Linearly polarized gluons and axial charge fluctuations in the Glasma

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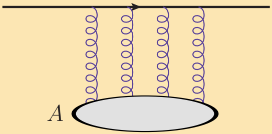
Outline

- From Wilson lines to glasma fields
- Parity violation, Chern-Simons
- Correlations and fluctuations of glasma fields
- Simple algorithm for initializing anomalous hydrodynamics

This talk:
"Linearly polarized gluons and axial charge fluctuations in the Glasma,"
T. L. and S. Schlichting, Phys. Rev. D **97** (2018) no.3, 034034 `[arXiv:1708.08625 [hep-ph]]`

- Generalizes B. Müller and A. Schäfer, Phys. Rev. D **85** (2012) 114030
- Rely on early time $\tau = 0$ expansion similarly to e.g.
  G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, Phys. Rev. C **92** (2015) no.6, 064912
- Gluon polarization in CGC:
  MV: A. Metz and J. Zhou, Phys. Rev. D **84** (2011) 051503
  JIMWLK: C. Marquet, E. Petreska and C. Roiesnel, JHEP **1610** (2016) 065

Eikonal scattering off target of glue



How to measure small-x glue?

- Dilute probe through target color field
- At high energy interaction is eikonal

Eikonal scattering amplitude: Wilson line V

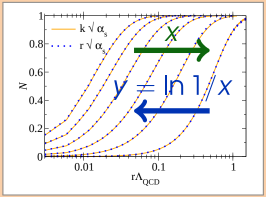
$$V = \mathbb{P}\exp\left\{ -ig\int^{x^+} \mathrm{d}y^+ A^-(y^+, x^-, \mathbf{x}) \right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_\mathrm{c})$$

- Amplitude for color dipole

$$\mathcal{N}\,(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

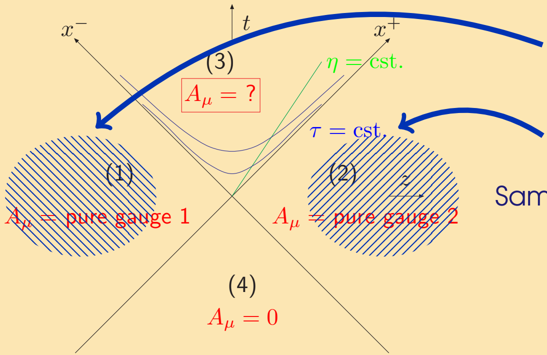
from color transparency to saturation

- $1/Q_\mathrm{s}$ is Wilson line **correlation length**



Classical Yang-Mills initial state

Classical Yang-Mills



Change to LC gauge:

$$\alpha^i = \frac{i}{g} V_{(1)}(\mathbf{x}) \partial_i V^\dagger_{(1)}(\mathbf{x})$$

$$\beta^i = \frac{i}{g} V_{(2)}(\mathbf{x}) \partial_i V^\dagger_{(2)}(\mathbf{x})$$

Same Wilson lines $V_{(1,2)}(\mathbf{x})$

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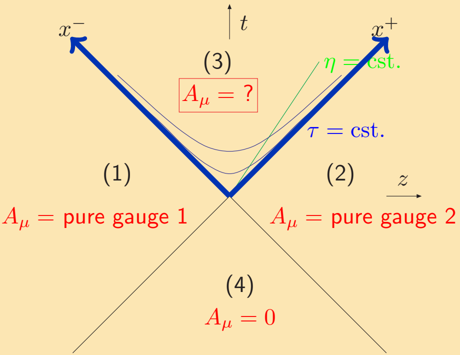

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Same Wilson lines $V_{(1,2)}(\mathbf{x})$

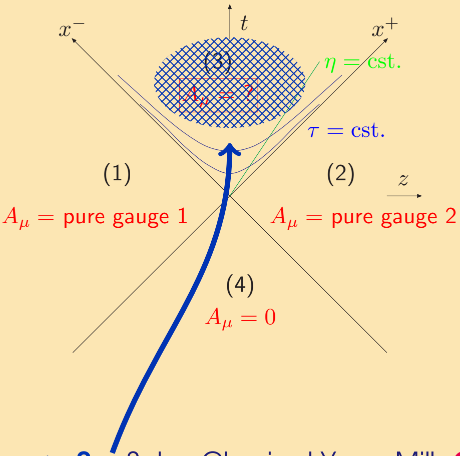
At $\tau = 0$:

