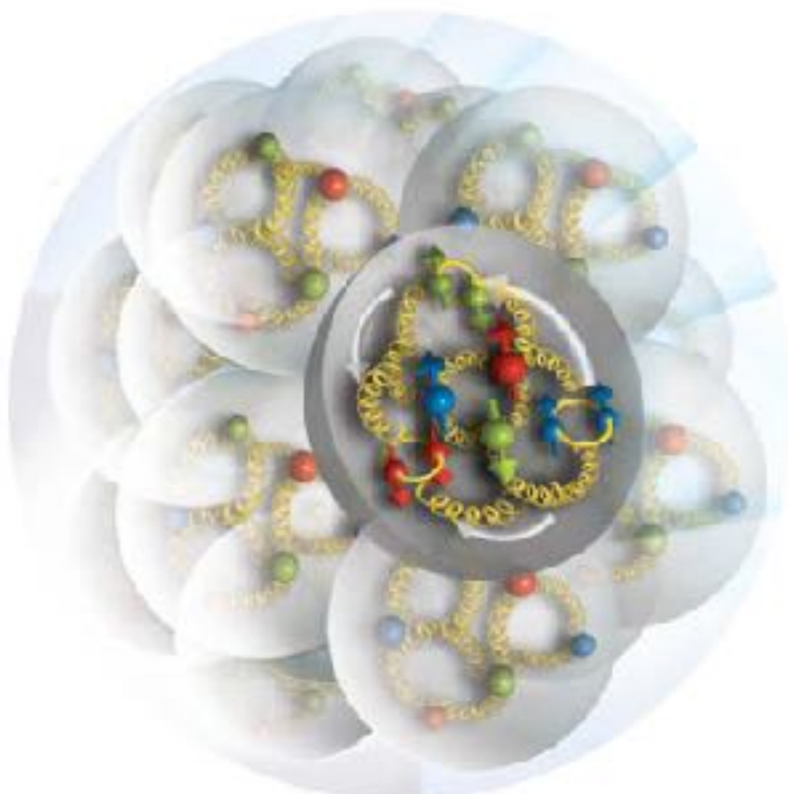
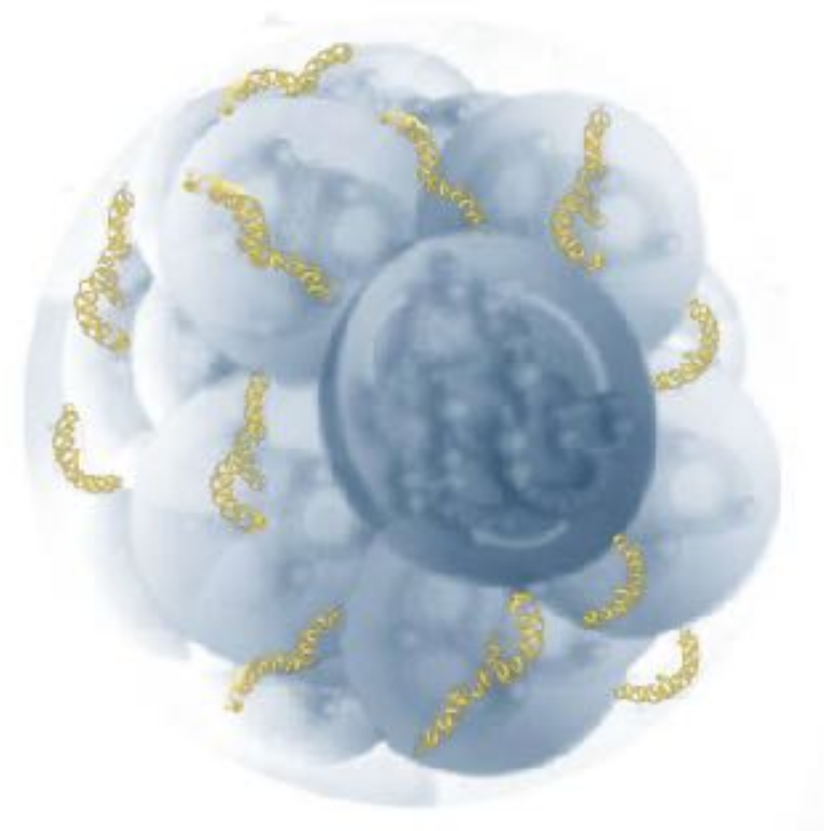
# Gluon structure from lattice QCD





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William and Mary
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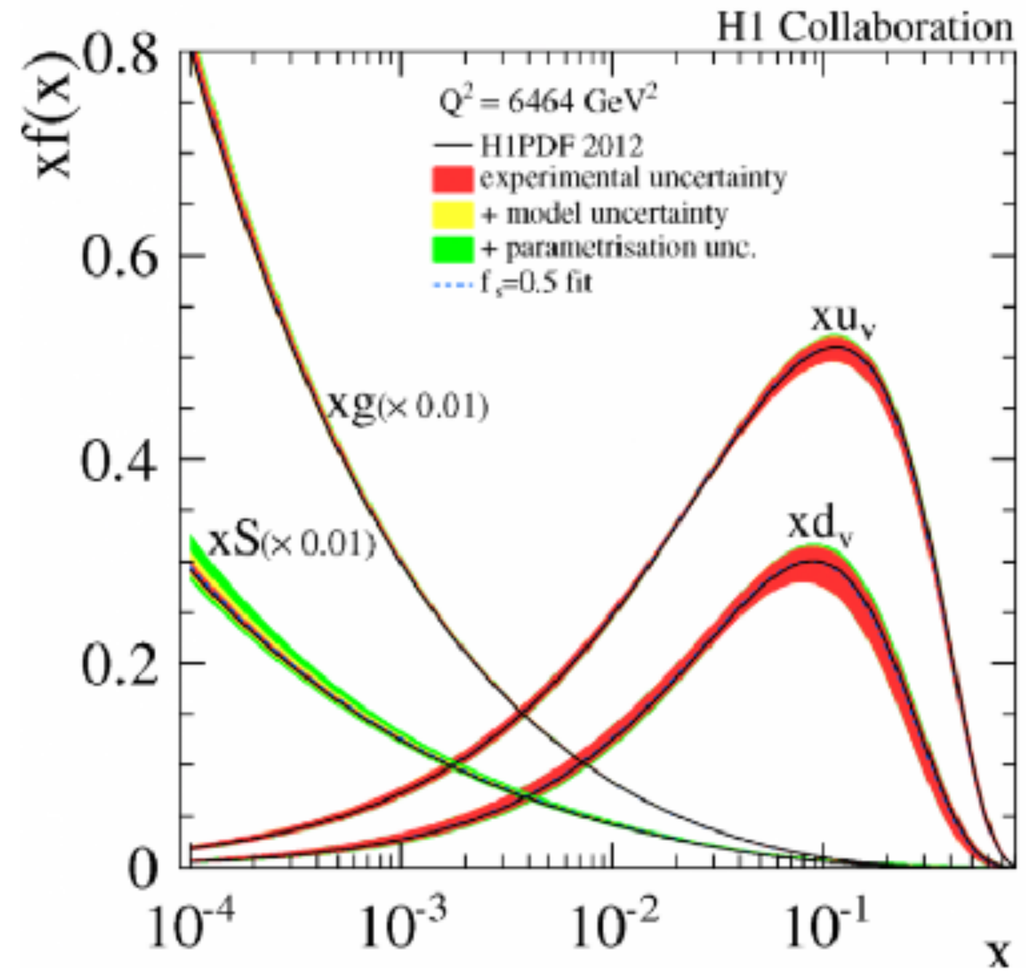
# Gluon structure

Gluons offer a new window on nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton




Longitudinal momentum fraction carried by parton

# Gluon Structure from LQCD

How much do gluons contribute to the proton's

- Momentum
- Mass
- Spin

1

What is the 3D gluon distribution of a proton

- PDFs
- TMDs
- GPDs
- 'Gluon radius'

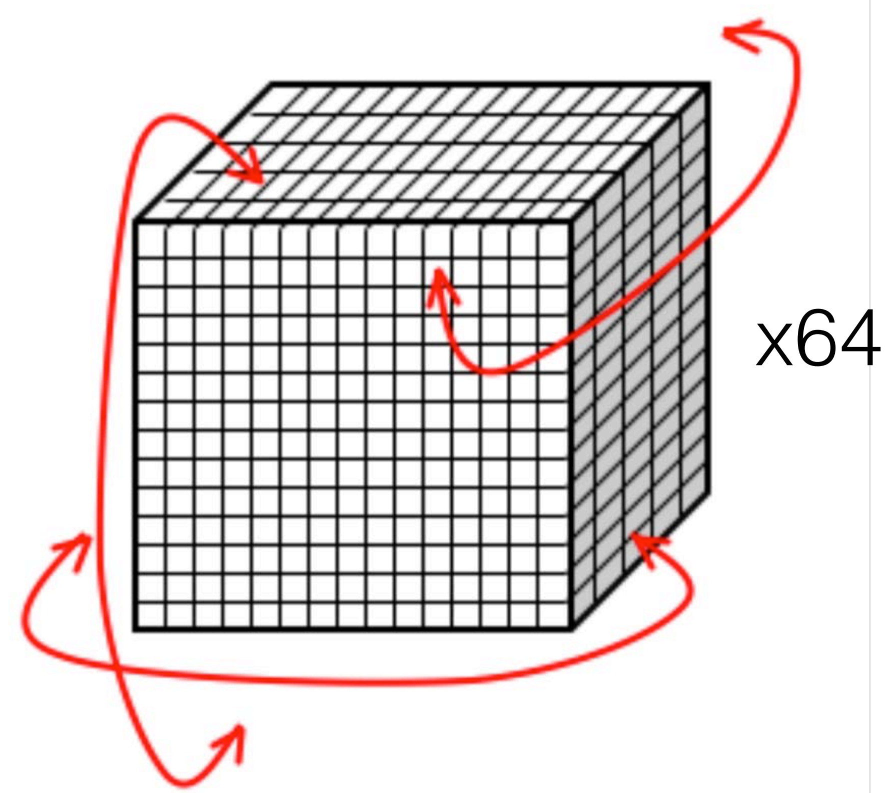
2

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue

# Lattice QCD

- Numerical first-principles approach to non-perturbative QCD
- Euclidean space-time $t \to i\tau$
  - Finite lattice spacing $a$
  - Volume $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Some calculations use larger-than-physical quark masses (cheaper)




Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \Rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_{i}^{N_{\text{conf}}} \mathcal{O}([U^i])$$

with field configurations $U^i$ distributed according to $e^{-S[U]}$

# Doing lattice QCD

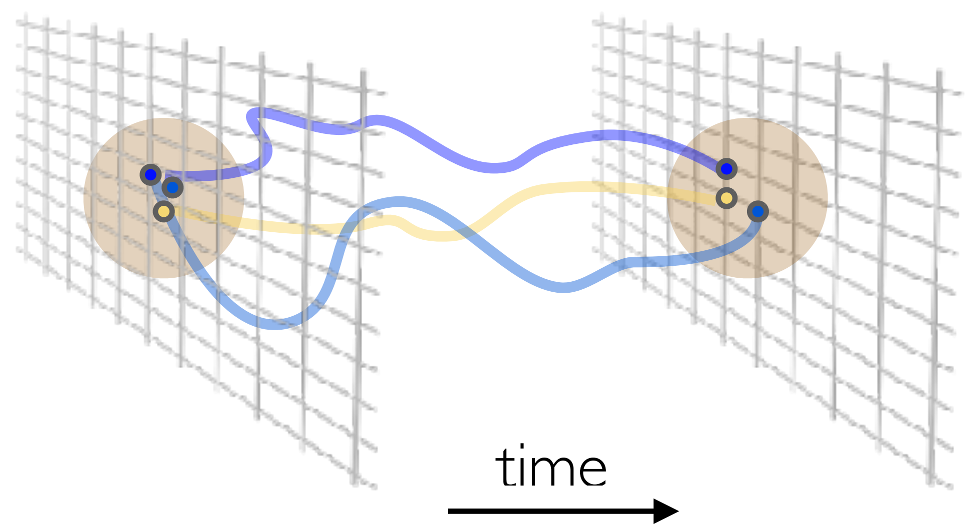
- Correlation decays exponentially with distance in time:

$$C_2(t) = \sum_n Z_n \exp(-E_n t)$$

$n$ ← all eigenstates with q#'s of proton

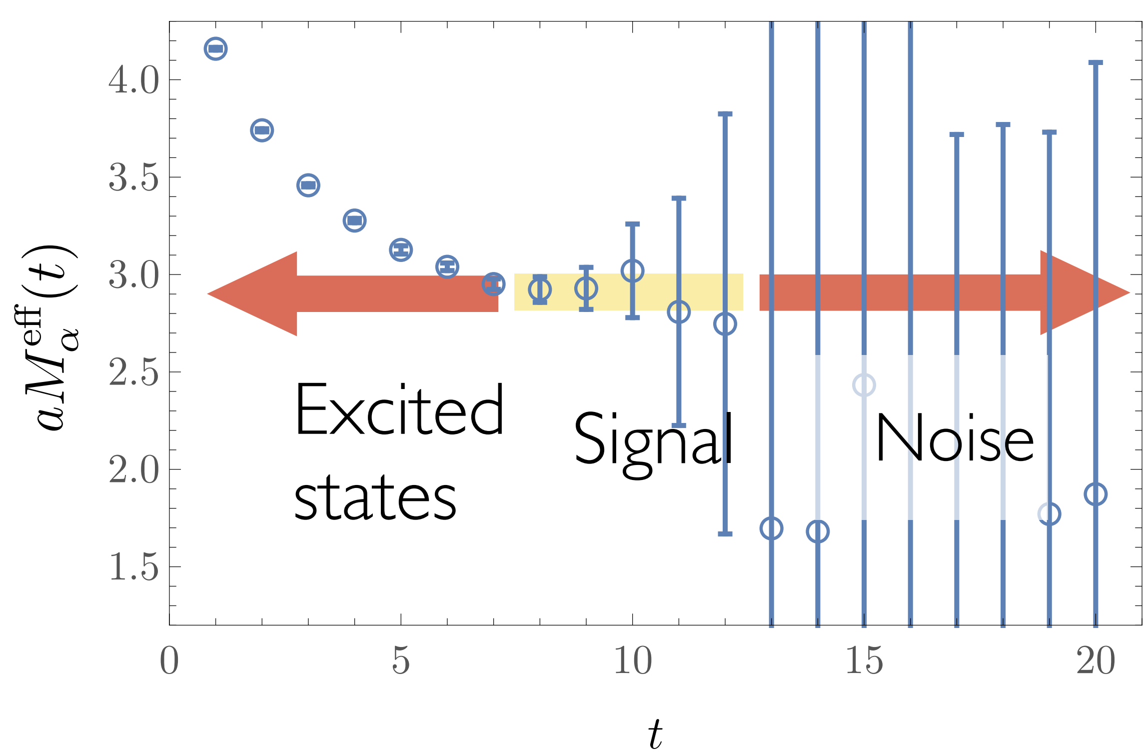
At late times:

$$\rightarrow Z_0 \exp(-E_0 t)$$




- Ground state mass revealed through "effective mass plot"

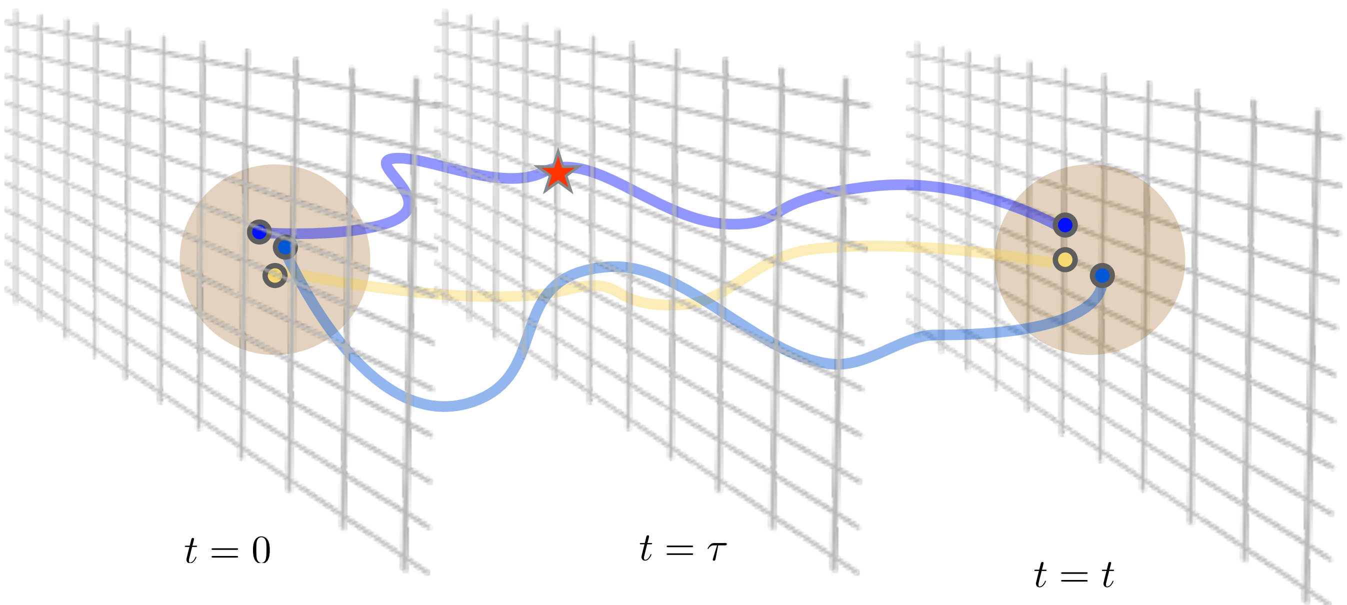
$$M(t) = \ln\left[\frac{C_2(t)}{C_2(t+1)}\right] \xrightarrow{t\rightarrow\infty} E_0$$



# LQCD matrix elements

## How do we calculate matrix elements?

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- Insert operator at intermediate timeslice



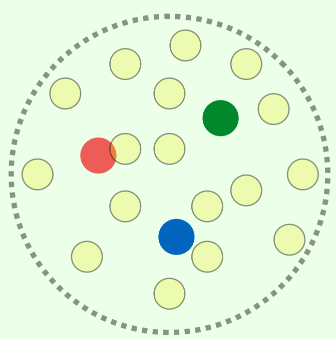

- Remove time-dependence by dividing out with two-point correlators: $\dfrac{C_3(t,\tau,\vec{p},\vec{q})}{C_2(\tau,\vec{p})} \xrightarrow{0 \ll \tau \ll t} \langle N(p')|\mathcal{O}(q)|N(p)\rangle$

# Gluon radii

1

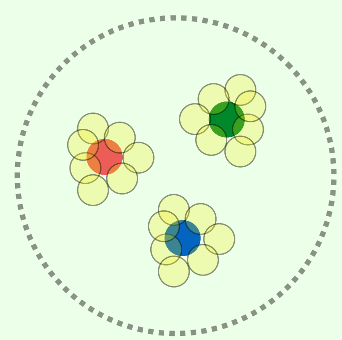
## How does the gluon radius of a proton compare to the quark/charge radius?

