Wigner, Husimi, and GTMD distributions for nucleon tomography

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Outline

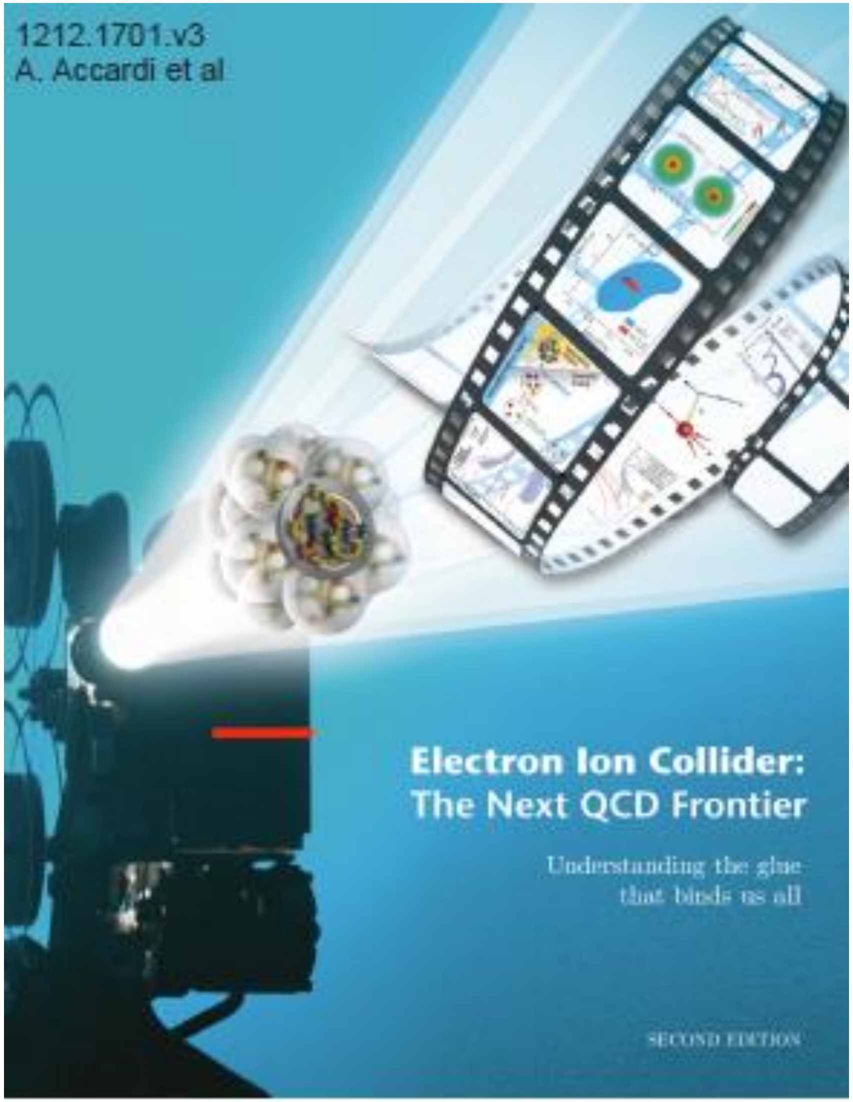
- Phase space distributions in QCD
- Connection to orbital angular momentum
- Accessing Wigner/GTMD in experiment
- Entropy of the nucleon?

Wigner distribution in 2012 EIC white paper

Almost no account.
Only briefly mentioned in two places.

Although there is no known way to measure Wigner distributions for quarks and gluons, they provide a unifying theoretical framework for the different aspects of hadron structure.

A lot of progress since then!



5D tomography: Wigner distribution— the “mother distribution”

Belitsky, Ji, Yuan (2003);
Lorce, Pasquini (2011)

$$W(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \frac{\Delta}{2}|\bar{q}(-z/2)\gamma^+ q(z/2)|P + \frac{\Delta}{2}\rangle$$

$$\begin{tikzcd}
 & W(x,\vec{k}_\perp,\vec{b}_\perp) \arrow[dl, "\int d\vec{b}_\perp"'] \arrow[dr, "\int d\vec{k}_\perp"] & & \\
\text{TMD}\quad f(x,\vec{k}_\perp) \arrow[dr, "\int d\vec{k}_\perp"'] & & \text{GPD}\quad f(x,\vec{b}_\perp) \arrow[dl, "\int d\vec{b}_\perp"] \arrow[dr, "\int dx"] & \\
 & \underset{\text{PDF}}{f(x)} \arrow[dr, "\int dx"'] & & F(\vec{b}_\perp)\quad \text{Form factor} \arrow[dl, "\int d\vec{b}_\perp"] \\
 & & \underset{\text{charge}}{Q} &
\end{tikzcd}$$

Generalized TMD (GTMD)

Meissner, Metz, Schlegel (2009)

Off-forward generalization of TMD

Fourier transform of Wigner
(Rather, Wigner is the Fourier transform of GTMD.)

$$W(x,\vec{k}_\perp,\vec{b}_\perp) \leftrightarrow G(x,\vec{k}_\perp,\vec{\Delta}_\perp)$$

$$G(x,\vec{k}_\perp,\vec{\Delta}_\perp) = \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\frac{\Delta}{2}|\bar{q}(-z/2)\gamma^+ q(z/2)|P+\frac{\Delta}{2}\rangle$$

More basic object, more directly related to experiments.