$$A^i\Big|_{\tau=0} = \alpha^i + \beta^i$$

$$A^\eta\big|_{\tau=0} = \frac{ig}{2}[\alpha^i, \beta^i]$$

Classical Yang-Mills initial state

Classical **Y**ang-**M**ills



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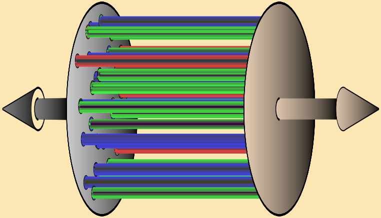
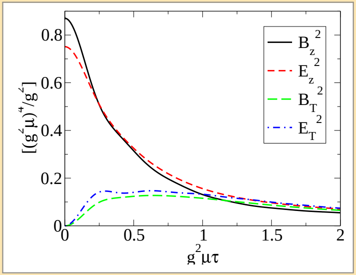
$\boldsymbol{\tau > 0}$ Solve Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over $V_{(1,2)}(\mathbf{x})$.

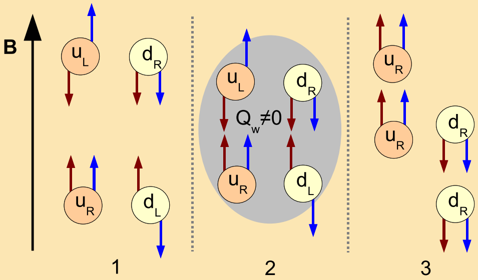
Initial glasma fields





- Initial condition is longitudinal E and B field,
  at $\tau \sim 1/Q_s$ evolves to $E_z^2 \sim B_z^2 \sim 2E_x^2 \sim 2B_x^2 \sim 2E_y^2 \sim 2B_y^2$
- Depend on transverse coordinate
  with correlation length $1/Q_s \implies$ gluon correlations
- Configuration naturally has Chern-Simons charge
  $N_{CS} \sim \varepsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} \sim E \cdot B$

Parity violation



- External (EM) magnetic field
- Parity violating color field i.e. Chern-Simons charge!
- Fermion pair creation

$\Longrightarrow$ Produced particles: Electric dipole || **B** Seen in experiment???

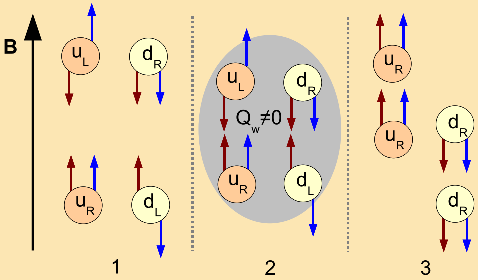
Sources of N_{CS}:

- $\tau \lesssim 1/Q_s$ boost-invariant 2-d fields

  Numerically Kharzeev, Krasnitz, Venugopalan 2001

- $\tau \gg 1/Q_s$ 3-d fields $\Longrightarrow$ sphalerons Mace et al 2016

Parity violation



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Initial energy density and Chern-Simons charge

Axial charge per unit area

(at $\tau \ll 1/Q_s$, neglecting time dependence of field)

$$\left.\frac{dN_5}{d^2\mathbf{x}\,d\eta}\right|_{\tau \lesssim 1/Q_s} \approx \frac{\tau^2}{2}\frac{g^2 N_f}{2\pi^2}\dot{\nu}(\mathbf{x}, \tau = 0^+)$$

$$\dot{\nu}(\mathbf{x}, \tau \approx 0) = \mathrm{Tr}\left[E^\eta(\tau = 0^+, \mathbf{x}) B^\eta(\tau = 0^+, \mathbf{x})\right] + \mathcal{O}(\tau^2)$$

$$\begin{aligned}
\langle \varepsilon(\mathbf{x}) \rangle &= (-ig)^2 \left(\delta^{ij}\delta^{kl} + \varepsilon^{ij}\varepsilon^{kl}\right) \left\langle \mathrm{Tr}\left([\alpha^i_{\mathbf{x}}, \beta^j_{\mathbf{x}}][\alpha^k_{\mathbf{x}}, \beta^l_{\mathbf{x}}]\right)\right\rangle \\
\langle \dot{\nu}(\mathbf{x}) \rangle &= (-ig)^2 \qquad \delta^{ij}\varepsilon^{kl} \qquad \left\langle \mathrm{Tr}\left([\alpha^i_{\mathbf{x}}, \beta^j_{\mathbf{x}}][\alpha^k_{\mathbf{x}}, \beta^l_{\mathbf{x}}]\right)\right\rangle
\end{aligned}$$