**MIT Bag Model**



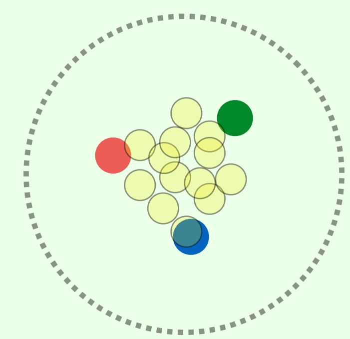
gluon radius > charge radius

**Constituent Quark Model**



gluon radius ~ charge radius

**LQCD with heavy quarks**



gluon radius < charge radius

**Charge radius:**
slope of electric form factor with respect to momentum transfer

↔

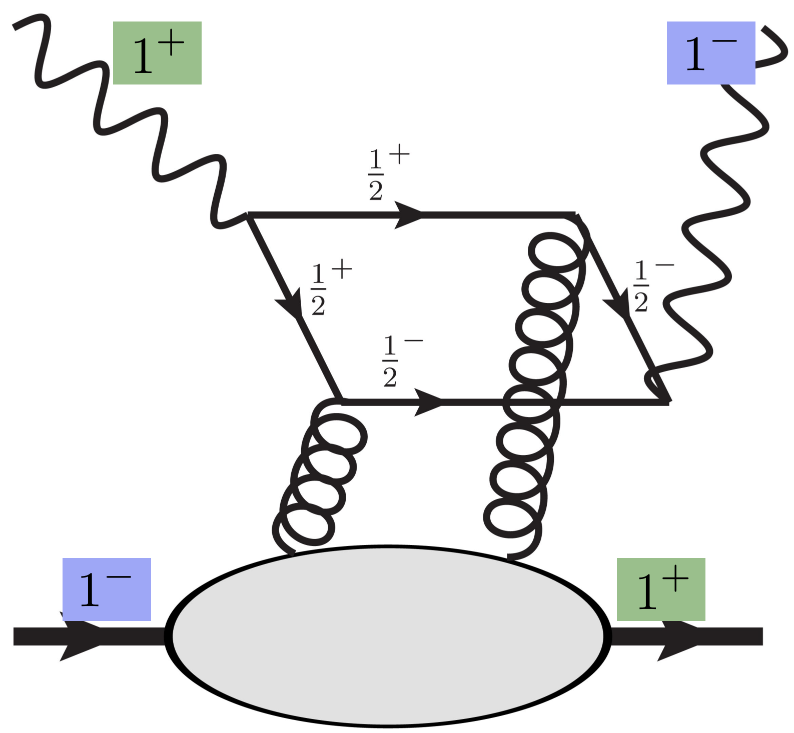
**Gluon radius:**
slope of gluon form factor with respect to momentum transfer

# Gluon transversity

**Leading twist gluon parton distribution Δ(x,Q²)**
[Jaffe & Manohar 1989]

- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- **Double helicity flip**: non-vanishing in forward limit for targets with spin≥ 1
- **Experimentally measurable** in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC

$$\Delta(x, Q^2) = A_{+-,-+}$$



# Gluon transversity

## Double helicity flip distribution $\Delta(x,Q^2)$

- **Hadrons:** Gluonic Transversity (parton model interpretation)

$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2 x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(y,Q^2)\right]$$

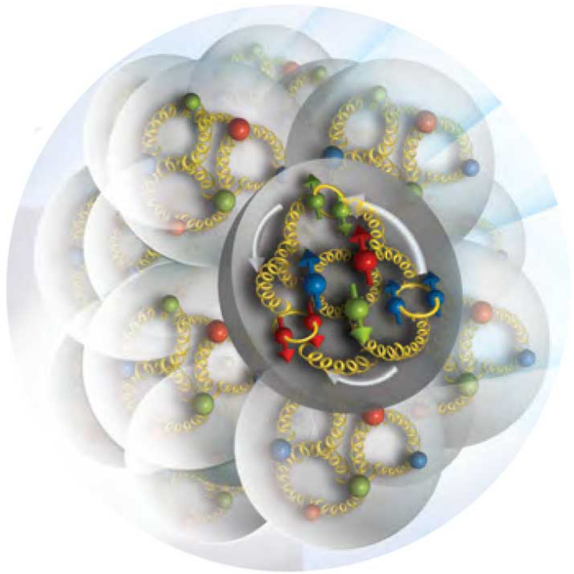
$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction $y$ linearly polarised in $\hat{x},\hat{y}$ direction in a target polarised in $\hat{x}$ direction

- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



# Gluon transversity

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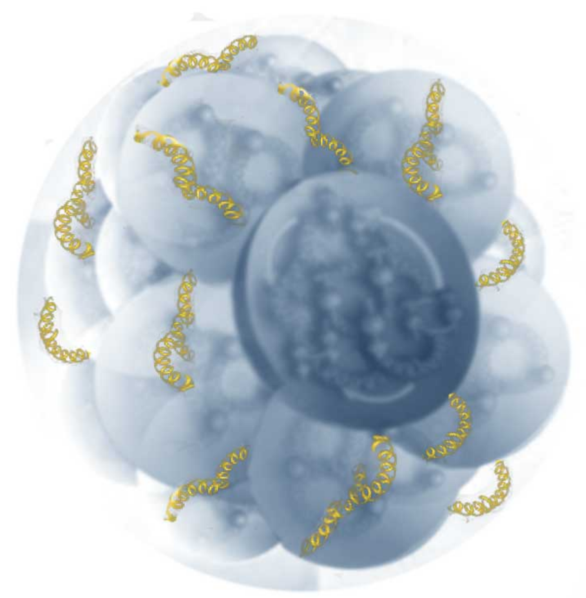


# Gluon transversity

Calculating lightcone distributions is challenging in Euclidean space

## Moments of Δ(x,Q²) are calculable in lattice QCD

Moments of distribution — Reduced Matrix Element

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

## Determined by matrix elements of local gluonic operators

Gluon field strength tensor — Polarisation vector (spin-1)

$$\langle p E' | S \left[ G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right] | p E \rangle$$

Symmetrise in $\mu_1, \ldots, \mu_n$, trace subtract in all free indices

$$= (-2i)^{n-2} S \left[ (p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} A_n(Q^2) \ldots,$$

Reduced Matrix Element

# Gluon radii

## Off-forward matrix elements (momentum transfer through operator)

- Moments of $\Delta(x,Q^2)$ related to many generalised form factors

$$
\begin{aligned}
&\left\langle p'E' \left| S\left[G_{\mu\mu_1} i\overleftrightarrow{D}_{\mu_3} \dots i\overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}\right]\right| pE \right\rangle \\
&= \sum_{\substack{m \text{ even} \\ m=2}}^{n} \Big\{ A^{(n)}_{1,m-2}(\Delta^2)\, S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\quad + A^{(n)}_{2,m-2}(\Delta^2)\, S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\quad + A^{(n)}_{3,m-2}(\Delta^2)\, S\Big[\big((\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) - (\Delta_\mu E'^*_{\mu_1} - E'^*_\mu \Delta_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\big) \\
&\qquad\qquad \times \Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&\quad + A^{(n)}_{4,m-2}(\Delta^2)\, S\left[(E_\mu E'^*_{\mu_1} - E_{\mu_1} E'^*_\mu)(P_\nu \Delta_{\mu_2} - P_{\mu_2}\Delta_\nu)\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\quad + \frac{A^{(n)}_{5,m-2}(\Delta^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2}) \\
&\qquad\qquad + (E'^*\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2} - E_\nu \Delta_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&\quad + \frac{A^{(n)}_{6,m-2}(\Delta^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) \\
&\qquad\qquad - (E'^*\cdot P)\,(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\Big] \\
&\quad + \frac{A^{(n)}_{7,m-2}(\Delta^2)}{M^2} (E'^*\cdot E) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad + \frac{A^{(n)}_{8,m-2}(\Delta^2)}{M^4} (E\cdot P)(E'^*\cdot P) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_m}P_{\mu_{m+1}}\dots P_{\mu_n}\right] \Big\}
\end{aligned}
$$

