Azimuthal distributions in the Drell-Yan process

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Univ. Tübingen

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Outline:

- Drell-Yan angular coefficients
- Fixed-order pQCD & phenomenology
- Resummation & phenomenology

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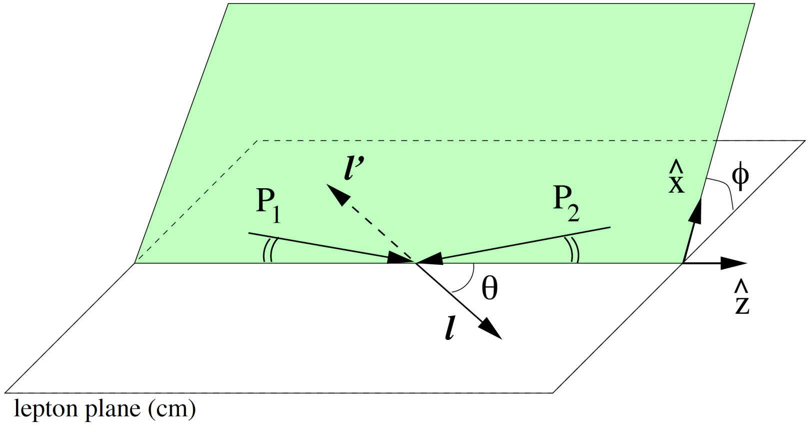
Collab. / discussions with M. Lambertsen, J. Steiglechner,
A. Bacchetta, G. Bozzi, F. Piacenza

Drell-Yan angular coefficients

Lepton angular distribution in Drell-Yan (photon exch.):

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha^2}{2\pi N_c Q^2 s^2}\Big(W_T\,(1+\cos^2\theta) + W_L\,(1-\cos^2\theta)$$

$$+W_\Delta \sin 2\theta\cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi\Big)$$



(Collins-Soper frame)

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$$= \frac{3\,\sigma_0}{4\pi}\,\frac{1}{\lambda+3}\left[1+\lambda\,\cos^2\theta+\mu\,\sin 2\theta\cos\phi+\frac{\nu}{2}\,\sin^2\theta\cos 2\phi\right]$$

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$$= \frac{3\,\sigma_0}{16\pi}\left[1+\cos^2\theta+\frac{A_0}{2}\,(1-3\cos^2\theta)+A_1\,\sin 2\theta\cos\phi+\frac{A_2}{2}\,\sin^2\theta\cos 2\phi\right]$$

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$$= \frac{3\,\sigma_0}{16\pi} \left[1 + \cos^2\theta + \frac{A_0}{2} \,(1 - 3\cos^2\theta) + A_1 \,\sin 2\theta \cos\phi + \frac{A_2}{2} \,\sin^2\theta \cos 2\phi \right]$$

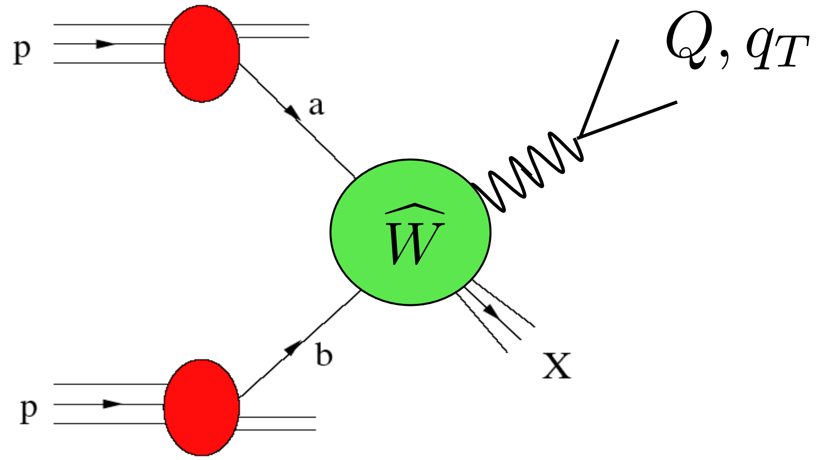
where:

$$\lambda = \frac{W_T - W_L}{W_T + W_L} \,, \quad \mu = \frac{W_\Delta}{W_T + W_L} \,, \quad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L}$$

$$A_0 = \frac{2W_L}{2W_T + W_L} \,, \quad A_1 = \frac{2W_\Delta}{2W_T + W_L} \,, \quad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L}$$

Fixed-order pQCD

Collinear factorization:



$$W_P = \sum_{a,b} \int dx_a dx_b \, f_a(x_a, \mu) \, f_b(x_b, \mu) \, \widehat{W}_{P,ab}(x_a P_a, x_b P_b, q, \alpha_s(\mu), \mu)$$

$$(P = T, L, \Delta, \Delta\Delta)$$

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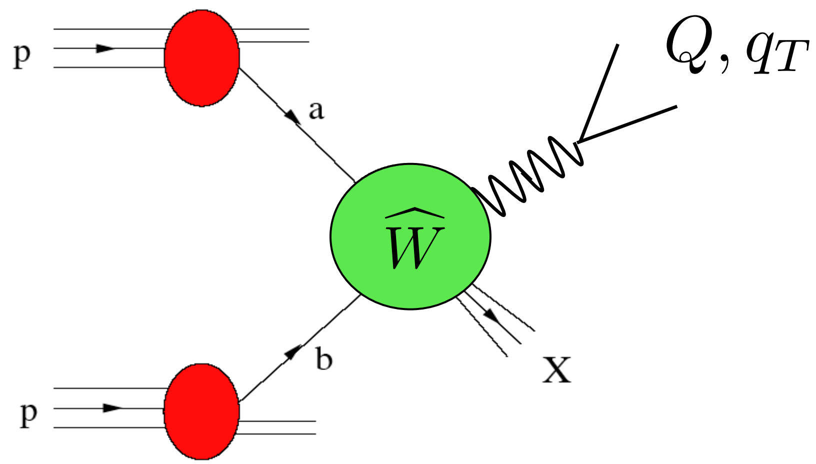

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- $\widehat{W}_P$ partonic structure fcts.: perturbative

$$\widehat{W}_P = \frac{\alpha_s}{\pi} \widehat{W}_P^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \widehat{W}_P^{(2)} + \ldots$$

(fixed-order)

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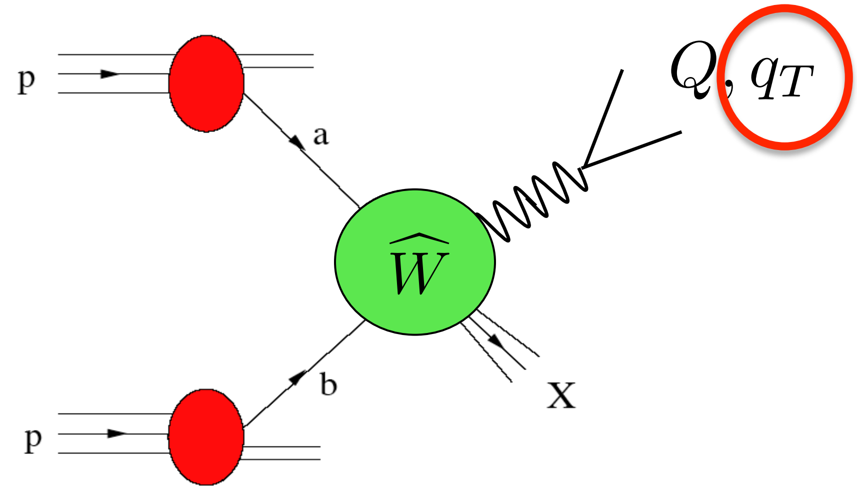

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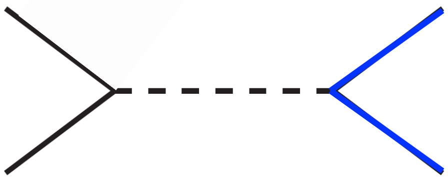
- zeroth order $q\bar{q} \to V \to \ell^+\ell^-$:



$q_T = 0$

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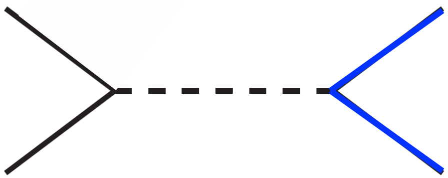

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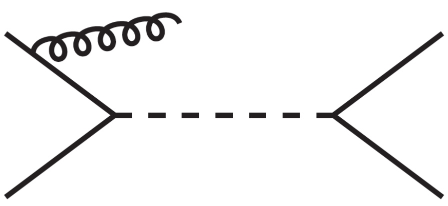
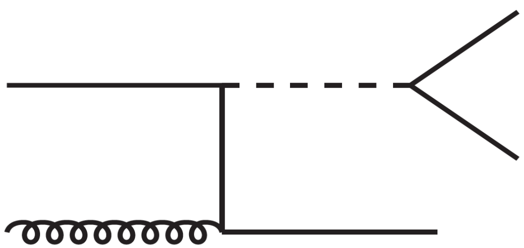
- $q_T \neq 0$: first non-trivial order (= LO) $\mathcal{O}(\alpha_s)$





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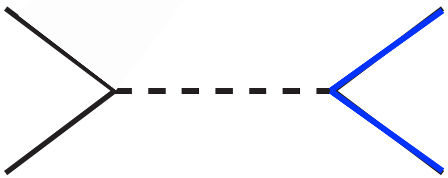

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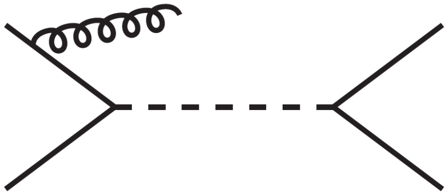
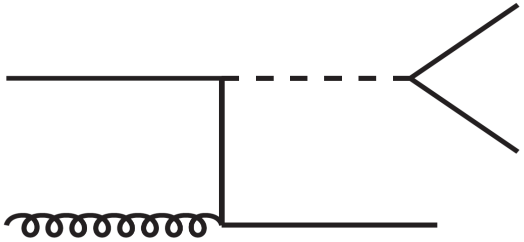


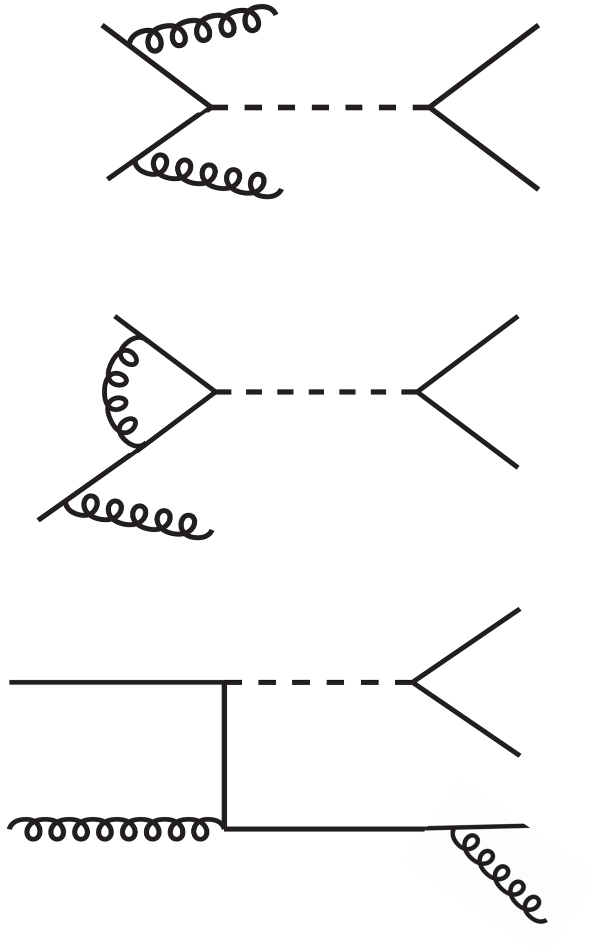


$$\lambda \neq 1, \ \mu \neq 0, \ \nu \neq 0$$

- but: $1 - \lambda - 2\nu \ = \ 0$ (Lam-Tung relation)

$$A_0 \ = \ A_2$$

NLO:



$+\ \ldots$

$+\ \ldots$

$+\ \ldots$

$\mathcal{O}(\alpha_s^2)$

first computed by Mirkes '92; Mirkes, Ohnemus '95

- a lot of work in recent 2 decades on $\mathcal{O}(\alpha_s^2)$ corrections for Drell-Yan process

Hamberg, van Neerven, Matsuura; Harlander, Kilgore;
Anastasiou, Dixon, Melnikov, Petriello; Melnikov, Petriello;
Li, Petriello, Quackenbusch; Catani, Cieri, Ferrera, de Florian, Grazzini;
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- especially: $\mathcal{O}(\alpha_s^2)$ Monte-Carlo codes

