TECHNISCHE
UNIVERSITÄT
WIEN
Vienna University of Technology

Introduction to Superconductivity

ESAS/EASITrain September 2018, Vienna

Franz Sauerzopf

What means Superconductivity For Us?

- A fascinating topic in solid state physics
- Highly relevant technology
- A huge field of scientific research
- (An exhausting playground for theoretical considerations)

Overview

- A first look at superconductivity
- Typ-I Superconductors
- BCS Theory
- Typ-II Superconductors

Overview

- A first look at superconductivity
- Typ-I Superconductors
  - critical temperature T_c, critcal field H_c magnetization and current transport
  - Thermodynamics
  - London Model
  - Josephson effect (leading to applications)
- BCS Theory
- Typ-II Superconductors

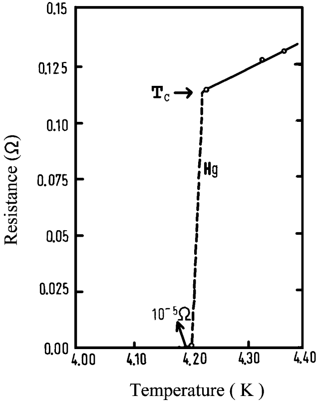
Overview

- A first look at superconductivity
- Typ-I Superconductors
- BCS Theory
  - brief overview
- Typ-II Superconductors

Overview

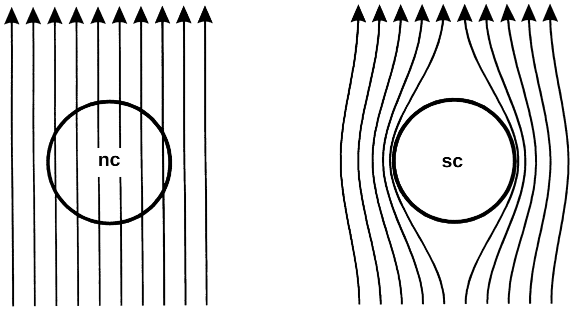
- A first look at superconductivity
- Typ-I Superconductors
- BCS Theory
- Typ-II Superconductors
  - magnetization and critcal fields H_{c1}, H_{c2}
  - Ginzburg-Landau theory
  - vortex lines
  - flux pinning and critical current J_c
  - Applications

First discovery of superconductivity



Heike Kamerlingh-Onnes: 1911
Nobel prize in Physics 1913

Meißner Effect

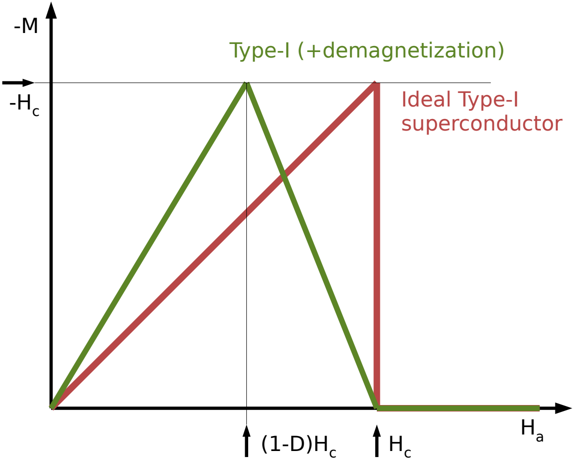


normal conducting

superconducting

Externally applied magnetic fields are completely expelled from the superconductor: *Type I superconductor*

Type I superconductor



Meißner effect: $M = -H_a$, **for** $0 < H_a < H_c$

Thermodynamic description

1. Free energy density

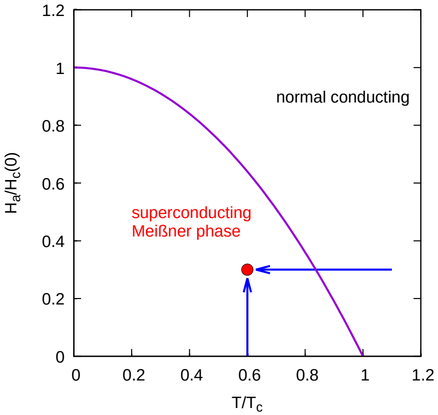
$$\delta G = -S\,\delta T - \mu_0 M\,\delta H$$

2. at the border between super- and normal conductivity:
   $G_N(H_c) = G_S(H_c)$

$\longrightarrow$ Condensation energy density of a superconductor:

$$G_S(0,T) - G_N(0,T) = -\frac{\mu_0 H_c^2}{2}$$

Phase diagram of a Typ-I superconductor



Meißner effect is not only due to induction for $R = 0$, but a basic property of superconductors!

