, 094017 (2017).

- 1 Deep Inelastic Scattering at LO and NLO in the dipole picture
- 2 Subtraction of the soft gluon divergence
- 3 Numerical evaluation of the NLO corrections to structure functions

Leading Order DIS in the dipole picture



Leading order virtual
photon fluctuation.

In leading order dipole picture $\gamma^* p$ cross section is of the form

$$\sigma_{L,T}^{\text{LO}}(x_{Bj}, Q^2) = 4N_c \alpha_{em} \sum_f e_f^2 \int_0^1 dz_1 \int_{\mathbf{x}_0, \mathbf{x}_1} \mathcal{K}_{L,T}^{\text{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, x_{Bj}),$$

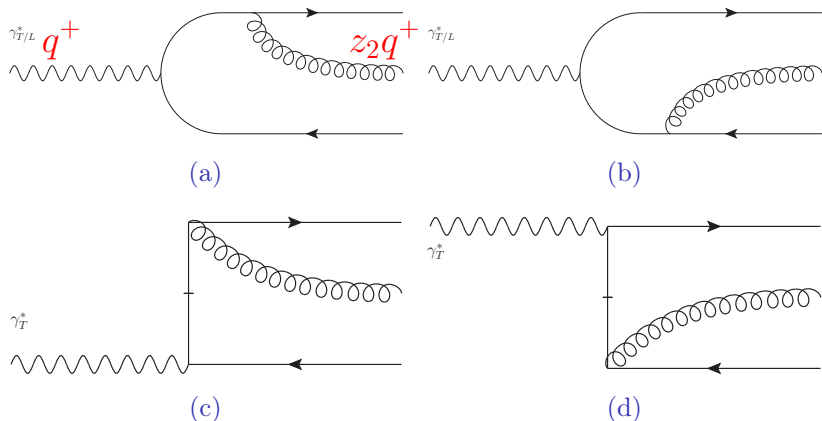
where

$$\mathcal{K}_L^{\text{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = 4Q^2 z_1^2 (1 - z_1)^2 K_0^2(QX_2) (1 - S_{01}(X)),$$

$$\mathcal{K}_T^{\text{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X) = Q^2 z_1 (1 - z_1) (z_1^2 + (1 - z_1)^2) K_1^2(QX_2) (1 - S_{01}(X))$$

with $\int_{\mathbf{x}_0} := \int \frac{d^2 \mathbf{x}_0}{2\pi}$, $X_2^2 := z_1(1 - z_1) \mathbf{x}_{01}^2$ and X is the momentum fraction at which the Wilson line correlator S_{01} is evaluated.

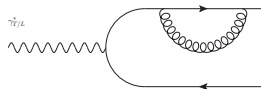
Next-to-Leading Order: Tree-level diagrams



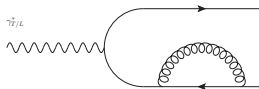
Virtual photon fluctuation diagrams relevant to the scattering at next-to-leading order. Cause a logarithmic divergence for $\sigma_{L,T}^{\text{NLO}}$ as $z_2 \rightarrow 0$.

Calculated by G. Beuf, Phys. Rev. D **96**, 074033 (2017).

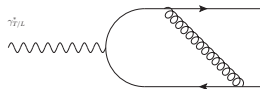
Next-to-Leading Order: One-gluon-loop diagrams



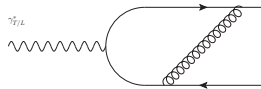
(a)



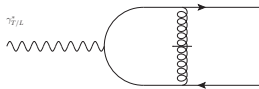
(b)



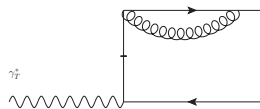
(c)



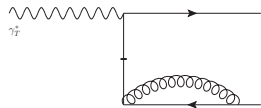
(d)



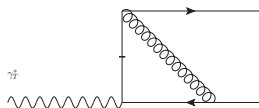
(e)



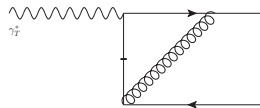
(f)



(g)



(h)



(i)

Loop diagrams relevant at next-to-leading order.

Calculated by G. Beuf, Phys. Rev. D **94**, 054016 (2016).