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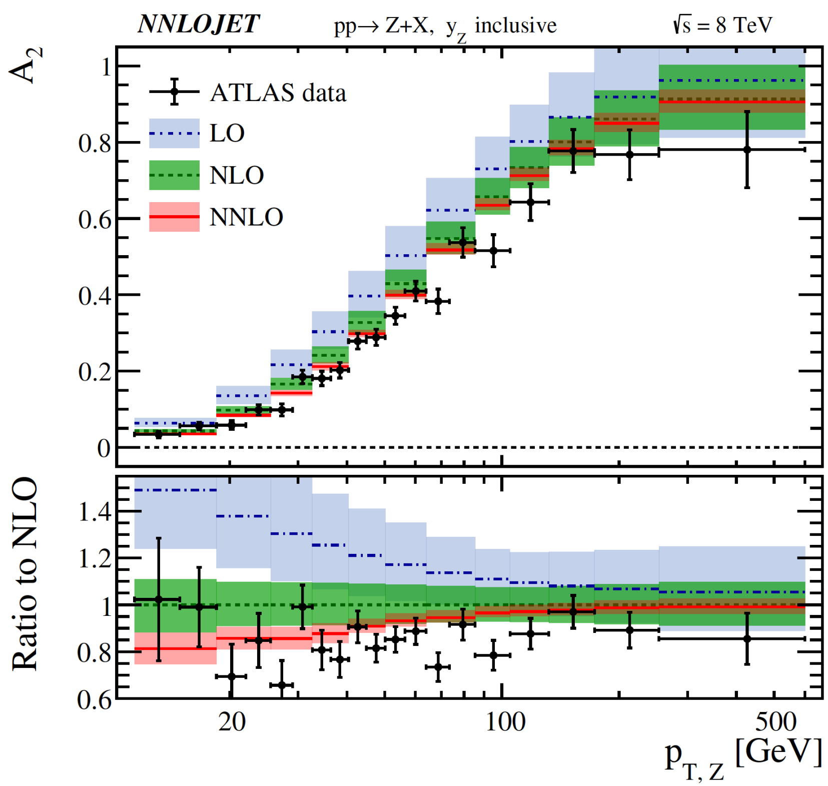
- most recently: $\mathcal{O}(\alpha_s^3)$

  Li, von Manteuffel, Schabinger, Zhu;
  Anastasiou, Duhr, Dulat, Herzog, Mistlberger;
  Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Fixed-order phenomenology

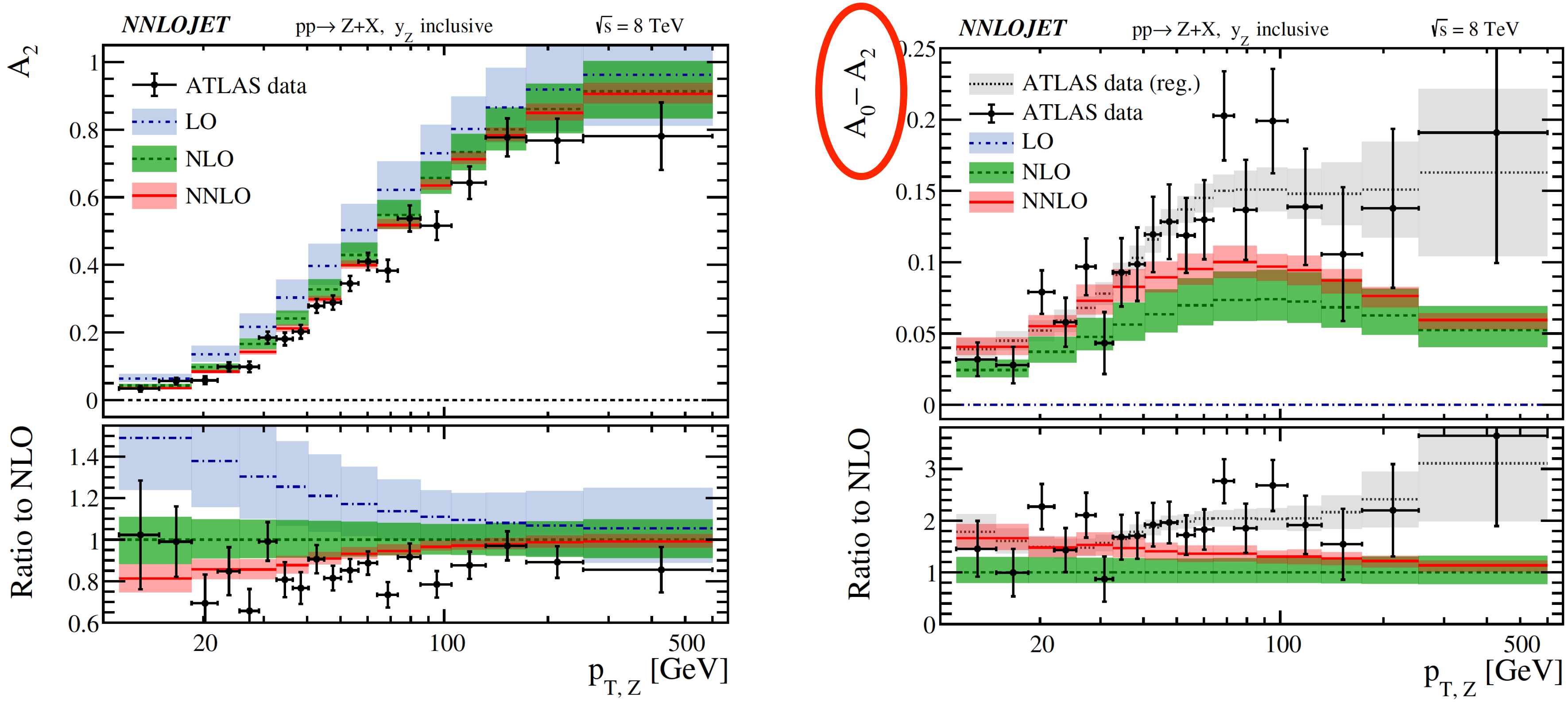
Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17



$$\mathcal{O}(\alpha_s^2) \quad \text{NLO (ATLAS):} \quad \chi^2/\mathrm{N_{data}} = 185.8/38 = 4.89$$

$$\mathcal{O}(\alpha_s^3) \quad \text{NNLO (ATLAS):} \quad \chi^2/\mathrm{N_{data}} = 68.3/38 = 1.80$$

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17



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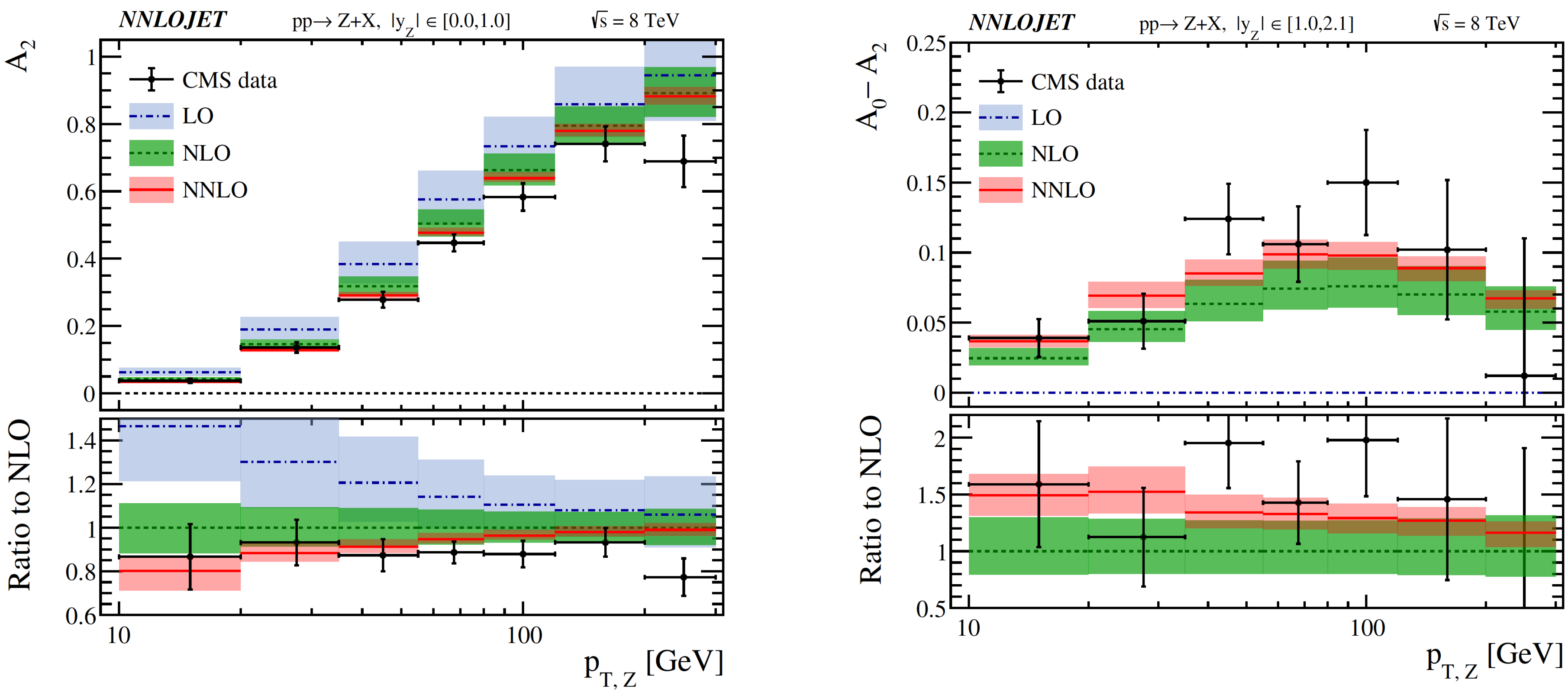
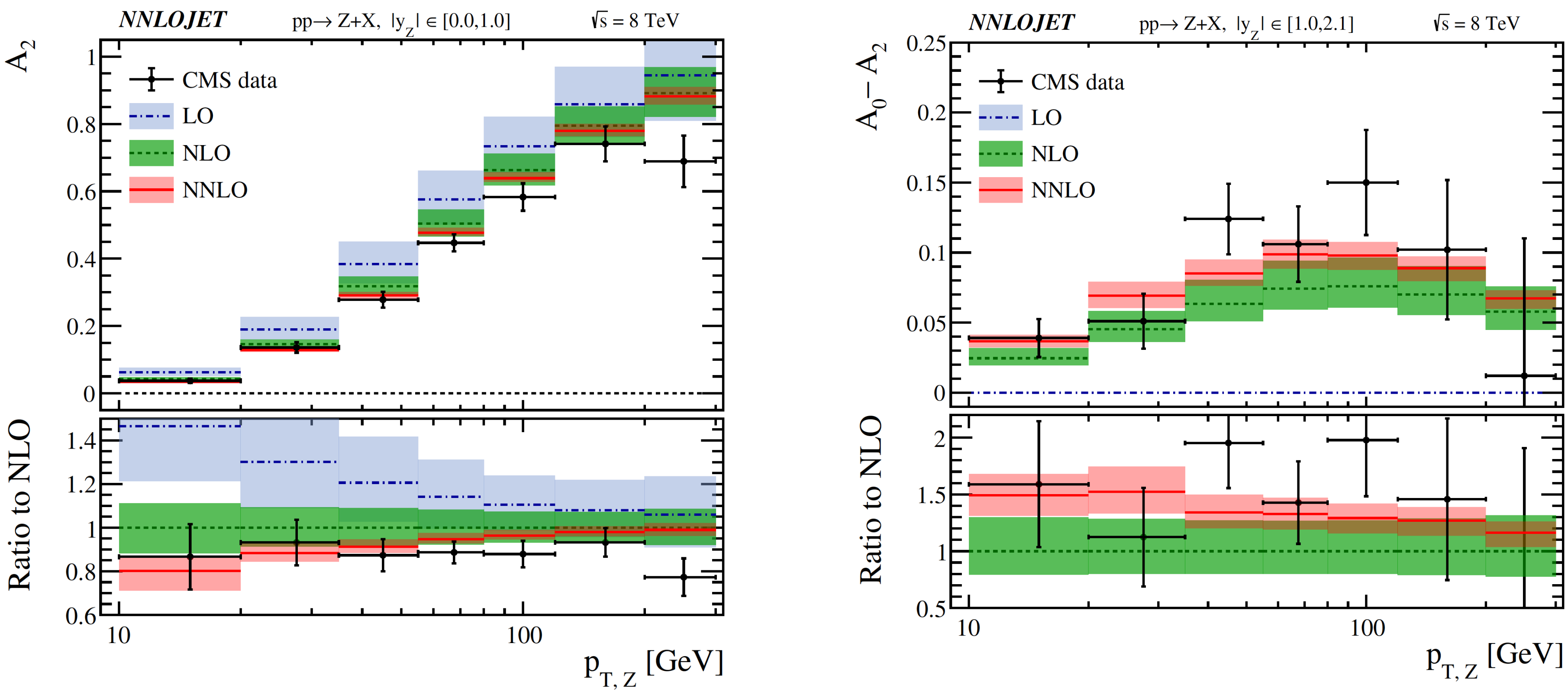