How much current can a superconductor take?

For zero resistance $R = 0$ we have to expect a natural limit for the current flowing in a superconductor.

- **Silsbee Rule:**
  the critical current I_c of a superconductor is reached when the field due to that current reaches the critical field H_c of the superconductor. -> geometry dependent!
- **depairing current:**
  the "binding energy" of the superconducting state (in the simplest case: $\frac{\mu_0 H_c^2}{2}$) is smaller than the kinetic energy of electrons due to this current.

Superconductivity Breaks Down at

- critical temperature T_c
- critical (magnetic) field H_c
- critical current (density) J_c, which is a consequence of H_c in Type-I superconductors

All of these values are rather small in Type-I superconductors.

→ Applications are only possible at low magnetic fields and temperatures

Search for theoretical explanations

In order to explain experimental data, the physics toolbox was used and people came up with a number of semi-empirical models, revealing a lot of the inner workings of superconductivity.

We already saw the thermodynamics approach, leading to the condensation energy density $-\frac{\mu_0 H_c^2}{2}$

Two-Fluid-Model (Gorter & Casimir)

1. fraction of superconducting electrons: ζ
2. fraction of normal conducting electrons: $1-\zeta$

lead to corresponding terms in the free energy:

1. condensation energy $G_S(T) = G(\zeta = 1, T) = \frac{\mu_0 H_c^2}{2}$
2. electronic specific heat (coeffizient γ):
   $G_N(T) = G(\zeta = 0, T) = \frac{\gamma}{2} T^2$

after some changes and some calculation we find:

$$\frac{H_c(T)}{H_{c0}} = 1 - \left(\frac{T}{T_c}\right)^2$$

London Model – two London equations

- defining current from movement of electrons: $\vec{j}_s = e\, n_s\, \vec{v}_s$ immediately leads to the first London equation:

$$\dot{\vec{j}}_s = \frac{n_s e^2}{m_e} \vec{E}$$

- Applying some Maxwell equations and some mathematics reveals the second London equation:

$$\Delta \vec{h} = \frac{1}{\lambda_L^2} \vec{h},$$

which defines the London penetration depth

$$\lambda_L^2 = \frac{m_e}{\mu_0 n_s e^2}$$

London Theory and Quantum Physics

Rewrite the 2nd London equation (...) to the form of the canonical impulse of a *superconductor particle*

$$\vec{p}_s = m_s\vec{v}_s + q_s\vec{A} = q_s\left(\frac{m_s}{q_s^2 n_{ss}}\vec{j}_s + \vec{A}\right) = \vec{\nabla}P$$

and you see that $m_s = 2m_e, q_s = 2e$, with $n_{ss} = n_s/2$

→ the superconducting charge carrier consists of two electrons

The Superconducting Quantum State

$$\vec{p}_s = m_s \vec{v}_s + q_s \vec{A} = q_s \left(\frac{m_s}{q_s^2 n_{ss}} \vec{j}_s + \vec{A} \right) = \vec{\nabla} P$$

The canonical impulse $\vec{p}_s$ is invariant, except for the gauge field P. Therefore, $\vec{p}_s$ is a sharp value, independent of $\vec{A}$. This implies that its extension in space can be infinitely large (c.f. the Heisenberg uncertainty relation).

What can we learn from London Theory?

- A magnetic field decays exponentionally from the surface into a superconductor, with the characteristic length λ_L.
- Superconductivity is performed by charge carriers consisting of two electrons.
- $\rightarrow$ A superconductor is a macroscopic quantum object

Flux Quantisation

From the canonical impulse we can calculate the phase difference between two points in a superconductor. In a calculation similar to the one leading to Bohr's atomic model, we can calculate this phase difference along a ring, and see that it has to be a multiple of 2π.
This implies that the flux inside of a massive superconducting ring can only be a multiple of the flux quantum Φ_0

$$\Phi_n = n\Phi_0, \text{with} \Phi_0 = 2.07 \cdot 10^{-15} \text{Wb}$$

Josephson Equations

If two superconductors are separated by a thin barrier, which allows a weak interaction between them, the system can be described by a set of two equations, the Josephson equations.