Full NLO DIS cross section in the dipole picture

Next-to-Leading Order γ^*p cross section can be partitioned as

$$\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{IC}} + \sigma_{L,T}^{qg} + \sigma_{L,T}^{\text{dip}},$$

where the NLO contributions are¹:

$$\begin{aligned}\sigma_{L,T}^{qg} &= 8N_c\alpha_{em}\frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 dz_1 \int^{1-z_1} \frac{dz_2}{z_2} \\ &\quad \times \int_{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2} \mathcal{K}_{L,T}^{\text{NLO}}(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)), \\ \sigma_{L,T}^{\text{dip}} &= 4N_c\alpha_{em}\frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 dz_1 \\ &\quad \times \int_{\mathbf{x}_0, \mathbf{x}_1} \mathcal{K}_{L,T}^{\text{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1, X^{\text{dip}}) \left[\frac{1}{2} \ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2} \right].\end{aligned}$$

$z_2 =$ gluon momentum fraction.

¹G. Beuf, Phys. Rev. D **96**, 074033 (2017).

Subtraction of the soft gluon divergence

- Find a natural lower limit for gluon fractional momentum z_2 .
- BK evolution of the target can be considered using two distinct variables: probe longitudinal momentum and target momentum fraction $X = \Delta k^- / P^-$.
- With k^- ordering one can recover a lower limit:
 $z_2 \gtrsim (x_{Bj}/x_0)/(k_T^2/Q^2)$, where k_T is gluon transverse momentum.
- For single inclusive pA dominant $k_T \sim pA$ hard scale.²

We assume the same for DIS, i.e. $k_T \sim Q$, and set $X(z_2) \equiv x_{Bj}/z_2$, kinematical limit to $X(z_2) < x_0$, i.e. $z_2 > x_{Bj}/x_0$.

²E. Iancu et al. , JHEP **12** (2016) 041; B. Ducloué et al., Phys. Rev. D **95** 114007 (2017).

First subtraction scheme: the 'unsubtracted' form

With the above assumptions for the subtraction scheme we get

$$\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{IC}} + \sigma_{L,T}^{qg,\text{unsub.}} + \sigma_{L,T}^{\text{dip}},$$

where $\sigma_{L,T}^{\text{IC}}$ is the LO result with non-evolved $S_{01}(X = x_0)$ and

$$\begin{aligned} \sigma_{L,T}^{qg,\text{unsub.}} &= 8N_c \alpha_{em} \frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 dz_1 \int_{x_{Bj}/x_0}^{1-z_1} \frac{dz_2}{z_2} \\ &\times \int_{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2} \mathcal{K}_{L,T}^{\text{NLO}}(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)), \end{aligned}$$

$$X(z_2) = x_{Bj}/z_2,$$

$z_2 =$ gluon momentum fraction.

Second subtraction scheme: the 'subtracted' form

$$\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{IC}} + \sigma_{L,T}^{qg,\text{unsub.}} + \sigma_{L,T}^{\text{dip}}$$

can be rewritten in an equivalent form:

$$\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{LO}} + \sigma_{L,T}^{qg,\text{sub.}} + \sigma_{L,T}^{\text{dip}},$$

where $\sigma_{L,T}^{\text{LO}}$ is the LO result with evolved $S_{01}(X = x_{Bj})$ and

$$\begin{aligned} \sigma_{L,T}^{qg,\text{sub.}} &= 8N_c \alpha_{em} \frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 dz_1 \int_{x_{Bj}/x_0}^1 \frac{dz_2}{z_2} \\ &\times \int_{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2} \left[\theta(1 - z_1 - z_2) \mathcal{K}_{L,T}^{\text{NLO}}(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)) \right. \\ &\quad \left. - \mathcal{K}_{L,T}^{\text{NLO}}(z_1, 0, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, X(z_2)) \right]. \end{aligned}$$

Few details about the published numerical results:

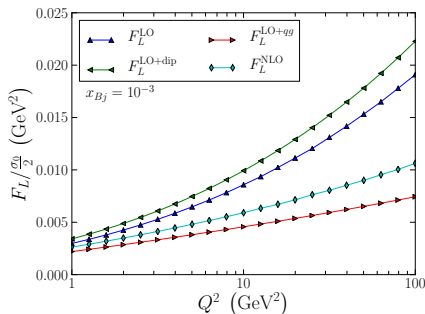
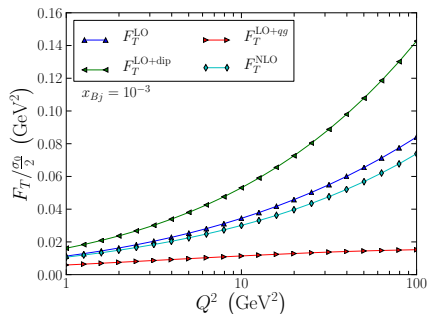
- Impact parameter dependence neglected \rightarrow plotting $F_{L,T}/(\sigma_0/2)$.
- Leading order BK, with McLerran-Venugopalan initial condition.