$$\text{NLO (CMS):} \qquad \chi^2/\text{N}_{\text{data}} = 24.5/14 = 1.75$$

$$\text{NNLO (CMS):} \qquad \chi^2/\text{N}_{\text{data}} = 14.2/14 = 1.01$$

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss '17



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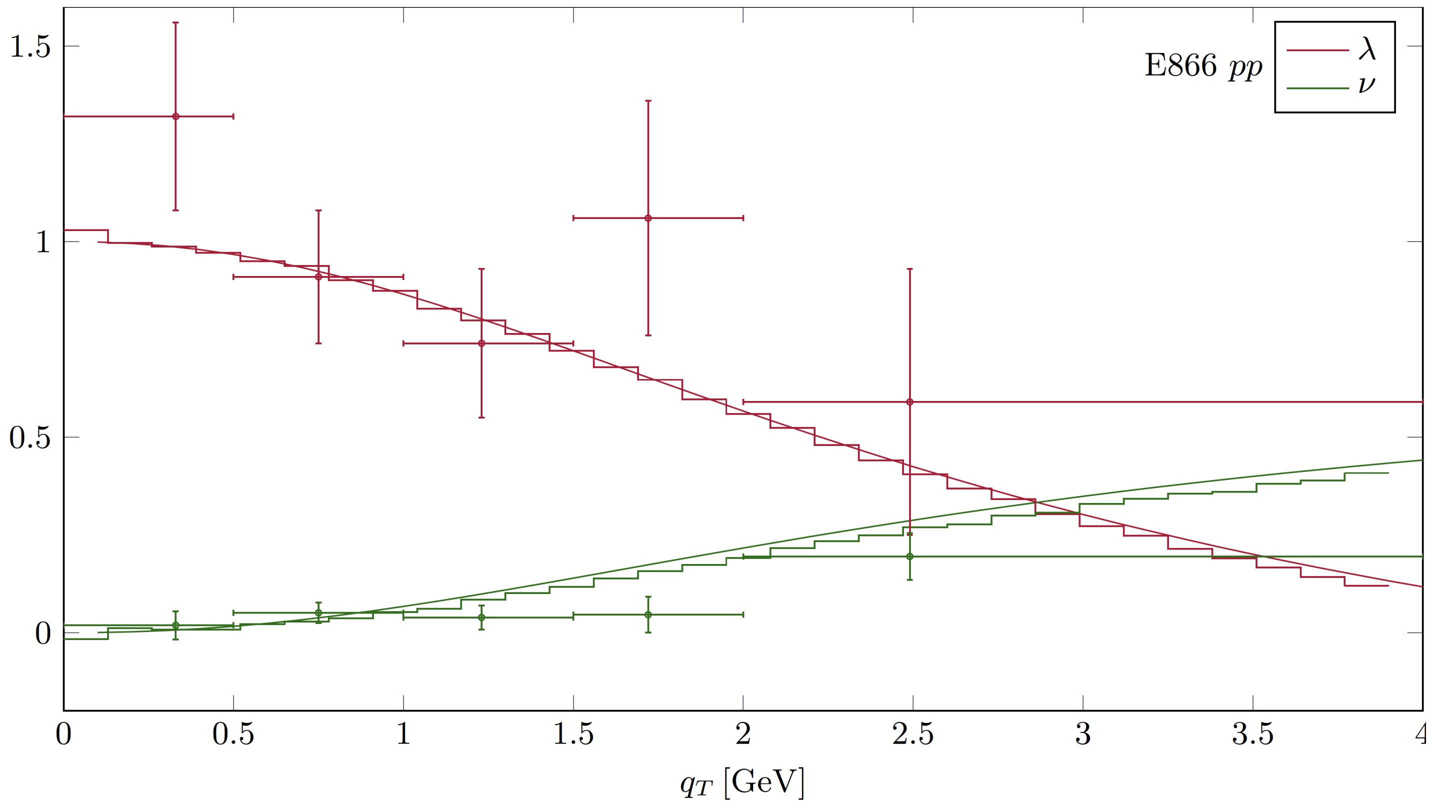
see also: Lambertsen, WV '16

Peng, Chang, McClellan, Teryaev '15

Lambertsen, WV '16

$pp, \ E = 800 \text{ GeV}$

E866

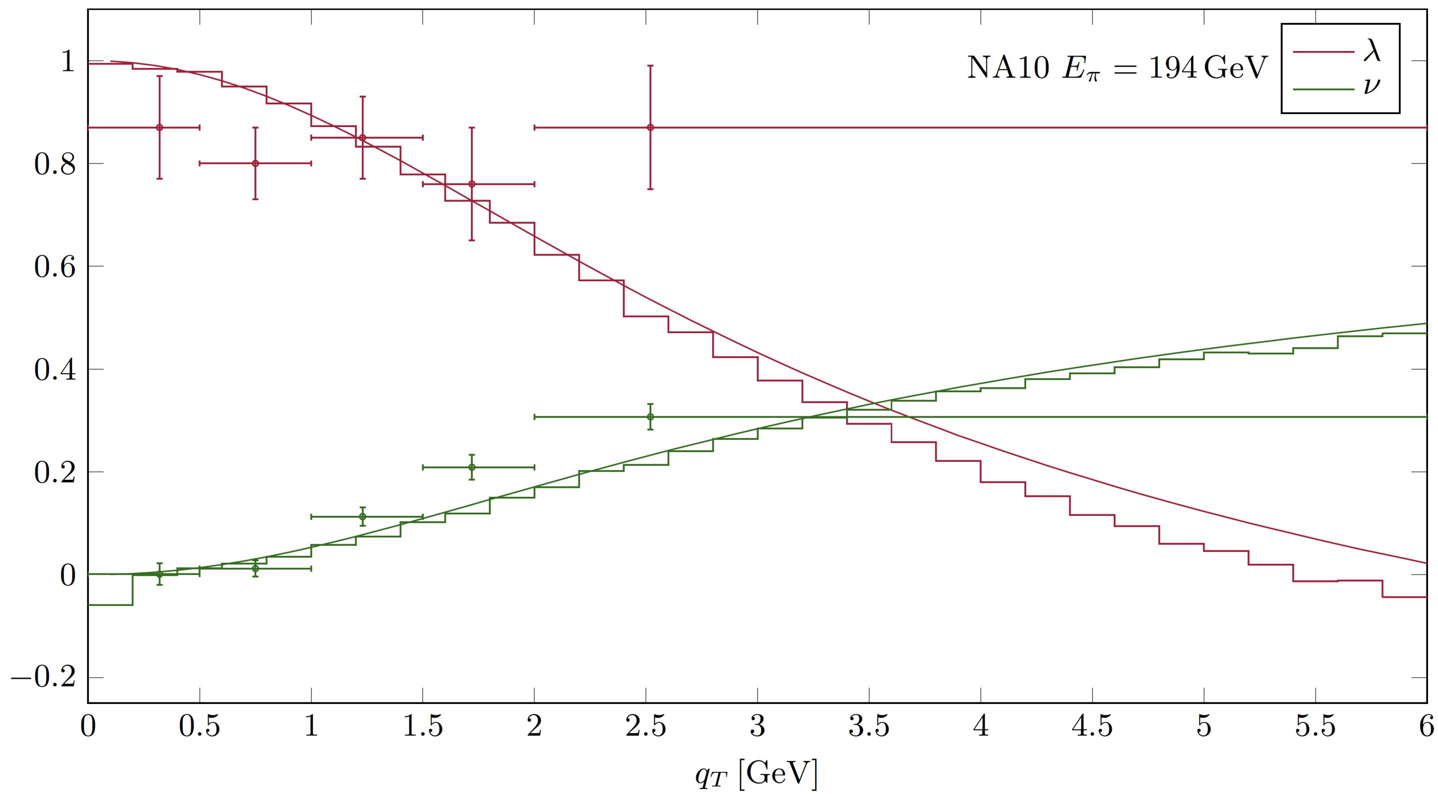


lines: LO $\mathcal{O}(\alpha_s)$

histograms: NLO $\mathcal{O}(\alpha_s^2)$

$\pi W,\ E_\pi = 194\,\mathrm{GeV}$

NA10



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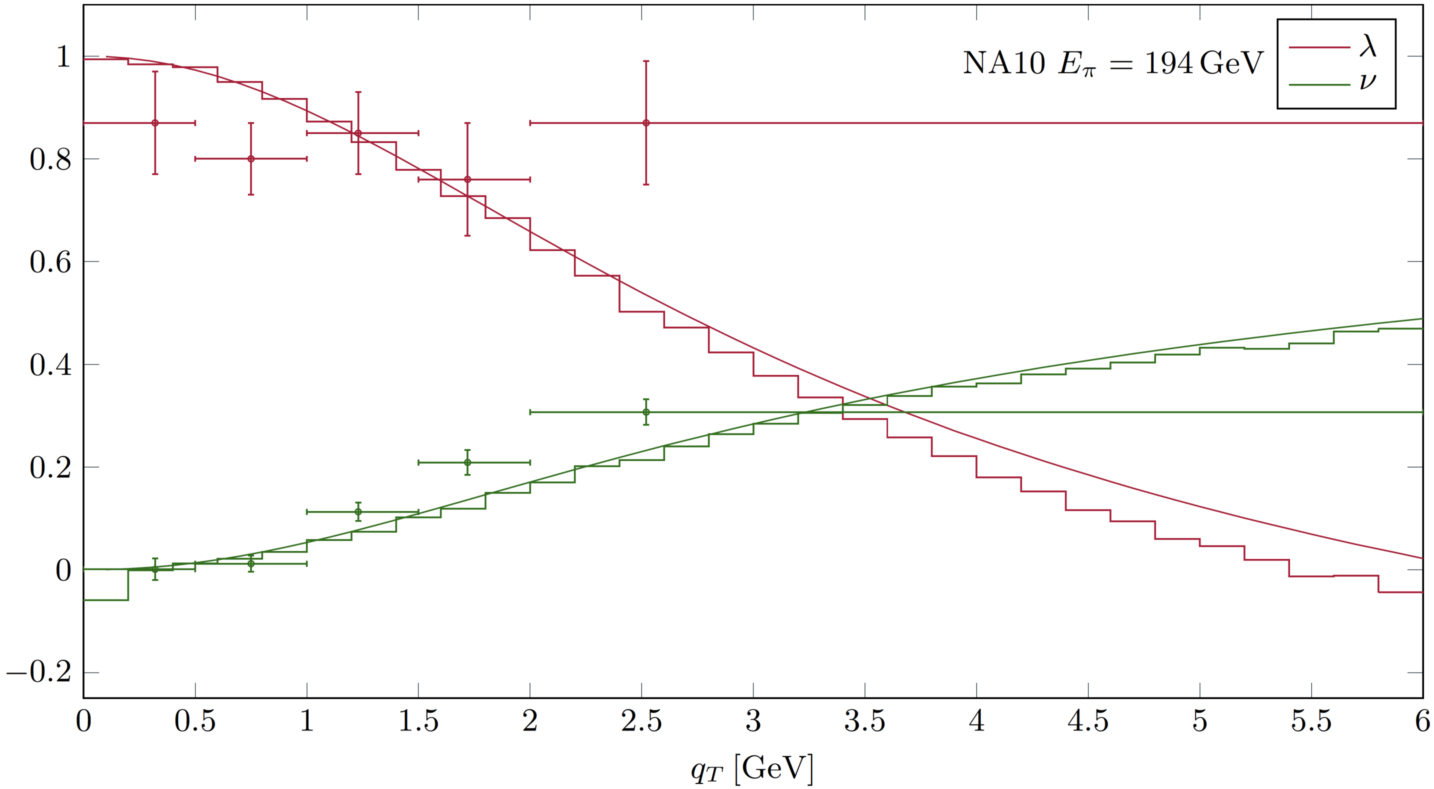