Nuclei are independent $\Longrightarrow$ factorize out 2-point functions

$$\left\langle \mathrm{Tr}\left([\alpha^i_{\mathbf{x}}, \beta^j_{\mathbf{x}}][\alpha^k_{\mathbf{x}}, \beta^l_{\mathbf{x}}]\right)\right\rangle = \frac{1}{2} i f^{abc} i f^{a'b'c} \left\langle \alpha^{i,a}_{\mathbf{x}} \alpha^{k,a'}_{\mathbf{x}} \right\rangle \left\langle \beta^{j,b}_{\mathbf{x}} \beta^{l,b'}_{\mathbf{x}} \right\rangle ,$$

Polarized and unpolarized gluon distributions

Need Weizsäcker-Williams gluon distributions in nuclei:

$$W^{ik}(\mathbf{x},\mathbf{y}) = \frac{1}{{N_c}^2 - 1}\left\langle \alpha^{i,a}_{\mathbf{x}} \alpha^{k,a}_{\mathbf{y}} \right\rangle$$

Decompose into unpolarized and linearly polarized:

$$\begin{aligned} W^{ij}(\mathbf{k}) &= \frac{1}{2}\delta^{ij} G(k_T) - \frac{1}{2}\left(\delta^{ij} - 2\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}\right) h(k_T) \\ W^{ij}(\mathbf{r}) &= \frac{1}{2}\delta^{ij} G(r) + \frac{1}{2}\left(\delta^{ij} - 2\frac{\mathbf{r}^i\mathbf{r}^j}{r^2}\right) h(r) \end{aligned}$$

$$G(r) = \int \frac{\mathrm{d}k_T k_T}{2\pi} J_0(k_T r) G(k_T) \qquad h(r) = \int \frac{\mathrm{d}k_T k_T}{2\pi} J_2(k_T r) h(k_T)$$

(Note Bessel index! Even for $G(k) = h(k)$ have $h(r=0) < g(r=0)$; will need this)

Result: $\langle \varepsilon(\mathbf{x}) \rangle = \frac{g^2\, N_c({N_c}^2 - 1)}{2} G_{(1)}(r=0) G_{(2)}(r=0)$, $\langle \dot{\nu}(\mathbf{x}) \rangle = 0$

Distributions in Gaussian approx

In Gaussian approximation (works very well!) can calculate

$$W^{ik}(\mathbf{x},\mathbf{y}) = \frac{1}{g^2 N_c}\left(\frac{\partial^i_{\mathbf{x}}\partial^k_{\mathbf{y}}\ln(D_{\mathbf{xy}})}{\ln(D_{\mathbf{xy}})}\right)\left(\left(D_{\mathbf{xy}}\right)^{C_A/C_F} - 1\right)$$

in terms of dipole $D_{\mathbf{xy}} = \frac{1}{N_c}\left\langle \mathrm{Tr}\left(V_{\mathbf{x}} V^\dagger_{\mathbf{y}}\right)\right\rangle$ (BK eq; DIS cross sections)

$$G(r) = \frac{1}{g^2 N_c}\frac{1-\left(D(r)\right)^{C_A/C_F}}{\ln\left(D(r)\right)}\left(\partial_r^2 + \frac{1}{r}\partial_r\right)\ln\left(D(r)\right)$$

$$h(r) = \frac{1}{g^2 N_c}\frac{1-\left(D(r)\right)^{C_A/C_F}}{\ln\left(D(r)\right)}\left(\partial_r^2 - \frac{1}{r}\partial_r\right)\ln\left(D(r)\right)$$

- GBW parametrization $G(r) = \frac{1-e^{-(C_A/C_F)Q_s^2 r^2/4}}{g^2 N_c r^2/4}$ & $h(r) = 0$
- Anomalous dimension $\ln D(r) \sim -r^{2\gamma}$ gives $h(r) \xrightarrow{r\to 0} \frac{1-\gamma}{\gamma} G(r)$