Both fixed ($\alpha_s = 0.2$) and parent dipole running coupling were used:

$$\alpha_s(\mathbf{x}_{01}^2) = \frac{12\pi}{(11N_c - 2n_f) \ln \left(\frac{4e^{-2\gamma_e}}{\mathbf{x}_{01}^2 \Lambda_{\text{QCD}}^2} \right)},$$

with $\Lambda_{\text{QCD}} = 0.241$ GeV.

Results: 'unsubtracted' scheme, fixed coupling



LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_s = 0.2$.

- Full NLO corrections are moderate.
- In this scheme there are large cancellations between the NLO contributions.

A possible approximative scheme: ' x_{Bj} -subtracted'

As an approximation set $X(z_2) \equiv x_{Bj}$ and $x_{Bj}/x_0 \rightarrow 0$.

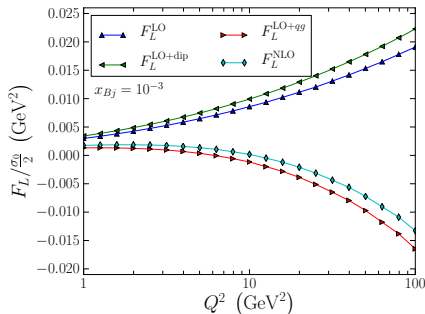
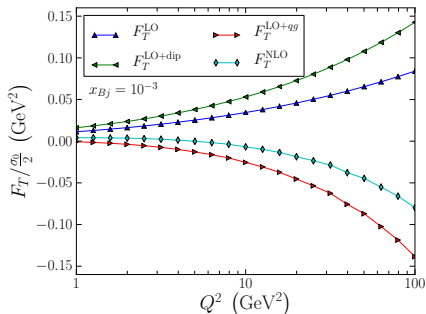
Motivation: S_{01} is evaluated only at x_{Bj} .

$$\begin{aligned}\sigma_{L,T}^{\text{NLO},x_{Bj}\text{-sub.}} &= \sigma_{L,T}^{\text{LO}} + \sigma_{L,T}^{qq,\text{sub.}*} + \sigma_{L,T}^{\text{dip}}, \\ \sigma_{L,T}^{qq,\text{sub.}*} &= 8N_c \alpha_{em} \frac{\alpha_s C_F}{\pi} \sum_f e_f^2 \int_0^1 dz_1 \int_0^1 \frac{dz_2}{z_2} \\ &\quad \times \int_{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2} \left[\theta(1-z_1-z_2) \mathcal{K}_{L,T}^{\text{NLO}}(z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, x_{Bj}) \right. \\ &\quad \left. - \mathcal{K}_{L,T}^{\text{NLO}}(z_1, 0, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, x_{Bj}) \right].\end{aligned}$$

Analogous to a subtraction scheme used for single inclusive particle production in pA .³

³G. A. Chirilli, B.-W. Xiao and F. Yuan, Phys. Rev. Lett. **108** (2012) 122301

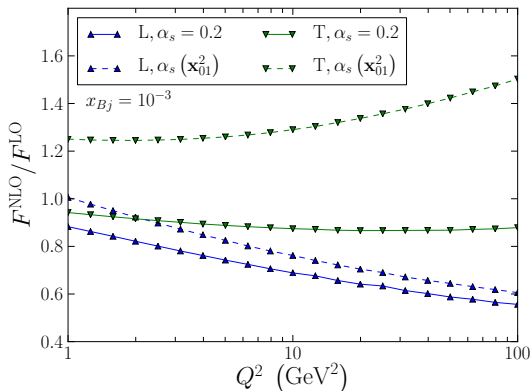
Results: approximative scheme, fixed coupling



LO and NLO contributions to F_T (left) and F_L (right) as a function of Q^2 at $x_{Bj} = 10^{-3}$ with $\alpha_s = 0.2$ and using the x_{Bj} -subtraction procedure.