Husimi distribution

Gaussian smearing of Wigner in the minimal uncertainty region $\delta q \delta p = \hbar/2$

$$H(q,p,t) = \frac{1}{\pi\hbar}\int dq' dp' e^{-m\omega(q'-q)^2/\hbar-(p'-p)^2/m\omega\hbar} W(q',p',t)$$

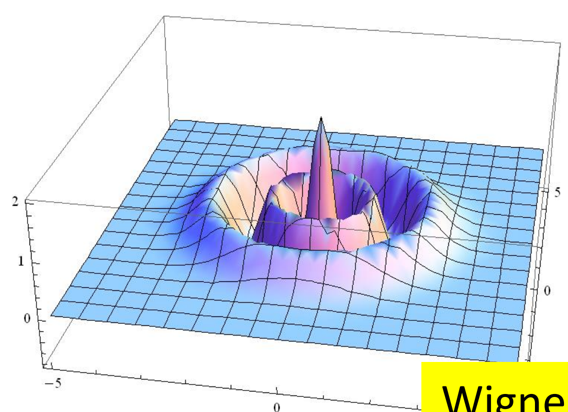
Husimi distribution is positive definite!

coherent state

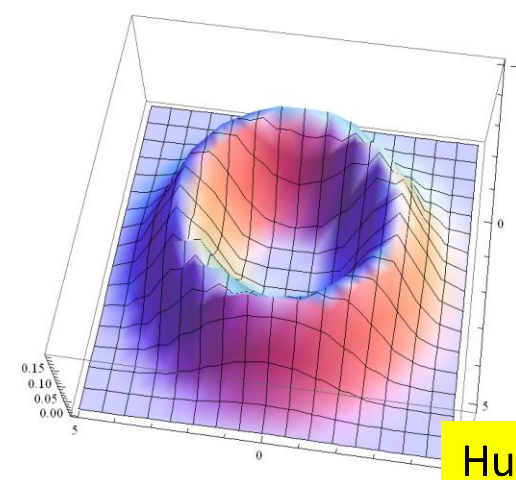
$$H(q,p,t) = \langle\lambda|\hat{\rho}|\lambda\rangle = |\langle\psi|\lambda\rangle|^2 \geq 0$$

Probabilistic interpretation possible. Well known in quantum optics.

4th excited state of
1D harmonic oscillator



Wigner



Husimi

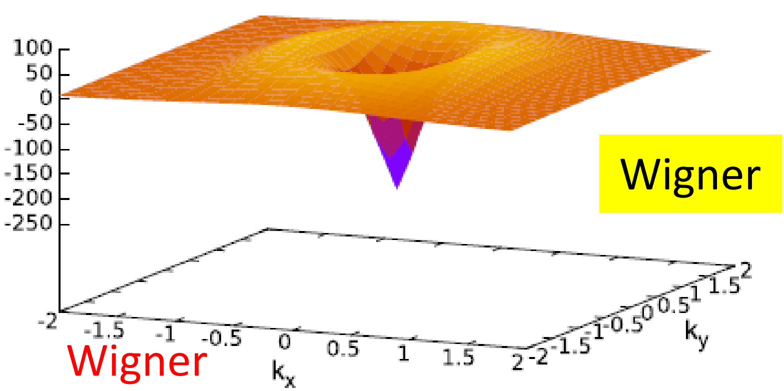
QCD Husimi distribution

Hagiwara, YH (2014)

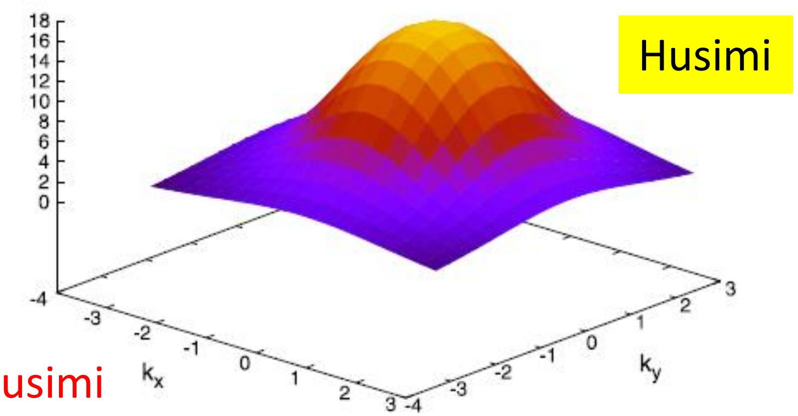
$$H(x,\vec{b}_\perp,\vec{k}_\perp)=\frac{1}{\pi^2}\int d^2b'_\perp d^2k'_\perp e^{-\frac{1}{\ell^2}(\vec{b}_\perp-\vec{b'}_\perp)^2-\ell^2(\vec{k}_\perp-\vec{k'}_\perp)^2}W(x,\vec{b'}_\perp,\vec{k'}_\perp)$$

Off-diagonal matrix element (relativistic effect).
Positivity not proven. But it is positive in all the models studied so far.