- “dispel myth” that pQCD cannot describe data: overall reasonable description

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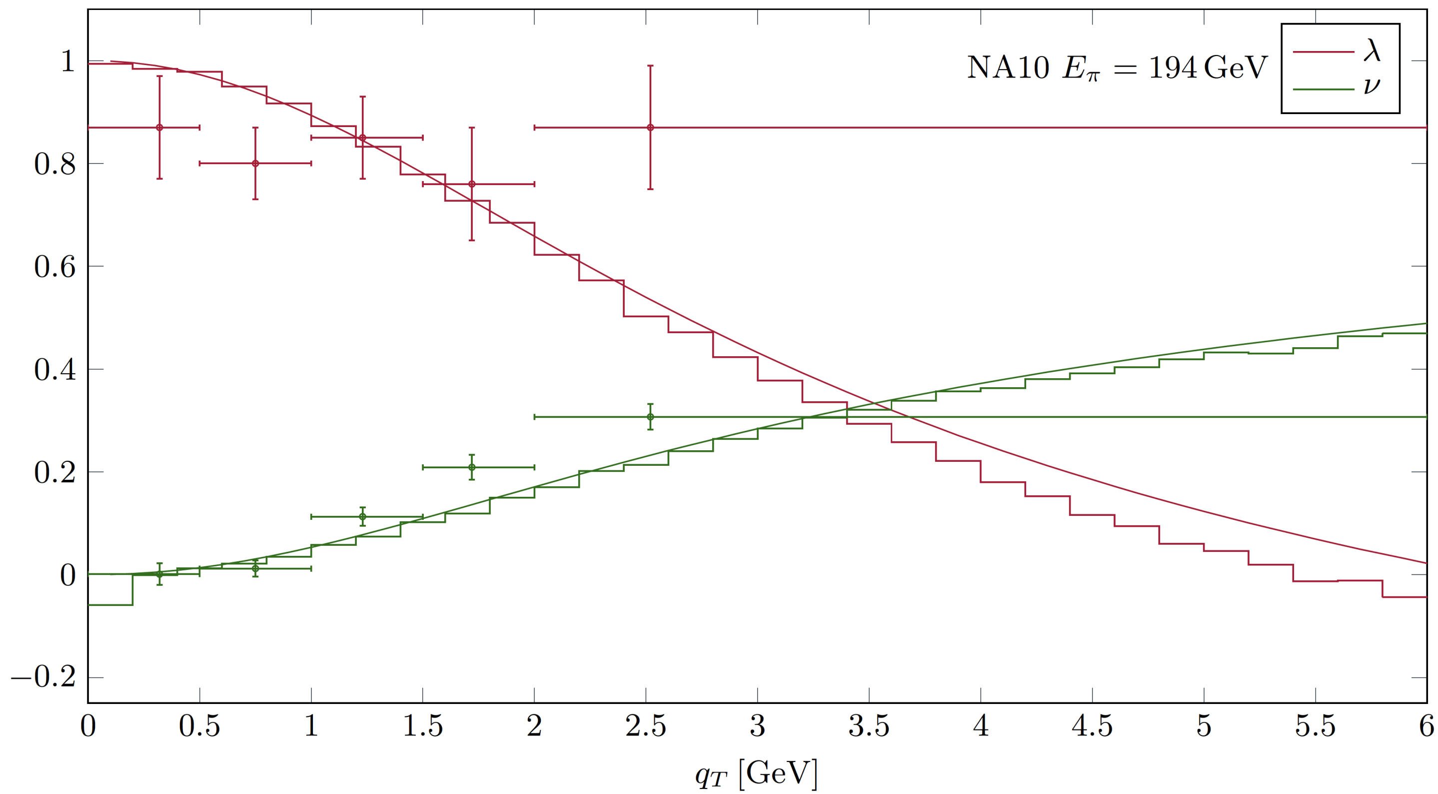
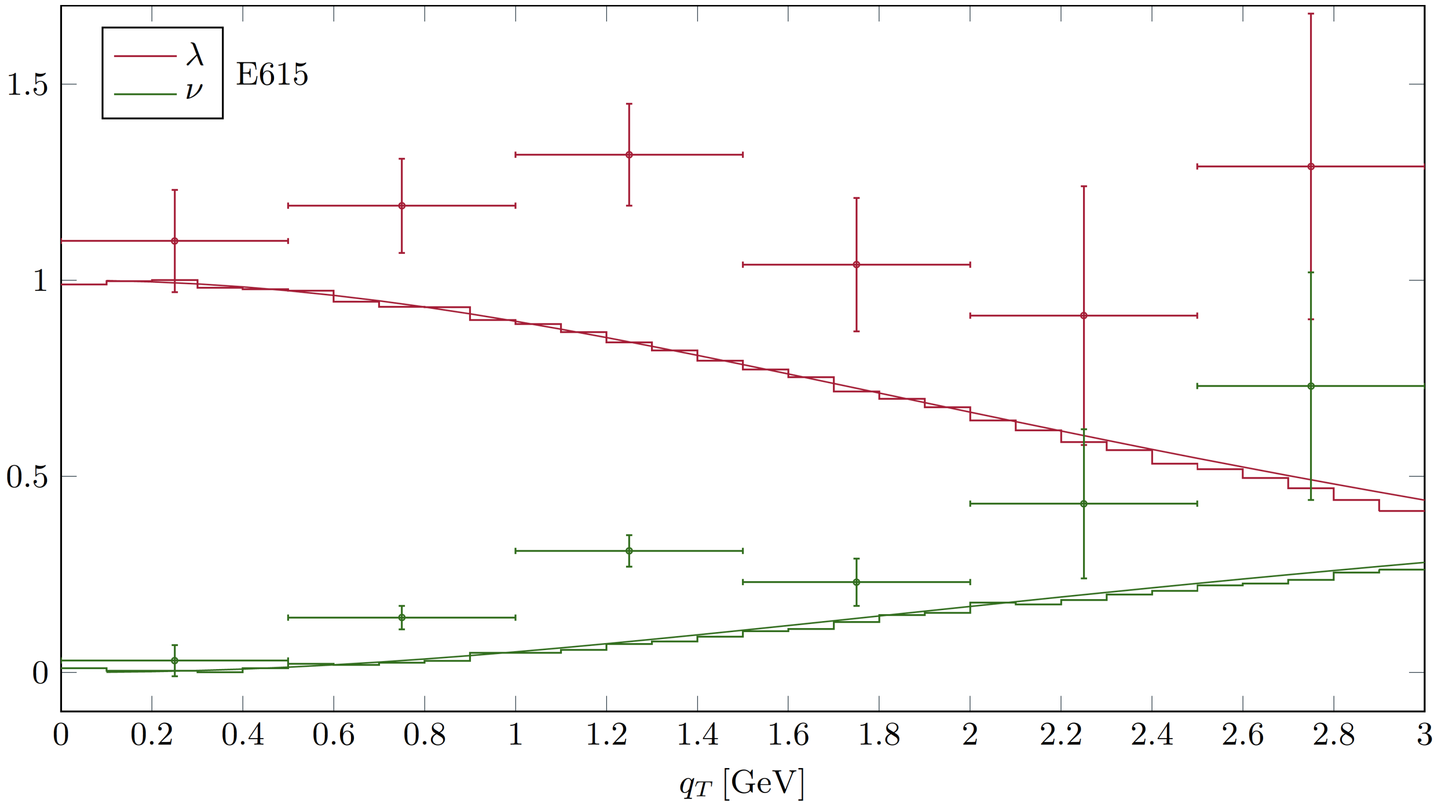
NA10



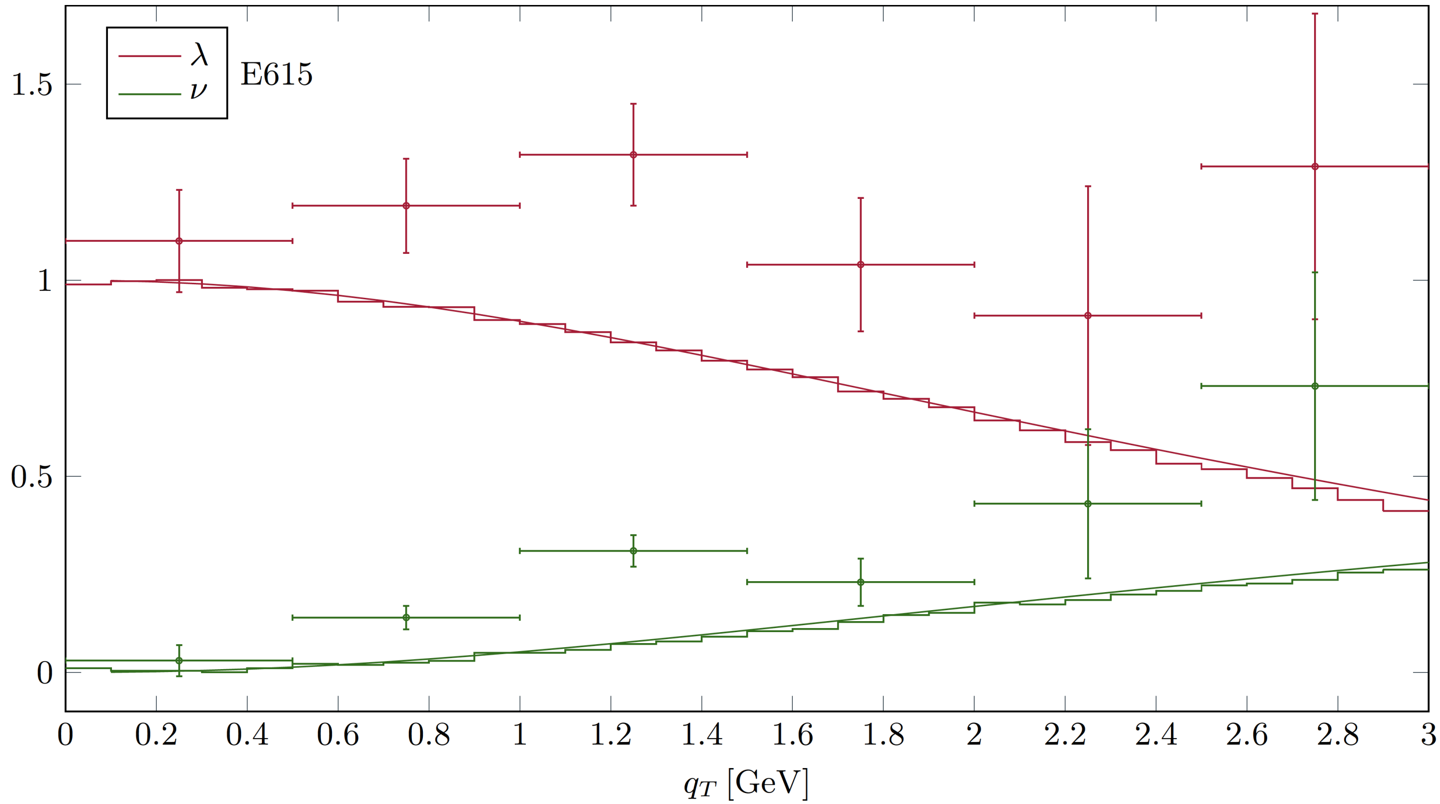
- “dispel myth” that pQCD cannot describe data: overall reasonable description
- relevant for extraction of Boer-Mulders fcts.

$\pi W,\ E_\pi = 252\,\mathrm{GeV}$

E615

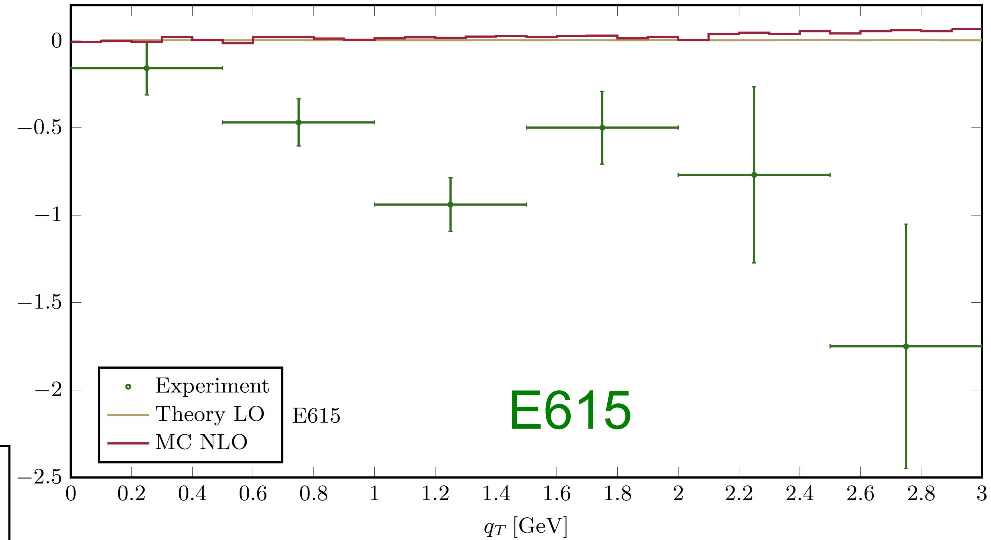


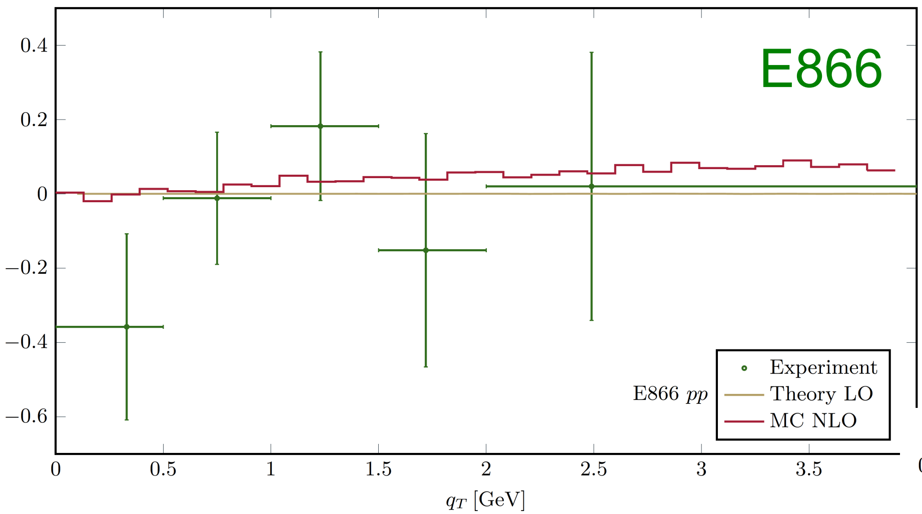
$\pi W,\ E_\pi = 252\,\mathrm{GeV}$ E615