1. JE. :
   $I_s = I_0 \sin \delta\varphi$
   The direct current flowing across the boundary is proportional to the sine of the phase difference across this boundary. This current flows without an electric voltage.
2. JE. :
   $I_s = I_0 \sin 2\pi \nu t$ with the frequency $\nu = V/\Phi_0$
   Applying a voltage V across the boundary leads to an alternating current with a freqency which is proportional to the applied voltage.

SQUIDs

Based on the two Josephson equations, we can construct extremely sensitive measuring instruments, which allow to measure into the regime governed by the Heisenberg uncertainty relationship.
Superconducting Quantum Interference Devices

BCS Theorie

In 1957, 56 years after the discovery of superconductivity, the first satisfying theory on the origin of the effect was given. John Bardeen, Leon Cooper und John Schrieffer described the so called BCS-Theory (Nobel prize 1972).

- A weak attractive interaction between electrons, mediated by vibrations in the crystal lattice
- At low temperatures, the attractive interaction can overcome thermal activation of electrons, and lead to the formation of electron pairs (Cooper pairs), which are Bosons.
- These Cooper pairs strongly overlap and form a single quantum state, similarly to a Bose-Einstein condensate.
- In this state, single electrons cannot interact with the lattice, and, therefore, can move in the crystal without resistance.

Some Basic Properties Of BCS Theory

- A narrow gap $2\Delta_0$ appears in the electronic density of states around the Fermi energy E_F. ($E_f \approx 5$ eV, und $2\Delta(0) \approx 10^{-4}$ eV)
- The density of states is strongly enhanced (diverges) below and above this gap.
- The superconductor's critical temperature is proportional to this gap. $\Delta(0) = 1.764\, k_B T_c$
- The electronic ground state is lowered by

$$W_n - W_s = N_0 \frac{\Delta^2(0)}{2}$$

from the normal conducting state (cf. condensation energy).

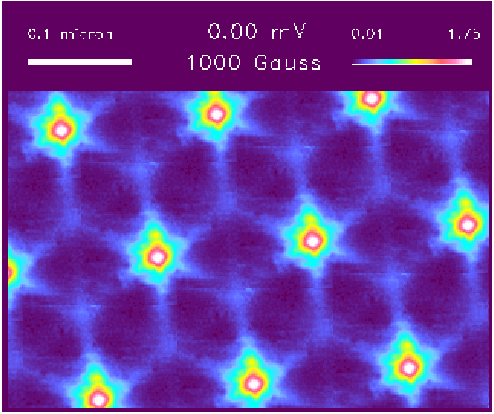
Type-II superconductors

- Magnetic flux penetrates into the superconductor
- Flux expulsion (Meißner state) ends at the field H_{c1}
- Superconductivity ends at the field H_{c2}

H_{c2} can be very high, allowing technical applications using these materials.

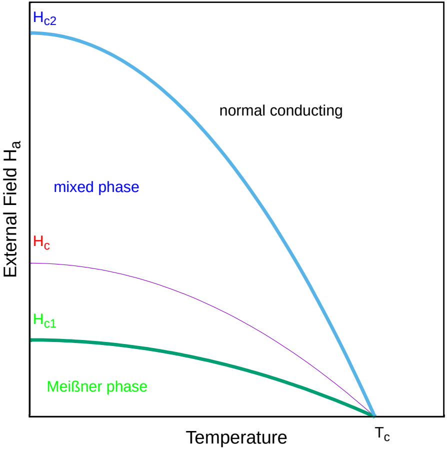
Flux Lines (Vortices) in Type-II superconductors

In a Type-II superconductor, the magnetic flux appears in cylindrical regions, the so called flux lines, or vortices.

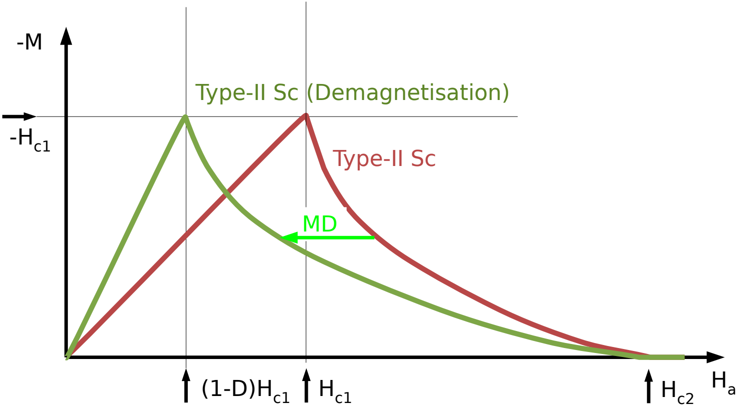


Each vortex contains the exactly one flux quantum Φ_0.