Wigner and Husimi of a quark at 1-loop.

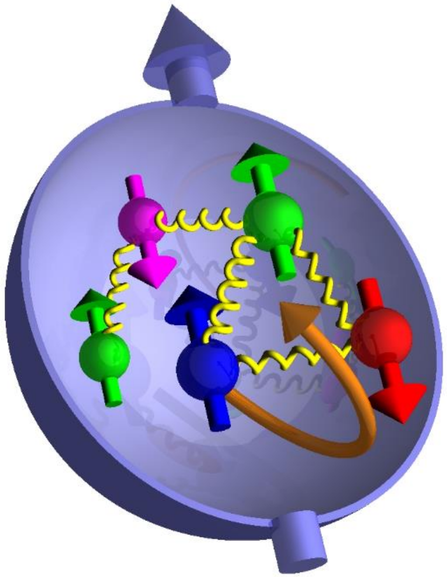


Wigner



Husimi

Wigner distribution and orbital angular momentum



Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Canonical Orbital angular momentum

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \begin{cases} W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \\ H^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \end{cases}$$

Lorce, Pasquini, (2011);
YH (2011)
Lorce, Pasquini, Xiong, Yuan

`PDF' for OAM

$$L^{q,g}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \begin{cases} W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \\ H^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \end{cases}$$

Canonical vs. kinetic orbital angular momentum

Canonical (Jaffe-Manohar) OAM from the light-cone Wilson line YH (2011)

$$\int \vec{b}\times\vec{k}W_{light-cone}(\vec{b},\vec{k}) = \langle\bar{\psi}\vec{b}\times i\overleftrightarrow{D}_{pure}\psi\rangle$$

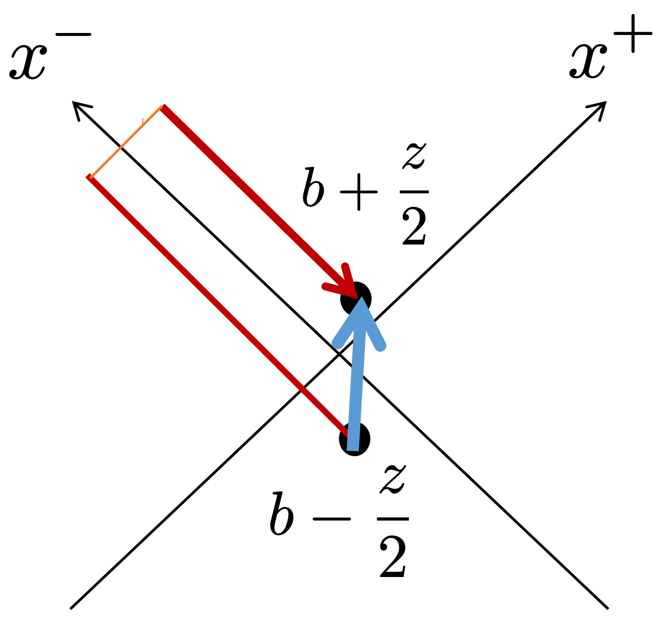
$$D^{\mu}_{pure} = D^{\mu} - \frac{i}{D^{+}}F^{+\mu}$$

Kinetic (Ji's) OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

$$\int \vec{b}\times\vec{k}W_{straight}(\vec{b},\vec{k})$$

$$= \langle\bar{\psi}\vec{b}\times i\overleftrightarrow{D}\psi\rangle$$



The difference numerically significant.

Engelhardt (2017)

Wigner distribution: Is it measurable?

In quantum optics, yes!

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Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