Glasma graph approximation

For correlations need

$$\langle \alpha_{\mathbf{x}}^{i,a} \alpha_{\mathbf{x}}^{k,c} \alpha_{\mathbf{y}}^{i',a'} \alpha_{\mathbf{y}}^{k',c'} \rangle$$

- In general calculable but difficult
- Here use "glasma graph" approx and assume Gaussian

$$\left\langle \alpha_{\mathbf{x}}^{i,a} \alpha_{\mathbf{x}}^{k,c} \alpha_{\mathbf{y}}^{i',a'} \alpha_{\mathbf{y}}^{k',c'} \right\rangle = \overbrace{\left\langle \alpha_{\mathbf{x}}^{i,a} \alpha_{\mathbf{x}}^{k,c} \right\rangle \left\langle \alpha_{\mathbf{y}}^{i',a'} \alpha_{\mathbf{y}}^{k',c'} \right\rangle}^{\text{disconnected}} \qquad [r = |\mathbf{x} - \mathbf{y}|]$$
$$\underbrace{+ \left\langle \alpha_{\mathbf{x}}^{i,a} \alpha_{\mathbf{y}}^{i',a'} \right\rangle \left\langle \alpha_{\mathbf{x}}^{k,c} \alpha_{\mathbf{y}}^{k',c'} \right\rangle + \left\langle \alpha_{\mathbf{x}}^{i,a} \alpha_{\mathbf{y}}^{k',c'} \right\rangle \left\langle \alpha_{\mathbf{x}}^{k,c} \alpha_{\mathbf{y}}^{i',a'} \right\rangle}_{\text{connected}}$$

- Subtle, actually approx not obvious beyond "pp" limit
  (also a bit of new insight on ridge correlation)
- In principle can do better; complicated but in progress

Result

$$\langle\varepsilon(\mathbf{x})\varepsilon(\mathbf{y})\rangle - \langle\varepsilon(\mathbf{x})\rangle\langle\varepsilon(\mathbf{y})\rangle = \frac{g^4 N_c{}^2(N_c{}^2-1)}{4}\Bigg[\qquad [r = |\mathbf{x}-\mathbf{y}|]$$

$$(G_{(1)}(0))^2 \left[\left(G_{(2)}(r)\right)^2 + \left(h_{(2)}(r)\right)^2\right] + [(1) \leftrightarrow (2)]$$

$$+ \left[\left(G_{(1)}(r)\right)^2\left(G_{(2)}(r)\right)^2 + \left(h_{(1)}(r)\right)^2\left(h_{(2)}(r)\right)^2\right]$$

$$+ \frac{1}{2}\left[\left(G_{(1)}(r)\right)^2\left(h_{(2)}(r)\right)^2 + \left(h_{(1)}(r)\right)^2\left(G_{(2)}(r)\right)^2\right]\Bigg]$$

$\Longrightarrow$ Depends on sum of unpolarized and polarized

$$\langle\dot\nu(\mathbf{x})\dot\nu(\mathbf{y})\rangle = \frac{3g^4 N_c{}^2(N_c{}^2-1)}{32}\Bigg[$$

$$\left(G_{(1)}(r)\right)^2\left(G_{(2)}(r)\right)^2 - \left(h_{(1)}(r)\right)^2\left(h_{(2)}(r)\right)^2\Bigg]$$

$\Longrightarrow$ Depends on **difference** $G^4 - h^4$.

Simple model for Glasma fluctuations

1. Use your favorite parametrization to calculate $\langle \varepsilon(\mathbf{x}, \tau_0) \rangle$
2. Estimate saturation scale Q_s from $\langle \varepsilon \rangle \approx \frac{1}{g^2} \frac{{N_c}^2}{C_F} Q_s^4$
3. Construct correlation functions; e.g. for GBW

$$\frac{\langle \varepsilon(\mathbf{x})\varepsilon(\mathbf{y}) \rangle}{\langle \varepsilon(\mathbf{x}) \rangle \langle \varepsilon(\mathbf{y}) \rangle} - 1 = \frac{3}{{N_c}^2 - 1} \Bigg[\frac{1}{3} \left(\frac{1 - e^{-\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2}}{\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2} \right)^4 + \frac{2}{3} \left(\frac{1 - e^{-\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2}}{\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2} \right)^2 \Bigg]$$

$$\frac{\langle \dot{\nu}(\mathbf{x})\dot{\nu}(\mathbf{y}) \rangle}{\langle \varepsilon(\mathbf{x}) \rangle \langle \varepsilon(\mathbf{y}) \rangle} = \frac{3}{8({N_c}^2 - 1)} \left(\frac{1 - e^{-\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2}}{\frac{N_c}{4C_F} Q_s^2 |\mathbf{x}-\mathbf{y}|^2} \right)^4$$