Phase Diagram Of A Type-II Superconductor



Type-II Magnetisation Curve



Ginzburg-Landau Theory

- Local theory (comparable to London theory)
- Derived from Landau's theory for 2^{nd} order phase transitions.
- Expand the free energy near the phase transition (at T_c) into a sum depending on an order parameter ψ_0, linearize by cutting off after first order in ψ_0.
- Identify $\psi_0 \rightarrow \psi$ as the effective wave function of the superconducting charge carriers, including coupling to the magnetic field.

Ginzburg-Landau Theory, first steps

The contributions of the first order in ψ and the consideration of special cases for zero and small magnetic fields lead to the Ginzburg-Landau equations, which can be solved and give a description of the superconducting state near the transition temperature T_c.

We can find the condition for the existence of Type-I and Type-II superconductors, as well as a description of the mixed state in Type-II superconductors.

Characteristic Lengths Of The Ginzburg-Landau Theory

$\xi(T)$, is the lengthscale for changes in the order parameter ψ. As was found out later (Gor'kov) that this length is strongly correlated with the BCS coherence length ξ_0.

$\lambda(T)$, the Ginzburg-Landau penetration length, which describes the distance over which the local field in the superconductor changes.

With these two length scales, we can describe the vortices inside a Type-II superconductor. They consist of a normal conducting core, with radius of about ξ, and around it a region with a radius of about λ in which the field in the core is decaying to zero due to circulating currents.

Ginzburg-Landau Parameter κ

An important parameter for the description of a superconductor is

$$\kappa = \frac{\lambda}{\xi}.$$

which determines whether the energy contribution of the normal- to superconductor interface is positive or negative.

- $\kappa < 1/\sqrt{2}$ ...positive interface energy...Type-I
- $\kappa > 1/\sqrt{2}$ ...negative interface energy...Type-II

Current Transport In Type-II superconductors

In an ideal Type-II superconductor, we observe an interaction between the magnetic moment of the vortex lines and a transport current. The resulting Lorentz force (which macroscopically is seen as $\vec{F} = \vec{J} \times \vec{B}$) moves the vortices orthogonal to field direction and current, which results in energy dissipation and a significant resistance.

Type-II Superconductors: Vortex Pinning

How to avoid movement of vortices?

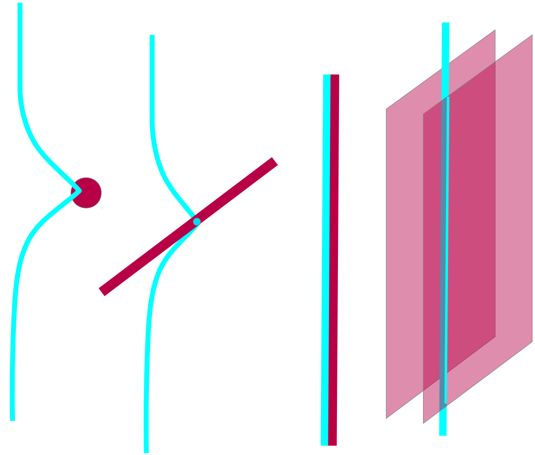
- Add some defects in the superconductor.
- Vortices will get caught on the defects (become pinned).
- Up to a certain force (depinning), vortices will not move.

How do vortices become pinned?

– Example: Core pinning

a vortex will sit on a normal conducting defect, in order to save the condensation energy to form its normal conducting core...

Geometrical Considerations: Vortex Pinning



Pinning the Vortex Lattice

- Interaction: one vortex – one defect
- many defects can pull simultaneously one one vortex
- vortices form a (soft) lattice, on which all available defects pull simultaneously

We have a severe mathematical problem...

Limits of Type-II superconductivity

- critical temperature T_c
- critical field H_{c2}
- critical current density J_c
  – here, J_c is not only determined by the basic properties T_c, H_{c2}, and H_c, but, very importantly, by the defect structure in the material...

Applications

- Magnets (motors, accelerators, fusion reactors)
- Current transport, network stability
- Levitation (trains!)
- ...
- + SQUID applications (measuring, detection, computing)