polarisation vectors

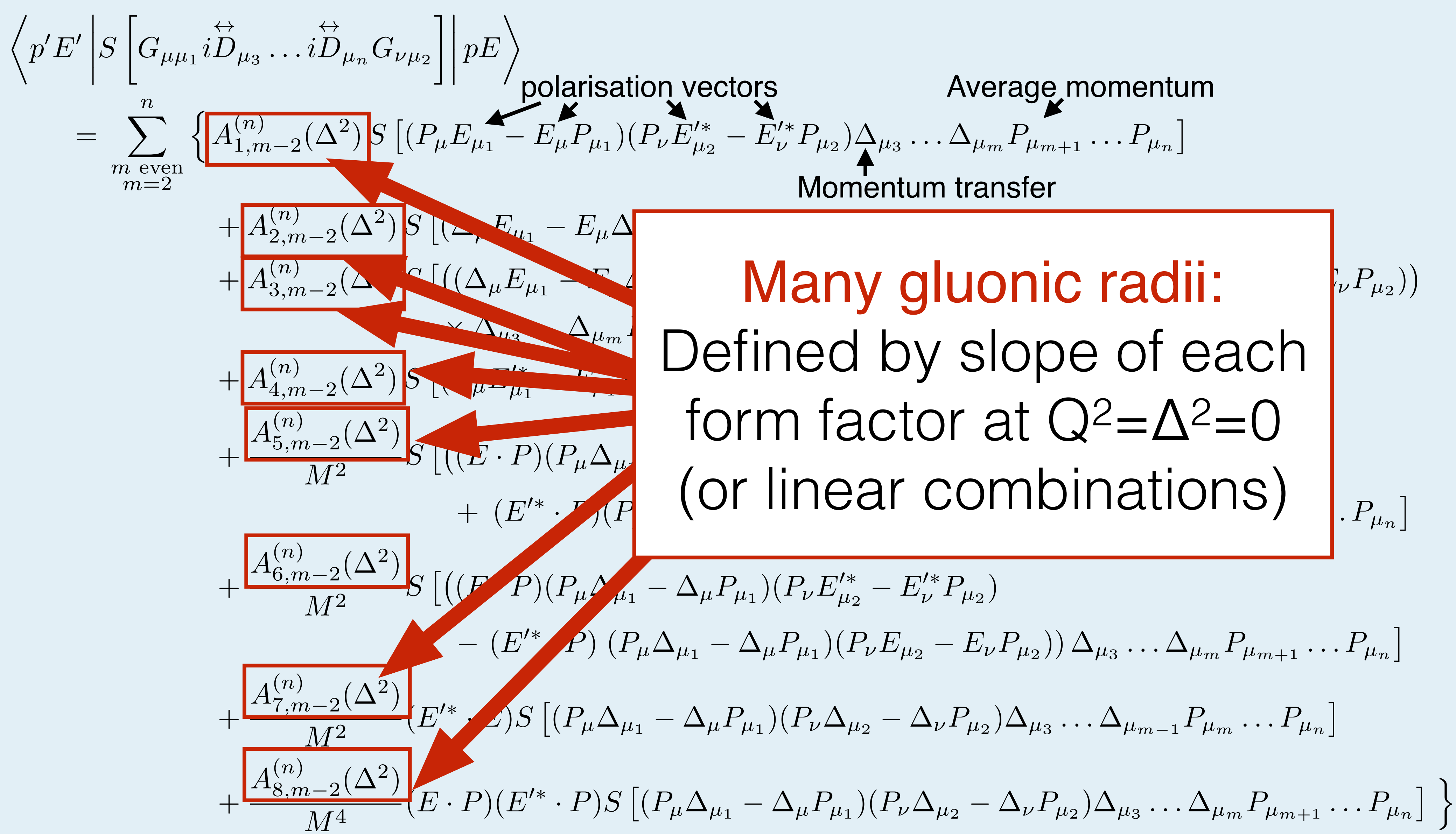
Average momentum

Momentum transfer

# Gluon radii

## Off-forward matrix elements (momentum transfer through operator)

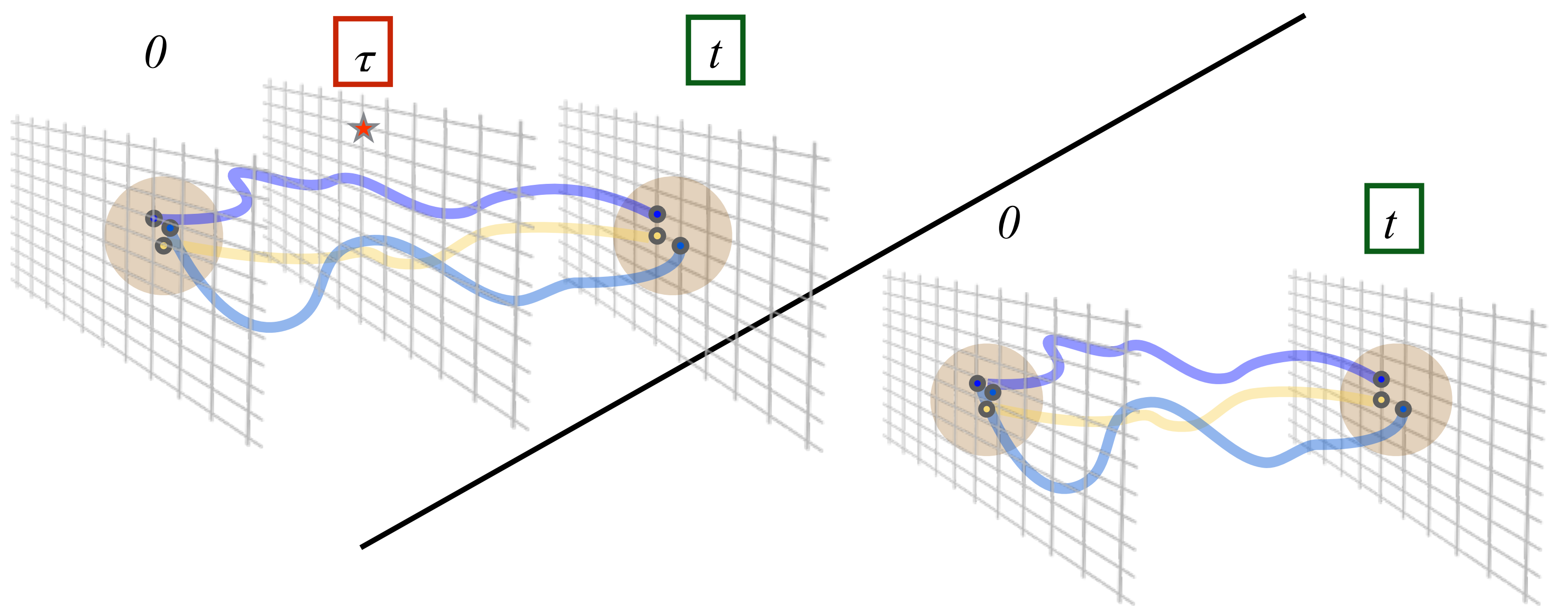
- Moments of Δ(x,Q²) related to many generalised form factors



# Gluon radii

- Discrete lattice: rotational symmetry ➡ hypercubic symmetry
- Take linear combinations of operators that transform irreducibly under hypercubic group

$R_{jk}(t, \tau, \vec{p}) =$




$\propto$ Matrix element

operator insertion time

$0 \ll \boxed{\tau} \ll \boxed{t}$

sink time

**Calculate ratio for**

- All source-sink polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- All operators in each hypercubic irrep.

# Gluon transversity GFFs

Detmold, Shanahan, PRD 94 (2016), 014507, Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

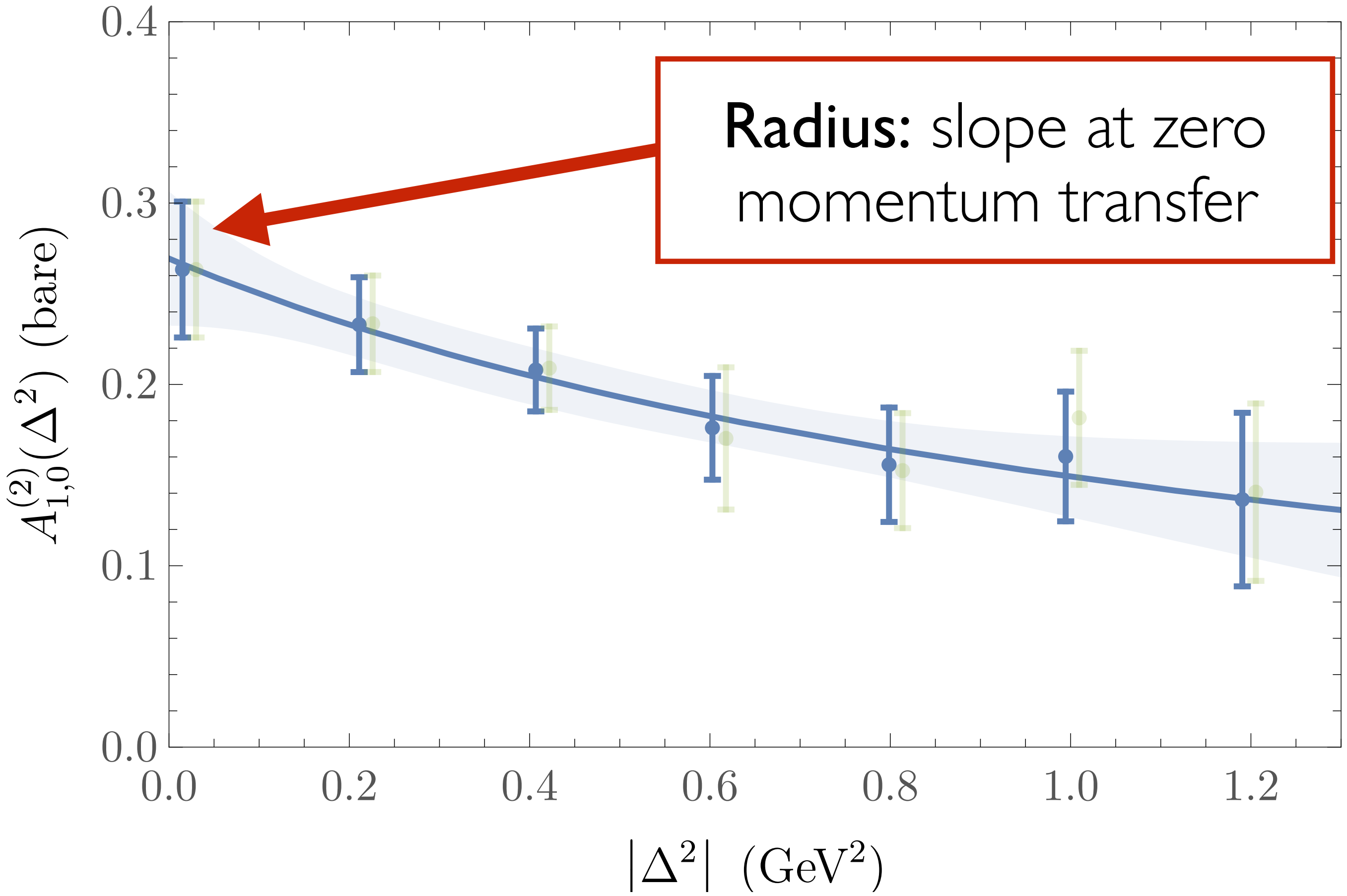
Calculate $\langle p'E'|\mathcal{O}|pE\rangle$ for all accessible $p, E, \mathcal{O}$ → Complicated analysis → Extract GFFs