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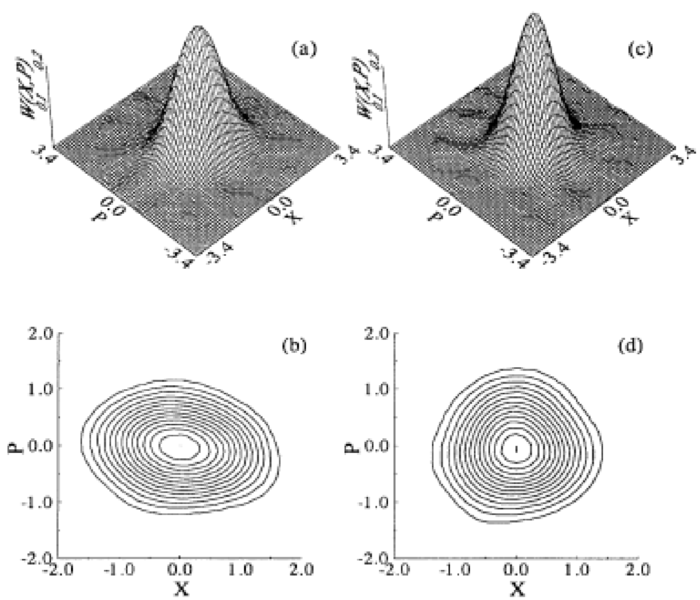
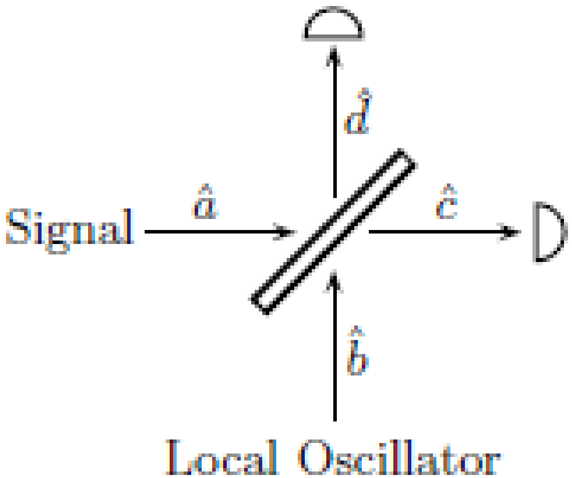


Homodyne Detection

Projection of Wigner : probability distribution

$$\int dx W(x,p) = P(p), \qquad \int dp W(x,p) = P(x)$$



(coherent state, phase ϕ)

Difference in photon counts at detectors C,D

$$\propto P(x_\phi) \qquad x_\phi = x\cos\phi + p\sin\phi$$

Reconstruct Wigner $W(x,p)$ via the inverse Radon transform

cf. computed tomography (CT)

What about in QCD?

Higher-dimensional distribution → More exclusive process.

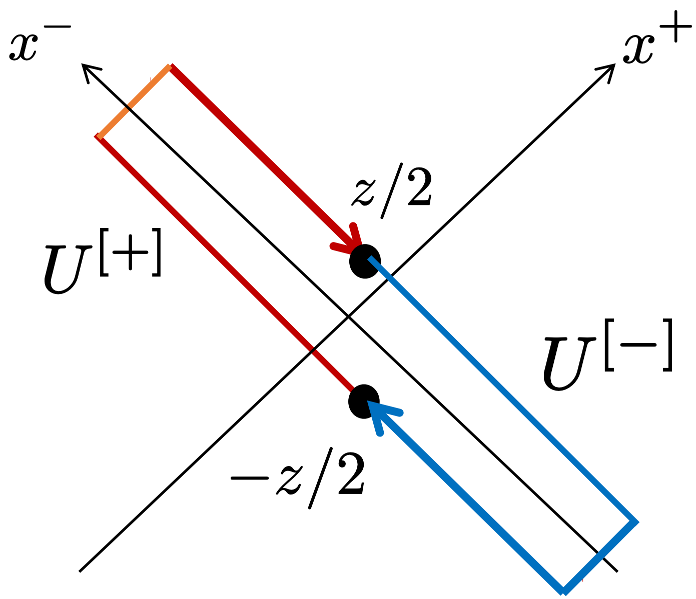
Tag at least three particles in the final state (e.g., dijet + recoiling proton)

Gluon Wigner distribution

$$xW(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \Delta/2 | F^{+i}(-z/2) F^{+}_{\ i}(z/2) | P + \Delta/2 \rangle$$

There are two ways to make it gauge invariant

Bomhof, Mulders (2008)
Dominguez, Marquet, Xiao, Yuan (2011)



Dipole distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[-]}]$$

Weizsacker-Williams (WW) distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[+]}]$$

Dipole gluon Wigner distribution at small-x

YH, Xiao, Yuan (2016)

Simply related to the dipole S-matrix at small-x

$$xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S_x(\vec{b}_\perp,\vec{r}_\perp)$$

$$S_x(\vec{b}_\perp,\vec{r}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$

$\cos 2\phi$ correlation expected

``Elliptic Wigner”

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = W_0(x,k_\perp,b_\perp) + 2\cos 2(\phi_k - \phi_b) W_1(x,k_\perp,b_\perp)$$

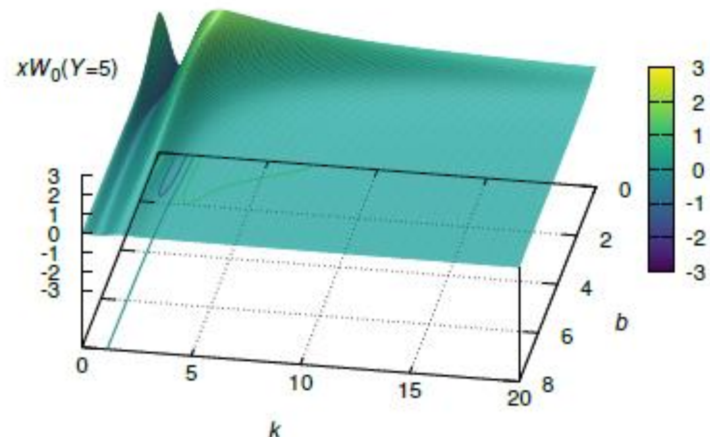
NB: Dependence on longitudinal spin drops out in this approximation.