- The approximations break the scheme and lead to negative NLO structure functions.
- Similar to the negativity issue seen with single inclusive pA.

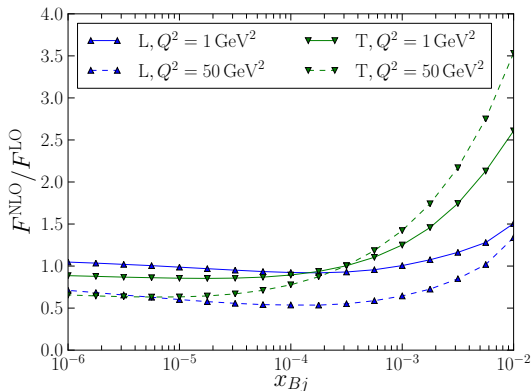
Results: 'unsubtracted' scheme, fixed v. running coupling



NLO/LO ratio for F_L and F_T as a function of Q^2 at $x_{Bj} = 10^{-3}$ with fixed (solid) and running (dashed) coupling.

- NLO corrections sensitive to subtraction scheme details e.g. running coupling due to the large cancellations.

Results: 'unsubtracted' scheme, running coupling



NLO/LO ratio for F_L and F_T as a function of x_{Bj} at $Q^2 = 1$ GeV 2 (solid) and $Q^2 = 50$ GeV 2 (dashed) with running coupling.

- By scheme construction $\sigma^{qg} \xrightarrow{x_{Bj} \rightarrow x_0} 0 \implies$ large transient effect at large x_{Bj} especially for F_T .

What was done:

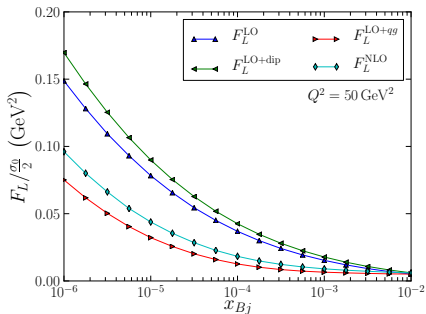
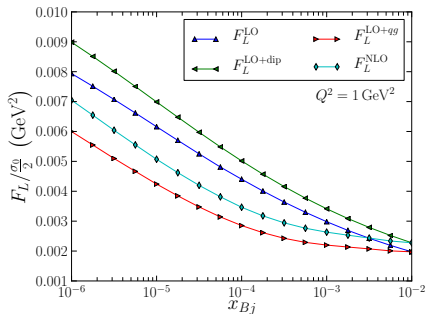
- Successfully evaluated NLO corrections to DIS structure functions for the first time.
- Choice of subtraction scheme can have strong effects.

To be done:

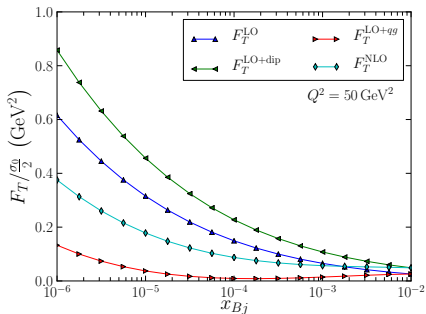
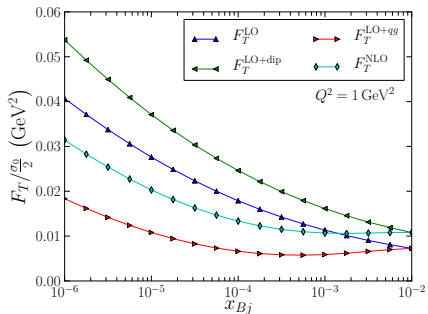
- Evaluation of NLO structure functions using both NLO impact factors and resummed/NLO BK equation.
- Careful treatment of the transient effect; choice of subtraction scheme.
- Fit to HERA data.
- Extension of impact factor results for massive quarks.

Thank you!

Backup slides



LO and NLO contributions to F_L as a function of x_{Bj} at $Q^2 = 1 \text{ GeV}^2$ (left) and $Q^2 = 50 \text{ GeV}^2$ (right) with $\alpha_s = 0.2$.



LO and NLO contributions to F_T as a function of x_{Bj} at $Q^2 = 1 \text{ GeV}^2$ (left) and $Q^2 = 50 \text{ GeV}^2$ (right) with $\alpha_s = 0.2$.