**ϕ($\bar{s}s$) meson (simplest spin-1 system):**

One transversity generalised form factor can be resolved at all momenta

Dipole-like fall-off vs $|\Delta^2|$

**$m_\pi$ ~450 MeV**



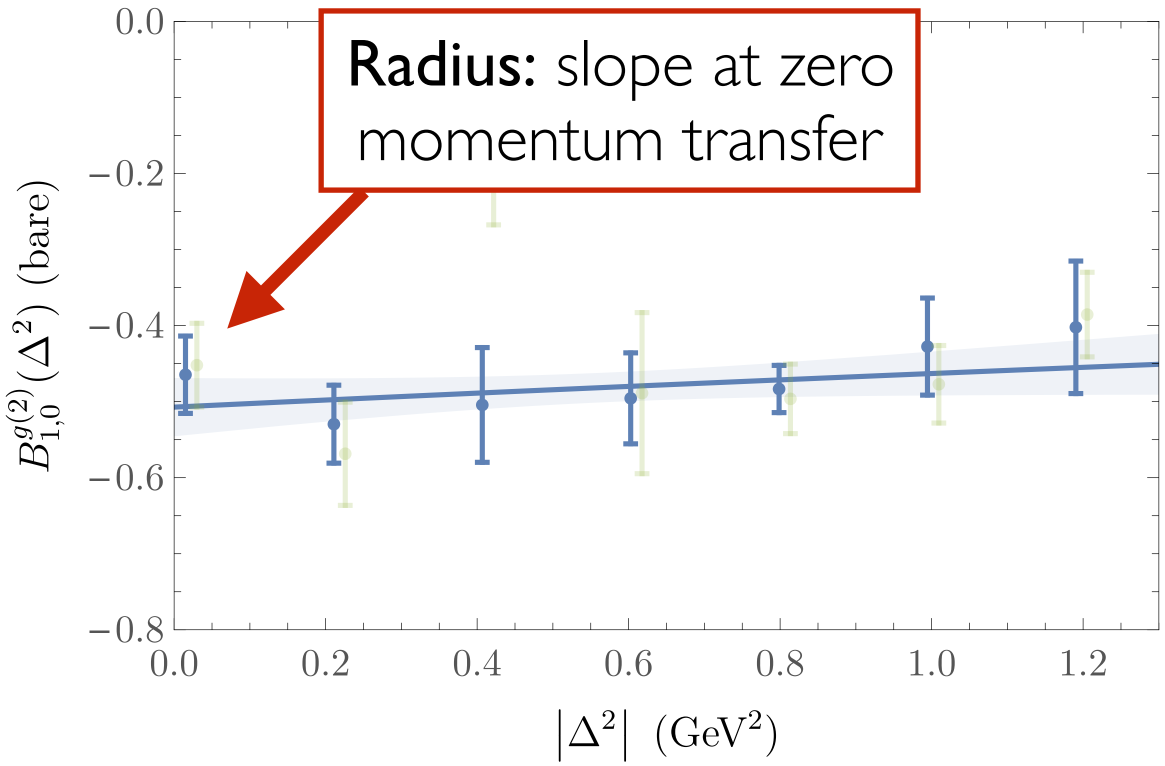

Squared momentum transfer

# Spin-indep. gluon GFFs

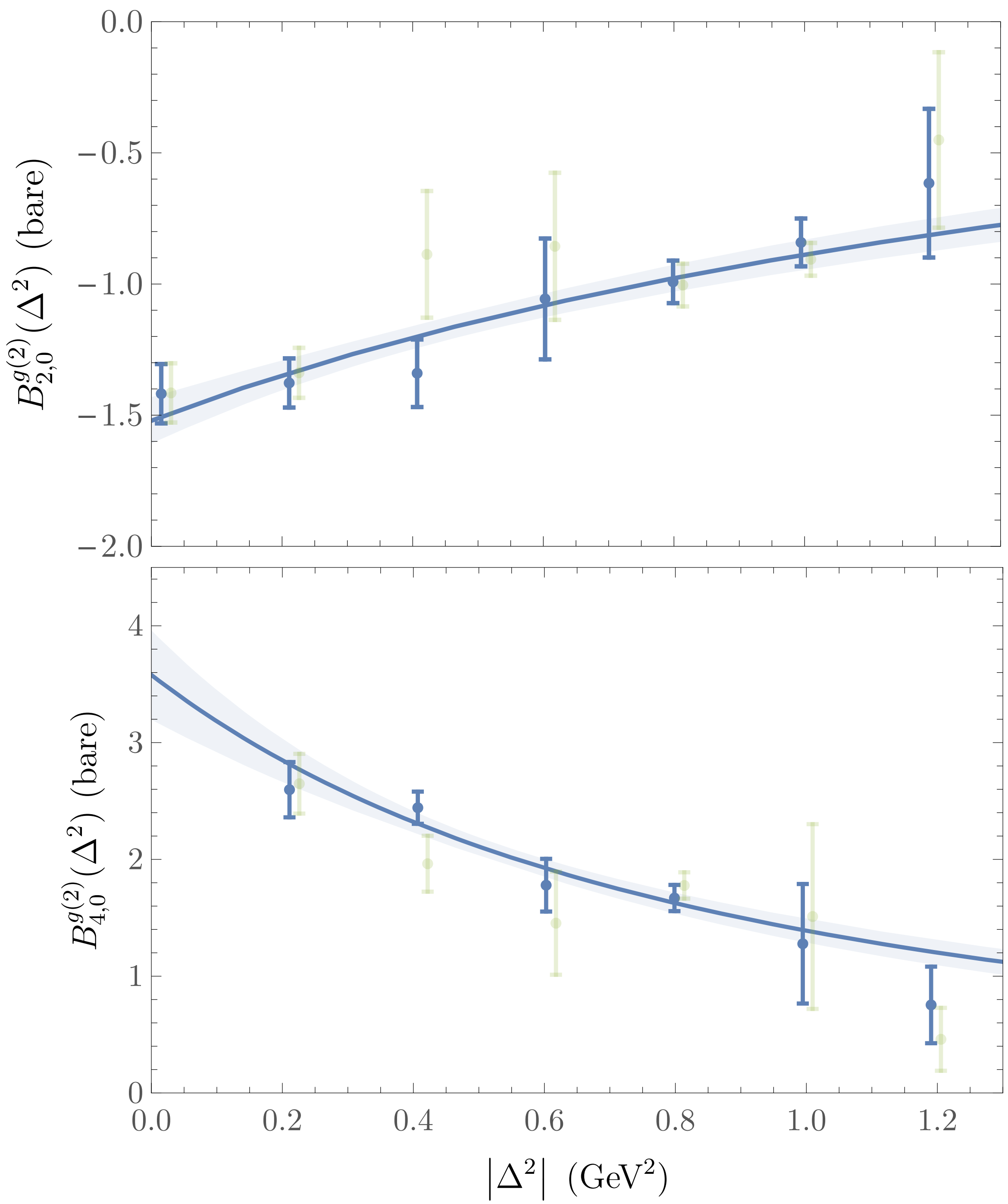
Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