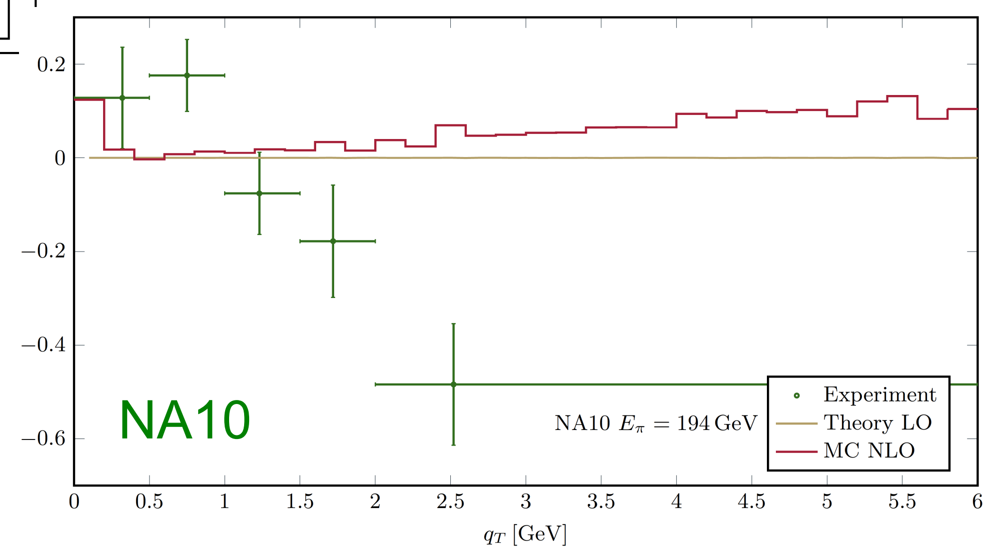


- note: positivity constraint $\lambda \leq 1$ Lam, Tung '78

Lam-Tung $1 - \lambda - 2\nu$

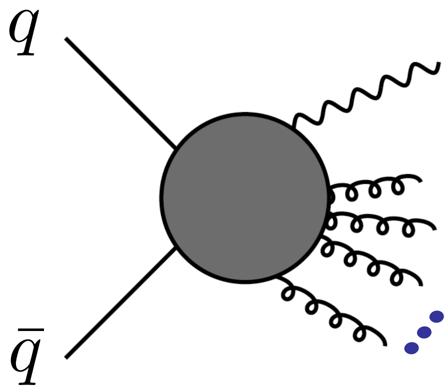






Resummation

- region $q_T \ll Q$:



$$\widehat{W}_T^{(k)} \propto \alpha_s^k \left(\frac{\log^{2k-1}(q_T^2/Q^2)}{q_T^2} \right)_+ + \ldots$$

"q_T logarithms"

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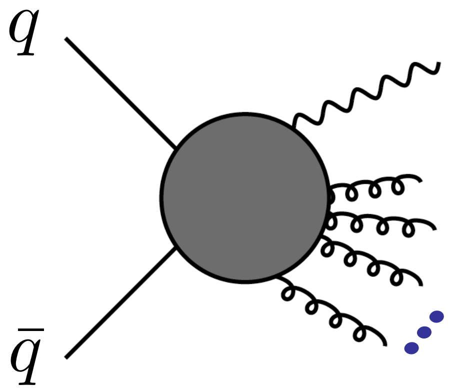

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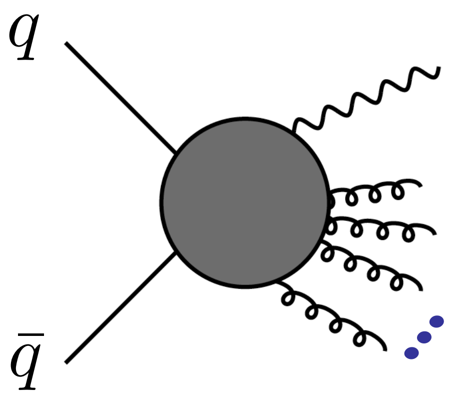

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- 1-1 correspondence to TMD evolution

  Collins, Mert Aybat, Rogers, Qiu; Echevarria, Melis, Scimemi, d'Alesio; Scimemi, Vladimirov; Kang, Prokudin, Sun, Yuan;…

- specifically structure functions at LO (CS frame):

$$\widehat{W}_T^{(1)} = -C_F \frac{\alpha_s}{2\pi} \frac{2\log(q_T^2/Q^2)+3}{q_T^2} + \ldots$$

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Boer, WV

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Berger, Qiu, Rodriguez

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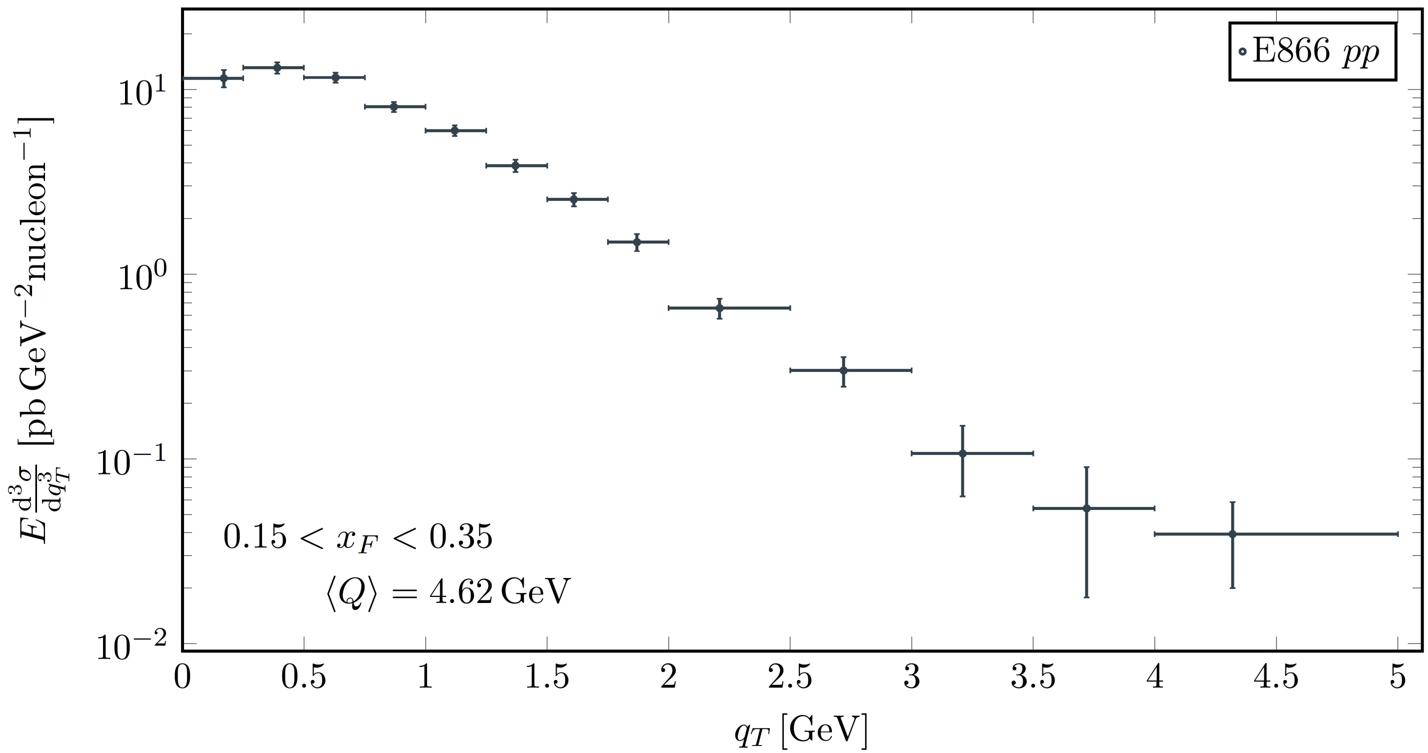
- at present, resummation for angular coeff. not fully understood. Vital for TMD phenomenology!

- in fact, would like to understand full q_T spectrum:



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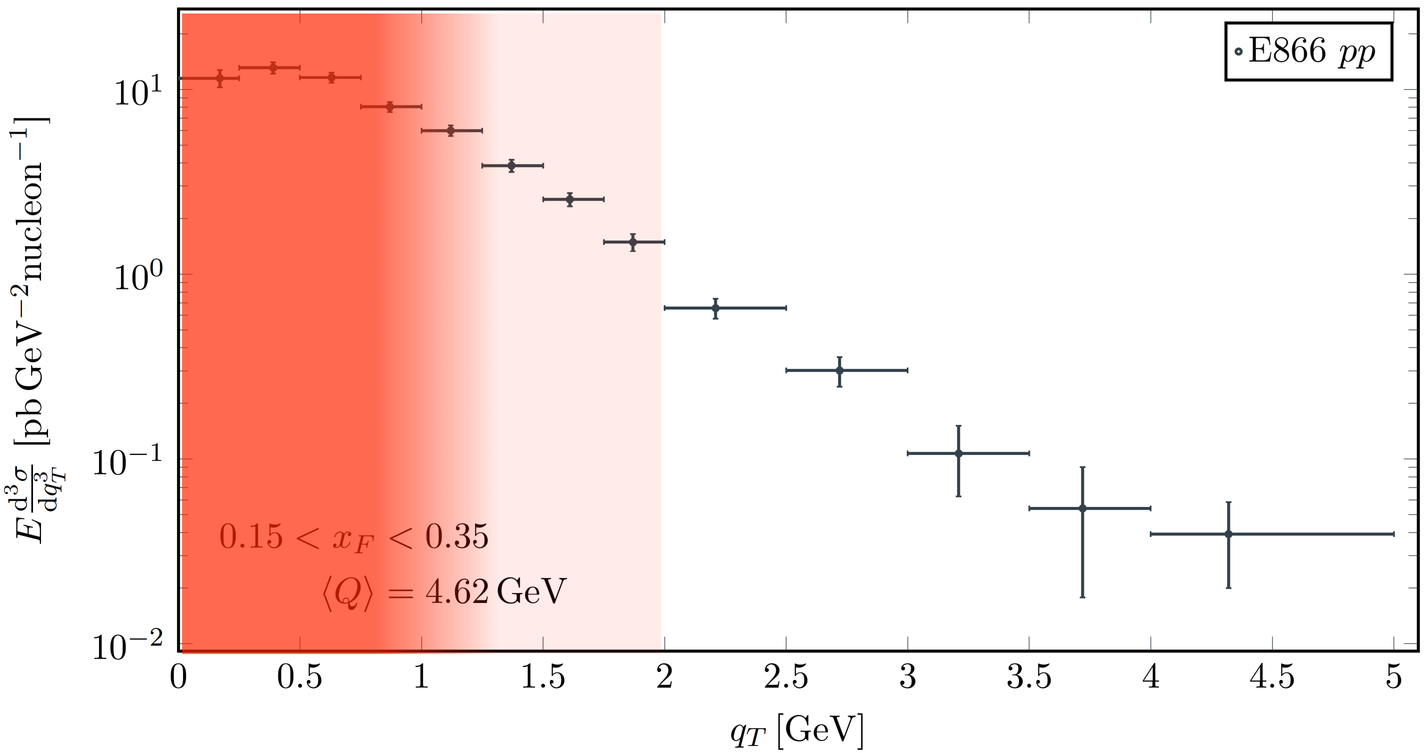

TMD regime

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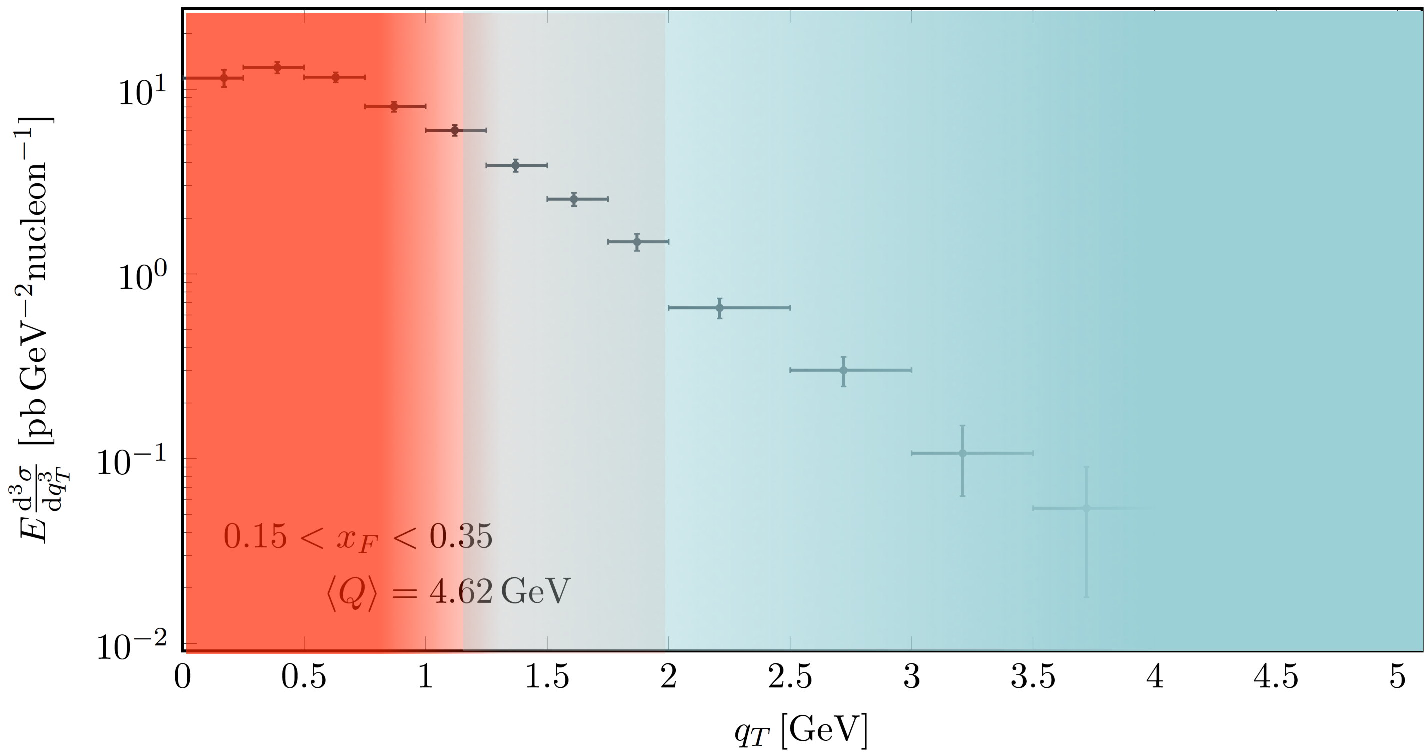