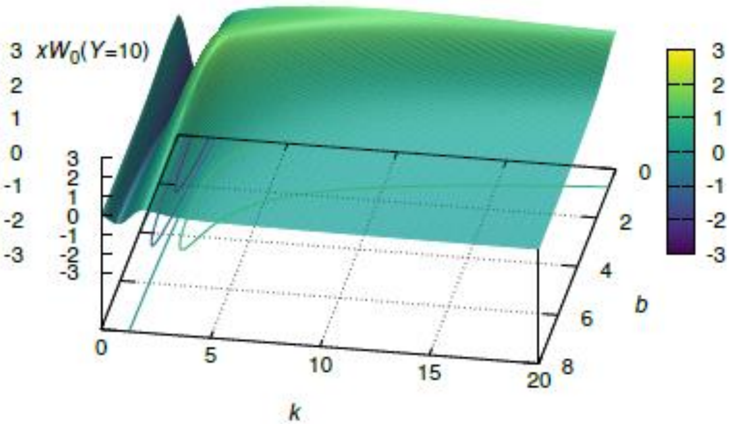
Dipole Wigner from Balitsky-Kovchegov equation

Hagiwara, YH, Ueda (2016)

Peak at the saturation momentum

W_0

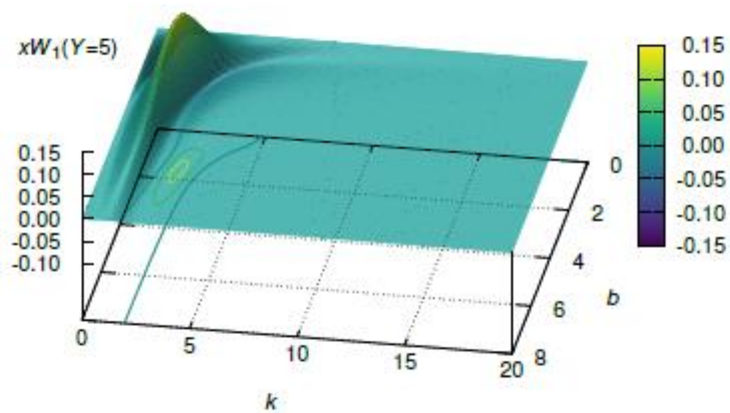


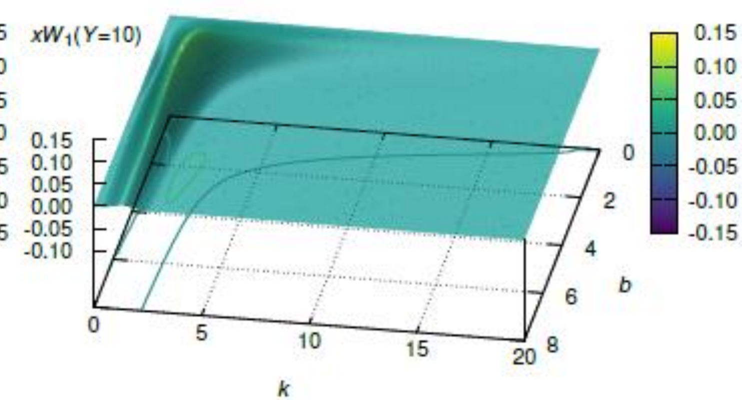


$$Y = \ln \frac{1}{x} = 5$$

$$Y = 10$$

W_1





Elliptic part small in magnitude (a few percent effect). No geometric scaling.

Elliptic Wigner in DVCS

YH, Xiao, Yuan (2017)

Gluon transversity GPD

$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+\cdots,$$

$$xE_{Tg}(x,\Delta_\perp)=\frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 S_1$$

Elliptic Wigner (GTMD)

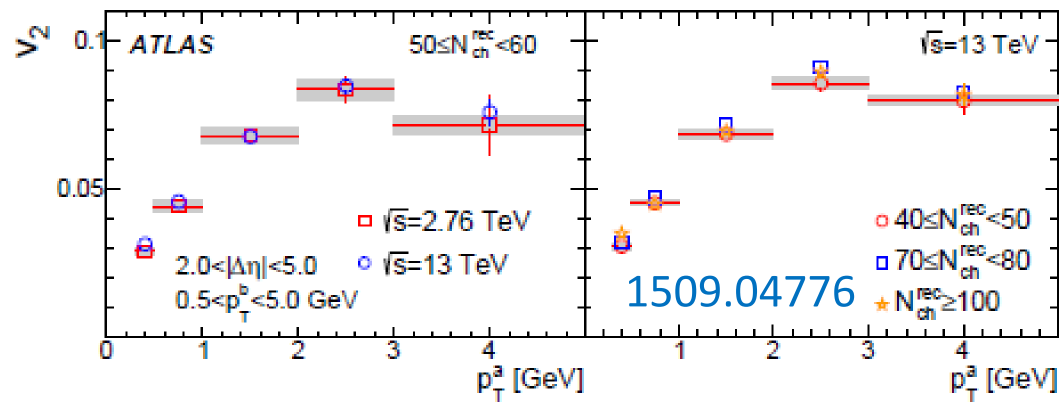
$$\frac{d\sigma(ep\to e'\gamma p')}{dx_B dQ^2 d^2\Delta_\perp}=\frac{\alpha_{em}^3}{\pi x_{Bj}Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)(\mathcal{A}_0^2+\mathcal{A}_2^2)+2(1-y)\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l})\right.$$
$$\left.+(2-y)\sqrt{1-y}(\mathcal{A}_0+\mathcal{A}_2)\mathcal{A}_L\cos\phi_{\Delta l}+(1-y)\mathcal{A}_L^2\right\}$$