## “Gravitational form factors”

Similarly complicated decomposition
Three GFFs can be resolved
for all momenta




Squared momentum transfer

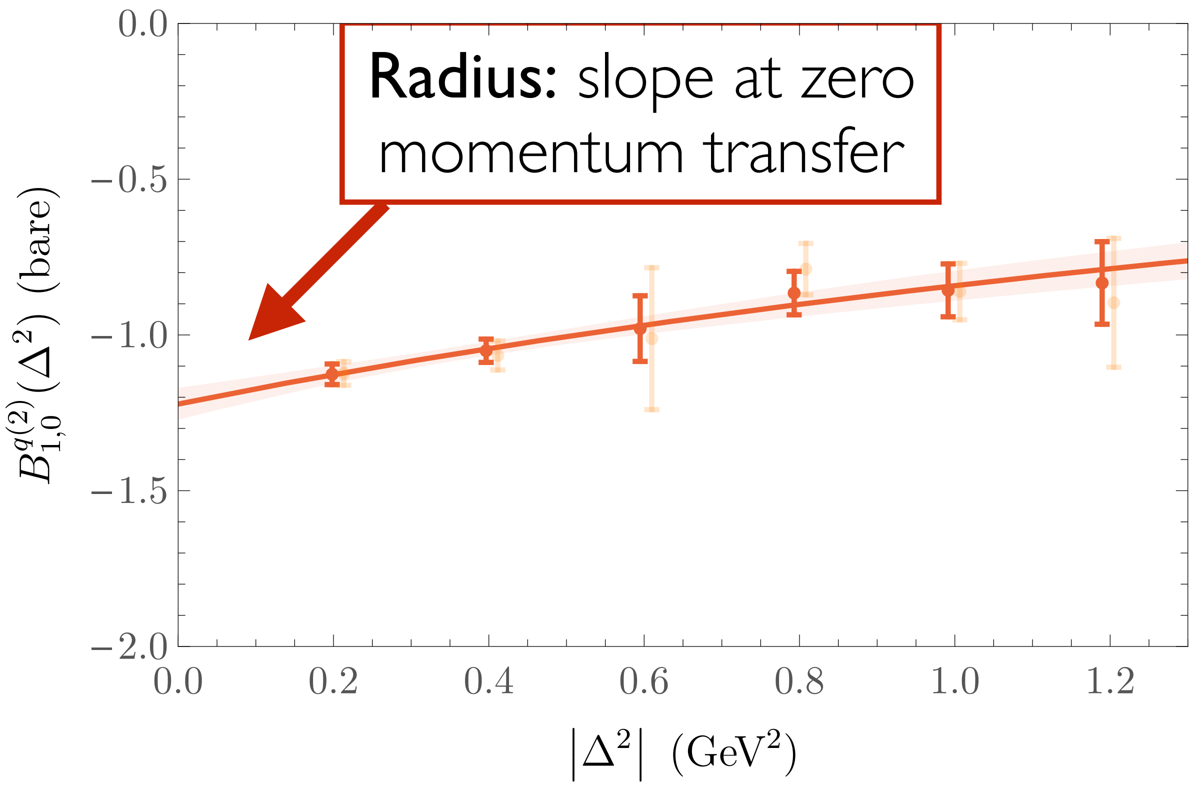



Squared momentum transfer

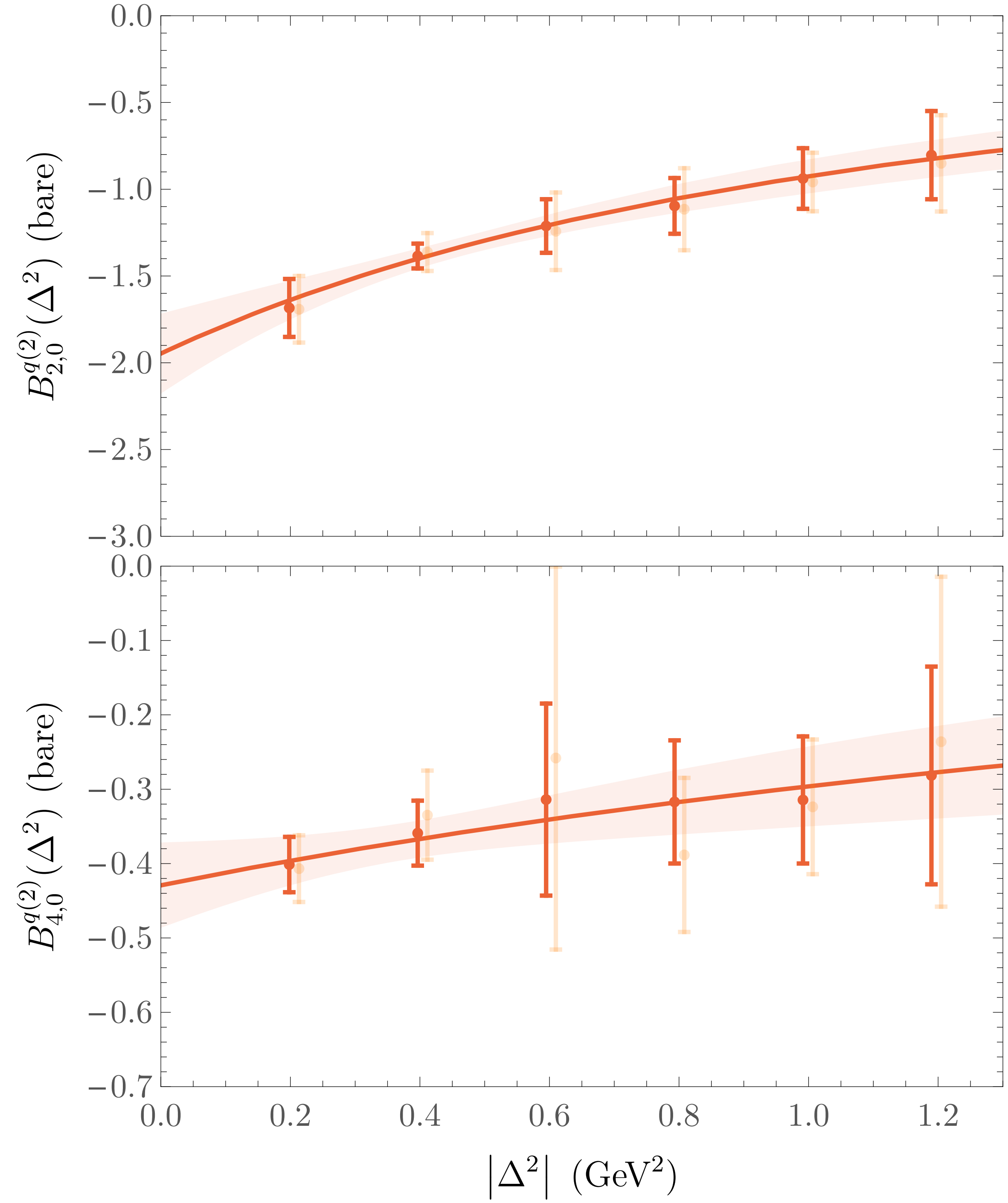
# Spin-indep. quark GFFs

Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

GFF decomposition has precisely the same structure as in the spin-independent gluon case




Squared momentum transfer

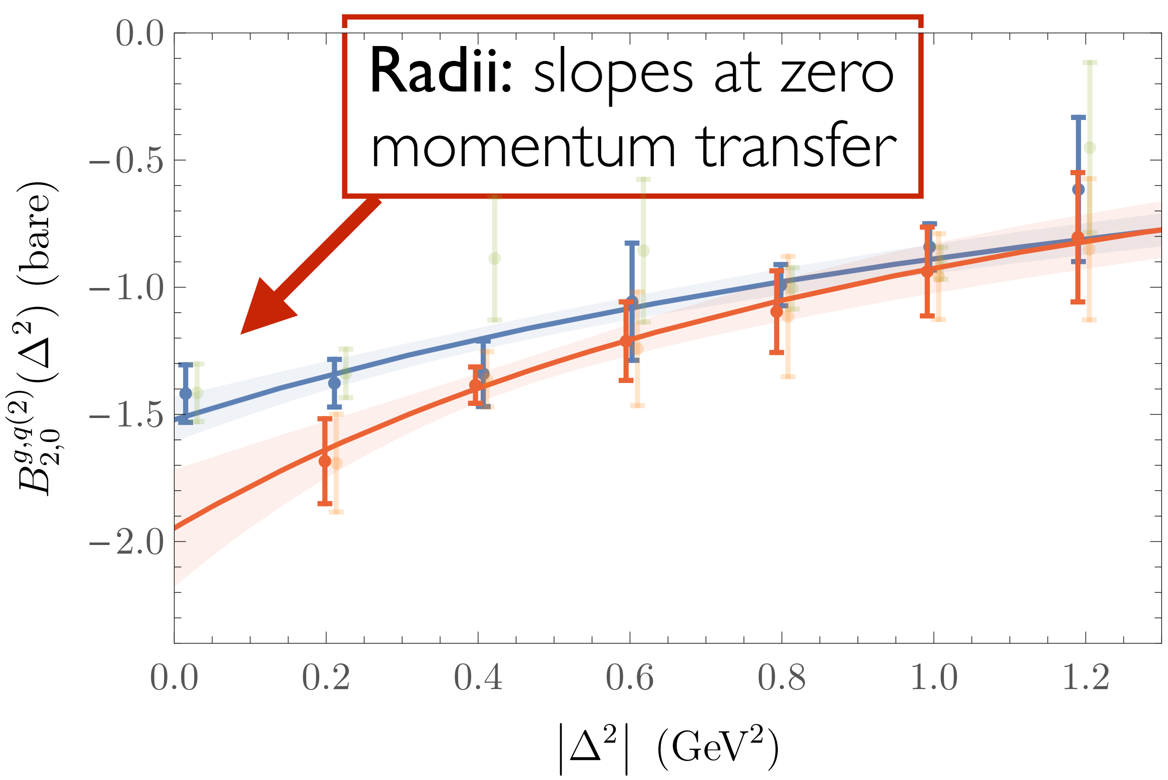



Squared momentum transfer

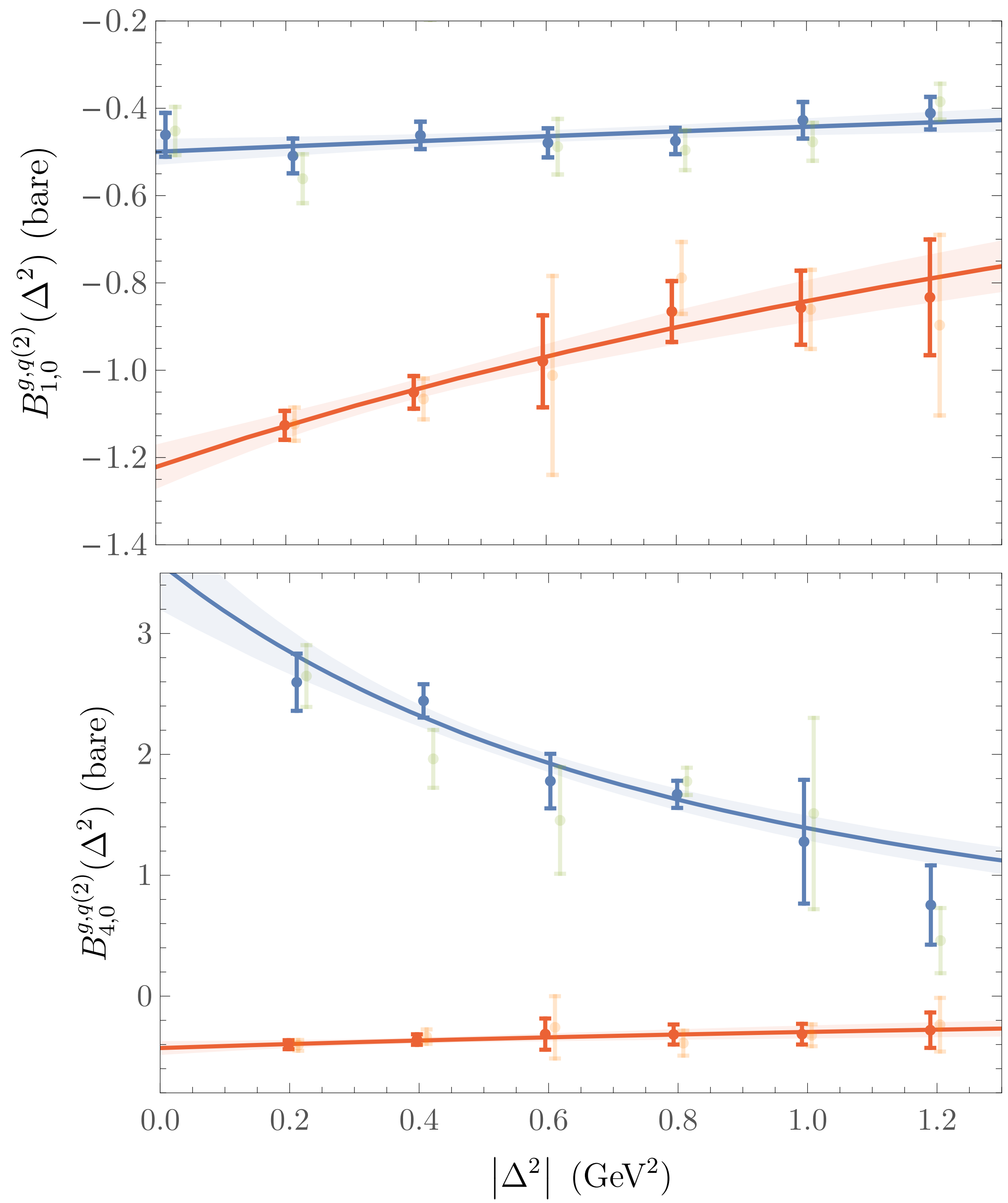
# Spin-indep. quark GFFs

Detmold, Pefkou, Shanahan, PRD 95 (2017), 114515

Gluon vs quark radius depends strongly on which aspect of structure is being probed