TMD regime collinear regime

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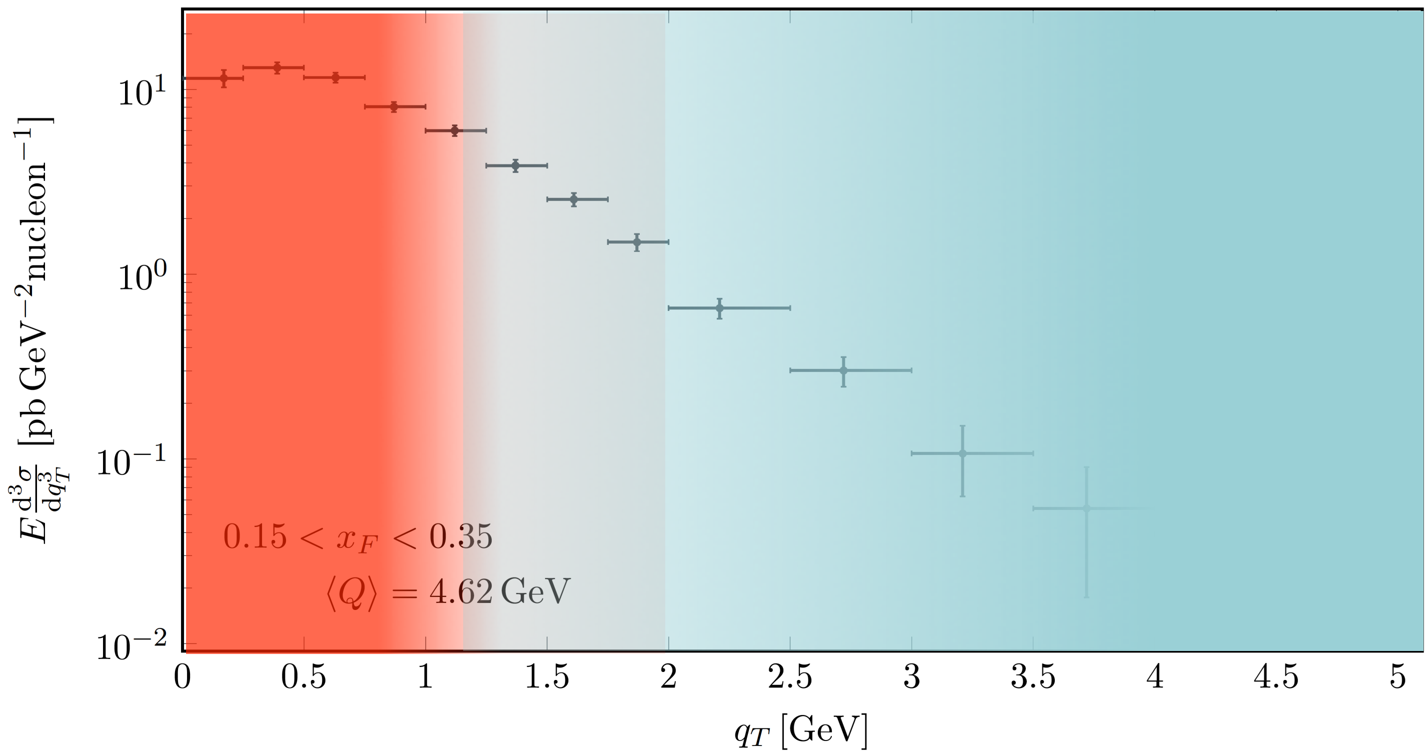


TMD regime

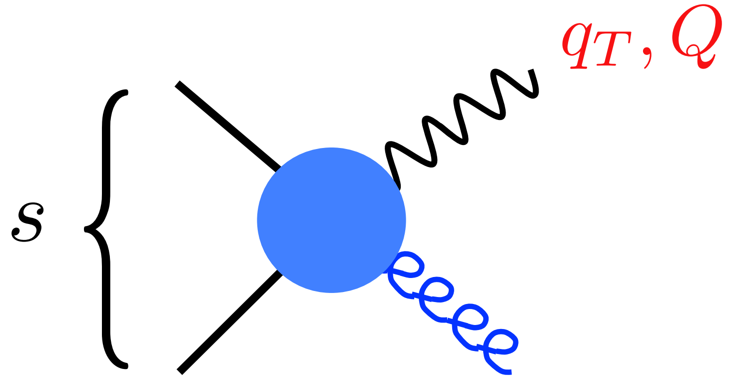
collinear regime

- crucial for matching ("Y term")

Vladimirov,…

- develops different set of large logs

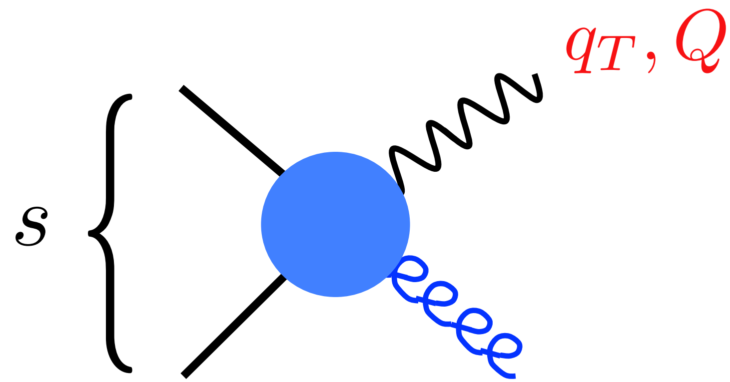
- LO :



$$\sqrt{s} \geq q_T + \sqrt{Q^2 + q_T^2}$$

$$y_T \equiv \frac{q_T + \sqrt{q_T^2 + Q^2}}{\sqrt{s}} \leq 1$$

- LO :



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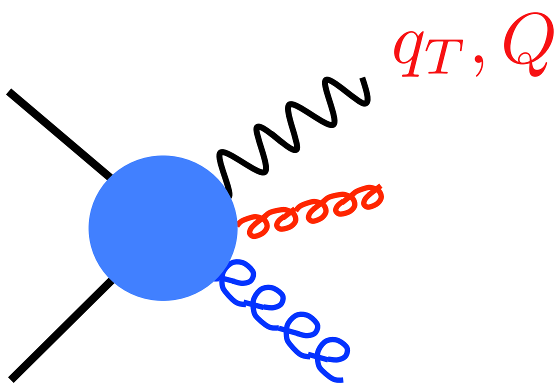
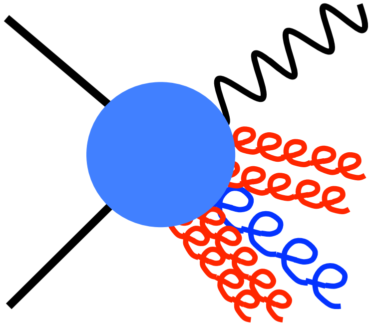
- NLO :



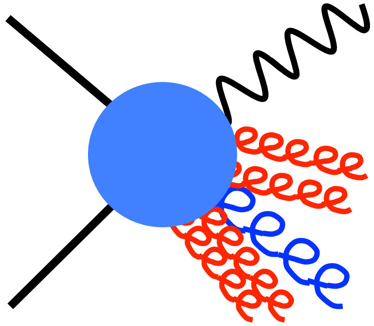
$$\frac{d\hat{\sigma}^{\rm NLO}}{dq_T} \propto \alpha_s \left[\mathcal{A} \log^2(1 - y_T^2) + \mathcal{B} \log(1 - y_T^2) + \mathcal{C} \right]$$

- N^{k}LO :



$$\frac{d\hat{\sigma}^{\mathrm{N}^k\mathrm{LO}}}{dq_T} \propto \alpha_s^k \log^{2k}(1 - y_T^2) + \dots$$

- $\mathrm{N^kLO}$:



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- threshold logarithms

de Florian, Kulesza, WV; Kidonakis, Gonsalves; …

- threshold resummation:

$$\hat{\sigma}_{q\bar{q}\to\gamma^* g}(N) \equiv \int_0^1 dy_T^2 (y_T^2)^{N-1} \frac{d\hat{\sigma}_{q\bar{q}\to\gamma^* g}}{dp_T}$$

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↑ LO cross sec.

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soft/coll. gluons

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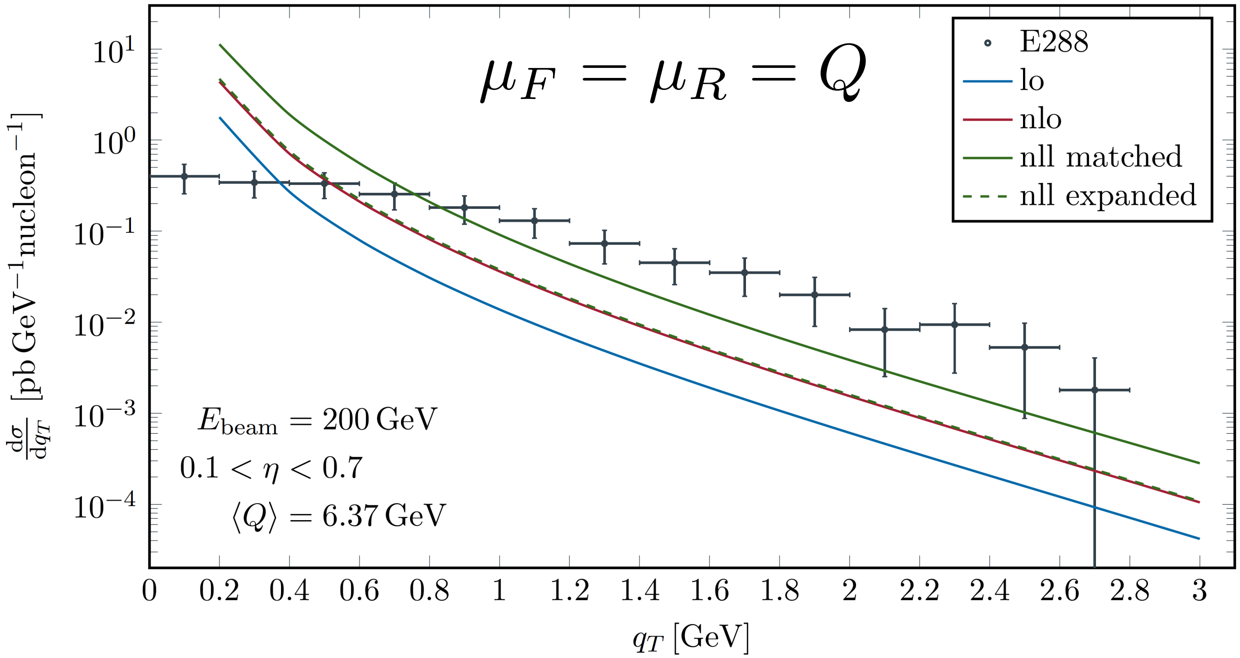
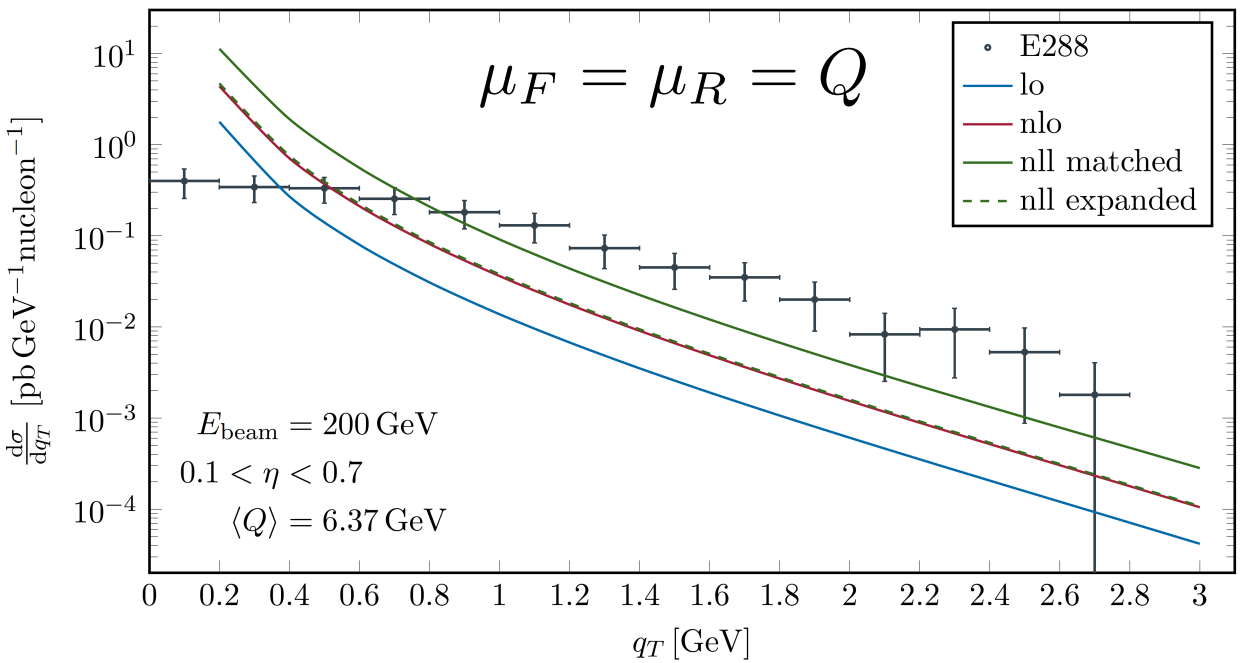
LO cross sec.

$$\ln \Delta_N^q = \int_0^1 dz \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_q(\alpha_s(q^2))$$



Lambertsen, Steiglechner, WV



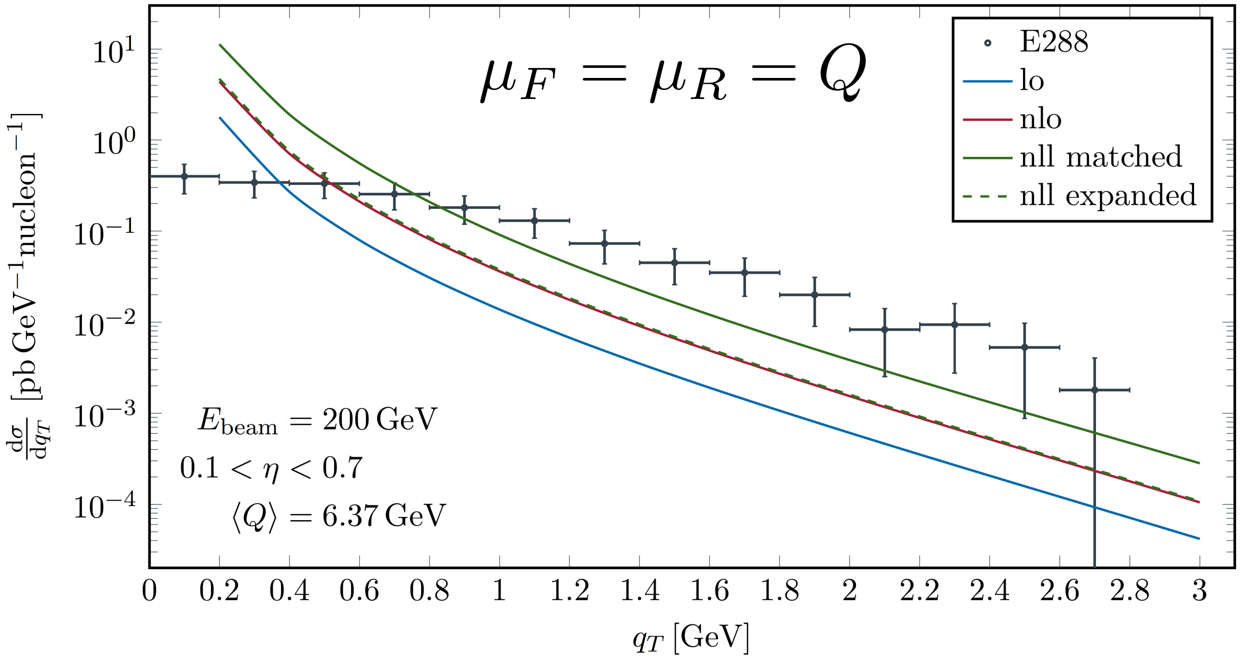
Lambertsen, Steiglechner, WV

$$\frac{Q}{\sqrt{2}} \leq \mu \leq \sqrt{2}Q$$

$\pm 7\%$

$\pm 25\%$

$\pm 40\%$



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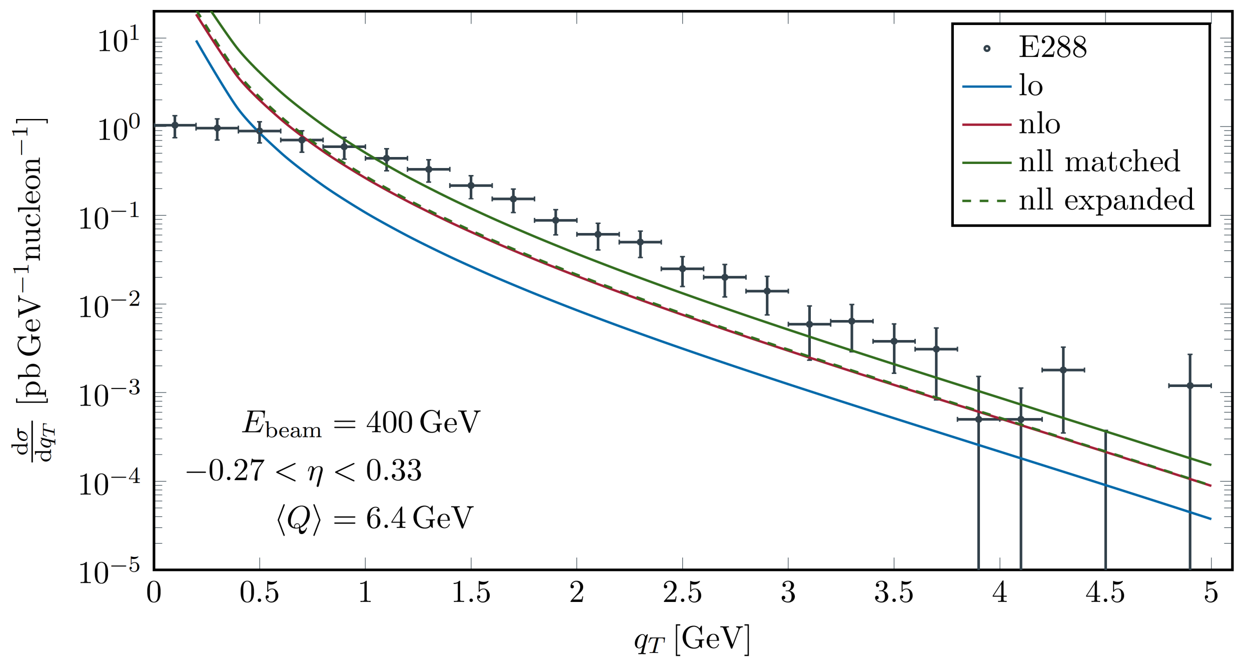

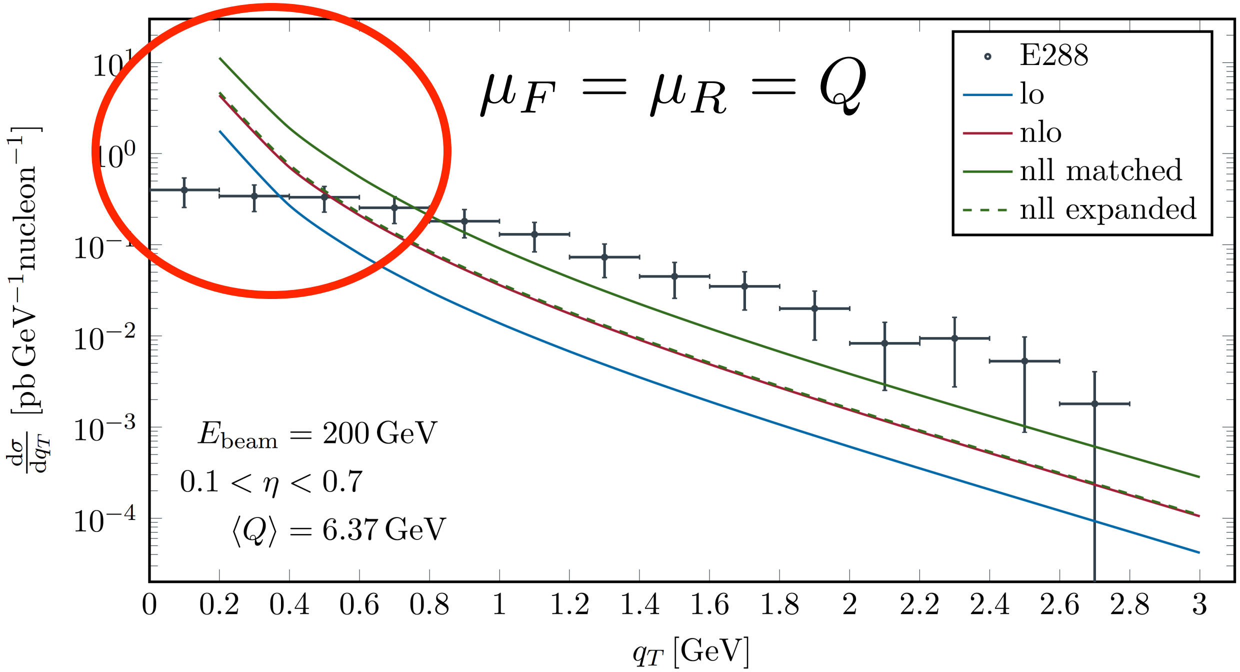

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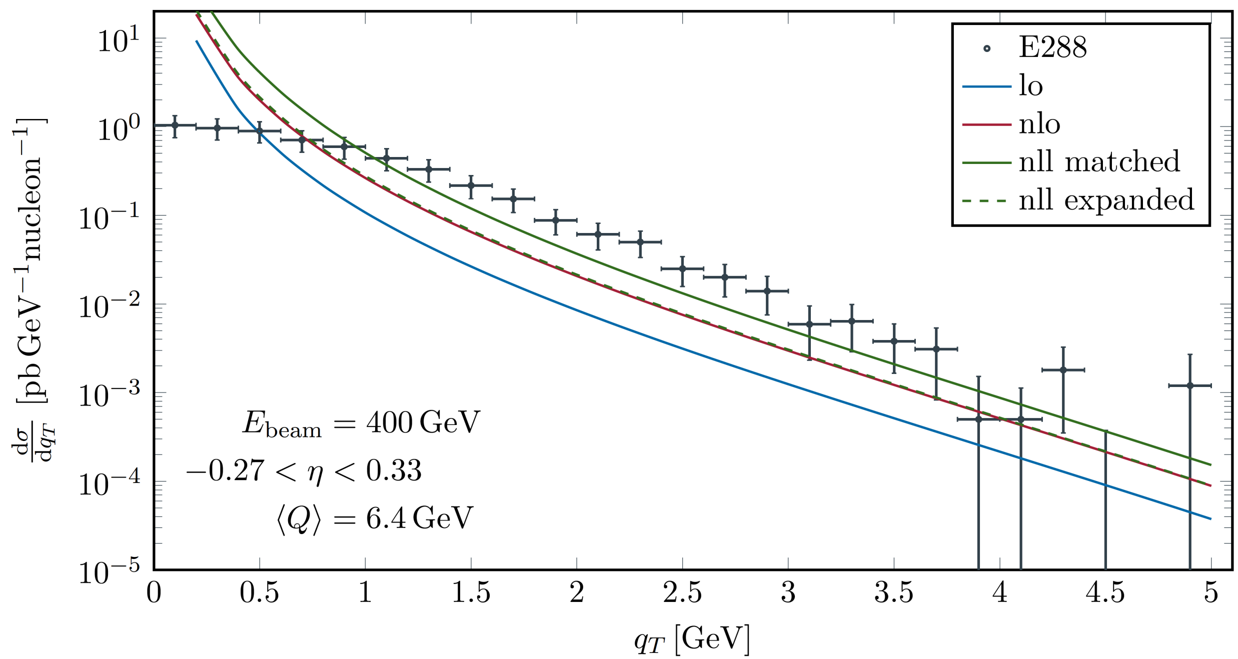
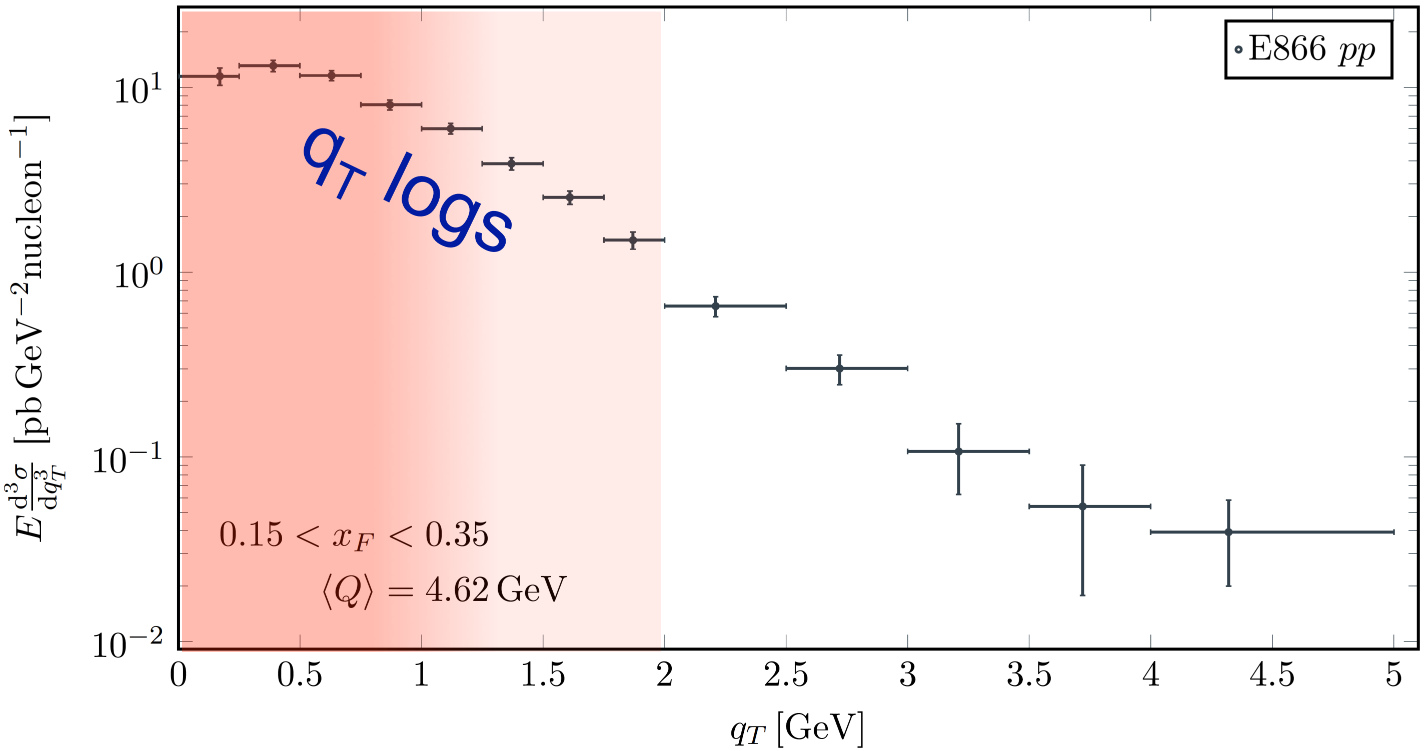
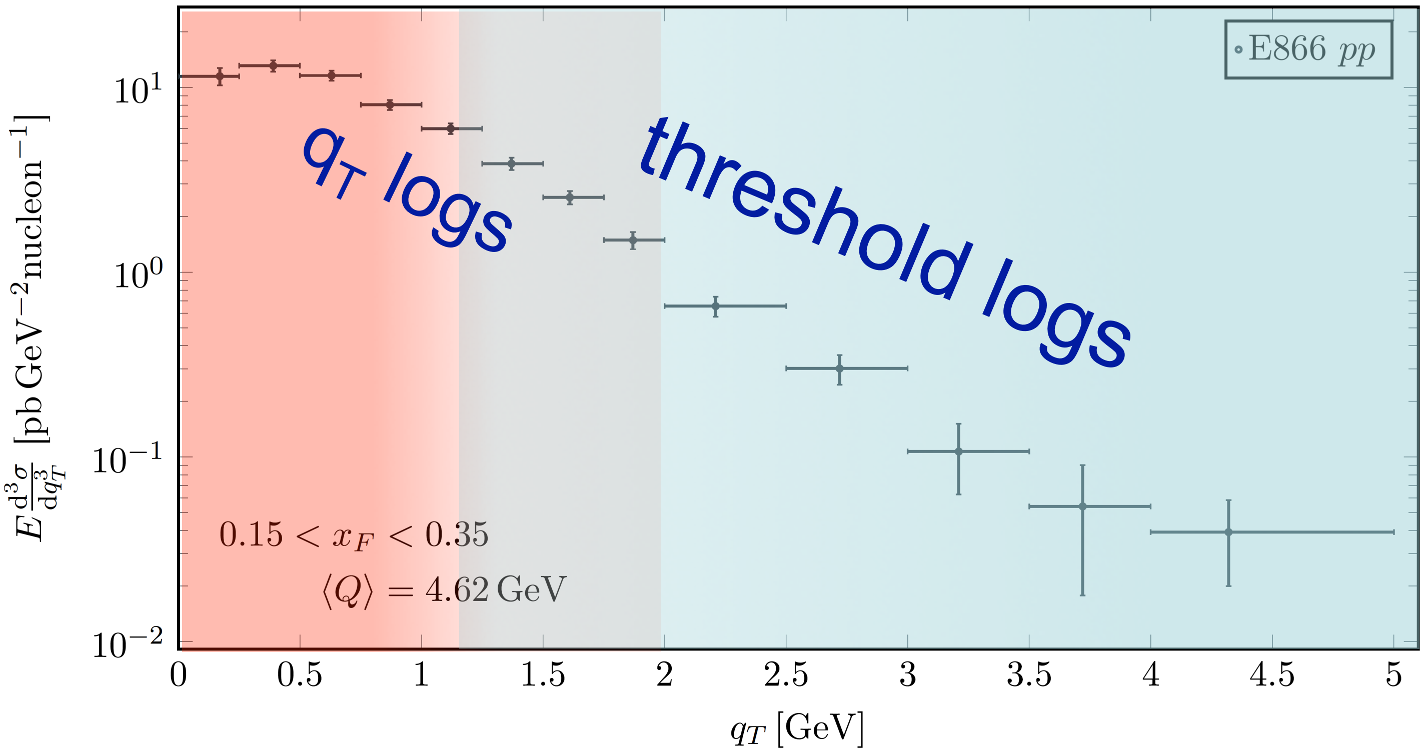




E866 pp
q_T logs
$0.15 < x_F < 0.35$
$\langle Q \rangle = 4.62\,\mathrm{GeV}$
$E\frac{d^3\sigma}{dq_T^3}$ [pb GeV^{-2}nucleon^{-1}]
q_T [GeV]



E866 pp
q_T logs
threshold logs
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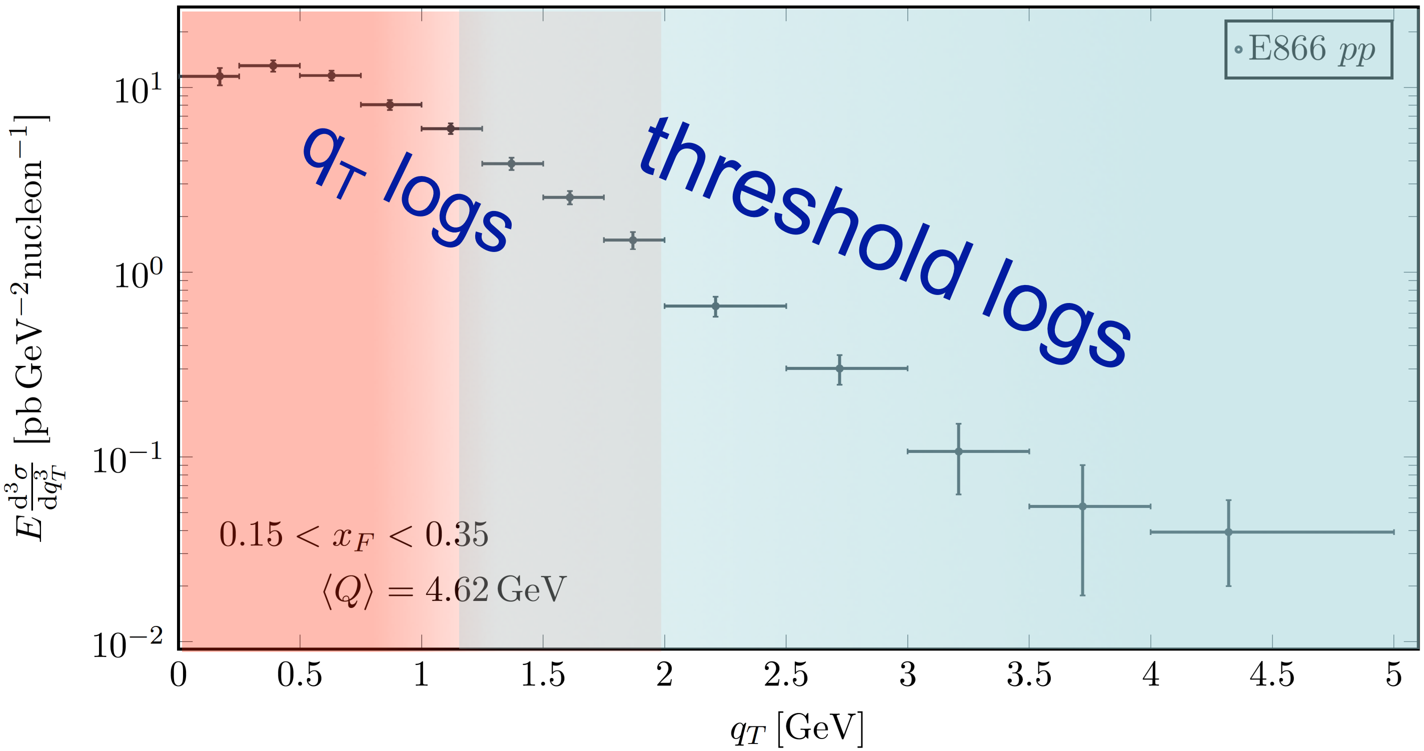
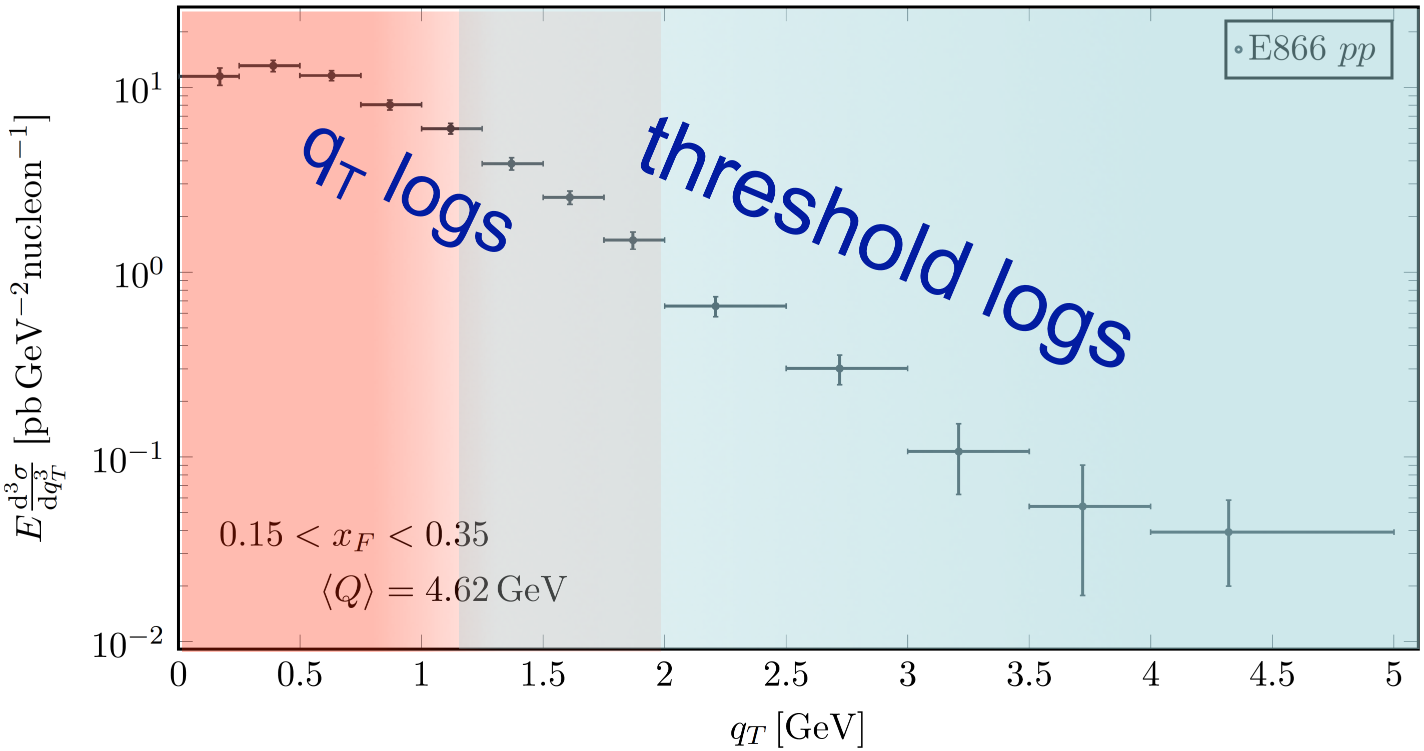


- note, threshold resummation for angular dependences not known



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- ultimately, will need joint resummation of both types of logs

- framework exists:

Laenen, Sterman, WV ’00
Lustermans, Waalewihn, Zeune ‘16
Muselli, Forte, Ridolfi ‘17

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Laenen,Sterman,WV
Kulesza,Sterman,WV

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- Drell-Yan at high q_T: Muselli, Forte, Ridolfi

Conclusions and outlook:

- overall reasonable pQCD description of λ, μ, ν
- *not* meant to argue that there are no effects beyond fixed-order pQCD
- presently not a really good understanding of Drell-Yan cross section at high q_T
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- same issues at EIC!

Koike, Nagashima, WV

$$\frac{d^5\sigma}{dQ^2 dx_{bj} dz_f dq_T^2 d\phi} = \sigma_0 + \cos(\phi)\sigma_1 + \cos(2\phi)\sigma_2$$

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- full QCD: NNLL: Hinderer, Ringer, Sterman, WV, to appear