Elliptic Wigner in high-multiplicity pp and pA

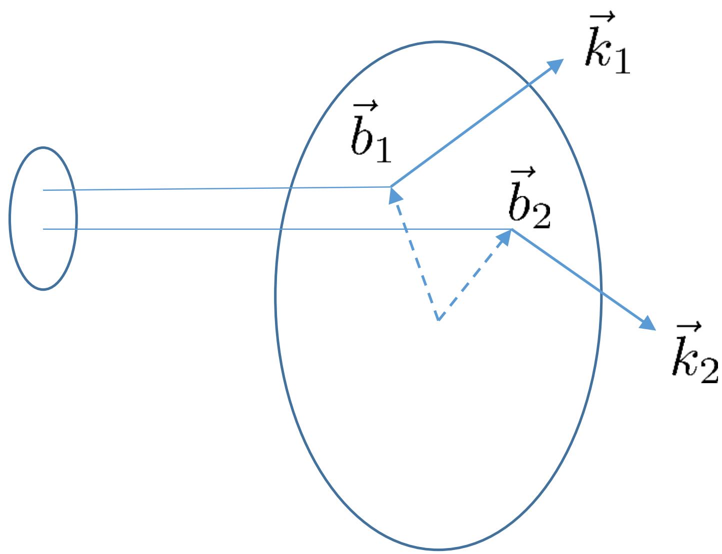
Kopeliovich et al. (2008),
Levin Rezaeian (2011),
Hagiwara, YH, Xiao, Yuan (2017)

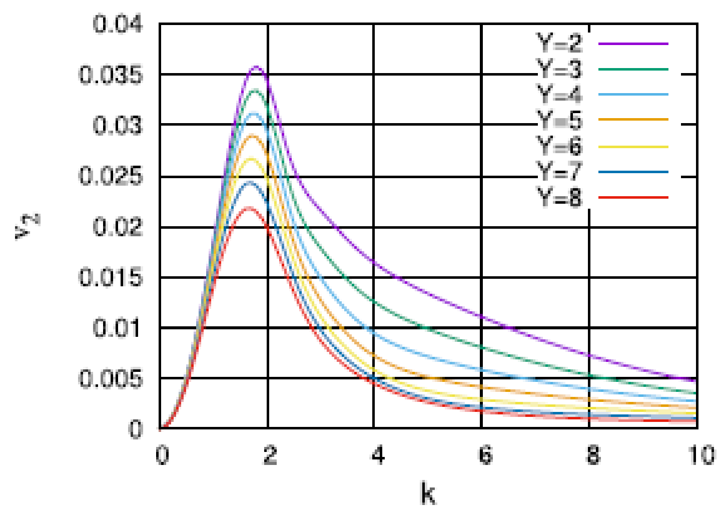
Elliptic flow v_2 observed in high-multiplicity pp and pA.

Initial state or final state effect?



Double parton scattering + elliptic Wigner = elliptic flow

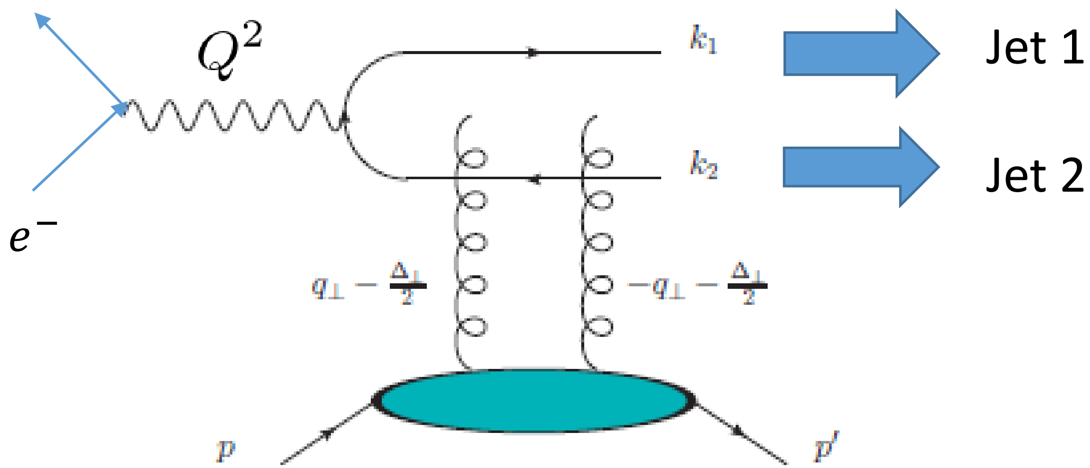




Probing Wigner (GTMD) in diffractive dijet production in ep

YH, Xiao, Yuan (2016), see also Altinoluk, Armesto, Beuf, Rezaeian (2015)