Squared momentum transfer




Squared momentum transfer

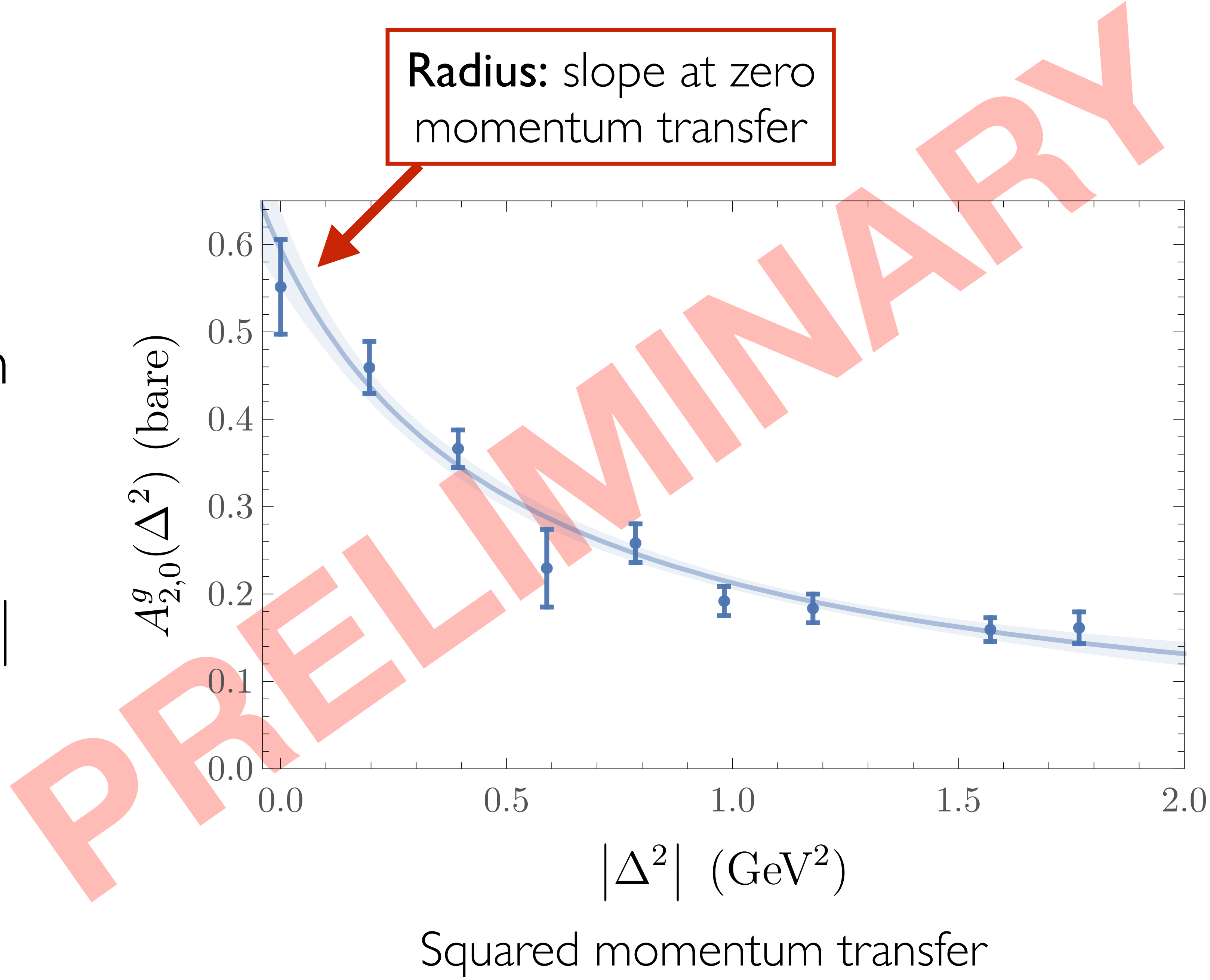
# Nucleon spin-indep. gluon GFFs

**Nucleon, $m_\pi$ ~450 MeV**

Three spin-independent generalised form factors, one can be resolved from zero at present statistics

Dipole-like fall-off vs $|\Delta^2|$

Renormalisation + comparison with quark GFFs in progress



# Gluon structure of nuclei

2

**How does the gluon structure of a nucleon change in a nucleus?**

Ratio of structure function $F_2$ per nucleon for iron and deuterium

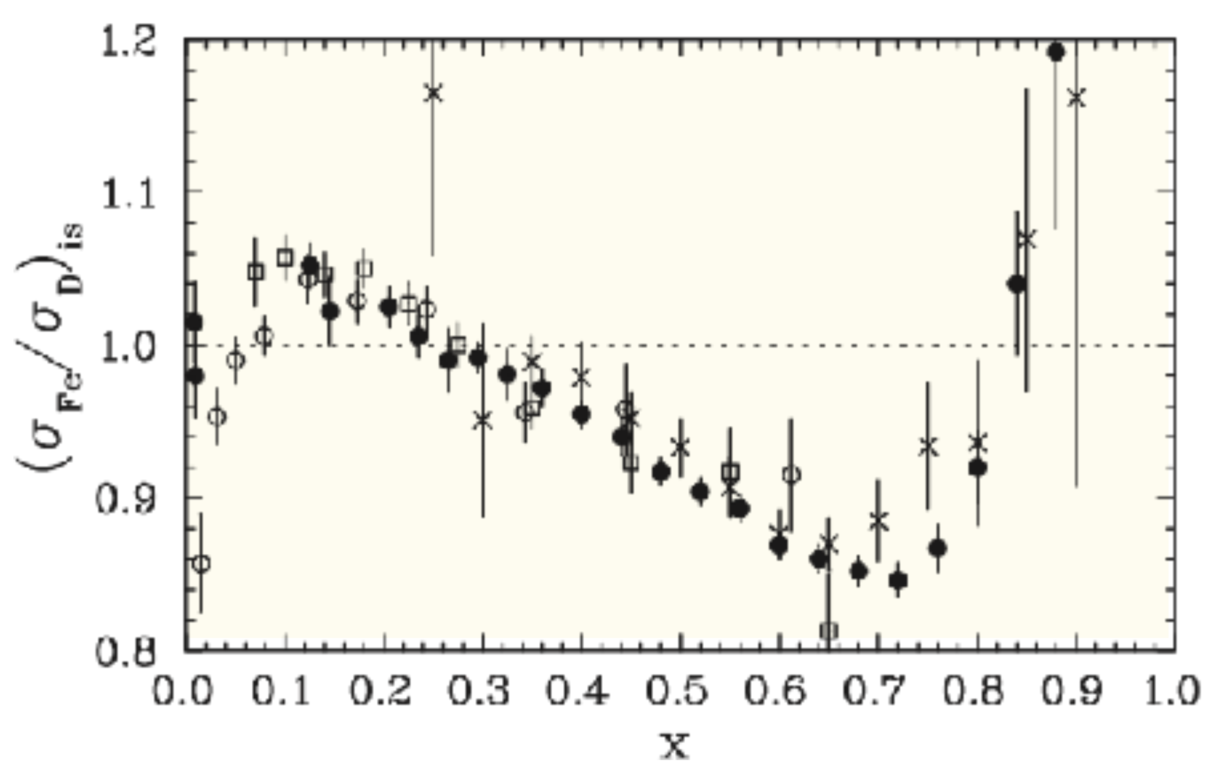
$$F_2(x, Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2 \left[q(x, Q^2) + \overline{q}(x, Q^2)\right]$$

Number density of partons of flavour q

**European Muon Collaboration (1983): "EMC effect"**

Modification of per-nucleon cross section of nucleons bound in nuclei

**Gluon analogue?**




Longitudinal momentum fraction

# Nuclear glue, $m_\pi$ ~450 MeV

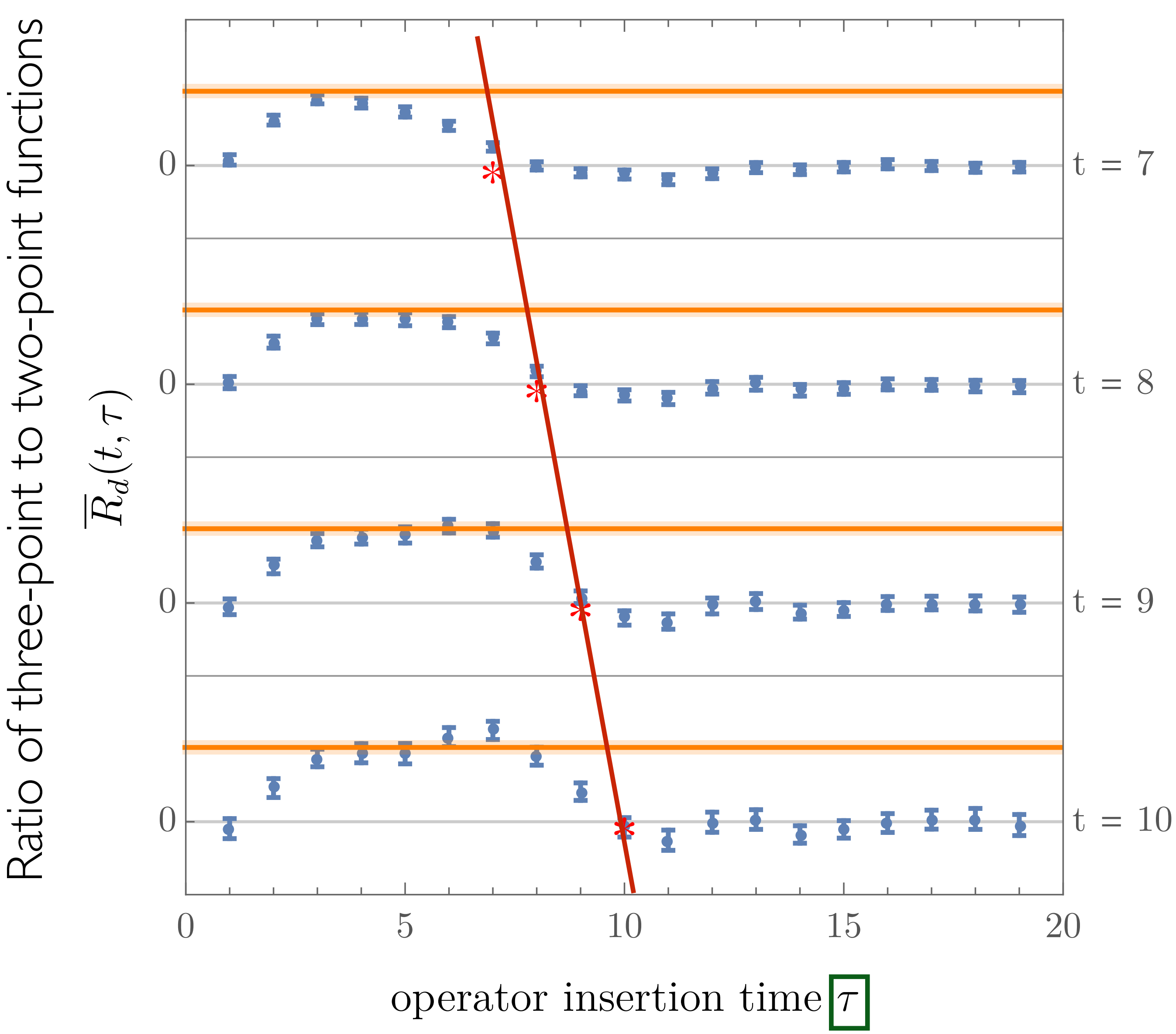
Look for **nuclear (EMC) effects** in the first moments of the spin-independent gluon structure function

### Doubly challenging

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)