4. Fluctuating $\varepsilon(\mathbf{x}, \tau_0)$ and $\dot{\nu}(\mathbf{x}, \tau_0)$ with Cholesky algorithm ...

Practical algorithm: Cholesky decomposition

How to reproduce given correlation function $C(\mathbf{x},\mathbf{y})$ in

$$\langle\dot{\nu}(\mathbf{x})\dot{\nu}(\mathbf{y})\rangle = C(\mathbf{x},\mathbf{y})\,\langle\epsilon(\mathbf{x})\rangle\,\langle\epsilon(\mathbf{y})\rangle$$

With random configurations $\dot{\nu}(\mathbf{x})$?

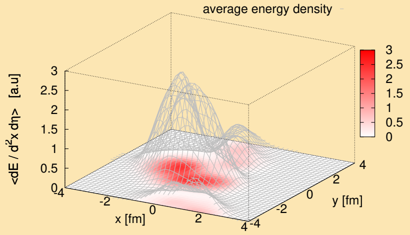
- Perform Cholesky decomposition for correlator

$$C(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{z}} L(\mathbf{x},\mathbf{z})L^T(\mathbf{z},\mathbf{y})\ .$$

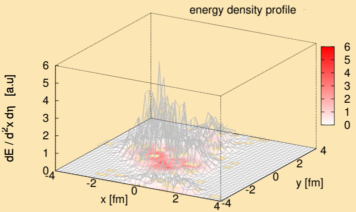
- Random noise: $\langle\xi(\mathbf{z})\rangle = 0 \quad \langle\xi(\mathbf{z})\xi(\mathbf{z}')\rangle = \delta_{\mathbf{z},\mathbf{z}'}$
- Take

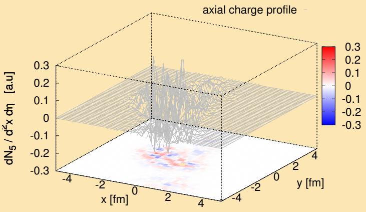
$$\dot{\nu}(\mathbf{x}) = \langle\epsilon(\mathbf{x})\rangle \sum_{\mathbf{z}} \xi(\mathbf{z})\ L(\mathbf{x},\mathbf{z})$$

Demonstration



Energy density profile from T_RENTO generator
$\Longrightarrow$ extract Q_s
$\Longrightarrow$ Q_s-scale fluctuations
(Poor man's IPglasma)





Energy density

Axial charge

Conclusions

- Analytic solution for glasma field at $\tau \lesssim 1/Q_s$
  $\Longrightarrow$ axial charge and energy density fluctuations
- N_{CS} related to gluon linear polarization:
  - ε fluctuations: sum of unpolarized and polarized distributions
  - N_{CS} fluctuations: difference
- In CGC polarized and unpolarized are related,
  but separately measurable at EIC
- Simple procedure for initial conditions of (anomalous) hydro
  Of course full CYM is a more complete way to do this
- Future work:
  - Full nonlinear gaussian instead of "glasma graph" approx.
  - Smaller systems?

Backup: glasma graph and ridge in pA

Want to use glasma graph approximation to get ridge in "pA"?
For gluon spectra transform to Coulomb gauge,
need 4-pt adjoint correlator on "A" side:

$$\left\langle U^{ab}_{\mathbf{p}} U^{\dagger ba'}_{\overline{\mathbf{p}}} U^{cd}_{\mathbf{q}} U^{\dagger dc'}_{\overline{\mathbf{q}}} \right\rangle$$

Would seem like natural "glasma graph":
decompose this into pairwise 2-pt functions of U's:

$$\langle UUUU \rangle \approx \langle UU \rangle \langle UU \rangle + \langle UU \rangle \langle UU \rangle + \langle UU \rangle \langle UU \rangle$$

But this is not actually the usual "glasma graph" approximation, which is constructed to give the right answer in the limits when momenta are pairwise close $\mathbf{p} \approx \overline{\mathbf{p}}$&$\mathbf{q} \approx \overline{\mathbf{q}}$ etc. E.g. would not give right N_c counting!