NLO calculation by Boussarie et al. (2016)



$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$
$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

GTMD (dipole S-matrix)

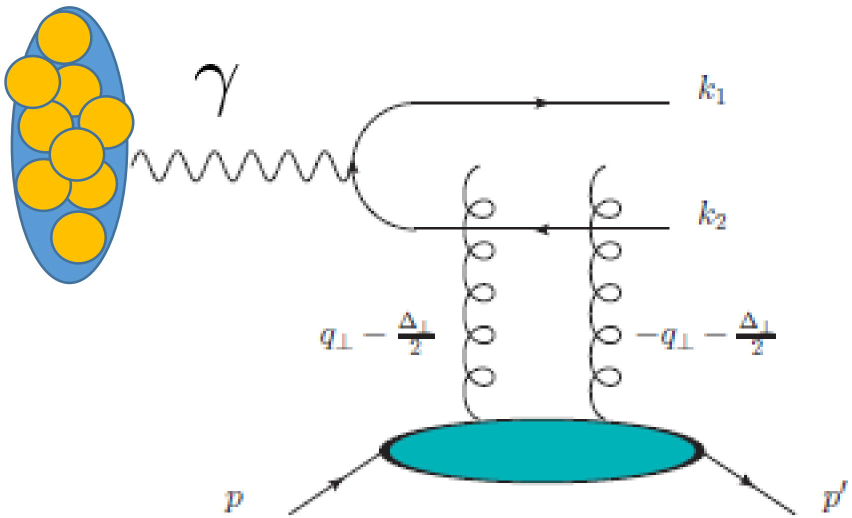
$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \propto z(1-z)[z^2+(1-z)^2]\int d^2q_\perp d^2q'_\perp S(q_\perp,\Delta_\perp)S(q'_\perp,\Delta_\perp)$$
$$\times \left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2+\epsilon^2}\right]\cdot\left[\frac{\vec{P}_\perp}{P_\perp^2+\epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2+\epsilon^2}\right]$$

$$\sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta)d\tilde{\sigma}$$

$$\epsilon^2 = z(1-z)Q^2$$

Probing dipole Wigner in ultra-peripheral pA collisions

Hagiwara, YH, Pasechnik, Tasevsky, Teryaev (2017)



Q^2 preferably small

Use the Weiszacker-Williams photons in UPC!

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega\frac{dN}{d\omega}\frac{N_c\alpha_{em}(2\pi)^4}{P_\perp^2}\sum_f e_f^2 2z(1-z)(z^2+(1-z)^2)\left(A^2 + 2\cos 2(\phi_P - \phi_\Delta)AB\right)$$

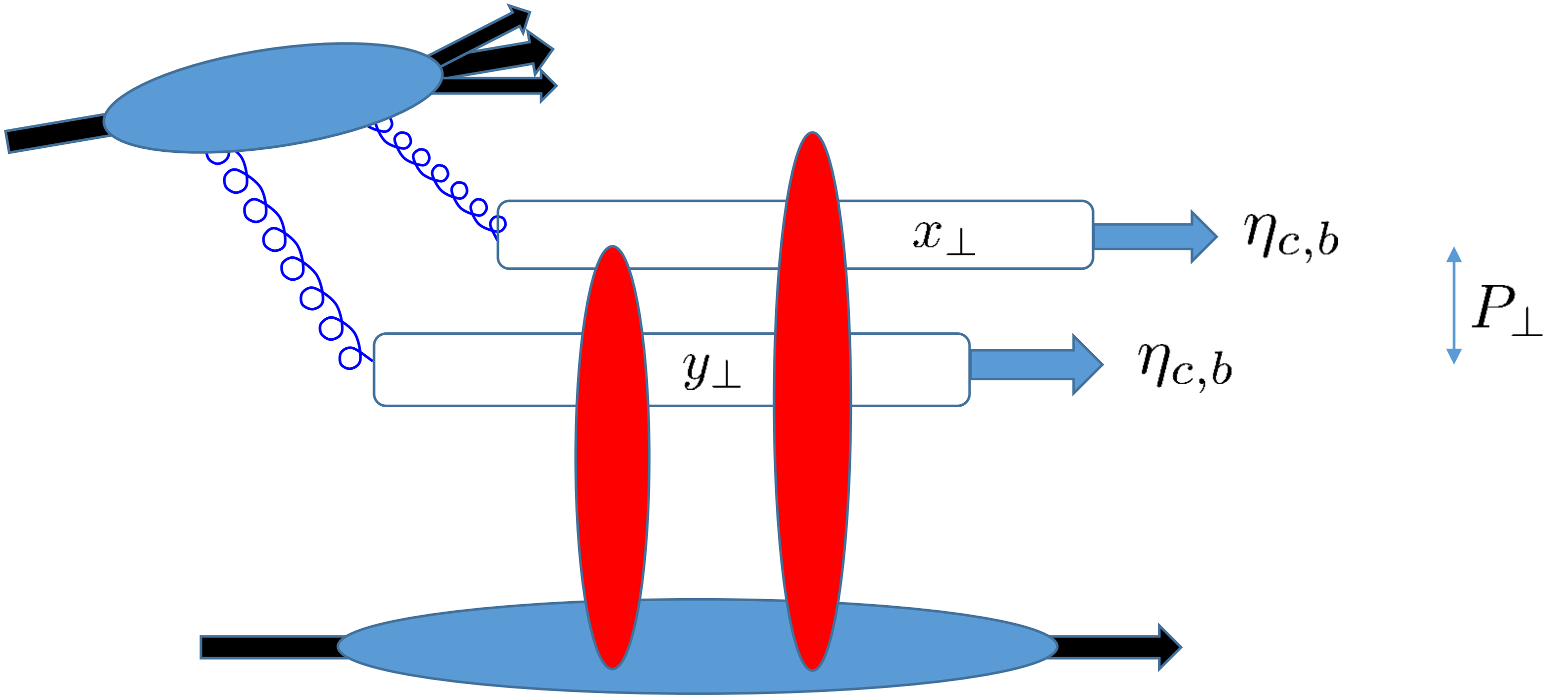
photon flux $\propto Z^2$

$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp}\frac{\partial}{\partial P_\perp}A(P_\perp, \Delta_\perp).$$

$$S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2}\int^{P_\perp^2}\frac{dP_\perp'^2}{P_\perp'^2}B(P_\perp', \Delta_\perp)$$

Probing the gluon WWW in pp

Boussarie, YH, Xiao, Yuan, in preparation



Amplitude proportional to

$$\int d^2(x_\perp - y_\perp) e^{iP_\perp\cdot(x_\perp - y_\perp)} \langle P'|U_x \vec{\partial} U_x^\dagger U_y \vec{\partial} U_y^\dagger |P\rangle$$

Similar idea by Bhattacarya, Metz, Ojha, Tsai, Zhou 1802.10550

`Entropy’ of partons?

A hadron in its ground state is a pure state → vanishing von Neumann entropy

Experiments probe only a part of the hadron wavefunction → entanglement entropy?

Recent interest in defining an (entanglement) `entropy’ of partons in the hadron wavefunction

→ Talks by Lublinsky and Florechinger

Remember, classically entropy is defined via the phase space distribution.

$$S = -\int \frac{dqdp}{2\pi\hbar} f(q,p) \ln f(q,p)$$

Wehrl entropy in quantum mechanics

von Neumann entropy

coherent state

$$S_{vN} = -\mathrm{Tr}\rho \ln \rho = -\int \frac{dqdp}{2\pi\hbar} \langle\lambda|\rho \ln \rho|\lambda\rangle$$

Replace $\langle\lambda|\rho \ln \rho|\lambda\rangle \to \langle\lambda|\rho|\lambda\rangle \ln \underline{\langle\lambda|\rho|\lambda\rangle}$

Husimi distribution

Wehrl entropy $$S_W = -\int \frac{dqdp}{2\pi\hbar} H(q,p) \ln H(q,p)$$

Nonvanishing even for a pure state because $S_W > S_{vN} \geq 0$

Wehrl entropy for the nucleon

Hagiwara, YH, Xiao, Yuan (2018)

Use the QCD Husimi distribution to define

$$S_W(x) \equiv -\int d^2b_\perp d^2k_\perp xH(x,b_\perp,k_\perp)\ln xH(x,b_\perp,k_\perp)$$

cf. Wehrl entropy for QGP; Kunihiro, Muller, Ohnishi, Schafer (2008)

$$S_W(x) \sim \frac{N_c}{\alpha_s}Q_s^2(x)S_\perp \propto A\left(\frac{1}{x}\right)^{\#\alpha_s} \qquad x \to 0$$

Parametrically the same result as in Kutak; Peschanski; Kovner-Lublinsky

Measure of `complexity' of the multiparton system.
Proportional to the final state multiplicity?
Saturation of entropy due to the Pomeron loop effect?

Conclusions

- Let’s get 5 dimensional. Even richer physics than TMD and GPD combined. A lot of progress since the 2012 EIC white paper.

- Wigner/GTMD measurable in ep, pp, pA, including the elliptic part and spin-dependent part (connection to OAM).

Bhattacharya, Metz, Zhou
Ji, Yuan, Zhao
YH, Nakagawa, Xiao, Yuan, Zhao

Experimentally challenging, but possible at the EIC!

- Phase space distributions naturally define an `entropy’ of nucleon. Communicate with other fields of physics