## Deuteron gluon momentum fraction

Ratio ∝ matrix element for $0 \ll \tau \ll t$



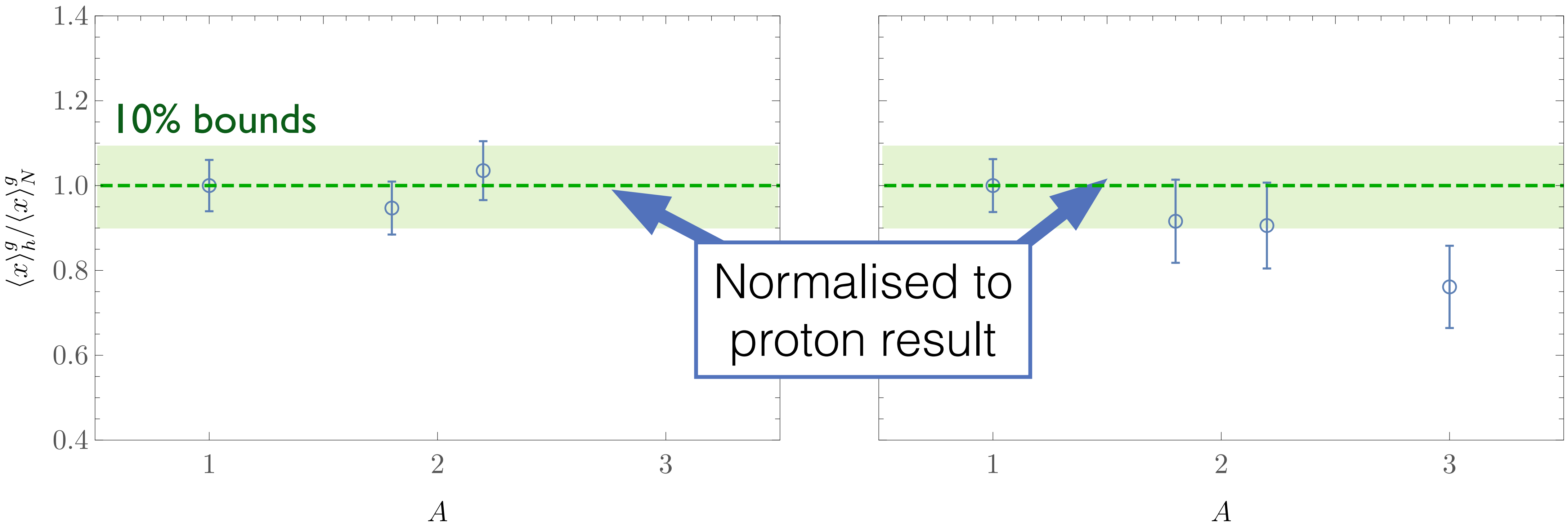

NPLQCD Collaboration PRD96 094512 (2017)

# Gluon momentum fraction

NPLQCD Collaboration PRD96 094512 (2017)

- Matrix elements of the **spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

**Ratio of gluon momentum fraction in nucleus to nucleon**




$m_\pi$ ~450 MeV $m_\pi$ ~800 MeV

# Gluon structure of nuclei

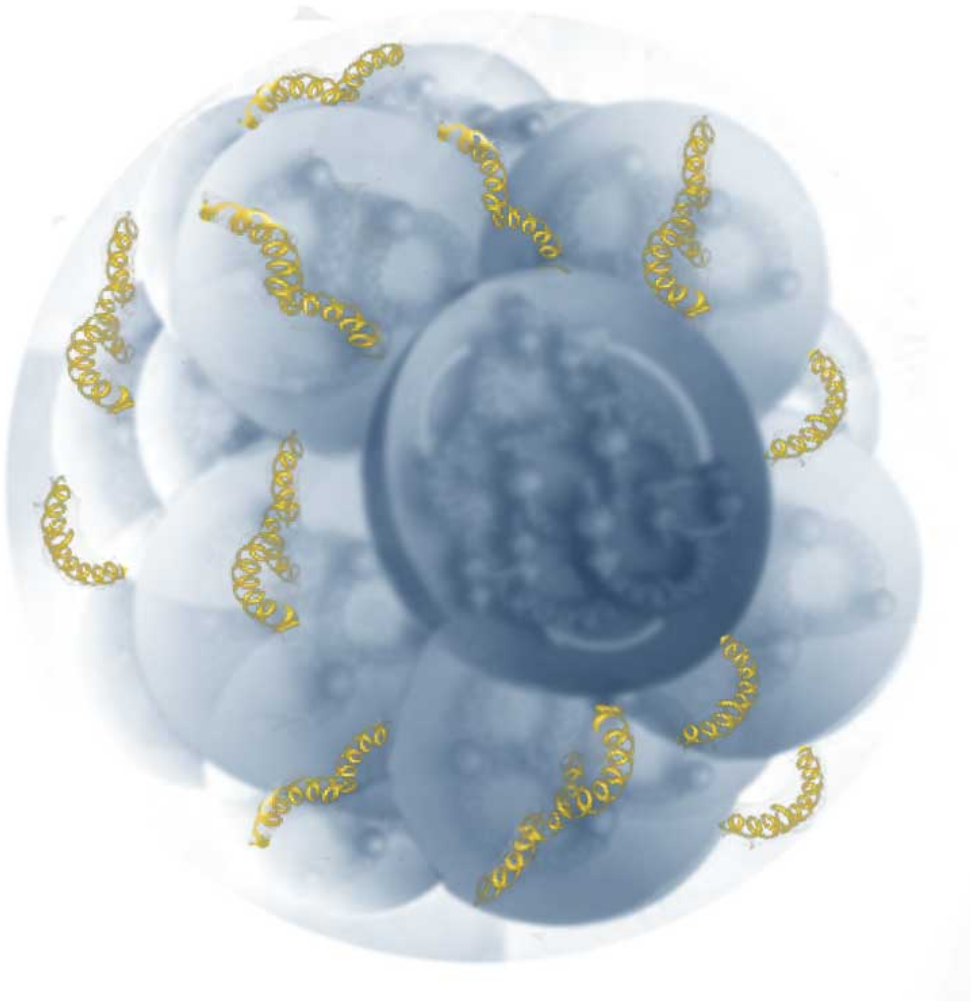
## Exotic Glue

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$



Jaffe and Manohar, "Nuclear Gluonometry"
Phys. Lett. B223 (1989) 218

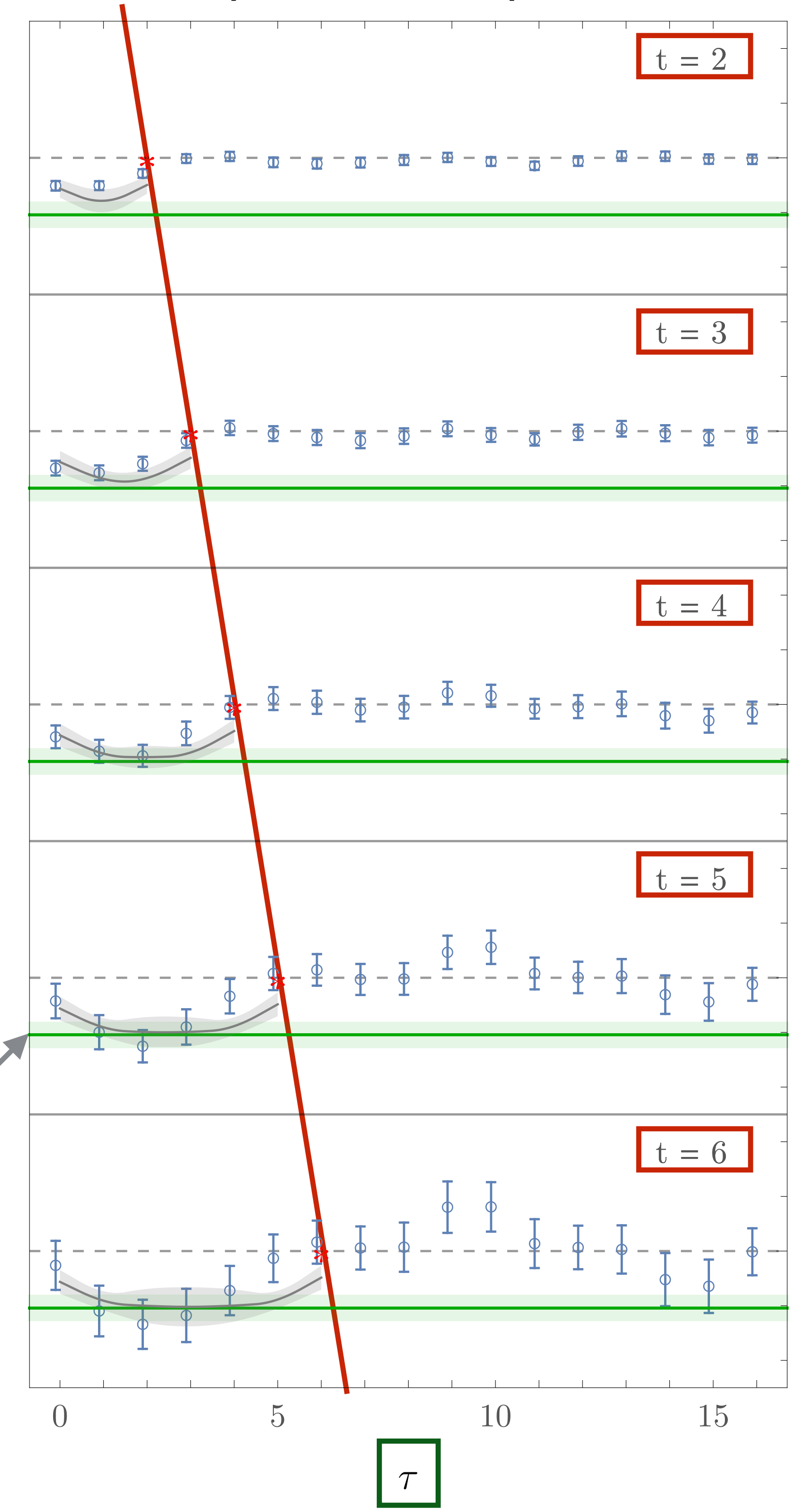
# Non-nucleonic glue in deuteron

NPLQCD Collaboration PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron,
$m_\pi$ ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- **Hypothesis of no signal ruled out to better than one part in $10^7$**
- Magnitude relative to momentum fraction as expected from large-$N_c$

Ratio of 3pt and 2pt functions




Ratio ∝ matrix element for $0 \ll \tau \ll t$

# Gluon structure from LQCD

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
  - $\Delta(x,Q^2)$ has an interesting role

    Purely gluonic

    Non-nucleonic: directly probe nuclear effects
  - Compare quark and gluon distributions in hadrons and nuclei

- Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature